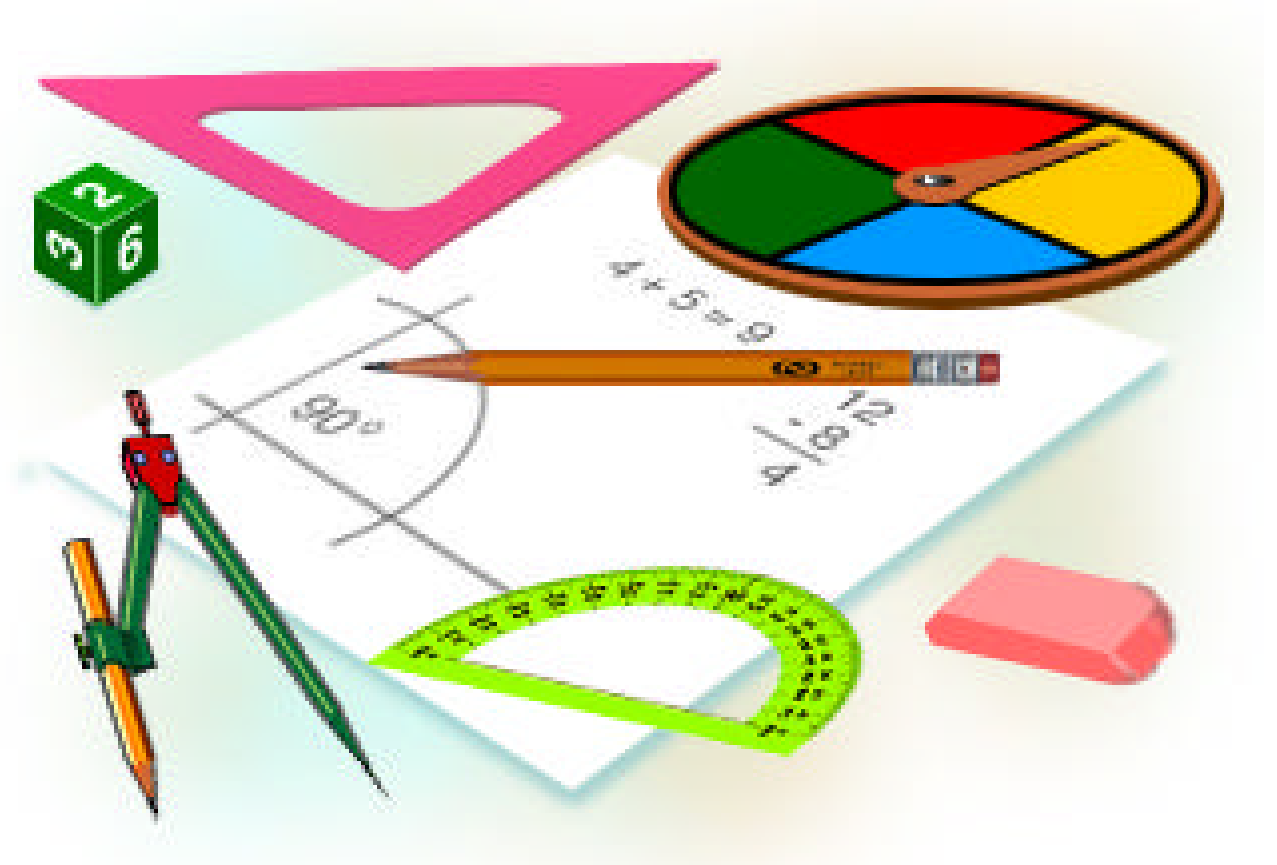


The Next Step

Mathematics Applications for Adults



Book 14015 - Fractions

OUTLINE

Mathematics - Book 14015

Fractions
<u>Understanding and Comparing Fractions</u>
explain proper and improper fractions
provide equivalent fractions.
recognize a fraction in its lowest terms.
demonstrate an understanding of reducing fractions to their lowest terms (e.g. $4/12=1/3$).
convert improper fractions to mixed numbers and vice versa.
find lowest common denominator (LCM) given 2 or 3 fractions with unlike denominators.

