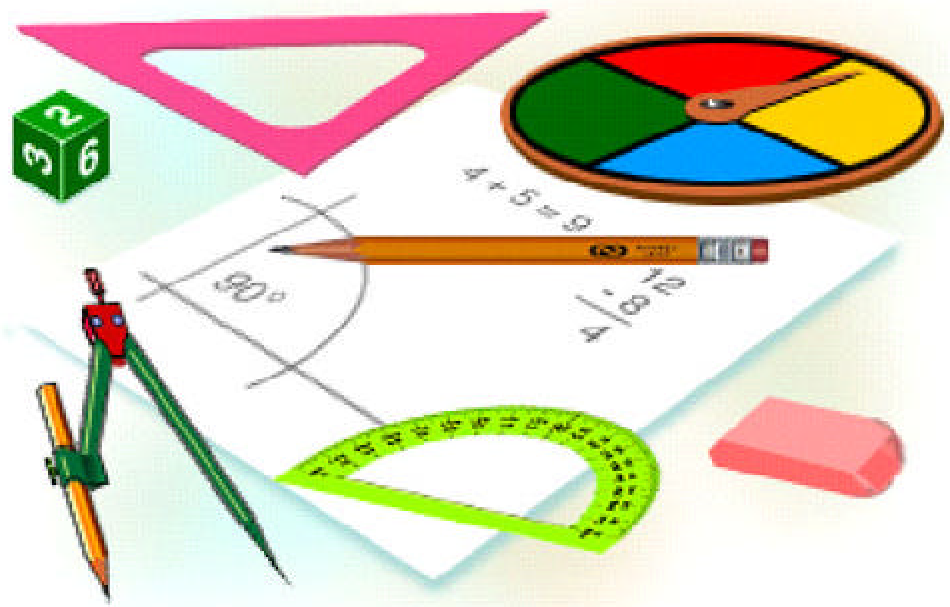
The Next Step

Mathematics Applications for Adults



<u>Book 14016</u>

INTRODUCTION

Why Math?

The most important reason for learning math is that it teaches us how to think. Math is more than adding and subtracting, which can easily be done on a calculator; it teaches us how to organize thoughts, analyze information, and better understand the world around us.

Employers often have to re-educate their employees to meet the demands of our more complex technological society. For example, more and more, we must be able to enter data into computers, read computer displays, and interpret results. These demands require math skills beyond simple arithmetic.

Everyone Is Capable of Learning Math

There is no **type** of person for whom math comes easily. Even mathematicians and scientists spend a lot of time working on a single problem. Success in math is related to practice, patience, confidence in ability, and hard work.

It is true that some people can solve problems or compute more quickly, but speed is not always a measure of understanding. Being “faster” is related to **more practice or experience.**

For example, the reason why math teachers can work problems quickly is because they've done them so many times before, not because they have "mathematical minds".

Working with something that is familiar is natural and easy. For example, when cooking from a recipe we have used many times before or playing a familiar game, we feel confident. We automatically know what we need to do and what to expect. Sometimes, we don't even need to think. However, when using a recipe for the **first** time or playing a game for the **first** time, we must concentrate on each step. We double-check that we have done everything right, and even then we fret about the outcome. The same is true with math. When encountering problems for the very first time, **everyone must have patience** to understand the problem and work through it correctly.

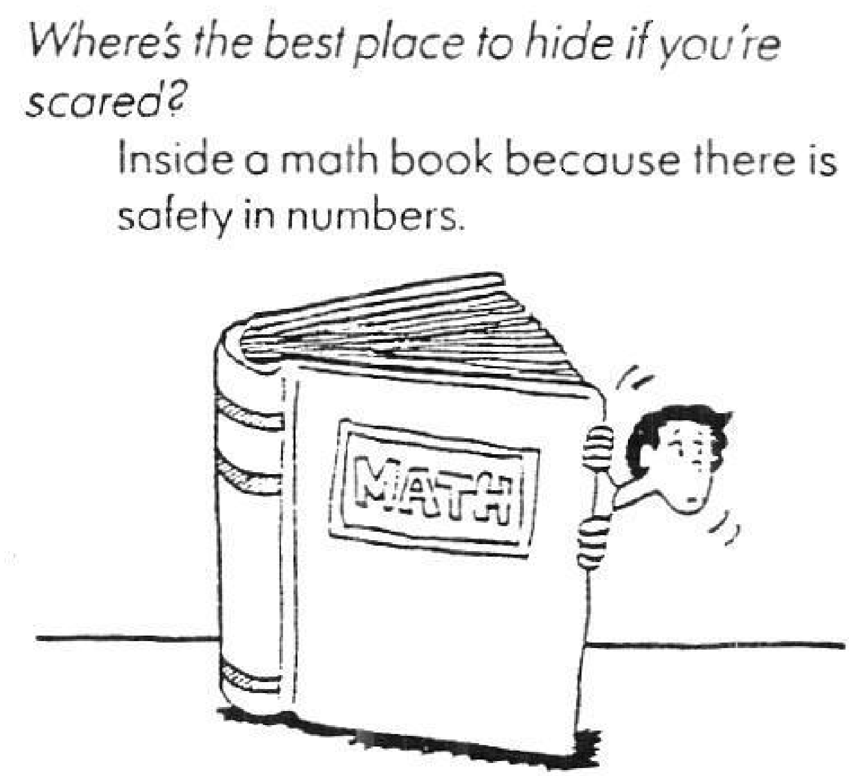
It's Never Too Late to Learn

One of the main reasons people don't succeed in math is that they don't start at the right place. **IMPORTANT! You must begin where *you* need to begin.** Could you hit a homerun if you hadn't figured out which end of the bat had to make contact with the ball? Why should learning math be any different?

If it has been a while since your last math class, **you must determine what level math you should take.** A teacher or trained tutor can help determine this with a few placement tests and questions.

Sometimes a few tutoring sessions can help you fill gaps in your knowledge or help you remember some of the things you have simply forgotten. It could also be the case where your foundations may be weak and it would be better for you to relearn the basics. **Get some help** to determine what is best for you.

Feeling good about ourselves is what all of us are ultimately striving for, and nothing feels better than conquering something that gives us difficulty. This takes a great deal of courage and the ability to rebound from many setbacks. This is a natural part of the learning process, and when the work is done and we can look back at our success, nothing feels better.



