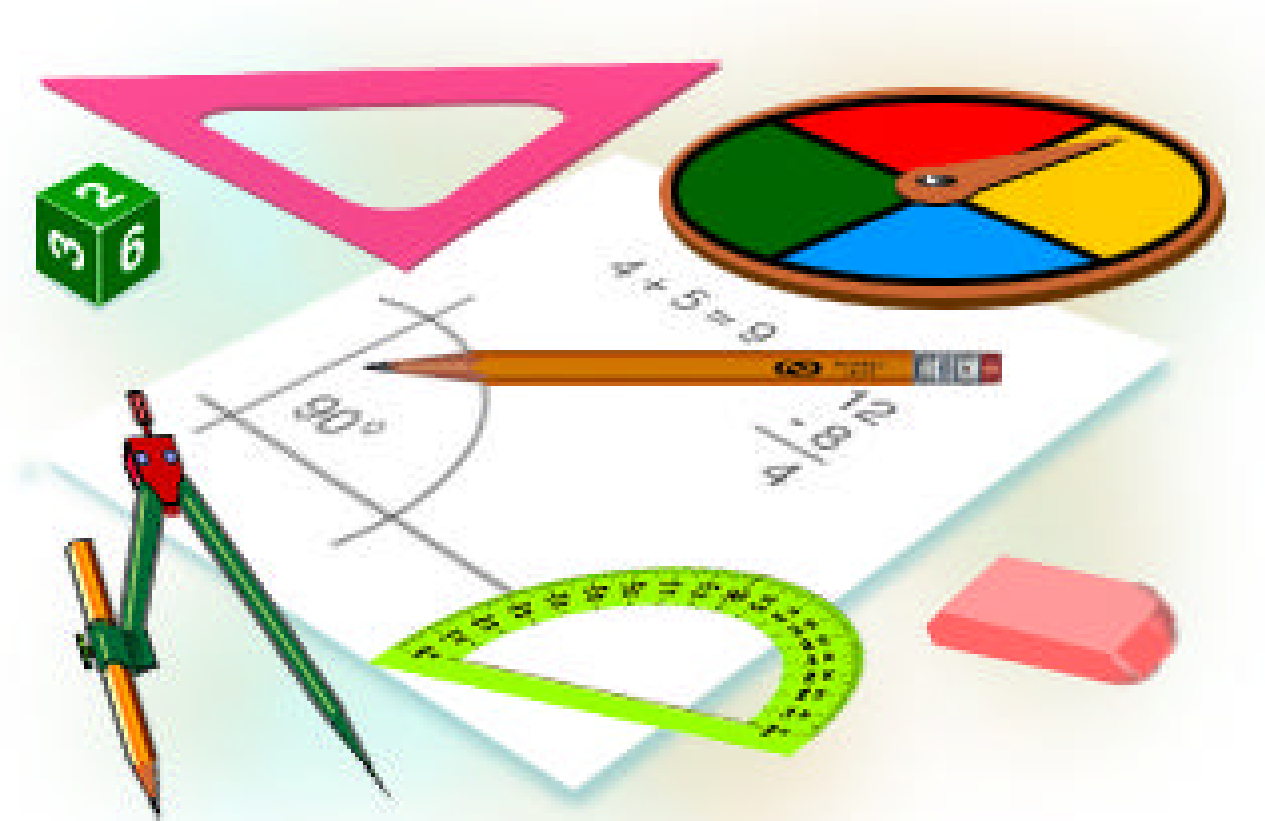


# The Next Step

## Mathematics Applications for Adults



**Book 14015 – Whole Numbers**

# INTRODUCTION

## Why Math?

The most important reason for learning math is that it teaches us how to think. Math is more than adding and subtracting, which can easily be done on a calculator; it teaches us how to organize thoughts, analyze information, and better understand the world around us.

Employers often have to re-educate their employees to meet the demands of our more complex technological society. For example, more and more, we must be able to enter data into computers, read computer displays, and interpret results. These demands require math skills beyond simple arithmetic.

## Everyone Is Capable of Learning Math

There is no **type** of person for whom math comes easily. Even mathematicians and scientists spend a lot of time working on a single problem. Success in math is related to practice, patience, confidence in ability, and hard work.

It is true that some people can solve problems or compute more quickly, but speed is not always a measure of understanding. Being “faster” is related to **more practice or experience**.

For example, the reason why math teachers can work problems quickly is because they've done them so many times before, not because they have "mathematical minds".

Working with something that is familiar is natural and easy. For example, when cooking from a recipe we have used many times before or playing a familiar game, we feel confident. We automatically know what we need to do and what to expect. Sometimes, we don't even need to think. However, when using a recipe for the **first** time or playing a game for the **first** time, we must concentrate on each step. We double-check that we have done everything right, and even then we fret about the outcome. The same is true with math. When encountering problems for the very first time, **everyone must have patience** to understand the problem and work through it correctly.

### **It's Never Too Late to Learn**

One of the main reasons people don't succeed in math is that they don't start at the right place. **IMPORTANT! You must begin where *you* need to begin.** Could you hit a homerun if you hadn't figured out which end of the bat had to make contact with the ball? Why should learning math be any different?

If it has been a while since your last math class, **you must determine what level math you should take.** A teacher or trained tutor can help determine this with a few placement tests and questions.

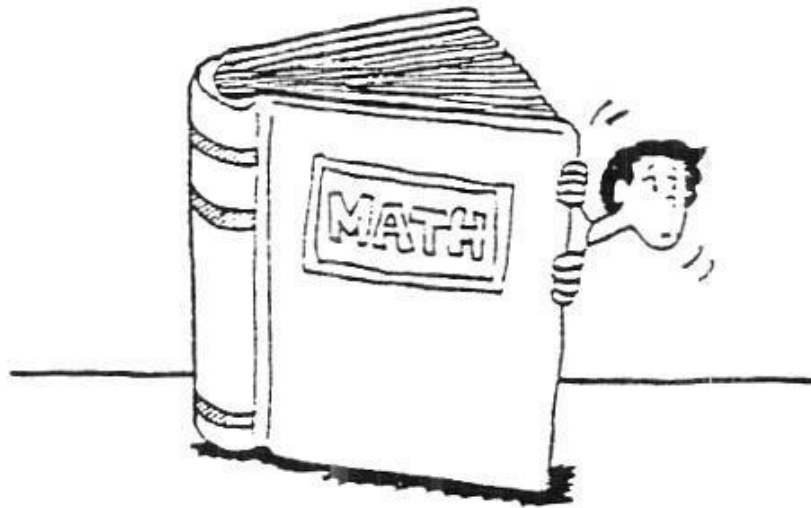
Sometimes a few tutoring sessions can help you fill gaps in your knowledge or help you remember some of the things you

have simply forgotten. It could also be the case where your foundations may be weak and it would be better for you to relearn the basics. **Get some help** to determine what is best for you.

Feeling good about ourselves is what all of us are ultimately striving for, and nothing feels better than conquering something that gives us difficulty. This takes a great deal of courage and the ability to rebound from many setbacks. This is a natural part of the learning process, and when the work is done and we can look back at our success, nothing feels better.

*Where's the best place to hide if you're scared?*

Inside a math book because there is safety in numbers.



*Artist Unknown*

## OUTLINE

### Mathematics - Book 14015

<b>Whole Numbers</b>
<b><u>Number/Word Recognition</u></b>
convert Arabic numbers to Roman numerals and vice versa (I – M...1 – 1,000).
correctly write the number words for Arabic numbers (0 – 1,000,000).
correctly write the Arabic numerals for any number word (0 – 1,000,000).
<b><u>Place Value</u></b>
recognize the place value of each digit of a number to the million's place.
determine how many hundred thousands, thousands, hundreds, tens and ones in any number (0 – 1,000,000).
<b><u>Counting</u></b>
count orally from 0 – 1,000,000 starting at any point in between those numbers.
count orally by 2's, 5's, and 10's to 100.
write all the even numbers from 2 - 100 and all the odd numbers from 1 - 99.
order numbers from greatest to least and least to greatest. (0 – 1,000,000).
<b><u>Addition</u></b>
find the sum of whole numbers up to 6 digits.
use addition facts to compute sums up to and including 18.
<b><u>Subtraction</u></b>
subtract two whole numbers up to 6 digits (using

borrowing/regrouping).
use subtraction facts to compute differences up to and including 18.
apply addition/subtraction skills by completing an incomplete equation (e.g. $14 + ? = 37$ ).
<b><u>Multiplication</u></b>
multiply 3 digit factors by 3 digit factors.
write the times tables to $12 \times 12$ (within a specified time).
multiply by 1, 10, 100 quickly (within a specified time).
<b><u>Division</u></b>
explain factoring.
find the factors of a given list of products.
identify prime numbers from a given list.
<b><u>Word Problems with Whole Numbers</u></b>
solve one/two step problems with addition, subtraction, multiplication or division of whole numbers.

## THE NEXT STEP

### Book 14015

#### Whole Numbers

#### Number Recognition



*Digit* is a counting word. A digit is any of the numerals from **1** to **9**. The word “digit” is also the name for a finger. So number digits can be counted on finger digits.

Our modern system of counting probably came from counting on fingers. Fingers and hands were among the earliest known calculators!

### **A CLOSER LOOK AT ROMAN NUMERALS**

Roman numerals are still used today, more than 2000 years after their introduction. The history of Roman numerals is not well

documented and written accounts are contradictory. Roman numerals are read from left to right.

It is likely that counting began on the fingers and that is why we count in tens.

A single stroke I represents one finger, five or a handful could possibly be represented by V and the X may have been used because if you stretch out two handfuls of fingers and place them close the two little fingers cross in an X. Alternatively, an X is like two Vs, one upside down.

The Roman numeral system uses seven letters to represent numbers. Combinations of these letters represent other numbers.