THE NEXT STEP

Book 14015

Fractions

Understanding and Comparing Fractions

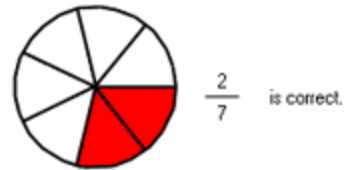
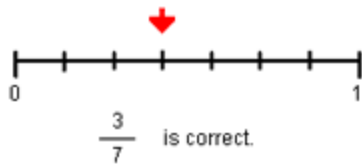
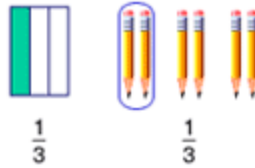
CLOSE TO HOME By John McPherson



“I’ll take a large pizza with half-onion, two-thirds olives, nine-fifteenths mushrooms, five-eighths pepperoni, one-eighth anchovies, and extra cheese on five-ninths of the onion half.”

The word *fraction* means “part of a whole.” The word comes from the Latin word *fractio*, meaning “to break into pieces.” In math, a fraction means one or more parts of a whole.

Example:



A fraction has two parts, a *denominator* and a *numerator*. The denominator is the numeral written under the bar and tells the number of parts a whole is divided into. The numerator is the numeral written above the bar. The numerator tells the number of parts of the whole that are being counted. A *proper fraction* has a numerator that is smaller than its denominator.

numerator	number of parts counted	1
denominator	total parts of the whole	17

Practice Exercise

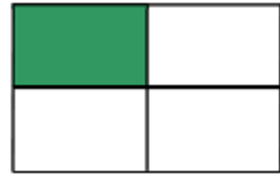
Write the fraction of the coloured part of the shapes on the next page.

1.



_____ of the circle
is coloured.

2.



_____ of the
rectangle is coloured.

3.



_____ of the
triangle is coloured.

4.



_____ of the
rectangle is coloured.

Fill in the blanks.

1. _____ and _____ make 1 whole.
2. _____ and _____ make 1 whole.
3. $\frac{1}{4}$ and _____ make 1 whole.

Improper Fractions

When the numerator of a fraction is greater than or equal to the denominator, the fraction is called an *improper fraction*.

$$\frac{3}{2} \quad \frac{4}{3} \quad \frac{5}{4} \quad \frac{6}{5} \quad \frac{7}{6} \quad \frac{8}{8}$$

P *The value of an improper fraction is always greater than or equal to one.*

Mixed Numerals

Mixed numerals combine whole numbers and fractions. The values of mixed numerals can also be written as *improper fractions*. To write a mixed numeral as an improper fraction, multiply the whole number by the denominator of the fraction, then add the numerator. Use your answer as the new numerator and keep the original denominator.

$$1 \frac{1}{2} = \frac{(2 \times 1) + 1}{2} = \frac{3}{2} \quad 2 \frac{3}{4} = \frac{(2 \times 4) + 3}{4} = \frac{11}{4}$$

To change an improper fraction to a mixed numeral, divide the numerator by the denominator. Then place the remainder over the old denominator.

$$\frac{3}{2} = 2 \frac{1}{3} = 1 \frac{1}{2}$$

$$\frac{11}{4} = 4 \frac{2}{11} = 2 \frac{3}{4}$$

Practice Exercise

Express each fraction as a whole number or as a mixed number.