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OUTLINE

Mathematics - Book 14016

Whole Numbers
Number/Word Recognition
convert Arabic numbers to Roman numerals and vice versa (I – M…1 – 1,000).
read dates written in Roman numerals.
correctly write the number words for Arabic numbers (0 – 1,000,000).
correctly write the Arabic numerals for any number word (0 – 1,000,000).
Place Value
recognize the place value of each digit of a number to the million's place.
determine how many hundred thousands, thousands, hundreds, tens and ones in any number (0 – 1,000,000).
round off whole numbers to the nearest million, thousand, hundred, ten, and one.
Counting
count orally from 0 – 1,000,000 starting at any point in between those numbers.
count orally by 2's, 5's, and 10's to 100.
write all the even numbers from 2 - 100 and all the odd numbers from 1 - 99.
order numbers from greatest to least and least to greatest. (0 – 1,000,000).
Addition

find the sum of whole numbers up to 6 digits each.
use addition facts to compute sums up to and including 18.
Subtraction
subtract two whole numbers up to 6 digits (using borrowing/regrouping).
use subtraction facts to compute differences up to and including 18.
apply addition/subtraction skills by completing an incomplete equation (e.g. 14 + ? = 37).
Multiplication
multiply 3 digit factors by 3 digit factors.
write the times tables to 12 x 12 (within a specified time).
multiply by 1, 10, 100 quickly (within a specified time).
Division
explain factoring.
find the factors of a given list of products.
identify prime numbers from a given list.
how to calculate average.
when to use averages.
Word Problems with Whole Numbers
solve one/two step problems with addition, subtraction, multiplication or division of whole numbers.
Fractions
Understanding and Comparing Fractions
explain proper and improper fractions.
explain equivalent fractions.

explain mixed number.
explain lowest common denominator (LCM).
demonstrate an understanding of fractions.
provide equivalent fractions.
recognize a fraction in its lowest terms.
demonstrate an understanding of reducing fractions to their lowest terms (e.g. 4/12=1/3).
convert improper fractions to mixed numbers and vice versa.
find lowest common denominator (LCM) given 2 or 3 fractions with unlike denominators.
Addition of Fractions
add fractions with like and unlike denominators.
Subtraction of Fractions
subtract fractions with like an unlike denominators.
Multiplication of Fractions
multiply common and improper fractions with a denominator up to and including 10.
find the greatest common factor (GCF).
find the lowest common denominator (LCM).
Division of Fractions
divide common and improper fractions with a denominator up to and including10.
Word Problems with Fractions
solve one/two step problems with addition, subtraction, multiplication, and division of fractions.
Decimals
Understanding and Comparing Decimals
organize a list of decimals and mixed decimals in ascending and descending order.

convert fractions to decimals.
convert decimals to fractions (excluding repeating decimals).
compare two decimals using “<” and “>” signs.
Addition of Decimals
add numbers containing decimals.
round off decimals to the hundredths place.
Subtraction of Decimals
subtract numbers containing decimals.
round off decimals to the hundredths place.
Multiplication of Decimals
multiply numbers containing decimals.
round off decimals to the hundredths place.
multiply decimals by 10, 100, 1,000 by moving decimal points required number of places.
Division of Decimals
divide numbers containing decimals.
round off decimals to the hundredths place.
divide decimals by 10, 100, 1,000 by moving decimal points required number of places.
Word Problems with Decimals
solve one/two step problems with addition, subtraction, multiplication, and division of decimals.
Percents
Understanding and Comparing Percents
demonstrate an ability to visualize percent.
compare percents by ordering them from greatest to least and vice versa.
convert percent to decimals and fractions.
convert decimals and fractions to percent.

Using Percents
find percent of a number by converting it to either a decimal or a fraction.
find the percentage that one number is of another number.
find the number when a percentage is given.
use the formula *r/100 = P/W* and cross multiplication.
Simple Interest
calculate simple interest.
Word Problems with Percent
solve one/two step word problems involving percent and simple interest.
Measurement
Time
add and subtract time.
Money
find unit cost, tax payable, discount amount, simple interest payable, rounding off to the nearest cent.
Charts and Graphs (bar, line, pictograph)
answer questions about information contained in a given chart.
answer questions about information contained in given graphs.
Metric Measurement
find area of rectangle, square, triangle and any multi-sided figure.
find volume of a rectangular prism.
Word Problems with Measurement
solve one/two step problems with addition, subtraction, multiplication and division of whole

numbers, fractions, decimals, percent, time, money, temperature, and metric measurement.

THE NEXT STEP

Book 14016

Whole Numbers

Number Recognition

Digit is a counting word. A digit is any of the numerals from **1** to **9.** The word "digit" is also the name for a finger. So number digits can be counted on finger digits.

Our modern system of counting probably came from counting on fingers. Fingers and hands were among the earliest known calculators!

A CLOSER LOOK AT ROMAN NUMERALS

Roman numerals are still used today, more than 2000 years after their introduction. The history of Roman numerals is not well documented and written accounts are contradictory. Roman numerals are read from left to right.

It is likely that counting began on the fingers and that is why we count in tens.

A single stroke I represents one finger, five or a handful could possibly be represented by V and the X may have been used because if you stretch out two handfuls of fingers and place them close the two little fingers cross in an X. Alternatively, an X is like two Vs, one upside down.

The Roman numeral system uses seven letters to represent numbers. Combinations of these letters represent other numbers.

I = 1	**C = 100**
V = 5	**D = 500**
X = 10	**M = 1,000**
L = 50	

Combining Roman Numerals

Here are the three rules for making numbers with Roman numerals:

1. If you put numbers of the same size together, then you add them.
   II=1+1=2
   XX=10+10=20
   XXX=10+10+10=30

2. If you put a small number to the right of a large number, then you add them, too.
   XV=10+5=15
   VIII=5+3=8
3. If you put a small number to the left of a large number, then you subtract the small one from the big one.

   IX=10-1=9
   CM=1000-100=900

Sample Numbers:

Letter	**Value**
I	1
II	2
III	3
IV	4
V	5
VI	6
VII	7
VIII	8
IX	9
X	10
XX	20
XXX	30

XL	40
L	50
LX	60
LXX	70
LXXX	80
XC	90
C	100
CC	200
CCC	300
CD	400
D	500
DC	600
DCC	700
DCCC	800
CM	900
M	1,000

How are larger numbers expressed using Roman numerals? A modern method has been developed.

Letter	**Value**
$\overline{V}$	5,000
$\overline{X}$	10,000
$\overline{L}$	50,000

$\overline{C}$	100,000
$\overline{D}$	500,000
$\overline{M}$	1,000,000

$\overline{M}$ represents 1,000,000—a small bar placed over the numeral multiplies the numeral by 1000. Thus, theoretically, it is possible, by using an infinite number of bars, to express the numbers from 1 to infinity. In practice, however, one bar is usually used; two are rarely used, and more than two are almost never used.

These are some examples of the use of Roman Numerals.