**I = 1**

**V = 5**

**X = 10**

**L = 50**

**C = 100**

**D = 500**

**M = 1,000**

## **Combining Roman Numerals**

Here are the three rules for making numbers with Roman numerals:

1. If you put numbers of the same size together, then you add them.

$$\text{II} = 1 + 1 = 2$$



$$XX=10+10=20$$

$$XXX=10+10+10=30$$

2. If you put a small number to the right of a large number, then you add them, too.

$$XV=10+5=15$$

$$VIII=5+3=8$$

3. If you put a small number to the left of a large number, then you subtract the small one from the big one.

$$IX=10-1=9$$

$$CM=1000-100=900$$

Sample Numbers:

Letter	Value
I	1
II	2
III	3
IV	4
V	5
VI	6
VII	7
VIII	8
IX	9
X	10
XX	20
XXX	30
XL	40
L	50

LX	60
LXX	70
LXXX	80
XC	90
C	100
CC	200
CCC	300
CD	400
D	500
DC	600
DCC	700
DCCC	800
CM	900
M	1,000

# Practice Exercise

Translate the numbers on the next page. E.g., X = 10;  
or 20 = XX.

$$102 = \underline{\hspace{2cm}} \quad 80 = \underline{\hspace{2cm}}$$

$$CXC = \underline{\hspace{2cm}} \quad CXXII = \underline{\hspace{2cm}}$$

$$CCCXCVII = \underline{\hspace{2cm}} \quad 132 = \underline{\hspace{2cm}}$$

$$CDXCVI = \underline{\hspace{2cm}} \quad 143 = \underline{\hspace{2cm}}$$

$$372 = \underline{\hspace{2cm}} \quad DLIV = \underline{\hspace{2cm}}$$

$$DI = \underline{\hspace{2cm}} \quad DLXXIV = \underline{\hspace{2cm}}$$

$$605 = \underline{\hspace{2cm}} \quad 394 = \underline{\hspace{2cm}}$$

$$DCXLII = \underline{\hspace{2cm}} \quad 583 = \underline{\hspace{2cm}}$$

$$DCCXV = \underline{\hspace{2cm}} \quad CMXLIX = \underline{\hspace{2cm}}$$

$$1,107 = \underline{\hspace{2cm}} \quad MLXXXI = \underline{\hspace{2cm}}$$

These are some examples of the use of Roman Numerals.

I. Chapters. Look at the chapter headings of any book you are using.

II. Buildings.

III. Movie copyright years.

IV. Clocks. Look at clocks closely, especially how they represent the number **four**.

## **A NEW THEORY ABOUT IIII**

A lot of people ask ‘Why is the number 4 on a clock-face depicted as IIII and not as IV?’ There is no certain answer to this question. One common suggestion is that around the circle the IIII balances the VIII which is in its mirror-symmetrical place – that is if a mirror was placed vertically between the XII and VI, the VIII and IIII would reflect on to each other. There are problems with this theory – the V does not balance the VII, nor the I the XI. Another plausible explanation might be that IV has three strokes and is more likely to be confused with the neighbouring III, as both are at unfamiliar angles to the reader. But neither really offers an adequate explanation of why the normal rules of Roman numerals have been broken.

The oldest surviving clock-face in its original condition is on the clock inside Wells Cathedral in Somerset, England. It dates from before 1392 and the original mechanism – now in the Science Museum – has some claim to be the oldest surviving clock works in the world. The current mechanism that drives it is Victorian, but the face has not been changed for more than 600 years.

The outermost circle is more than six feet (1.93m) in diameter and around it, in Roman numerals, are the twenty-four hours of the day with the 4 indicated by IIII.

### **Exceptions**

The practice of using IIII rather than IV on clock-faces, although common, is not universal. The well-known clock, commonly called Big Ben, at the Palace of Westminster in London (where

Parliament meets) has gothic style Roman numerals round its face and the 4 is depicted as iv. (Strictly speaking, the hour bell is Big Ben, the clock is the Great Clock, and the tower is the Clock Tower and although the building is correctly called the Palace of Westminster, most people refer to it as the Houses of Parliament). Other examples of an iv on a clock-face are rare in England - but the clock in the South Transept of Norwich Cathedral is one example. Others are found in Spain. San Sebastian in northern Spain has at least two clocks – one on the cathedral and one on another church – which both have clear plain Roman numerals on the dial and which use IV for the 4.

### Number/Word Recognition

Every number can be written two ways.

It can be written as a numeral.

Or it can be written as a word.

The numeral and word stand for the same thing.

<b>Numeral</b>	<b>Word</b>
0	zero
1	one
2	two
3	three
4	four
5	five
6	six

7	seven
8	eight
9	nine

Learn to say these 2-place numbers:

10	ten
11	eleven
12	twelve
13	thirteen
14	fourteen
15	fifteen
16	sixteen
17	seventeen
18	eighteen
19	nineteen

The 2-place numbers go from 10 (ten) to 99 (ninety-nine).  
We have just learned about the 2-place numbers from 10 to 19.  
Now learn these 2-place numbers:

20	twenty
21	twenty-one
22	twenty-two
23	twenty-three
24	twenty-four
25	twenty-five
26	twenty-six
27	twenty-seven
28	twenty-eight
29	twenty-nine

30	thirty
31	thirty-one
32	thirty-two
33	thirty-three
34	thirty-four
35	thirty-five
36	thirty-six
37	thirty-seven
38	thirty-eight
39	thirty-nine
40	forty
41	forty-one
42	forty-two
43	forty-three
44	forty-four
45	forty-five
46	forty-six
47	forty-seven
48	forty-eight
49	forty-nine
50	fifty
51	fifty-one
52	fifty-two
53	fifty-three
54	fifty-four
55	fifty-five
56	fifty-six
57	fifty-seven
58	fifty-eight
59	fifty-nine
60	sixty

61	sixty-one
62	sixty-two
63	sixty-three
64	sixty-four
65	sixty-five
66	sixty-six
67	sixty-seven
68	sixty-eight
69	sixty-nine
70	seventy
71	seventy-one
72	seventy-two
73	seventy-three
74	seventy-four
75	seventy-five
76	seventy-six
77	seventy-seven
78	seventy-eight
79	seventy-nine
80	eighty
81	eighty-one
82	eighty-two
83	eighty-three
84	eighty-four
85	eighty-five
86	eighty-six
87	eighty-seven
88	eighty-eight
89	eighty-nine
90	ninety
91	ninety-one



92	ninety-two
93	ninety-three
94	ninety-four
95	ninety-five
96	ninety-six
97	ninety-seven
98	ninety-eight
99	ninety-nine

The number 99 is the greatest 2-place number.  
The next number in order is 100 (one hundred).

100 is one more than 99.  
It is a 3-place number.  
It has three numerals: 1, 0, and 0.  
They stand for 1 hundred, 0 tens, and 0 ones

The greatest 3-place number is 999 (nine hundred ninety-nine).  
It stands for 9 hundreds, 9 tens, and 9 ones.

Every 3-place number tells how many hundreds, tens, and ones  
the number stands for.

The number 999 is the greatest 3-place number.  
The next number in order is 1,000 (one thousand).  
It is one more than 999.  
It is a 4-place number.  
It has four numerals: 1, 0, 0, and 0.  
They stand for 1 thousand, 0 hundreds, 0 tens, and 0 ones.

The greatest 4-place number is 9,999  
(nine thousand, nine hundred ninety-nine).  
The number after 9,999 is 10,000 (ten thousand).  
Ten thousand is a 5-place number.

The greatest 5-place number is 99,999 (ninety-nine thousand,  
nine hundred ninety-nine).  
The number after 99,999 is 100,000  
(one hundred thousand).  
One hundred thousand is a 6-place number.

The greatest 6-place number is 999,999 (nine hundred ninety-  
nine thousand, nine hundred ninety-nine).  
The number after 999,999 is 1,000,000 (one million).  
One million is a 7-place number

We use a comma after the number in the thousands' and  
millions' places.  
The comma makes large numbers easier to read.

## Practice Exercise

Read the number word and write the number.

1. seventy-four thousand, six hundred ninety-three

**74,63**

2. eighty-five thousand, seven hundred twenty-three  

---
3. three thousand, six hundred sixteen  

---
4. seventy-one thousand, seven hundred fifty-three  

---
5. fifty-nine thousand, five hundred eighty-four  

---
6. thirty-three thousand  

---
7. nine thousand, sixty-two  

---
8. one million, seventy-eight thousand, three hundred eleven  

---
9. one million, twenty-five thousand, fifty  

---
10. one million, twenty-four thousand, nine hundred sixty-eight  

---

11. one million, six hundred fifty thousand

---

12. one million, one hundred sixty-six thousand, six hundred seventy-three

---

13. one million, twenty-eight thousand, nine hundred eighteen

---

14. one million, nine hundred forty-nine thousand, five hundred eighty-four

---

### Number Words

Write the numeral as a number word.

The first one is already done for you.

1. 96,106 **ninety-six thousand, one hundred six**

2. 10,671

---

3. 7,470

---

4. 89,012

---

5. 4,047

---

6. 11,126

---

7. 66,129

---

8. 1,000,004

---

9. 1,031,561

---

10.	1,026,814	_____
11.	1,240,000	_____
12.	1,366,077	_____
13.	1,543,654	_____
14.	1,520,086	_____
15.	1,621,723	_____
16.	1,631,618	_____
17.	1,054,827	_____

## Place Value

To read the place value of numerals in a number, read from left to right.

Each column has a value 10 times greater than the column to its right.