- | | | | |
|------------------------|------------------------|-----------------------|----------------------|
| 1. $\frac{52}{9} =$ | 2. $\frac{31}{10} =$ | 3. $\frac{119}{11} =$ | 4. $\frac{66}{6} =$ |
| 5. $\frac{22}{2} =$ | 6. $\frac{9}{4} =$ | 7. $\frac{34}{5} =$ | 8. $\frac{67}{12} =$ |
| 9. $\frac{80}{7} =$ | 10. $\frac{35}{4} =$ | 11. $\frac{48}{12} =$ | 12. $\frac{55}{6} =$ |
| 13. $\frac{8}{2} =$ | 14. $\frac{32}{11} =$ | 15. $\frac{51}{5} =$ | 16. $\frac{28}{3} =$ |
| 17. $\frac{72}{10} =$ | 18. $\frac{48}{9} =$ | 19. $\frac{64}{9} =$ | 20. $\frac{17}{8} =$ |
| 21. $\frac{101}{11} =$ | 22. $\frac{131}{12} =$ | 23. $\frac{5}{2} =$ | 24. $\frac{33}{3} =$ |

$$25. \frac{73}{6} = \quad 26. \frac{68}{10} = \quad 27. \frac{41}{7} = \quad 28. \frac{36}{4} =$$

$$29. \frac{64}{10} = \quad 30. \frac{78}{7} = \quad 31. \frac{18}{3} = \quad 32. \frac{43}{5} =$$

Express each mixed numeral as an improper fraction.

1. $2 \frac{3}{4} =$
2. $9 \frac{1}{2} =$
3. $8 \frac{3}{4} =$
4. $12 \frac{1}{4} =$
5. $1 =$
6. $5 =$
7. $6 =$
8. $4 =$
9. $7 =$
10. $2 \frac{1}{2} =$

Common Denominators

Many fractions have *common denominators*. That means that the numbers in their denominators are the same.

$$\frac{1}{2} \quad \frac{3}{2} \quad \frac{5}{2}$$

To find common denominators, $\textcircled{1}$ find the *least common multiple* for the denominators of the fractions you are

comparing.

Find the *multiples* of a number by multiplying it by other whole numbers. The multiples of 2, for example, are:

$$0 \times 2 = 0$$

$$2 \times 3 = 6$$

$$1 \times 2 = 2$$

$$2 \times 4 = 8$$

$$2 \times 2 = \underline{4}$$

$$2 \times 5 = \underline{10}$$

... and so on.

As you can see, the multiples of 2 include 0, 2, 4, 6, 8, and 10. The list continues into infinity!

Some numbers share the same multiples. Those multiples are known as *common multiples*.

Number Multiples

0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25
	0	1	2	3	4	5

The least multiple of two or more numbers is the least common multiple. For example, the least common multiple of 2 and 3 is 6.

$2 \times 1 = 2$

$2 \times 2 = 4$

$2 \times 3 = 6$

$3 \times 1 = 3$

$3 \times 2 = 6$

Compare:

$$\frac{1}{2} \text{ and } \frac{2}{3}$$

Answer: least common multiple is 6

② Divide the common multiple by the denominators.

$$2 \overline{) 6}$$

$$3 \overline{) 6}$$

③ Multiply the quotients by the old numerators to calculate the new numerators.

$$\begin{array}{r} 3 \\ \times 1 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 2 \\ \times 2 \\ \hline 4 \end{array}$$

④ Place the new numerators over the common denominator.

$$\frac{3}{6}$$

$$\frac{4}{6}$$

P *To reduce a fraction to its lowest terms, divide both the numerator and the denominator by their*

greatest common denominator or greatest common factor.

$$\frac{4}{8} \cdot \frac{4}{4} = \frac{1}{2}$$

Practice Exercise

Find the least common multiple for the given numbers.

1. 4, 8 **8**
2. 7, 5
3. 3, 2
4. 5, 12
5. 9, 10
6. 10, 5
7. 8, 10
8. 9, 8
9. 3, 12
10. 10, 25
11. 8, 2
12. 15, 19
13. 10, 12
14. 20, 24
15. 24, 18

16. 5, 6
17. 23, 28
18. 12, 6
19. 24, 48
20. 9, 18
21. 40, 4
22. 36, 30
23. 5, 27
24. 17, 23
25. 140, 125
26. 10, 6
27. 80, 144
28. 18, 45
29. 4, 54
30. 60, 36

Rewrite each set of fractions using the least common denominator (least common multiple).