I. Chapters. Look at the chapter headings of any book you are using.

II. Buildings.

III. Movie copyright years.

IV. Tombstones

IV. Clocks and watches. Look at clocks and watches closely, especially how they represent the number **four**.

A NEW THEORY ABOUT IIII

A lot of people ask ‘Why is the number 4 on a clock-face depicted as IIII and not as IV?’ There is no certain answer to this question. One common suggestion is that around the circle the IIII balances the VIII which is in its mirror-symmetrical place – that is if a mirror was placed vertically between the XII and VI, the VIII and IIII would reflect on to each other. There are problems with this theory – the V does not balance the VII, nor the I the XI. Another plausible explanation might be that IV has three strokes and is more likely to be confused with the neighbouring III, as both are at unfamiliar angles to the reader. But neither really offers an adequate explanation of why the normal rules of Roman numerals have been broken.

The oldest surviving clock-face in its original condition is on the clock inside Wells Cathedral in Somerset, England. It dates from before 1392 and the original mechanism – now in the Science Museum – has some claim to be the oldest surviving clock works in the world. The current mechanism that drives it is Victorian, but the face has not been changed for more than 600 years.

The outermost circle is more than six feet (1.93m) in diameter and around it, in Roman numerals, are the twenty four hours of the day with the 4 indicated by IIII.

Exceptions

The practice of using IIII rather than IV on clock-faces, although common, is not universal. The well-known clock, commonly called Big Ben, at the Palace of Westminster in London (where

Parliament meets) has gothic style Roman numerals round its face and the 4 is depicted as iv. (Strictly speaking, the hour bell is Big Ben, the clock is the Great Clock, and the tower is the Clock Tower and although the building is correctly called the Palace of Westminster, most people refer to it as the Houses of Parliament). Other examples of an IV on a clock-face are rare in England - but the clock in the South Transept of Norwich Cathedral is one example. Others are found in Spain. San Sebastian in northern Spain has at least two clocks – one on the cathedral and one on another church – which both have clear plain Roman numerals on the dial and which use IV for the 4.

Let's say that you wanted to read a date on a tombstone or building cornerstone. Most likely the date would have been etched into the stone as a Roman numeral.

What would the number MDCCXLVIII represent?

MDCCXLVIII =	M +	D +	C +	C +	XL +	V +	III
	1,000	500	100	100	(50-10)	5	3

MDCCXLVIII = 1748

Practice Exercise

What number does each of the following Roman numerals represent?

1. XXXVI
2. CLXV
3. MDCLIX
4. MCCLIV
5. MCMXCIII
6. MMXLVII

Write a Roman numeral for each of the following numbers:

7. 19
8. 299
9. 847
10. 1492
11. 1776
12. 2015

13. A cornerstone is marked MCMXIX. What date does it represent?

What number is represented by the Roman numeral in each of the following:

14. Page VII
15. Unit XLVI

16. Item CXC

Write the Roman numeral that comes next after each of the following:

17. XIII
18. XXIV
19. XCIX
20. XLVIII

Number/Word Recognition

Every number can be written two ways.
It can be written as a numeral.
Or it can be written as a word.
The numeral and word stand for the same thing.

Numeral	**Word**
0	zero
1	one
2	two
3	three
4	four
5	five
6	six
7	seven
8	eight
9	nine

Learn to say these 2-place numbers:

10	ten
11	eleven
12	twelve
13	thirteen
14	fourteen
15	fifteen
16	sixteen
17	seventeen
18	eighteen
19	nineteen

The 2-place numbers go from 10 (ten) to 99 (ninety-nine).
We have just learned about the 2-place numbers from 10 to 19.
Now learn these 2-place numbers:

20	twenty
21	twenty-one
22	twenty-two
23	twenty-three
24	twenty-four
25	twenty-five
26	twenty-six
27	twenty-seven
28	twenty-eight
29	twenty-nine
30	thirty
31	thirty-one
32	thirty-two

33	thirty-three
34	thirty-four
35	thirty-five
36	thirty-six
37	thirty-seven
38	thirty-eight
39	thirty-nine
40	forty
41	forty-one
42	forty-two
43	forty-three
44	forty-four
45	forty-five
46	forty-six
47	forty-seven
48	forty-eight
49	forty-nine
50	fifty
51	fifty-one
52	fifty-two
53	fifty-three
54	fifty-four
55	fifty-five
56	fifty-six
57	fifty-seven
58	fifty-eight
59	fifty-nine
60	sixty
61	sixty-one
62	sixty-two
63	sixty-three

64	sixty-four
65	sixty-five
66	sixty-six
67	sixty-seven
68	sixty-eight
69	sixty-nine
70	seventy
71	seventy-one
72	seventy-two
73	seventy-three
74	seventy-four
75	seventy-five
76	seventy-six
77	seventy-seven
78	seventy-eight
79	seventy-nine
80	eighty
81	eighty-one
82	eighty-two
83	eighty-three
84	eighty-four
85	eighty-five
86	eighty-six
87	eighty-seven
88	eighty-eight
89	eighty-nine
90	ninety
91	ninety-one
92	ninety-two
93	ninety-three
94	ninety-four

95	ninety-five
96	ninety-six
97	ninety-seven
98	ninety-eight
99	ninety-nine

The number 99 is the greatest 2-place number.
The next number in order is 100 (one hundred).

100 is one more than 99.
It is a 3-place number.
It has three numerals: 1, 0, and 0.
They stand for 1 hundred, 0 tens, and 0 ones

The greatest 3-place number is 999 (nine hundred ninety-nine).
It stands for 9 hundreds, 9 tens, and 9 ones.

Every 3-place number tells how many hundreds, tens, and ones the number stands for.

The number 999 is the greatest 3-place number.
The next number in order is 1,000 (one thousand).
It is one more than 999.
It is a 4-place number.
It has four numerals: 1, 0, 0, and 0.
They stand for 1 thousand, 0 hundreds, 0 tens, and 0 ones.

The greatest 4-place number is 9,999
(nine thousand, nine hundred ninety-nine).
The number after 9,999 is 10,000 (ten thousand).
Ten thousand is a 5-place number.

The greatest 5-place number is 99,999 (ninety-nine thousand, nine hundred ninety-nine).
The number after 99,999 is 100,000
(one hundred thousand).
One hundred thousand is a 6-place number.

The greatest 6-place number is 999,999 (nine hundred ninety-nine thousand, nine hundred ninety-nine).
The number after 999,999 is 1,000,000 (one million).
One million is a 7-place number

We use a comma after the number in the thousands' place.
The comma makes large numbers easier to read.

Practice Exercise

Write the numeral as a number word.
The first one has been done for you.

1. 37,215 **thirty-seven thousand, two hundred fifteen**

2. 42,499 ____

3. 45,291 ____

4. 10,148 ____

5. 9,030 ____

6. 33,000 ______________________________________

7. 51,935 ______________________________________

8. 1,099,868 ______________________________________

9. 1,094,442 ______________________________________

10. 1,570,000 ______________________________________

11. 1,098,329 ______________________________________

12. 1,964,835 ______________________________________

13. 1,668,100 ______________________________________

14. 1,010,521 ______________________________________

15. 1,456,142 ______________________________________

16. 1,942,818 ______________________________________

17. 1,362,825 ______________________________________

18. 1,011,431 ______________________________________

Read the number word and write the number.