### Place Value

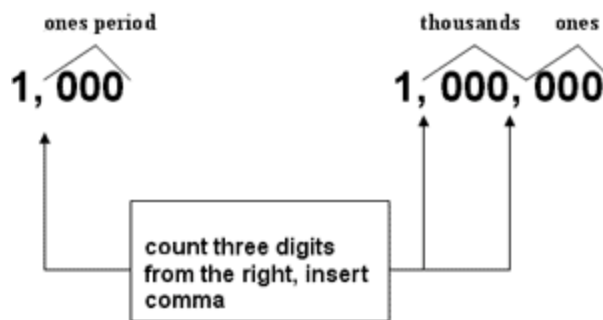
The value of a digit as determined by its position in a number

*Example:*

	PLACE VALUE									
	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
1, 623, 051 →	1	6	2	3	0	5	1	.		
0.053 →							0	0	5	3
32.4 →						3	2	.	4	

## Periods

Three places in the place value chart make up a *period*. Periods are always counted from the right---from the “ones” column---of a number. Periods are separated in numerals by commas.



Millions Period			Thousands Period			Ones Period		
Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
100,000,000	10,000,000	1,000,000	100,000	10,000	1,000	100	10	1
900,000,000	90,000,000	9,000,000	900,000	90,000	9,000	900	90	9

# Practice Exercise

## Millions, Thousands, Hundreds, Tens, and Ones

Write the place value of the bold number in each numeral below.

- |                                   |                                   |
|-----------------------------------|-----------------------------------|
| 1. 8,526,75 <u>8</u> Ones         | 2. <u>18</u> 9,758    _____       |
| 3. 933,95 <u>5</u> ,341    _____  | 4. <u>63</u> ,229,798    _____    |
| 5. 2,85 <u>3</u> _____            | 6. <u>26</u> ,574    _____        |
| 7. <u>7</u> ,945    _____         | 8. 42,12 <u>1</u> ,831    _____   |
| 9. 236,595, <u>2</u> 65    _____  | 10. <u>82</u> 5,617    _____      |
| 11. 38, <u>1</u> 28    _____      | 12. 6,39 <u>1</u> ,112    _____   |
| 13. 28,9 <u>9</u> 8,455    _____  | 14. 71, <u>9</u> 46    _____      |
| 15. 3,529,5 <u>1</u> 1    _____   | 16. 298,76 <u>7</u> ,487    _____ |
| 17. <u>93</u> 7,328    _____      | 18. <u>7</u> ,477    _____        |
| 19. <u>92</u> ,813,133    _____   | 20. <u>4</u> 1,875    _____       |
| 21. 135, <u>5</u> 47,569    _____ | 22. 9,225, <u>6</u> 24    _____   |
| 23. <u>77</u> 7,278    _____      | 24. 7,3 <u>5</u> 8    _____       |
| 25. 791,527, <u>5</u> 17    _____ | 26. 3,6 <u>7</u> 1    _____       |
| 27. <u>93</u> ,952,737    _____   | 28. 3,46 <u>4</u> ,574    _____   |
| 29. 831,3 <u>7</u> 5    _____     | 30. <u>11</u> ,232    _____       |



## Expanded Notation

Each of us knows how to read the number 463. In words we say “four hundred sixty-three.” Our number system suggests that the position or place of a digit determines its value. Thus, “four hundred sixty-three” really means four hundreds plus six tens plus three units or ones.

$$463 = 4 \times 100 + 6 \times 10 + 3$$

Any number, no matter how large, can be written in *expanded notation* by simply using decreasing multiples of 10, and working from left to right.

Example: Write 3,962,514 in expanded form.

$$3,962,514 = 3 \times 1,000,000 + 9 \times 100,000 + 6 \times 10,000 + 2 \times 1,000 + 5 \times 100 + 1 \times 10 + 4$$

## Counting

The set of counting numbers has no end. It can go on forever. The idea that counting numbers can go on and on is called *infinity*.

The set of *counting numbers*, or *natural numbers*, begins with the number 1 and continues into infinity.

$$\{1,2,3,4,5,6,7,8,9,10...\}$$

The set of *whole numbers* is the same as the set of counting numbers, except that it begins with **0**.

**{0,1,2,3,4,5,6,7,8,9,10...}**

*All counting numbers are whole numbers. Zero is the only whole number that is not a counting number.*

*Even numbers* include the numbers **0** and **2** and all numbers that can be divided evenly by **2**. *Odd numbers* are all numbers that cannot be divided evenly by **2**.

### Odd and Even Numbers to 100

1	3	5	7	9	11	13	15	17	19	21
0	2	4	6	8	10	12	14	16	18	20
23	25	27	29	31	33	35	37	39	41	
22	24	26	28	30	32	34	36	38	40	
43	45	47	49	51	53	55	57	59	61	
42	44	46	48	50	52	54	56	58	60	
63	65	67	69	71	73	75	77	79	81	
62	64	66	68	70	72	74	76	78	80	
83	85	87	89	91	93	95	97	99		
82	84	86	88	90	92	94	96	98	100	

### Skip Counting

To count by 2's, simply count all the **even** numbers: **0, 2, 4, 6, 8, 10...**and so on.

To count by 5's: **0, 5, 10, 15, 20...**and so on.

To count by 10's: **0, 10, 20, 30, 40...**and so on.

To count by 100's: **0, 100, 200, 300, 400...**and so on.

# Practice Exercise

Complete the number patterns.

1. 58 274 , 60 274 , \_\_\_\_\_ , 64 274
2. 305 472 , \_\_\_\_\_ , 405 472 , 455 472
3. \_\_\_\_\_ , 1 200 115 , 1 600 115 , 2 000 115
4. 2 000 , 20 000 , \_\_\_\_\_ , 2 000 000
5. 24 250 , 36 250 , \_\_\_\_\_ , 60 250
6. 300 000 , \_\_\_\_\_ , 180 000 , 120 000
7. \_\_\_\_\_ , 2 481 215 , 2 581 215 , 2 681 215
8. 3 020 , 103 020 , \_\_\_\_\_ , 303 020
9. 50 125 , 45 125 , \_\_\_\_\_ , 35 125
10. 80 000 , \_\_\_\_\_ , 240 000 , 320 000
11. \_\_\_\_\_ , 1 400 000 , 2 100 000 , 2 800 000
12. 23 020 , 123 020 , \_\_\_\_\_ , 323 020

**Ordering** numbers means listing numbers from least to greatest, or from greatest to least. Two symbols are used in ordering.

<

is less than

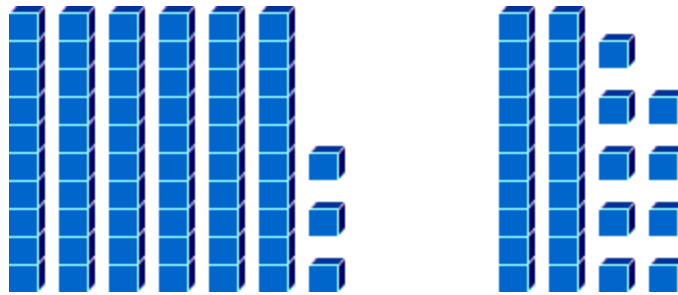
$$2 < 10$$

>

is greater

$$10 > 2$$

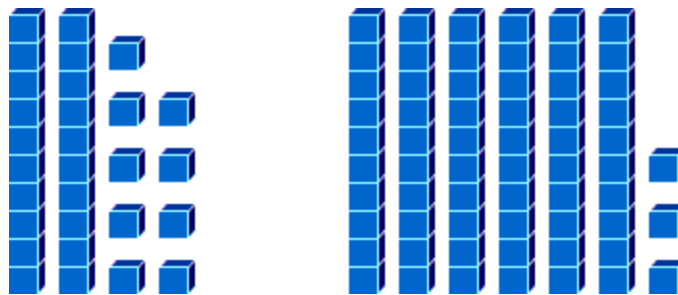
Greater Than >



63 is **greater than** 29.

$$63 > 29$$

Less Than <



29 is **less than** 63.

$$29 < 63$$

# Practice Exercise

Arrange the numbers from greatest to least.

1. 225 266 , 226 662 , 216 226 , 225 626
2. 1 395 090 , 1 359 890 , 1 953 092 , 1 595 190
3. 320 822 , 220 882 , 302 228 , 202 882

Arrange the numbers from least to greatest.

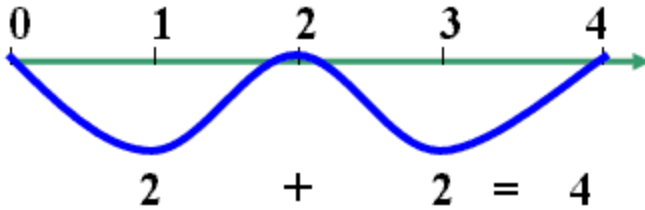
1. 639 892 , 693 800 , 619 892 , 683 899
2. 1 854 190 , 1 485 190 , 1 584 190 , 1 584 901
3. 109 120 , 190 210 , 189 120 , 150 , 102

## Addition

Combining two or more numbers is called *addition*. The term for addition is *plus*, and the symbol for plus is +. The numbers that are combined in addition are called *addends* and together they form a new number called a *sum*.

$$\begin{array}{r} 2 \text{ ---- addends ---- } 3 \\ + 2 \quad \quad \quad + 1 \\ \hline 4 \text{ ---- sum ----- } 4 \end{array}$$

Adding whole numbers is as simple as  $2 + 2$ ! To add two whole numbers, you can simply follow the number line and complete the addition fact.