1. $\frac{1}{2}$, $\frac{4}{6}$	2. $\frac{3}{4}$, $\frac{2}{5}$
3. $\frac{3}{4}$, $\frac{1}{2}$	4. $\frac{3}{4}$, $\frac{3}{6}$
5. $\frac{3}{7}$, $\frac{1}{8}$	6. $\frac{5}{7}$, $\frac{4}{6}$

7. $\frac{5}{7}$, $\frac{4}{10}$	8. $\frac{6}{10}$, $\frac{1}{6}$
9. $\frac{4}{5}$, $\frac{7}{10}$	10. $\frac{4}{9}$, $\frac{5}{6}$
11. $\frac{2}{8}$, $\frac{7}{9}$	12. $\frac{4}{5}$, $\frac{5}{6}$
13. $\frac{2}{8}$, $\frac{1}{5}$	14. $\frac{1}{7}$, $\frac{2}{5}$
15. $\frac{8}{9}$, $\frac{6}{7}$	16. $\frac{2}{10}$, $\frac{2}{9}$
17. $\frac{6}{9}$, $\frac{7}{10}$, $\frac{8}{12}$	18. $\frac{2}{5}$, $\frac{10}{11}$, $\frac{6}{10}$

Reduce each fraction to lowest terms.

(Hint: Divide its numerator and denominator by their Greatest Common Denominator or Greatest Common Factor)

1. $\frac{15}{25} =$ 2. $\frac{10}{60} =$ 3. $\frac{24}{30} =$ 4. $\frac{5}{15} =$

5. $\frac{5}{40} =$ 6. $\frac{15}{33} =$ 7. $\frac{3}{12} =$ 8. $\frac{20}{30} =$

9. $\frac{15}{66} =$	10. $\frac{30}{33} =$	11. $\frac{6}{24} =$	12. $\frac{18}{48} =$
13. $\frac{22}{21} =$	14. $\frac{8}{80} =$	15. $\frac{52}{77} =$	16. $\frac{15}{27} =$
17. $\frac{13}{32} =$	18. $\frac{23}{38} =$	19. $\frac{9}{99} =$	20. $\frac{24}{32} =$
21. $\frac{45}{32} =$	22. $\frac{24}{48} =$	23. $\frac{46}{52} =$	24. $\frac{8}{40} =$

Equivalent Fractions

You know from experience that different fractions can have the same value.

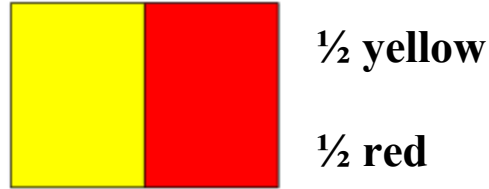
Since there are 100 pennies in a dollar, 25 pennies is equal to $\frac{25}{100}$ of a dollar. The same amount also equals a quarter, or $\frac{1}{4}$ of a dollar.

On a measuring cup, $\frac{1}{2}$ cup is the same amount as $\frac{2}{4}$ cup.

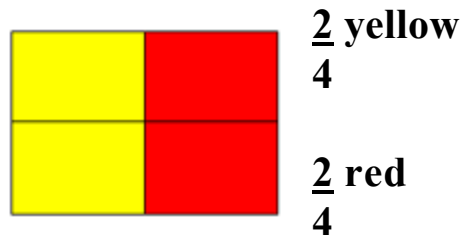
On an odometer, $\frac{5}{10}$ of a mile is the same as $\frac{1}{2}$ mile.

Out of a dozen doughnuts, six doughnuts equal $\frac{6}{12}$, or $\frac{1}{2}$ dozen.

A napkin is folded into two parts. One part is yellow, the other red.



Then the napkin is folded again. Now there are two yellow parts and two red parts.