1. eighty thousand, thirty-seven

   80,037

2. four thousand, five hundred sixty-seven

   ____

3. ninety-two thousand, seven hundred thirty-four

   ____

4. two thousand, seventy-six

   ____

5. thirty-four thousand, six hundred seventy-six

   ____

6. twenty-three thousand

   ____

7. seventy-seven thousand, nine hundred

   ____

8. one million, sixty-three thousand, five hundred sixty-one

   ____

9. one million, sixty-six thousand, four hundred eleven

   ____

10. one million, nine hundred sixty thousand

    ____

11. one million, sixty thousand, seven hundred eight

    ____

12. one million, six hundred seventy-one thousand, eight hundred forty-three

    ____

13. one million, forty-eight thousand, five hundred thirty-four

    ____

14. one million, four hundred ninety-seven thousand, three hundred two

    ____

Place Value

To read the place value of numerals in a number, read from left to right.
Each column has a value 10 times greater than the column to its right.

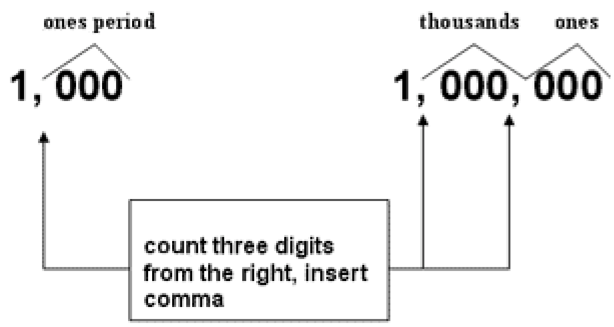
Place Value

The value of a digit as determined by its position in a number
Example:

	PLACE VALUE									
	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
1, 623, 051→	1	6	2	3	0	5	1.			
0.053→							0.	0	5	3
32.4→						3	2.	4		

Periods

Three places in the place value chart make up a ***period***. Periods are always counted from the right---from the "ones" column---of a number. Periods are separated in numerals by commas.



Millions Period			Thousands Period			Ones Period		
Hundred Millions	**Ten Millions**	**Millions**	**Hundred Thousands**	**Ten Thousands**	**Thousands**	**Hundreds**	**Tens**	**Ones**
100,000,000 **900,000,000**	**10,000,000** **90,000,000**	**1,000,000** **9,000,000**	**100,000** **900,000**	**10,000** **90,000**	**1,000** **9,000**	**100** **900**	**10** **90**	**1** **9**

Practice Exercise

Millions, Thousands, Hundreds, Tens, and Ones

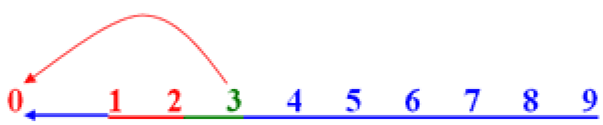
Write the place value of the bold number in each numeral.

1. 11,513,5**1**2	Tens	2. 97**2**,329,932	________________
3. 2,388,7**1**8	________________	4. 4,57**3**	________________
5. 24,1**7**7	________________	6. **7**74,465	________________
7. 496,283,6**2**2	________________	8. 7,**4**78,314	________________
9. 33,74**6**	________________	10. 5**4**5,383	________________
11. 7,7**1**8	________________	12. 55,793,5**5**8	________________
13. **3**,211	________________	14. 20**3**,244,825	________________
15. 42,66**6**,497	________________	16. 2**5**9,154	________________
17. 1,2**1**7,992	________________	18. 86,85**9**	________________
19. 52,318,6**4**7	________________	20. 5,312,**1**48	________________
21. 3,98**5**	________________	22. 1**3**4,293	________________
23. **6**2,885	________________	24. 578,59**9**,999	________________
25. 9,19**4**	________________	26. 92,375,46**9**	________________
27. 637,5**5**2	________________	28. 92,**8**27	________________

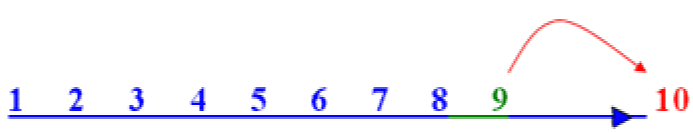
What's Rounding?

Rounding means to express a number to the nearest given place. The number in the given place is increased by one if the digit to its right is 5 or greater. The number in the given place remains the same if the digit to its right is less than 5. When rounding whole numbers, the digits to the right of the given place become zeros (digits to the left remain the same). When rounding decimal numbers, the digits to the right of the given place are dropped (digits to the left remain the same).

If you are rounding 3 to the nearest tens place, you would round down to 0, because 3 is closer to 0 than it is to 10.



If you were rounding 9, you would round up to 10.



General Rule for Rounding to the Nearest 10, 100, 1,000, and Higher!

Round down from numbers under 5 and round up from numbers 5 and greater.

The same holds true for multiples of 10. Round to the nearest 100 by rounding down from 49 or less and up from 50 or greater. Round to the nearest 1,000 by rounding down from 499 or less and up from 500 or greater.

Why Round?

Sometimes you have to figure out a math problem without using a pencil and paper or a calculator. Rounding numbers makes them easier to work with. Check out the shopping lists below. One list tells the prices of five items. The other list shows the same prices, rounded to the nearest ten cents. Which list is easier to add in your head?

Real Prices		Prices Rounded	
Pencil	**.29**	**Pencil**	**.30**
Eraser	**.19**	**Eraser**	**.20**
Stickers	**1.39**	**Stickers**	**1.40**

Notepad	**.89**	**Notepad**	**.90**
Pen	**<u>.59</u>**	**Pen**	**<u>.60</u>**
Total	**3.35**	**Total**	**3.40**

Practice Exercise

Round to the nearest <u>thousands place</u>.

1. 13,260	13,000	2. 9,459	________________
3. 3,113	________________	4. 23,175	________________
5. 95,276	________________	6. 13,705	________________
7. 81,724	________________	8. 7,848	________________
9. 7,590	________________	10. 7,094	________________
11. 9,072	________________	12. 13,893	________________
13. 2,942	________________	14. 16,168	________________
15. 1,093	________________	16. 2,295	________________

Round to the nearest <u>hundreds place</u>.