### Table of Addition Facts

<b>+</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
<b>1</b>	2	3	4	5	6	7	8	9	10	11
<b>2</b>	3	4	5	6	7	8	9	10	11	12
<b>3</b>	4	5	6	7	8	9	10	11	12	13
<b>4</b>	5	6	7	8	9	10	11	12	13	14
<b>5</b>	6	7	8	9	10	11	12	13	14	15
<b>6</b>	7	8	9	10	11	12	13	14	15	16
<b>7</b>	8	9	10	11	12	13	14	15	16	17
<b>8</b>	9	10	11	12	13	14	15	16	17	18
<b>9</b>	10	11	12	13	14	15	16	17	18	19
<b>10</b>	11	12	13	14	15	16	17	18	19	20

## Regrouping Numbers in Addition

Addition often produces sums with a value greater than **9** in a given place. The value of ten is then *regrouped* (or *carried*) to the next place.

	tens	ones
		1
+		9
<hr/>		
	1	0

	tens	ones
	1	3
+		9
<hr/>		
	2	2

	hundreds	tens	ones
	4	1	3
+			8
<hr/>			
	4	2	1

	hundreds	tens	ones
	4	9	6
+			5
<hr/>			
	5	0	1

1 thousands	1 hundreds	1 tens	ones
1,	3	4	3
+3,	7	9	8
5,	1	4	1

To explain addition another way, it can be done by adding the place value amounts separately.

e.g.  $69$   
 $+ 8$   
 $\hline 77$   
 $\underline{60}$  (the 6 in the tens place means 6 tens or “60”)  
 $77$

⇒ If there are not enough digits in each number to make even columns under each place value, then zeros may be used **before** a given number to make adding easier. Do not add zeros **after** a number because it changes the value of the whole number.

e.g.  $69 + 8 + 125$  could be added as:

$$\begin{array}{r} 069 \\ 008 \\ +125 \\ \hline \end{array}$$



## Commutative Property of Addition

The property which states that addends can be added in any order. The sum is always the same

*Example:*

$$267 + 132 = 132 + 267$$

$$399 = 399$$

# Practice Exercise

Solve for each of the given problems.

1. 
$$\begin{array}{r} 818 \\ + 975 \\ \hline \end{array}$$

2. 
$$\begin{array}{r} 109 \\ + 330 \\ \hline \end{array}$$

3. 
$$\begin{array}{r} 626 \\ + 451 \\ \hline \end{array}$$

4. 
$$\begin{array}{r} 321 \\ + 871 \\ \hline \end{array}$$

5. 
$$\begin{array}{r} 74,845 \\ + 84,104 \\ \hline \end{array}$$

6. 
$$\begin{array}{r} 77,295 \\ + 99,985 \\ \hline \end{array}$$

7. 
$$\begin{array}{r} 37,369 \\ + 31,083 \\ \hline \end{array}$$

8. 
$$\begin{array}{r} 89,670 \\ + 53,730 \\ \hline \end{array}$$

9. 
$$\begin{array}{r} 81,579 \\ + 5,654 \\ \hline \end{array}$$

10. 
$$\begin{array}{r} 86,642 \\ + 52,448 \\ \hline \end{array}$$

11. 
$$\begin{array}{r} 47,124 \\ + 24,972 \\ \hline \end{array}$$

12. 
$$\begin{array}{r} 98,455 \\ + 78,977 \\ \hline \end{array}$$

13. 
$$\begin{array}{r} 359,916 \\ + 187,596 \\ \hline \end{array}$$

14. 
$$\begin{array}{r} 937,902 \\ + 665,891 \\ \hline \end{array}$$

15. 
$$\begin{array}{r} 799,688 \\ + 911,286 \\ \hline \end{array}$$

16. 
$$\begin{array}{r} 813,840 \\ + 632,998 \\ \hline \end{array}$$

17. 
$$\begin{array}{r} 912,647 \\ + 431,439 \\ \hline \end{array}$$

18. 
$$\begin{array}{r} 211,152 \\ + 638,672 \\ \hline \end{array}$$

19. 
$$\begin{array}{r} 938,521 \\ + 880,467 \\ \hline \end{array}$$

20. 
$$\begin{array}{r} 918,772 \\ + 593,857 \\ \hline \end{array}$$

## Subtraction

“Taking away” one or more numbers from another number is called *subtraction*. The term for subtraction is *minus*, and the symbol for minus is -. The number being subtracted is called a *subtrahend*. The number being subtracted from is called a *minuend*. The new number left after subtracting is called a *remainder* or *difference*.

$$\begin{array}{r} 4 \text{ ---- } \text{minuend} \text{ ---- } 4 \\ - 2 \text{ --subtrahend - } - 1 \\ \hline 2 \text{ - difference ---- } 3 \end{array}$$

The complete addition or subtraction “sentence” is called an *equation*. An equation has two parts. The two parts are separated by the *equal sign*, =. For example, *the minuend minus the subtrahend equals the difference*. An *addition fact* or a *subtraction fact* is the name given to specific addition or subtraction equations.

$0 + 1 = 1$

$1 + 1 = 2$

$2 + 1 = 3$

$3 + 1 = 4$

$4 + 1 = 5$

$5 + 1 = 6$

$6 + 1 = 7$

$7 + 1 = 8$

$8 + 1 = 9$

$1 - 1 = 0$

$2 - 1 = 1$

$3 - 1 = 2$

$4 - 1 = 3$

$5 - 1 = 4$

$6 - 1 = 5$

$7 - 1 = 6$

$8 - 1 = 7$

$9 - 1 = 8$

## Regrouping in Subtraction

**Regrouping**, sometimes called **borrowing**, is used when the subtrahend is greater than the minuend in a given place. Regrouping means to take a group of tens from the next greatest place to make a minuend great enough to complete the subtraction process.

	tens	ones
21	1	2
- 3		3
<hr/>		
18	1	8

*Note: In the diagram, a red arrow points from the 2 in the tens place to the 1 in the ones place, and the 2 is crossed out. The 1 in the ones place is highlighted in orange.*

	tens	ones
46	3	4
- 9		9
<hr/>		
37	3	7

*Note: In the diagram, a red arrow points from the 4 in the tens place to the 3 in the ones place, and the 4 is crossed out. The 3 in the ones place is highlighted in orange.*

	hundreds	tens	ones
343	3	3	4
- 9			9
<hr/>			
334	3	3	4

*Note: In the diagram, a red arrow points from the 4 in the tens place to the 3 in the ones place, and the 4 is crossed out. The 3 in the ones place is highlighted in orange.*

	hundreds	tens	ones
521	<del>4</del> 5	1 1 2	1 1
- 62	-	6	2
-----	4	5	9
459	4	5	9

	hundreds	tens	ones
506	<del>4</del> 5	9 1 0	1 6
- 8	-		8
-----	4	9	8
498	4	9	8

# Practice Exercise

Solve for each of the given problems.

1. 
$$\begin{array}{r} 919 \\ - 793 \\ \hline \end{array}$$

2. 
$$\begin{array}{r} 65 \\ - 36 \\ \hline \end{array}$$

3. 
$$\begin{array}{r} 81 \\ - 39 \\ \hline \end{array}$$

4. 
$$\begin{array}{r} 72 \\ - 62 \\ \hline \end{array}$$

5. 
$$\begin{array}{r} 9,744 \\ - 3,278 \\ \hline \end{array}$$

6. 
$$\begin{array}{r} 93,054 \\ - 45,155 \\ \hline \end{array}$$

7. 
$$\begin{array}{r} 83,864 \\ - 69,908 \\ \hline \end{array}$$

8. 
$$\begin{array}{r} 79,618 \\ - 47,796 \\ \hline \end{array}$$

9.	$\begin{array}{r} 14,220 \\ - 6,933 \\ \hline \end{array}$	10.	$\begin{array}{r} 95,808 \\ - 11,029 \\ \hline \end{array}$	11.	$\begin{array}{r} 96,002 \\ - 21,707 \\ \hline \end{array}$	12.	$\begin{array}{r} 19,732 \\ - 8,758 \\ \hline \end{array}$
13.	$\begin{array}{r} 844,446 \\ - 711,970 \\ \hline \end{array}$	14.	$\begin{array}{r} 456,443 \\ - 152,986 \\ \hline \end{array}$	15.	$\begin{array}{r} 903,331 \\ - 168,442 \\ \hline \end{array}$	16.	$\begin{array}{r} 813,341 \\ - 249,750 \\ \hline \end{array}$
17.	$\begin{array}{r} 451,752 \\ - 286,817 \\ \hline \end{array}$	18.	$\begin{array}{r} 973,309 \\ - 792,959 \\ \hline \end{array}$	19.	$\begin{array}{r} 142,133 \\ - 113,663 \\ \hline \end{array}$	20.	$\begin{array}{r} 963,116 \\ - 960,180 \\ \hline \end{array}$

Inverse (opposite) operations are used to simplify an equation for solving.

One operation is involved with the unknown and the inverse operation is used to solve the equation.

**Addition and subtraction are inverse operations.**

Given an equation such as  $7 + x = 10$ , the unknown  $x$  and  $7$  are *added*. Use the inverse operation subtraction. To solve for  $n$ , subtract  $7$  from  $10$ . The unknown value is therefore  $3$ .

Examples for addition and subtraction

Addition Problem

$$x + 15 = 20$$

Solution

$$x = 20 - 15 = 5$$

Subtraction Problem

$$x - 15 = 20$$

Solution

$$x = 20 + 15 = 35$$

# Practice Exercise

Solve each equation.