In this example, the red part of the napkin can be described as $\frac{1}{2}$ red or $\frac{2}{4}$ red. That makes $\frac{1}{2}$ and $\frac{2}{4}$ *equivalent fractions*.

When solving math problems, reduce fractions to their lowest equivalent. Rather than describing the napkin as $\frac{2}{4}$ yellow, call it $\frac{1}{2}$ yellow.

Some Equivalent Fractions

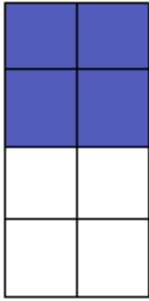
$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20}$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$$

You can tell if two fractions are equal by finding cross products.

Example Are $\frac{4}{8}$ and $\frac{3}{6}$ equal fractions?



Multiply diagonally as shown by the arrows below. If the cross products are equal, the fractions are equal.

$$\begin{array}{cc} \frac{4}{8} & \frac{3}{6} \\ \swarrow & \searrow \\ & \end{array} \quad \begin{array}{l} 4 \times 6 = 24 \\ 8 \times 3 = 24 \end{array}$$

Since the cross products are equal, $\frac{4}{8} = \frac{3}{6}$.

Sometimes you need to find an equal fraction with higher terms. You raise a fraction to higher terms by multiplying both the numerator and the denominator by the same number (except 0).

$$\frac{5}{8} \text{ and } \frac{20}{32} \text{ are equal fractions because } \frac{5 \times 4 = 20}{8 \times 4 = 32}$$

Often you will need to find an equal fraction with a specific denominator. To do this, think, “What number multiplied by the original denominator will result in the new denominator?” Then multiply the original numerator by the same number.

Example $\frac{3}{4} = \frac{?}{24}$

Since $4 \times 6 = 24$, multiply the numerator 3 by 6. $\frac{3 \times 6 = 18}{4 \times 6 \quad 24}$

The fractions $\frac{3}{4}$ and $\frac{18}{24}$ are equal fractions.

Comparing Fractions

When two fractions have the same number as the denominator, they are said to have a common denominator, and the fractions are called like fractions. When you compare like fractions, the fraction with the greater numerator is the greater fraction.

Example 1 Which fraction is greater, $\frac{3}{5}$ or $\frac{4}{5}$?

The fractions $\frac{3}{5}$ and $\frac{4}{5}$ are like fractions because they have a common denominator, 5. Compare the numerators.

Since 4 is greater than 3, $\frac{4}{5}$ is greater than $\frac{3}{5}$.

Fractions with different denominators are called unlike fractions. To compare unlike fractions, you must change them to fractions with a common denominator.

The common denominator will always be a multiple of both of the original denominators. The multiples of a number are found by going through the times tables for the number. For instance, the multiples of 3 are 3, 6, 9, 12, 15, 18, and so on.

You can often find a common denominator by using mental math. If not, try these methods:

1. See whether the larger denominator could be a common denominator. In other words, if the smaller denominator can divide into the larger denominator evenly, use the larger denominator as the common denominator.
2. Go through the multiples of the larger denominator. The first one that can be divided evenly by the smaller denominator is the lowest common denominator.

Example 2 Which is greater, $\frac{5}{6}$ or $\frac{3}{4}$?

Go through the multiples of the larger denominator: 6, 12, 18, 24, 30.... Since 12 can be divided evenly by both 4 and 6, 12 is the lowest common denominator.

Build equal fractions, each with the
9
denominator 12: $\frac{5 \times 2 = 10}{6 \times 2 = 12}$ $\frac{3 \times 3 = 9}{4 \times 3 = 12}$

Compare the like fractions. Since $\frac{10}{12} > \frac{9}{12}$, the fraction $\frac{5}{6} > \frac{3}{4}$.

Practice Exercise

Equal Fractions

Fill in the missing numerator or denominator.