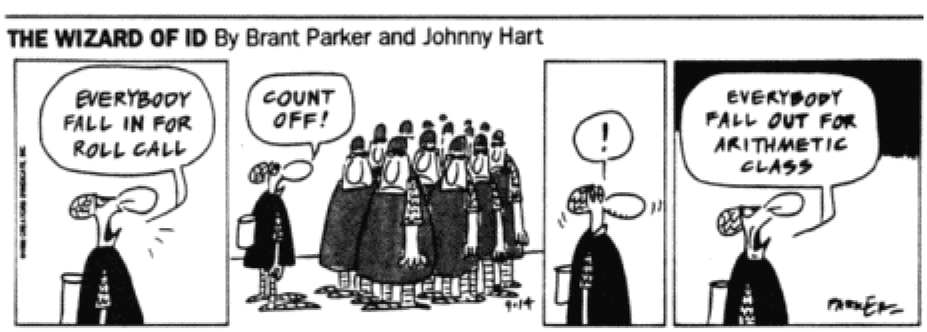
17. 361	400	18. 17,341	________________
19. 2,362	________________	20. 322	________________
21. 2,664	________________	22. 16,392	________________
23. 948	________________	24. 291	________________
25. 984	________________	26. 49,744	________________

27. 7,118	________________	28. 977	________________
29. 339	________________	30. 70,356	________________
31. 54,973	________________	32. 627	________________

Round to the nearest tens place.

33. 2,293	2,290	34. 669	________________
35. 38	________________	36. 845	________________
37. 58	________________	38. 26,322	________________
39. 47,958	________________	40. 95	________________
41. 79,313	________________	42. 712	________________
43. 66	________________	44. 24,092	________________
45. 94	________________	46. 50,398	________________
47. 67,284	________________	48. 523	________________

Counting



Wizard of ID by *Brant Parker & Johnny Hart*

The set of counting numbers has no end. It can go on forever. The idea that counting numbers can go on and on is called ***infinity***. Infinity has a special symbol:

$$\infty$$

There is no such thing as the "largest number." You can always add to or multiply a large number to make an even bigger number.

$$\infty + 3 = \infty$$

$$\infty \times 10 = \infty$$

If you began writing all the counting numbers today, you could continue writing every moment of every day for every day of the rest of your life and never be finished!

What's a googol?

A googol is a 1 with a hundred zeroes behind it. We can write a googol using exponents by saying a googol is 10^100 or 10 to the 100^{th} power.

The biggest named number that we know is googolplex, ten to the googol power, or (10)^(10^100). That's written as a one followed by googol zeroes.

It's funny that no one ever seems to ask, "What is the smallest number?" Again, there is really no such thing. You could always subtract from or divide a small number to make an even smaller number. As the number gets smaller and smaller, you would be approaching, but never reaching, negative infinity.

$$-\infty$$

The set of ***counting numbers***, or ***natural numbers***, begins with the number **1** and continues into infinity.

{1,2,3,4,5,6,7,8,9,10...}

The set of ***whole numbers*** is the same as the set of counting numbers, except that it begins with **0**.

{0,1,2,3,4,5,6,7,8,9,10...}

☞ ***All counting numbers are whole numbers. Zero is the only whole number that is not a counting number.***

Even numbers include the numbers ***0*** and ***2*** and all numbers that can be divided evenly by ***2***. ***Odd numbers*** are all numbers that cannot be divided evenly by ***2***.

Odd and Even Numbers to 100

1	3	5	7	9	11	13	15	17	19	21
0	2	4	6	8	10	12	14	16	18	20
23	25	27	29	31	33	35	37	39	41	
22	24	26	28	30	32	34	36	38	40	
43	45	47	49	51	53	55	57	59	61	
42	44	46	48	50	52	54	56	58	60	
63	65	67	69	71	73	75	77	79	81	
62	64	66	68	70	72	74	76	78	80	
83	85	87	89	91	93	95	97	99		
82	84	86	88	90	92	94	96	98	100	

Skip Counting

To count by 2's, simply count all the even numbers: 0, 2, 4, 6, 8, 10…and so on.

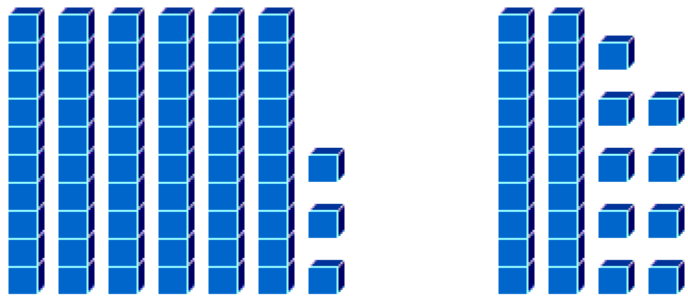
To count by 5's: 0, 5, 10, 15, 20…and so on.

To count by 10's: 0, 10, 20, 30, 40…and so on.

To count by 100's: 0, 100, 200, 300, 400…and so on.
Ordering numbers means listing numbers from least to greatest, or from greatest to least. Two symbols are used in ordering.

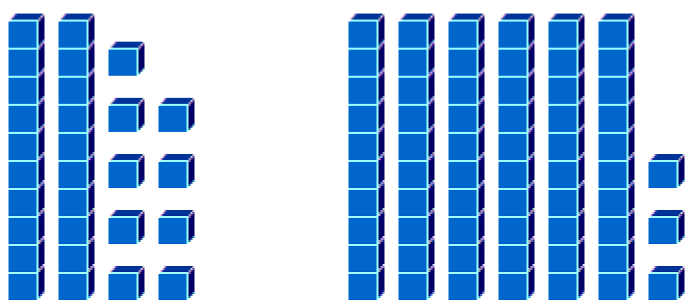
<	>
is less than	**is greater**
2 < 10	**10 > 2**

Greater Than >



63 is **greater than** 29.
63 > 29

Less Than <



29 is **less than** 63.
29 < 63

Practice Exercise

Fill in the blanks to complete the numerical sequence.
The first one has been done for you.

1.	8	9	10	11	**12**	**13**	
2.	5	10	_______	20	_______	30	
3.	18	20	_______	24	26	30	_______
4.	10	20	30	_______	50	_______	70
5.	106	107	_______	109	_______	111	112
6.	3,214	3,216	3,218	_______	3,222	_______	3,226
7.	45,625	45,630	45,635	45,640	_______	45,650	_______
8.	789,060	789,070	789,080	789,090	789,100	_______	_______

9.	95	96	97	98	_______	100
	_______					
10.	1,000	998	996	_______	992	_______
11.	210	205	_______	195	190	185
	_______	_______				

Compare the two numbers.
In the middle of the two numbers, write either **>** (greater than), **<** (less than), or = (equals) to complete the problem.

1.	604	>	367
2.	933	_____	356
3.	two hundred fifty-three	_____	eight hundred thirty-four
4.	510	_____	293
5.	299	_____	390
6.	673	_____	357
7.	two hundred ninety-nine	_____	eight hundred nineteen
8.	eight hundred fifty-nine	_____	652
9.	one hundred seventy-two	_____	699
10.	16	_____	903
11.	191	_____	470

12.	419	_____	580
13.	877	_____	286
14.	207	_____	454
15.	55	_____	83
16.	29	_____	702
17.	27	_____	65
18.	459	_____	895
19.	26	_____	39
20.	822	_____	330
21.	eight hundred twenty-five	_____	
22.	41	_____	224
23.	895	_____	28
24.	eight hundred eighty-three	_____	

Arrange the numbers from greatest to least.