(Hint: Use inverse operation rules to solve

1.  $49 = a - 36$     85

2.  $x - 73 = 25$  \_\_\_\_\_

3.  $34 + y = 55$  \_\_\_\_\_

4.  $x + 88 = 92$  \_\_\_\_\_

5.  $9 = a - 67$  \_\_\_\_\_

6.  $x + 24 =$   
103 \_\_\_\_\_

7.  $39 + y = 137$  \_\_\_\_\_

8.  $x - 19 = 37$  \_\_\_\_\_

9.  $3 + y = 68$  \_\_\_\_\_

10.  $x + 94 =$   
167 \_\_\_\_\_

11.  $93 + y = 151$  \_\_\_\_\_

12.  $16 = a - 28$  \_\_\_\_\_

13.  $x - 29 = 22$  \_\_\_\_\_

14.  $28 = a - 51$  \_\_\_\_\_

15.  $13 = a - 43$  \_\_\_\_\_

16.  $x + 21 = 74$  \_\_\_\_\_

17.  $719 + y =$   
1181 \_\_\_\_\_

18.  $77 + y =$   
135 \_\_\_\_\_

## Multiplication

**Multiplication** is a quick form of addition. By multiplying numbers together, you are really adding a series of one number to itself. For example, you can add 2 plus 2. Both **2 plus 2** and **2 times 2** equal 4.

$$\begin{array}{r} 2 + 2 = 4 \\ 2 \times 2 = 4 \end{array} \qquad \begin{array}{r} 2 \\ + 2 \\ \hline 4 \end{array} \qquad \begin{array}{r} 2 \\ \times 2 \\ \hline 4 \end{array}$$

But what if you wanted to calculate the number of days in five weeks? You could add 7 days + 7 days + 7 days + 7 days + 7 days or you could multiply 7 days times 5. Either way you arrive at **35**, the number of days in five weeks.

$$\begin{array}{r} 7 + 7 + 7 + 7 + 7 = 35 \\ 5 \times 7 = 35 \end{array}$$

Although multiplication is related to addition, the parts of multiplication are not known as addends. Instead, the parts are known as **multiplicands** and **multipliers**. A multiplication sentence, like an addition sentence, is called an **equation**. But a multiplication sentence results in a **product**, not a sum.

Diagram illustrating the parts of the multiplication equation  $2 \times 2 = 4$ . The first 2 is labeled as the **multiplicand**, the second 2 is labeled as the **multiplier**, and the result 4 is labeled as the **product**.

2 — **multiplicand**

x 2 — **multiplier**

-----  
4 — **product**

X	0	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
<del>5</del>	<del>0</del>	<del>5</del>	<del>10</del>	<del>15</del>	<del>20</del>	<del>25</del>	<del>30</del>	<del>35</del>	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

## Multiplication, Step-by-Step

When the multiplicand and the multiplier are numbers with two or more digits, multiplication becomes a step-by-step process.

Look at  $15 \times 13$ :

$$\begin{array}{r}
 15 \\
 \times 3 \\
 \hline
 15
 \end{array}$$

First, multiply the ones –  $3 \times 5$ . Write down the product so the ones columns line up.



$$\begin{array}{r}
 15 \\
 \times 3 \\
 \hline
 15
 \end{array}$$

Next, multiply the tens – 3 x 1 ten.  
Line up the product with the tens column.

$$\begin{array}{r}
 30 \\
 \hline
 \end{array}$$

— Zero is the place holder.

$$\begin{array}{r}
 15 \\
 \times 3 \\
 \hline
 15
 \end{array}$$

Last, add the ones and tens to find the product of the equation.

$$\begin{array}{r}
 15 \\
 + 30 \\
 \hline
 45
 \end{array}$$

Here is a shorter way:

$$\begin{array}{r}
 1 \\
 15 \\
 \times 3 \\
 \hline
 45
 \end{array}$$

1. Multiply the ones:  $3 \times 5 = 15$ .  
Put the 5 in the ones column and regroup the 1 to the tens column.

2. Multiply the tens:  $3 \times 1 = 3$ .

3. Add the 1 that you regrouped to the 3, put the sum in the tens column.

Look at  $265 \times 23$ :

$$\begin{array}{r} 265 \\ \times 23 \\ \hline 15 \\ 180 \\ 600 \end{array}$$

First, multiply the multiplicand by the ones in the multiplier –  $3 \times 5$ ,  $3 \times 6$ , and  $3 \times 2$ . Zero is the place holder.

$$\begin{array}{r} 265 \\ \times 23 \\ \hline 15 \\ 180 \\ 600 \\ \hline 100 \\ 1,200 \\ 4,000 \end{array}$$

Next, multiply by the tens -  $2 \times 5$ ,  $2 \times 6$ , and  $2 \times 2$ . Zero is the place holder.

$$\begin{array}{r} 265 \\ \times 23 \\ \hline 15 \\ + 180 \\ + 600 \\ \hline + 100 \\ + 1,200 \\ + 4,000 \\ \hline 6,095 \end{array}$$

Last, add.

Here is a shorter way:

$$\begin{array}{r} 11 \\ 11 \\ 265 \\ \times 23 \\ \hline 795 \end{array}$$

1. Multiply the ones:  $3 \times 265$   
 $3 \times 5 = 15$  regroup the 1  
 $3 \times 6 = 18$  plus the regrouped 1 =  
regroup the 1  
 $3 \times 2 = 6$  plus the regrouped 1 = 7

$$\begin{array}{r} 5300 \\ \hline 6,095 \\ 13; \end{array}$$

2. Multiply the tens:  $2 \times 265$   
0 is the place holder  
 $2 \times 5 = 10$  regroup the 1  
 $2 \times 6 = 12$  plus the regrouped 1 =  
regroup the 1  
 $2 \times 2 = 4$  plus the regrouped 1 = 5

3. Add  $795 + 5300 = 6,095$

### Partial Product

A method of multiplying where the ones, tens, hundreds, and so on are multiplied separately and then the products added together

*Example:*

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 12 \\ + 60 \\ \hline 72 \end{array}$$

← Multiply the ones:  $3 \times 4 = 12$   
← Multiply the tens:  $3 \times 20 = 60$

$$36 \times 17 = 42 + 210 + 60 + 300 = 612$$

When you multiply whole numbers, the *product* usually has a greater value than either the *multiplicand* or the *multiplier*.

But there are exceptions:

A number multiplied by *1* is always equal to itself.

$$\begin{array}{r} 1 \\ \times 1 \\ \hline 1 \end{array} \quad 21 \times 1 = 21 \quad \begin{array}{r} 36 \\ \times 1 \\ \hline 36 \end{array}$$

A number multiplied by *0* is always equal to *0*.

$$\begin{array}{r} 1 \\ \times 0 \\ \hline 0 \end{array} \quad 21 \times 0 = 0 \quad \begin{array}{r} 36 \\ \times 0 \\ \hline 0 \end{array}$$

To multiply a number by 10, add a 0 to the right of the number.

EXAMPLE

$$25 \times 10 = 250 \quad \text{or} \quad \begin{array}{r} 25 \\ \times 10 \\ \hline 250 \end{array}$$

To multiply a number by 100, add two 0's to the right of the number.

EXAMPLE

$$36 \times 100 = 3,600 \quad \text{or} \quad \begin{array}{r} 36 \\ \times 100 \\ \hline 3,600 \end{array}$$

## Commutative Property of Multiplication

The property which states that **factors** can be multiplied in any order. The **product** is always the same.

*Example:*

$$5 \times 7 = 7 \times 5$$

$$35 = 35$$

## Associative Property of Multiplication

The property which states that when **multiplying** three or more **factors**, any two of the factors can be multiplied, and the remaining factors may then be multiplied without changing the total **product**

*Example:*

$$(3 \times 4) \times 5 = 3 \times (4 \times 5)$$

$$12 \times 5 = 3 \times 20$$

$$60 = 60$$

# Practice Exercise

Solve for each of the given problems.

1.  $\begin{array}{r} 46 \\ \times 5 \\ \hline \end{array}$     2.  $\begin{array}{r} 65 \\ \times 4 \\ \hline \end{array}$     3.  $\begin{array}{r} 60 \\ \times 2 \\ \hline \end{array}$     4.  $\begin{array}{r} 81 \\ \times 5 \\ \hline \end{array}$     5.  $\begin{array}{r} 80 \\ \times 5 \\ \hline \end{array}$

6.  $\begin{array}{r} 97 \\ \times 9 \\ \hline \end{array}$     7.  $\begin{array}{r} 16 \\ \times 3 \\ \hline \end{array}$     8.  $\begin{array}{r} 41 \\ \times 3 \\ \hline \end{array}$     9.  $\begin{array}{r} 17 \\ \times 5 \\ \hline \end{array}$     10.  $\begin{array}{r} 31 \\ \times 4 \\ \hline \end{array}$

$$\begin{array}{r} 11. \quad 58 \\ \times 5 \\ \hline \end{array} \quad \begin{array}{r} 12. \quad 78 \\ \times 5 \\ \hline \end{array} \quad \begin{array}{r} 13. \quad 56 \\ \times 3 \\ \hline \end{array} \quad \begin{array}{r} 14. \quad 12 \\ \times 4 \\ \hline \end{array} \quad \begin{array}{r} 15. \quad 10 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 16. \quad 776 \\ \times 3 \\ \hline \end{array} \quad \begin{array}{r} 17. \quad 101 \\ \times 2 \\ \hline \end{array} \quad \begin{array}{r} 18. \quad 750 \\ \times 2 \\ \hline \end{array} \quad \begin{array}{r} 19. \quad 174 \\ \times 6 \\ \hline \end{array} \quad \begin{array}{r} 20. \quad 429 \\ \times 6 \\ \hline \end{array}$$

$$\begin{array}{r} 21. \quad 1819 \\ \times 7 \\ \hline \end{array} \quad \begin{array}{r} 22. \quad 9178 \\ \times 3 \\ \hline \end{array} \quad \begin{array}{r} 23. \quad 2459 \\ \times 4 \\ \hline \end{array} \quad \begin{array}{r} 24. \quad 8553 \\ \times 5 \\ \hline \end{array} \quad \begin{array}{r} 25. \quad 3652 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 26. \quad 7423 \\ \times 8 \\ \hline \end{array} \quad \begin{array}{r} 27. \quad 4123 \\ \times 4 \\ \hline \end{array} \quad \begin{array}{r} 28. \quad 6812 \\ \times 4 \\ \hline \end{array} \quad \begin{array}{r} 29. \quad 5336 \\ \times 4 \\ \hline \end{array} \quad \begin{array}{r} 30. \quad 1438 \\ \times 6 \\ \hline \end{array}$$

## Division

**Division** is the process of finding out how many times one number, the **divisor**, will fit into another number, the **dividend**. The division sentence results in a **quotient**. The signs of division  $\div$  and  $\sqrt{\quad}$ , and mean **divided by**. You can think of division as a series of repeated subtractions. For example,  $40 \div 10$  could also be solved by subtracting  $10$  from  $40$  four times:

$$40 - 10 - 10 - 10 - 10 = 0$$

Because **10** can be subtracted four times, you can say that **40** can be divided by **10** four times, or  $40 \div 10 = 4$ .