1. $\frac{2}{4} = \frac{16}{\quad}$	2. $\frac{5}{8} = \frac{20}{\quad}$
3. $\frac{1}{11} = \frac{\quad}{22}$	4. $\frac{5}{9} = \frac{\quad}{72}$
5. $\frac{10}{12} = \frac{40}{\quad}$	6. $\frac{1}{2} = \frac{\quad}{10}$
7. $\frac{2}{3} = \frac{18}{\quad}$	8. $\frac{3}{60} = \frac{\quad}{10}$
9. $\frac{2}{5} = \frac{\quad}{10}$	10. $\frac{4}{12} = \frac{40}{\quad}$
11. $\frac{2}{6} = \frac{\quad}{72}$	12. $\frac{2}{9} = \frac{16}{\quad}$
13. $\frac{2}{7} = \frac{22}{\quad}$	14. $\frac{7}{8} = \frac{35}{\quad}$
15. $\frac{4}{10} = \frac{\quad}{60}$	16. $\frac{1}{3} = \frac{\quad}{9}$

17. $\frac{4}{8} = \frac{40}{\quad}$	18. $\frac{7}{11} = \frac{\quad}{132}$
--------------------------------------	--

Fraction Comparison (Same Denominator)

1. $\frac{1}{18} < \frac{2}{18}$	2. $\frac{1}{5} \text{ — } \frac{2}{5}$
3. $\frac{2}{13} \text{ — } \frac{4}{13}$	4. $\frac{9}{14} \text{ — } \frac{11}{14}$
5. $\frac{7}{10} \text{ — } \frac{12}{10}$	6. $\frac{8}{16} \text{ — } \frac{12}{16}$
7. $\frac{1}{17} \text{ — } \frac{3}{17}$	8. $\frac{10}{11} \text{ — } \frac{2}{11}$
9. $\frac{7}{14} \text{ — } \frac{6}{14}$	10. $\frac{3}{12} \text{ — } \frac{7}{12}$
11. $\frac{2}{4} \text{ — } \frac{1}{4}$	12. $\frac{1}{19} \text{ — } \frac{8}{19}$
13. $\frac{6}{9} \text{ — } \frac{3}{9}$	14. $\frac{1}{2} \text{ — } \frac{1}{2}$
15. $\frac{8}{10} \text{ — } \frac{1}{10}$	16. $\frac{3}{7} \text{ — } \frac{1}{7}$

17. $\frac{1}{12} \quad \text{---} \quad \frac{11}{12}$	18. $\frac{5}{8} \quad \text{---} \quad \frac{5}{8}$
---	--

Compare Fractions

Compare the given fractions. In the middle of the two fractions, write either $>$ (greater than), $<$ (less than), or $=$ (equals).

- | | | |
|---|--|--|
| (1) $\frac{32}{49} \quad \frac{55}{225}$ | (2) $\frac{4}{2} \quad \frac{205}{95}$ | (3) $\frac{9}{39} \quad \frac{24}{45}$ |
| (4) $\frac{112}{196} \quad \frac{28}{49}$ | (5) $\frac{1125}{550} \quad \frac{225}{110}$ | (6) $\frac{75}{75} \quad \frac{20}{43}$ |
| (7) $\frac{215}{30} \quad \frac{8}{12}$ | (8) $\frac{60}{60} \quad \frac{28}{88}$ | (9) $\frac{14}{16} \quad \frac{6}{6}$ |
| (10) $\frac{100}{500} \quad \frac{20}{100}$ | (11) $\frac{55}{10} \quad \frac{75}{195}$ | (12) $\frac{34}{3} \quad \frac{66}{66}$ |
| (13) $\frac{100}{40} \quad \frac{20}{8}$ | (14) $\frac{330}{735} \quad \frac{110}{245}$ | (15) $\frac{36}{19} \quad \frac{18}{21}$ |

Answer Key

Book 14015 - Fractions

Page 5 1. $\frac{4}{8}$ or $\frac{1}{2}$ 2. $\frac{1}{4}$ 3. $\frac{2}{4}$ or $\frac{1}{2}$
3. $\frac{3}{6}$ or $\frac{1}{2}$