1. 172 909 , 279 000 , 160 909 , 20 999
2. 135 620 , 513 527 , 315 427 , 315 727
3. 802 250 , 802 520 , 999 999 , 255 000

Arrange the numbers from least to greatest.

1. 266 225 , 662 226 , 226 216 , 626 225
2. 90 395 , 890 359 , 92 953 , 190 595
3. 822 320 , 882 220 , 228 302 , 882 202

Addition

Combining two or more numbers is called ***addition***. The term for addition is ***plus***, and the symbol for plus is +. The numbers that are combined in addition are called ***addends*** and together they form a new number called a ***sum***.

2 ---- addends ---- 3
+ 2 + 1
4 ---- sum --------- 4

Adding whole numbers is as simple as 2 + 2! To add two whole numbers, you can simply follow the number line and complete the addition fact.

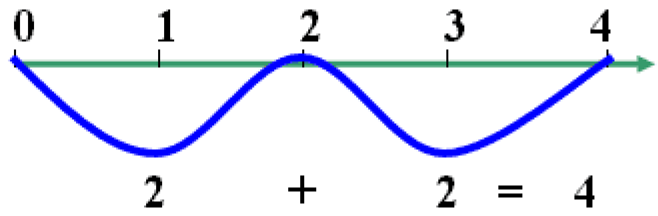
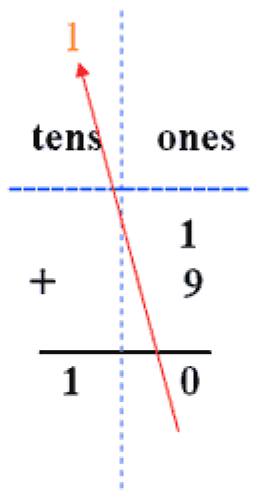


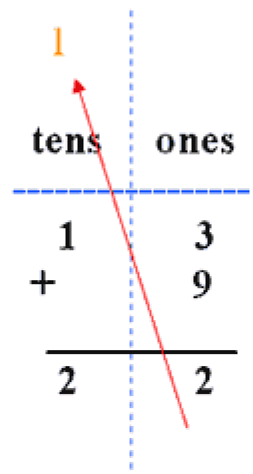
Table of Addition Facts

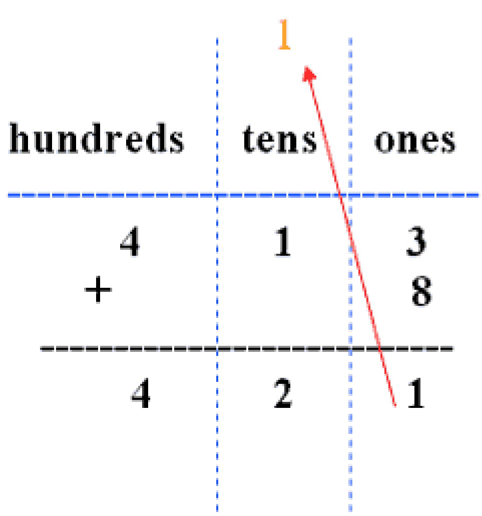
+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

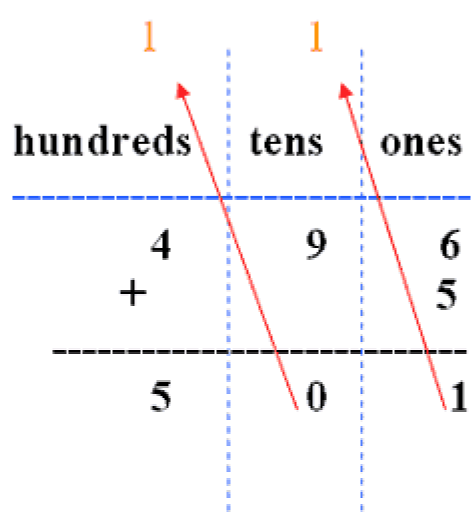
Regrouping Numbers in Addition

Addition often produces sums with a value greater than ***9*** in a given place. The value of ten is then ***regrouped*** (or ***carried***) to the next place.









1	1	1	
thousands	**hundreds**	**tens**	**ones**
1,	**3**	**4**	**3**
+3,	**7**	**9**	**8**
5,	**1**	**4**	**1**

To explain addition another way, it can be done by adding the place value amounts separately.

e.g.
```
  69
+ 8
  17
  60 (the 6 in the tens place means 6 tens or "60")
  77
```

⟹ If there are not enough digits in each number to make even columns under each place value, then zeros may be used **before** a given number to make adding easier. Do **not** add zeros **after** a number because it changes the value of the whole number.

e.g. 69 + 8 + 125 could be added as:

```
 069
 008
+125
```

Commutative Property of Addition

The property which states that addends can be added in any order. The sum is always the same

Example:

$$2.67 + 1.32 = 1.32 + 2.67$$

$$3.99 = 3.99$$

Associative Property

Addends can be grouped differently; the sum is always the same.

Example:

$$(8 + 7) + 4 = 8 + (7 + 4)$$

Practice Exercise

1.	**26 +12**	**2.**	**481 +23**	**3.**	**4321 +103**	**4.**	**32452 +2667**
5.	**33 +32**	**6.**	**49 + 9**	**7.**	**3283 + 31**	**8.**	**3694 +270**
9.	**23 +15**	**10.**	**221 +13**	**11.**	**4625 +403**	**12.**	**176 + 9**
13.	**21 +18**	**14.**	**354 +19**	**15.**	**4757 +226**	**16.**	**38788 +1290**

17.	**389**	**18.**	**80**	**19.**	**1011**	**20.**	**1022**
	13		**12**		**23**		**123**
	+33		**+18**		**+18**		**+15**

21.	**3643**	**22.**	**394**	**23.**	**205**	**24.**	**276**
	115		**5**		**30**		**19**
	+386		**+ 9**		**+ 8**		**+18**

25.	**397877**	**26.**	**349080**
	368901		**331234**
	234567		**123456**
	+118901		**+956789**

27. 99 + 2 + 24 + 16 = _______

28. 270 + 22 + 12 + 14 = _______

29. 131 + 0 + 20 = _______

30. 192 + 4 + 13 + 0 = _______

31. 18 + 834 + 2256 + 478 = ___

32. 3143 + 20 + 20 = _______

33. 179054 + 1712 + 3534 = _____

34. 3365 + 13 + 11 + 9 = ______

35. 1378 + 1490 + 6123 = _______

36. 34 + 165 + 26 + 297 = ______

37. 246 + 18 + 2 = _______

38. 8 + 269 + 350 + 221 = ______

39. 27 + 23 + 2 + 19 = _______

40. 1128 + 193 + 114 = _______

41. 319 + 11 + 9 + 28 = _______

42. 1 + 236 + 277 + 128 = ______

Subtraction

"Taking away" one or more numbers from another number is called ***subtraction***. The term for subtraction is ***minus***, and the symbol for minus is **-**. The number being subtracted is called a ***subtrahend***. The number being subtracted from is called a ***minuend***. The new number left after subtracting is called a ***remainder*** or ***difference***.