$40 \div 10 = 4$   
 dividend      divisor      quotient  
 divisor      dividend      quotient

Many numbers do not fit evenly into other numbers. They are **not evenly divisible by** those numbers, and the number left over is called the **remainder**.

$3 \overline{)10}$   
 $-9$   
 $\hline 1$   
 10 is not evenly divisible by 3

$7 \overline{)20}$   
 $-14$   
 $\hline 6$   
 20 is not evenly divisible by 7

remainder

We would record the answer for the first question as 3 r 1 and for the second question as 2 r 6. The “r” stands for remainder.

To divide whole numbers, reverse the process of multiplication. For example, if  $2 \times 7 = 14$  in a multiplication equation, then in a division sentence, **14** is the **dividend** and **7** is the **divisor** with a **quotient** of **2**.

$$\begin{array}{c}
 \text{divisor} \\
 \swarrow \\
 14 \div 7 = 2 \\
 \swarrow \quad \searrow \\
 \text{dividend} \quad \text{quotient}
 \end{array}
 \qquad
 \begin{array}{c}
 \text{quotient} \\
 \swarrow \\
 7 \overline{) 14} \\
 \swarrow \quad \searrow \\
 \text{divisor} \quad \text{dividend}
 \end{array}$$

A whole number divided by  $1$  will always equal itself.

$$1 \ , \ 1 = 1 \qquad 1 \overline{) 21} \qquad 36 \ , \ 1 = 36$$

Zero divided by a whole number will always equal  $0$ .

$$0 \ , \ 12 = 0 \qquad 3 \overline{) 0} \qquad 0/7 = 0$$

## Division, Step-by-Step

Where the dividend and divisor are numbers with two or more digits, division becomes a step-by-step process.

$$\begin{array}{r}
 2 \\
 8 \overline{) 208} \\
 -16 \phantom{0} \\
 \hline
 4
 \end{array}$$

First, round the divisor up - 8 rounds up to 10 - and estimate the number of 10s in 20. Answer: 2. Multiply the divisor - 8 x 2 - and subtract the product from the dividend.



$$\begin{array}{r}
 26 \\
 8 \overline{) 208} \\
 - 16 \phantom{0} \\
 \hline
 48 \\
 - 48 \\
 \hline
 0
 \end{array}$$

Next, pull down the next digit from the dividend – 8 – and repeat the estimation and subtraction process.

$$\begin{array}{r}
 26 \\
 8 \overline{) 208} \\
 - 16 \phantom{0} \\
 \hline
 48 \\
 - 48 \\
 \hline
 0
 \end{array}$$

Last, when you can subtract no more you've found the quotient.

0 ——— No remainder

$$\begin{array}{r}
 1 \\
 23 \overline{) 276} \\
 - 23 \phantom{0} \\
 \hline
 4
 \end{array}$$

First, round 23 to 25 and estimate the number of 25s in 27. Answer: 1.

Multiply the divisor by 1 – 23 x 1 – and subtract.

$$\begin{array}{r}
 12 \\
 23 \overline{) 276} \\
 - 23 \phantom{0} \\
 \hline
 46 \\
 - 46 \\
 \hline
 0
 \end{array}$$

Next, pull down the next digit from the dividend – 6 – and repeat the estimation and subtraction process.

$$\begin{array}{r}
 23 \overline{)276} \\
 \underline{-23} \phantom{0} \\
 46 \\
 \underline{-46} \\
 0
 \end{array}$$

Then, pull down the next digit, estimate, and subtract, until you can subtract no more.

0 ——— No remainder

# Practice Exercise

Solve each problem.

- |                                      |                                       |  |
|--------------------------------------|---------------------------------------|--|
| 1. $45 \div 9 = 5$                   | 2. $721 \div 9 = \underline{\quad}$   | 3. $2423 \div 4 = \underline{\quad}$   |
| 4. $14 \div 2 = \underline{\quad}$   | 5. $60 \div 12 = \underline{\quad}$   | 6. $994 \div 11 = \underline{\quad}$   |
| 7. $55 \div 5 = \underline{\quad}$   | 8. $100 \div 10 = \underline{\quad}$  | 9. $185 \div 6 = \underline{\quad}$    |
| 10. $20 \div 4 = \underline{\quad}$  | 11. $166 \div 8 = \underline{\quad}$  | 12. $1278 \div 3 = \underline{\quad}$  |
| 13. $56 \div 8 = \underline{\quad}$  | 14. $109 \div 5 = \underline{\quad}$  | 15. $9601 \div 8 = \underline{\quad}$  |
| 16. $80 \div 10 = \underline{\quad}$ | 17. $502 \div 10 = \underline{\quad}$ | 18. $108 \div 12 = \underline{\quad}$  |
| 19. $36 \div 12 = \underline{\quad}$ | 20. $30 \div 3 = \underline{\quad}$   | 21. $132 \div 12 = \underline{\quad}$  |
| 22. $28 \div 7 = \underline{\quad}$  | 23. $903 \div 9 = \underline{\quad}$  | 24. $2045 \div 2 = \underline{\quad}$  |
| 25. $63 \div 7 = \underline{\quad}$  | 26. $326 \div 8 = \underline{\quad}$  | 27. $8789 \div 4 = \underline{\quad}$  |
| 28. $60 \div 2 = \underline{\quad}$  | 29. $120 \div 12 = \underline{\quad}$ | 30. $4812 \div 12 = \underline{\quad}$ |
| 31. $77 \div 11 = \underline{\quad}$ | 32. $88 \div 8 = \underline{\quad}$   | 33. $403 \div 10 = \underline{\quad}$  |
| 34. $36 \div 4 = \underline{\quad}$  | 35. $814 \div 9 = \underline{\quad}$  | 36. $24 \div 12 = \underline{\quad}$   |
| 37. $48 \div 8 = \underline{\quad}$  | 38. $425 \div 6 = \underline{\quad}$  | 39. $2467 \div 8 = \underline{\quad}$  |
| 40. $16 \div 4 = \underline{\quad}$  | 41. $72 \div 12 = \underline{\quad}$  | 42. $308 \div 6 = \underline{\quad}$   |
| 43. $54 \div 9 = \underline{\quad}$  | 44. $110 \div 10 = \underline{\quad}$ | 45. $359 \div 7 = \underline{\quad}$   |
| 46. $44 \div 4 = \underline{\quad}$  | 47. $121 \div 11 = \underline{\quad}$ | 48. $9012 \div 3 = \underline{\quad}$  |

$$\begin{array}{lll}
 49. 333 \div 11 = \underline{\quad\quad} & 50. 704 \div 7 = \underline{\quad\quad} & 51. 225 \div 11 = \underline{\quad\quad} \\
 52. 846 \div 12 = \underline{\quad\quad} & 53. 667 \div 6 = \underline{\quad\quad} & 54. 608 \div 10 = \underline{\quad\quad} \\
 55. 409 \div 5 = \underline{\quad\quad} & 56. 120 \div 2 = \underline{\quad\quad} & 57. 181 \div 9 = \underline{\quad\quad}
 \end{array}$$

**Factors** are numbers that, when multiplied together, form a new number called a **product**. For example, **1** and **2** are factors of **2**, and **3** and **4** are factors of **12**.

Every number except **1** has at least two factors: **1** and itself.

### **Common Factor**

A number that is a **factor** of two or more numbers

*Example:*

factors of 6: 1, 2, 3, 6

factors of 12: 1, 2, 3, 4, 6, 12

The common factors of 6 and 12 are  
1, 2, 3, and 6.

### **Greatest Common Factor (GCF)**

The greatest **factor** that two or more numbers have in  
common

*Example:*

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

5 is the GCF of 18 and 30.

# Practice Exercise

Find the greatest common factor (GCF) for the given numbers.

1. 5, 3            **1**

2. 4, 6

15. 30, 4

3. 12, 9

16. 20, 12

4. 3, 4

17. 6, 3

5. 9, 6

18. 28, 21

6. 8, 12

19. 40, 5

7. 6, 12

20. 12, 30

8. 6, 10

21. 14, 12

9. 12, 4

22. 44, 48

10. 21, 5

23. 24, 40

11. 18, 6

24. 36, 6

12. 20, 30

25. 70, 110

13. 24, 10

26. 14, 17

14. 21, 20

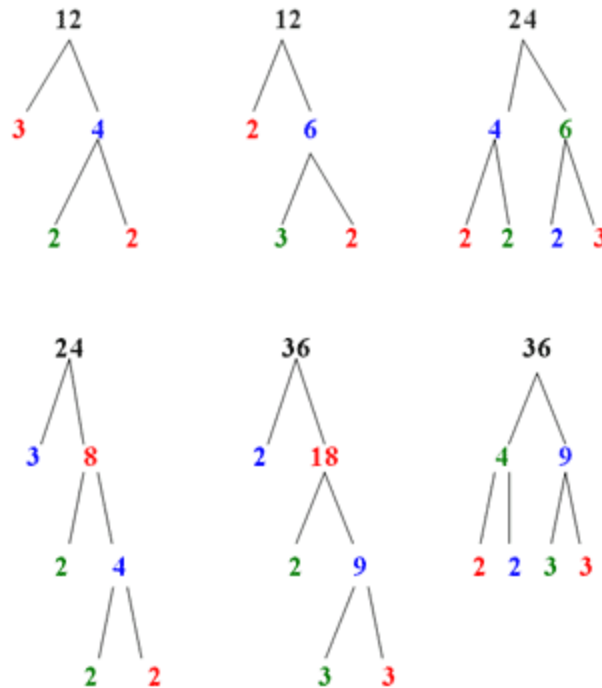
27. 60, 30

28. 120, 48

29. 42, 105

30. 9, 15

**Composite numbers** have more than two factors. In fact, every composite number can be written as the product of **prime numbers**. You can see this on a **factor tree**.