Page 5 (fill in the blanks) 1. $\frac{1}{8}$ 2. $\frac{1}{3}$ 3. $\frac{3}{4}$

Page 7 1. $5\frac{7}{9}$ 2. $3\frac{1}{10}$ 3. $10\frac{9}{11}$ 4. 11
5. 11 6. $2\frac{1}{4}$ 7. $6\frac{4}{5}$ 8. $5\frac{7}{12}$
9. $11\frac{3}{7}$ 10. $8\frac{3}{4}$ 11. 4 12. $9\frac{1}{6}$
13. 4 14. $2\frac{10}{11}$ 15. $10\frac{1}{5}$ 16. $9\frac{1}{3}$
17. $7\frac{1}{5}$ 18. $5\frac{1}{3}$ 19. $7\frac{1}{9}$ 20. $2\frac{1}{8}$
21. $9\frac{2}{11}$ 22. $10\frac{11}{12}$ 23. $2\frac{1}{2}$ 24. 11
25. $12\frac{1}{6}$ 26. $6\frac{4}{5}$ 27. $5\frac{6}{7}$ 28. 9
29. $6\frac{2}{5}$ 30. $11\frac{1}{7}$ 31. 6 32. $8\frac{3}{5}$

Page 8 1. $11\frac{1}{4}$ 2. $19\frac{1}{2}$ 3. $35\frac{1}{4}$ 4. $49\frac{1}{4}$
5. $\frac{4}{3}$ 6. $17\frac{1}{3}$ 7. $49\frac{1}{8}$ 8. $35\frac{1}{8}$
9. $61\frac{1}{8}$ 10. $5\frac{1}{2}$

Page 11 2. 35 3. 6 4. 60 5. 90 6. 10 7. 40
8. 72 9. 12 10. 50 11. 8 12. 285
13. 60 14. 120 15. 72 16. 30 17. 644
18. 12 19. 48 20. 18 21. 40 22. 180
23. 135 24. 391 25. 3500 26. 30
27. 720 28. 90 29. 108 30. 180

Page 12 1. $\frac{3}{6}$, $\frac{4}{6}$ 2. $\frac{15}{20}$, $\frac{8}{20}$ 3. $\frac{3}{4}$ $\frac{2}{4}$

4. $9/12, 6/12$ 5. $24/56, 7/56$
6. $30/42, 28/42$ 7. $50/70, 28/70$
8. $18/30, 5/30$ 9. $8/10, 7/10$
10. $8/18, 15/18$ 11. $18/72, 56/72$
12. $24/30, 25/30$ 13. $10/40, 8/40$
14. $5/35, 14/35$ 15. $56/63, 54/63$
15. $18/90, 20/90$
16. $720/1080, 756/1080, 720/1080$
17. $220/550, 500/550, 330/550$

Page 13

1. $3/5$ 2. $1/6$ 3. $4/5$ 4. $1/3$ 5. $1/8$
6. $5/11$ 7. $1/4$ 8. $2/3$ 9. $5/22$
10. $10/11$ 11. $1/4$ 12. $3/8$ 13. $1\ 1/21$
14. $1/10$ 15. $52/77$ 16. $5/9$ 17. $13/32$
18. $23/38$ 19. $1/11$ 20. $3/4$ 21. $1\ 13/32$
22. $1/2$ 23. $23/26$ 24. $1/5$

Page 19

1. 32 2. 32 3. 2 4. 40 5. 48 6. 5
7. 27 8. 21 9. 4 10. 120 11. 24
12. 72 13. 77 14. 40 15. 24 16. 3
17. 80 18. 84

Page 20

2. < 3. < 4. < 5. = 6. < 7. <
8. > 9. > 10. < 11. > 12. < 13. >
14. = 15. > 16. > 17. < 18. =

Page 21

1. > 2. < 3. < 4. = 5. < 6. >
7. > 8. > 9. < 10. = 11. > 12. >
13. = 14. = 15. >