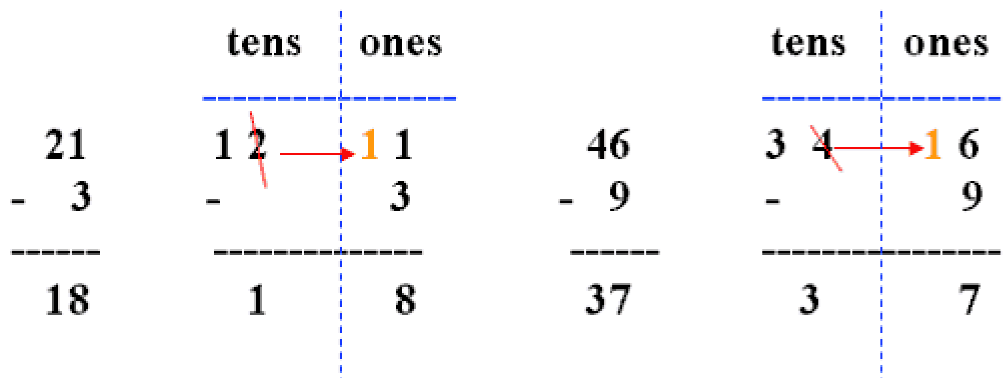
4 ---- minuend ---- 4
- 2 --subtrahend - - 1
2 – difference ---- 3

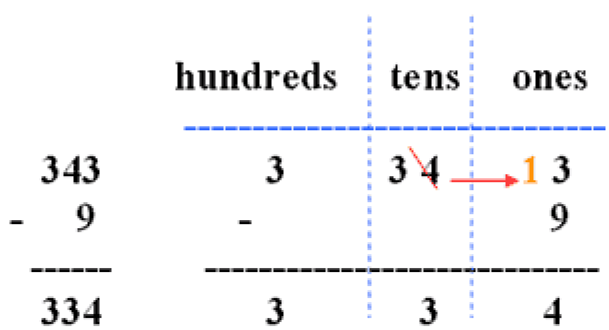
The complete addition or subtraction "sentence" is called an ***equation***. An equation has two parts. The two parts are separated by the ***equal sign***, **=**. For example, ***the minuend minus the subtrahend equals the difference***. An ***addition fact*** or a ***subtraction fact*** is the name given to specific addition or subtraction equations.

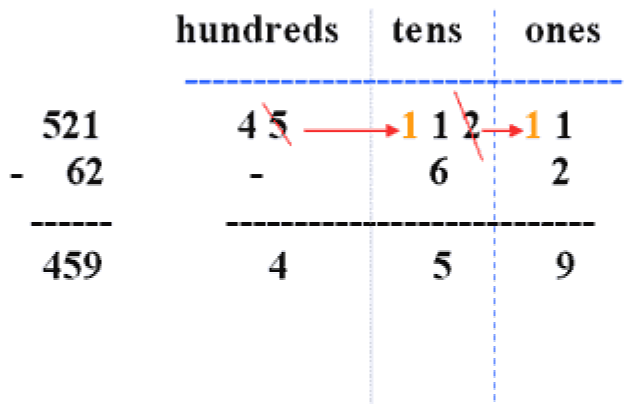
0 + 1 = 1	**1 – 1 = 0**
1 + 1 = 2	**2 – 1 = 1**
2 + 1 = 3	**3 – 1 = 2**
3 + 1 = 4	**4 – 1 = 3**
4 + 1 = 5	**5 – 1 = 4**
5 + 1 = 6	**6 – 1 = 5**
6 + 1 = 7	**7 – 1 = 6**
7 + 1 = 8	**8 – 1 = 7**
8 + 1 = 9	**9 – 1 = 8**

Regrouping in Subtraction

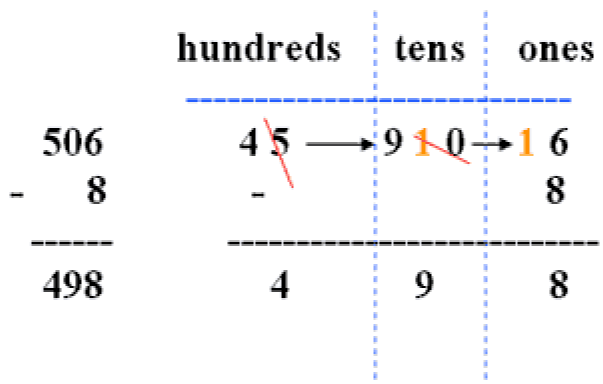
Regrouping, sometimes called ***borrowing***, is used when the subtrahend is greater than the minuend in a given place. Regrouping means to take a group of tens from the next greatest place to make a minuend great enough to complete the subtraction process.






hundreds
tens
ones
521
- 62

459
4 5 → 1 1 2 → 1 1
- 6 2
4 5 9


hundreds
tens
ones
506
- 8

498
4 5 → 9 1 0 → 1 6
- 8
4 9 8

Practice Exercise

Solve for each of the given problems.

1. 291 - 64	**2.** 756 – 220	**3.** 889 - 380	**4.** 842 - 529
5. 7,110 - 5,431	**6.** 12,733 -5,812	**7.** 40,782 - 38,073	**8.** 22,235 - 12,757
9. 41,969 - 19,756	**10.** 68,932 - 63,034	**11.** 46,775 - 42,402	**12.** 17,882 - 2,416
13. 767,851 - 649,634	**14.** 834,101 - 648,103	**15.** 430,492 - 272,645	**16.** 774,056 - 160,412
17. 535,823 - 232,711	**18.** 649,153 - 149,334	**19.** 629,063 - 511,007	**20.** 878,709 - 400,376

Solving Incomplete Addition or Subtraction Equations

Inverse (opposite) operations are used to simplify an equation for solving.
One operation is involved with the unknown and the inverse operation is used to solve the equation.

Addition and subtraction are inverse operations.
Given an equation such as 7 + x = 10, the unknown x and 7 are *added*. Use the inverse operation subtraction. To solve for n, subtract 7 from 10. The unknown value is therefore 3.