**Prime numbers** are counting numbers that can be divided by only two numbers---**1** and themselves. A prime number can also be described as a counting number with only two factors, **1** and itself. The number **1**, because it can be divided only by itself, is **not** a prime number.

### Prime Numbers to 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,

53, 59, 61, 67, 71, 73, 79, 83, 89, 97

# Practice Exercise

Find the prime factorization of each number.

1. 18                      **2, 3, 3**
2. 20
3. 6
4. 28
5. 30
6. 10
7. 96
8. 92
9. 16
10. 34
11. 12
12. 42
13. 88
14. 72
15. 74
16. 64
17. 44
18. 120

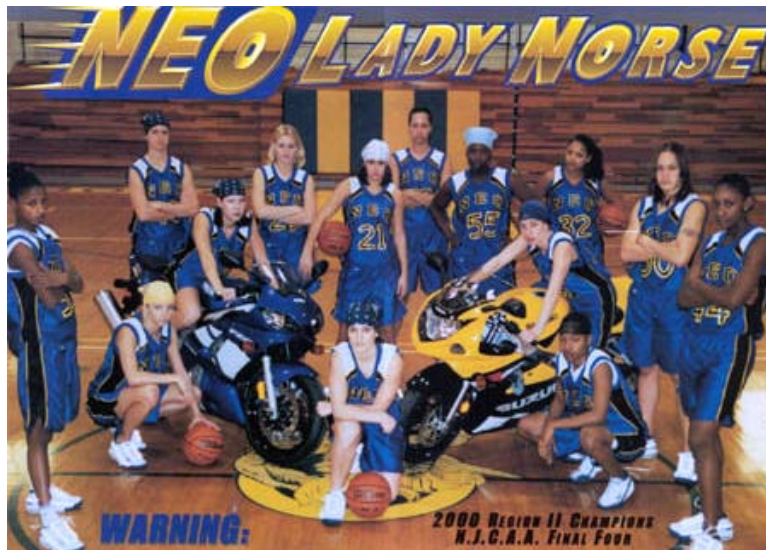
## Averages

The most common way to find an *average* is to add up a list of numbers and divide the sum by the number of items on the list. Another word for average is *mean*.

$$3 + 4 + 6 + 8 + 9 = 30 \quad \text{number of addends}$$

**sum** —  $30 \div 5 = 6$  So, the average of the numbers 3, 4, 6, 8, and 9 is 6.

When do you need to calculate an average? Your grades may be based on the average of all your test scores. In sports, you might want to find out the average height of players on your favorite basketball team.



**The height of the starters for this team is:**

**Anita      60"**

**Jane        58"**

**Caitlin    57"**

**Josie       52"**

**Tanya      48"**

**The average height of these  
players is 55 inches.**





## Word Problems with Whole Numbers

Within every story (word) problem are several *clue words*. These words tell you the kind of math sentence (equation) to write to solve the problem.

### **Addition Clue Words**

add  
sum  
total  
plus  
in all  
both  
together  
increased by  
all together  
combined

### **Subtraction Clue Words**

subtract  
difference  
take away  
less than  
are not  
remain  
decreased by  
have or are left  
change (money problems)  
more  
fewer

### **Multiplication Clue Words**

times  
product of  
multiplied by  
by (dimension)

### **Division Clue Words**

quotient of  
divided by  
half [or a fraction]  
split  
separated  
cut up  
parts  
shared equally

⇒ *Division clue words are often the same as subtraction clue words. Divide when you know the total and are asked to find the size or number of “one part” or “each part.”*

Following a system of steps can increase your ability to accurately solve problems. Use these steps to solve word problems.

1. Read the problem carefully. Look up the meanings of unfamiliar words.
2. Organize or restate the given information.
3. State what is to be found.
4. Select a strategy (such as making a chart of working backward) and plan the steps to solve the problem.
5. Decide on an approximate answer before solving the problem.
6. Work the steps to solve the problem.
7. Check the final result. Does your answer seem reasonable?

The Problem Solving System was used to solve the following problem:

**Mary has ten marbles. Lennie has thirteen. How many marbles do they have in all?**

**1. Mary has ten marbles. Lennie has thirteen.  
How many marbles do they have in all?**

**2. Mary – 10 marbles  
Lennie – 13 marbles**

**3. How many marbles in all?**

**4. Add**

**5. A little over 20 marbles ( $10 + 10 = 20$ )**

**6.   10  
   +13  
   —  
   23 marbles**

**7. The final sum of 23 marbles is close to the estimated answer of 20 marbles. The final result is reasonable.**

***P*** *Be sure to label answers whenever possible. For example: marbles, grams, pounds, feet, dogs, etc.*

***P*** *Some problems may require several steps to solve. Some may have more than one correct answer. And some problems may not have a solution.*

Have you ever tried to help someone else work out a word problem? Think about what you do. Often, you read the problem with the person, then discuss it or put it in your own words to help the person see what is happening. You can use this method---restating the problem---on your own as a form of “talking to yourself.”

Restating a problem can be especially helpful when the word problem contains no key words. Look at the following example:

**Example:** Susan has already driven her car 2,700 miles since its last oil change. She still plans to drive 600 miles before changing the oil. How many miles does she plan to drive between oil changes?

**Step 1:** *question:* How many miles does she plan to drive between oil changes?

**Step 2:** *necessary information:* 2,700 miles, 600 miles

**Step 3:** *decide what arithmetic to use:* Restate the problem in your own words: “You are given the number of miles Susan has already driven and the number of miles more that she plans to drive. You need to add these together to find the total number of miles between oil changes.”

**Step 4:**  $2,700 \text{ miles} + 600 \text{ miles} = \mathbf{3,300 \text{ miles}}$  between oil changes.

**Step 5:** It makes sense that she will drive 3,300 miles between oil changes, since you are looking for a number larger than the 2,700 miles that she has already driven.

For some problems, you have to write two or three equations to solve the problem. For others, you may need to make charts or lists of information, draw pictures, find a pattern, or even guess and check. Sometimes you have to work backwards from a sum, product, difference, or quotient, or simply use your best logical thinking.

### List/Chart

**Marty's library book was six days overdue. The fine is \$.05 the first day, \$.10, the second, \$.20 the third day, and so on. How much does Marty owe?**

**Marty's library book was six days overdue. The fine is \$.05 the first day, \$.10, the second, \$.20 the third day, and so on. How much does Marty owe?**

<b>Days</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>Fine</b>	<b>\$.05</b>	<b>\$.10</b>	<b>\$.20</b>	<b>\$.40</b>	<b>\$.80</b>	<b>\$1.60</b>

**Answer: \$1.60**

**Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?**

**Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?**

Veronica	Veronica	Archie	Archie
Archie	Betty	Veronica	Betty
Betty	Archie	Betty	Veronica
Betty	Betty		
Veronica	Archie		
Archie	Veronica		

**Answer: 6 ways**

### Find a Pattern

Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X\_\_\_\_\_. How did Jenny fill in the blanks?

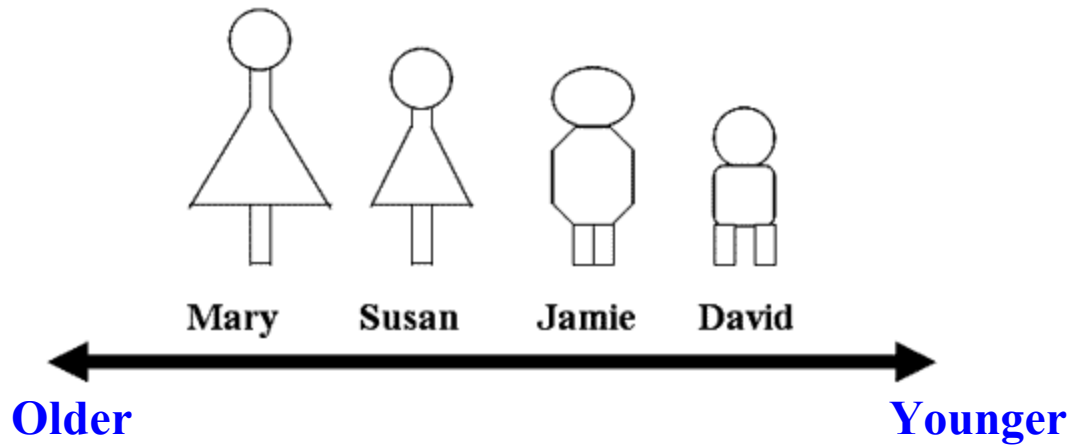
Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X\_\_\_\_\_. How did Jenny fill in the blanks?

**Answer: 10, 11, 12 W**

### Draw a Picture

Mary is older than Jamie. Susan is older than Jamie, but younger than Mary. David is younger than Jamie. Who is oldest?

Mary is older than Jamie. Susan is older than Jamie, but younger than Mary. David is younger than Jamie. Who is oldest?



**Answer: Mary is oldest.**

### Guess and Check

Farmer Joe keeps cows and chickens in the farmyard. All together, Joe can count 14 heads and 42 legs. How many cows and how many chickens does Joe have in the farmyard?

Farmer Joe keeps cows and chickens in the farmyard. All together, Joe can count 14 heads and 42 legs. How many cows and how many chickens does Joe have in the farmyard?

6 cows      Guess a number of      6 cows      = 24 legs  
+8 chickens      cows. Then add      +8 chickens = 16 legs

**14 heads**      the number of      **40 legs**  
 chickens to arrive  
 at the sum of 14  
 heads. Then check  
 the total legs.

**7 cows**      Adjust your      **7 cows**      = **28 legs**  
**+7 chickens** guesses. Then check **+7 chickens** = **14 legs**  
**14 heads** again until you      **42 legs**  
 solve the problem.

**Answer: 7 cows and 7 chickens**

### Work Backwards

Marsha was banker for the school play. She took in \$175 in ticket sales. She gave Wendy \$75 for sets and costumes and Paul \$17.75 for advertising and publicity. After paying for the props, Marsha had \$32.25 left. How much did the props cost?

Marsha was banker for the school play. She **took in \$175** in ticket sales. She **gave Wendy \$75** for sets and costumes **and Paul \$17.75** for advertising and publicity. **After paying for the props, Marsha had \$32.25 left.** How much did the props cost?

\$ 175.00 tickets	\$ 82.25
- 75.00 costumes	- 32.25
<u>\$ 100.00</u>	<u>\$ 50.00 cost of props</u>



- 17.75 advertising  
\$ 82.25

## Logical Reasoning

Jean challenged Sheila to guess her grandmother's age in ten questions or less. It took her six. Here's what Sheila asked:

Jean challenged Sheila to **guess her grandmother's age** in ten questions or less. It took her six. Here's what Sheila asked:

“Is she less than fifty?” “No.”	<b>50+ years old</b>
“Less than seventy-five?” “Yes.”	<b>50 to 74 years old</b>
“Is her age an odd or even number?”	
“Odd.”	<b>ends in 1, 3, 5, 7 or 9</b>
“Is the last number less than or equal to five?” “No.”	<b>ends in 7 or 9</b>
“Is it nine?” “No.”	<b>ends in 7 – 57 or 67</b>
“Is she in her sixties?” “No.”	<b>57 years old</b>

# Practice Exercise

Solve for each of the given problems.

1. The new theater has a capacity of 900 people. The theater has 36 rows of seats. How many seats are on each row?
2. A crate of bananas weighs 225 pounds. A crate of tangerines weighs 200 pounds. How many pounds do both crates weigh?
3. A math book has 285 pages. If Brad has finished reading 74 pages, how many more pages are left to read?
4. Brad drove 909 miles on Tuesday. Michael drove 423 miles on Monday. How many more miles did Brad drive?
5. The pizza delivery consisted of 8 pizzas. Brad cut 10 slices in each of 3 pizzas. Jane cut each of the rest of the pizzas into 8 pieces each. How many slices are there?
6. Amy was 25 when Greg was born. How old will Amy be when Greg is fourteen?
7. The Brewer family pays a car loan payment of \$269 a month. How much will they pay on the loan in a 12-

month period?

8. Last month the Smiths paid \$137 to heat their home. This month they paid \$124. What is the total cost of heating their home for the two months?
9. Six friends travel to work together in Saint John. Each week, the gas, parking, and tolls cost \$114. If they split the costs equally, how much does each friend pay per week toward the traveling costs?
10. Emma worked 68 hours in 2 weeks. She earns \$7 per hour. How much did she earn in the 2 weeks?

## Answer Key

### Book 14015 – Whole Numbers

- Page 10** Row 1: CII, LXXX  
Row 2: 190, 122  
Row 3: 397, CXXXII  
Row 4: 496, CXLIII  
Row 5: CCCLXXII, 554  
Row 6: 501, 574  
Row 7: DCV, CCCXCIV  
Row 8: 642, DLXXXIII  
Row 9: 715, 949  
Row 10: MCVII, 1081

- Page 18** 2. 85723 3. 3616 4. 71753 5. 59584  
6. 33000 7. 9062 8. 1078311  
9. 1025050 10. 1024968 11. 1650000  
12. 1166673 13. 1028918 14. 1949584

- Page 20** 2. ten thousand, six hundred seventy-one  
3. seventy thousand, four hundred seventy  
4. eighty-nine thousand, twelve 5. four  
thousand, forty-seven 6. eleven thousand,  
one hundred twenty-six 7. sixty-six thousand,  
one hundred twenty-nine 8. one million, four  
9. one million, thirty-one thousand, five  
hundred sixty-one 10. one million, twenty-  
six thousand, eight hundred fourteen 11. one  
million, two hundred forty thousand 12. one

million, three hundred sixty-six thousand,  
seventy-seven    **13.** one million, five hundred  
forty-three thousand, six hundred fifty-four  
**14.** one million, five hundred twenty thousand,  
eighty-six    **15.** one million, six hundred  
twenty-one thousand, seven hundred twenty-  
three    **16.** one million, six hundred thirty-one  
thousand, six hundred eighteen    **17.** one  
million, fifty-four thousand, eight hundred  
twenty-seven    **18.** one million, eight hundred  
fifty thousand, four hundred fifty-six.

**Page 24**

- 2.** Ten Thousands    **3.** Thousands  
**4.** Millions    **5.** Ones    **6.** Ten Thousands  
**7.** Thousands    **8.** Thousands    **9.** Hundreds  
**10.** Ten Thousands    **11.** Hundreds  
**12.** Thousands    **13.** Ten Thousands  
**14.** Hundreds    **15.** Tens    **16.** Thousands  
**17.** Ten Thousands    **18.** Thousands  
**19.** Millions    **20.** Ten Thousands  
**21.** Hundred Thousands    **22.** Hundreds  
**23.** Ten Thousands    **24.** Tens    **25.** Hundreds  
**26.** Ones    **27.** Millions    **28.** Thousands  
**29.** Ones    **30.** Thousands

**Page 27**

- 1.** 62274    **2.** 355472    **3.** 800115  
**4.** 200000    **5.** 48250    **6.** 240000  
**7.** 2381215    **8.** 203020    **9.** 40125  
**10.** 160000    **11.** 700000    **12.** 223020

**Page 29** (greatest to least) 1. 226662, 225626, 225266,  
216226 2. 1953092, 1595190, 1395090,  
1359890 3. 320822, 302228, 220882, 202882

**Page 29** (least to greatest) 1. 619892, 639892, 683899,  
693800 2. 1485190, 1584190, 1584901,  
1854190 3. 150102, 109120, 189120, 190210

**Page 33** 1. 1793 2. 439 3. 1077 4. 1192  
5. 158949 6. 177280 7. 68452  
8. 143400 9. 87233 10. 139090  
11. 72096 12. 177432 13. 547512  
14. 1603793 15. 1710974 16. 1446838  
17. 1344086 18. 849824 19. 1818988  
17. 1512629

**Page 36** 1. 126 2. 29 3. 42 4. 10 5. 16466  
6. 47899 7. 13956 8. 31822 9. 7287  
10. 84799 11. 74295 12. 10974  
13. 132476 14. 303457 15. 734889  
16. 563591 17. 164935 18. 180350  
19. 28470 20. 2936

**Page 38** 2. 98 3. 21 4. 4 5. 76 6. 79 7. 98  
8. 56 9. 65 10. 73 11. 58 12. 44  
13. 51 14. 79 15. 56 16. 53 17. 462  
18. 58

**Page 45** 1. 230 2. 260 3. 120 4. 405 5. 400  
6. 873 7. 48 8. 123 9. 85 10. 124  
11. 290 12. 390 13. 168 14. 48

15. 50   16. 2328   17. 202   18. 1500  
 19. 1044   20. 2574   21. 12733   22. 27534  
 23. 9836   24. 42765   25. 14608  
 26. 59384   27. 16492   28. 27248  
 29. 21344   30. 8628

**Page 50**

2. 80 r 1   3. 605 r 3   4. 7   5. 5  
 6. 90 r 4   7. 11   8. 10   9. 30 r 5   10. 5  
 11. 20 r 6   12. 426   13. 7   14. 21 r 4  
 15. 1200 r 1   16. 8   17. 50 r 2   18. 9  
 19. 3   20. 10   21. 11   22. 4   23. 100 r 3  
 24. 1022 r 1   25. 9   26. 40 r 6  
 27. 2197 r 1   28. 30   29. 10   30. 401  
 31. 7   32. 11   33. 40 r 3   34. 9  
 35. 90 r 4   36. 2   37. 6   38. 70 r 5  
 39. 308 r 3   40. 4   41. 6   42. 51 r 2  
 43. 6   44. 11   45. 51 r 2   46. 11   47. 11  
 48. 3004   49. 30 r 3   50. 100 r 4  
 51. 20 r 5   52. 70 r 6   53. 111 r 1  
 54. 60 r 8   55. 81 r 4   56. 60   57. 20 r 1

**Page 52**

2. 2   3. 3   4. 1   5. 3   6. 4   7. 6  
 8. 2   9. 4   10. 1   11. 6   12. 10   13. 2  
 14. 1   15. 2   16. 4   17. 3   18. 7   19. 5  
 20. 6   21. 2   22. 4   23. 8   24. 6  
 25. 10   26. 1   27. 30   28. 24   29. 21  
 30. 3

**Page 54**

2. 2, 2, 5   3. 2, 3   4. 2, 2, 7   5. 2, 3, 5  
 6. 2, 5   7. 2, 2, 2, 2, 3   8. 2, 2, 23  
 9. 2, 2, 2, 2   10. 2, 17   11. 2, 2, 3

- 12.** 2, 3, 7    **13.** 2, 2, 2, 11    **14.** 2, 2, 2, 3, 3  
**15.** 2, 37    **16.** 2, 2, 2, 2, 2, 2    **17.** 2, 2, 11  
**18.** 2, 2, 2, 3, 5

**Page 66**

- 1.** 25 seats    **2.** 425 pounds    **3.** 211 pages  
**4.** 486 miles    **5.** 70 slices    **6.** 39 years old