Examples for addition and subtraction

Addition Problem	Solution
x + 15 = 20	x = 20 - 15 = 5

Subtraction Problem	Solution
x - 15 = 20	x = 20 + 15 = 35

Practice Exercise

Solve each equation.
(Hint: Use inverse operation rules to solve)

1.	**41 = a - 16 57**	**2.**	**x + 31 = 51 ______**
3.	**78 = a - 22 ______**	**4.**	**x - 67 = 18 ______**
5.	**x - 27 = 59 ______**	**6.**	**60 + y = 141 ______**
7.	**19 = a - 57 ______**	**8.**	**x - 59 = 38 ______**
9.	**57 + y = 119 ______**	**10.**	**3 = a - 36 ______**
11.	**x + 80 = 124 ______**	**12.**	**92 + y = 171 ______**
13.	**x + 81 = 102 ______**	**14.**	**x + 83 = 100 ______**
15.	**83 = a - 3 ______**	**16.**	**31 = a - 26 ______**
17.	**x + 67 = 108 ______**	**18.**	**x - 64 = 28 ______**

Multiplication

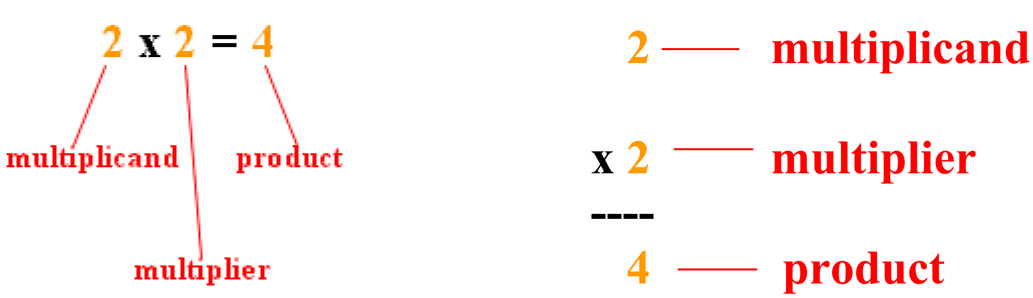
Multiplication is a quick form of addition. By multiplying numbers together, you are really adding a series of one number to itself. For example, you can add ***2*** plus ***2***. Both ***2 plus 2*** and ***2 times 2*** equal ***4***.

$$2 + 2 = 4 \qquad \begin{array}{r} 2 \\ +\,2 \\ \hline 4 \end{array} \qquad \begin{array}{r} 2 \\ \times\,2 \\ \hline 4 \end{array}$$
$$2 \times 2 = 4$$

But what if you wanted to calculate the number of days in five weeks? You could add *7* days + *7* days + *7* days + *7* days + *7* days or you could multiply *7* days times ***5***. Either way you arrive at ***35***, the number of days in five weeks.

$$7 + 7 + 7 + 7 + 7 = 35$$
$$5 \times 7 = 35$$

Although multiplication is related to addition, the parts of multiplication are not known as addends. Instead, the parts are known as ***multiplicands*** and ***multipliers***. A multiplication sentence, like an addition sentence, is called an ***equation***. But a multiplication sentence results in a ***product***, not a sum.



X	0	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Multiplication, Step-by-Step

When the multiplicand and the multiplier are numbers with two or more digits, multiplication becomes a step-by-step process.

Look at 15 x 13:

```
  1 5
x   3
-----
  1 5
```

First, multiply the ones – 3 x 5. Write down the product so the ones columns line up.

```
  1   5        Next, multiply the
               tens – 3 x 1 ten.
x     3        Line up the product
----------     with the tens
  1   5        column.

  3   0   —    Zero is the place
----------     holder.


  1   5        Last, add the ones
               and tens to find the
x     3        product of the
----------     equation.
  1   5

+ 3   0
----------
  4   5
```

Here is a shorter way:

```
  1
  1   5
x     3
-----------
  4   5
```

1. Multiply the ones: 3 x 5 = 15. Put the 5 in the ones column and regroup the 1 to the tens column.

2. Multiply the tens: 3 x 1 = 3.

3. Add the 1 that you regrouped to the 3, put the sum in the tens column.

Look at 265 x 23:

```
   265
x   23
---------
    15
   180
   600
```

First, multiply the multiplicand by the ones in the multiplier – 3 x 5, 3 x 6, and 3 x 2. Zero is the place holder.

```
   265
x   23
--------
    15
   180
   600
--------
   100
 1,200
 4,000
```

Next, multiply by the tens – 2 x 5, 2 x 6, and 2 x 2. Zero is the place holder.

```
     265
x     23
---------
      15
+    180
+    600
---------
+    100
+  1,200
+  4,000
---------
   6,095
```

Last, add.

Here is a shorter way:

```
  11
  11
  265
x  23
---------
  795
 5300
---------
6,095
```

1. Multiply the ones: 3 x 265
   3 x 5 = 15 regroup the 1
   3 x 6 = 18 plus the regrouped 1 = 19; regroup the 1
   3 x 2 = 6 plus the regrouped 1 = 7

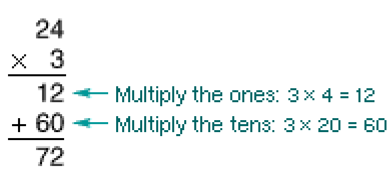
2. Multiply the tens: 2 x 265
   0 is the place holder
   2 x 5 = 10 regroup the 1
   2 x 6 = 12 plus the regrouped 1 = 13; regroup the 1
   2 x 2 – 4 plus the regrouped 1 = 5

3. Add 795 + 5300 = 6,095

Partial Product

A method of multiplying where the ones, tens, hundreds, and so on are multiplied separately and then the products added together

Example:



36 x **17 = 42 + 210 + 60 + 300 = 612**

When you multiply whole numbers, the ***product*** usually has a greater value than either the ***multiplicand*** or the ***multiplier***.

But there are exceptions:

A number multiplied by ***1*** is always equal to itself.

$$\begin{array}{r} 1 \\ \underline{\times 1} \\ 1 \end{array} \qquad 21 \times 1 = 21 \qquad \begin{array}{r} 36 \\ \underline{\times 1} \\ 36 \end{array}$$

A number multiplied by ***0*** is always equal to ***0***.

$$\begin{array}{r} 1 \\ \underline{\times 0} \\ 0 \end{array} \qquad 21 \times 0 = 0 \qquad \begin{array}{r} 36 \\ \underline{\times 0} \\ 0 \end{array}$$

To multiply a number by 10, add a 0 to the right of the number.

EXAMPLE

$$25 \text{ X } 10 = 250 \qquad \text{or} \qquad \begin{array}{r} 25 \\ \underline{\text{x}10} \\ 250 \end{array}$$

To multiply a number by 100, add two 0’s to the right of the number.

EXAMPLE

$$36 \text{ X } 100 = 3{,}600 \qquad \text{or} \qquad \begin{array}{r} 36 \\ \underline{\text{x}100} \\ 3{,}600 \end{array}$$

Commutative Property of Multiplication

The property which states that factors can be multiplied in any order. The product is always the same.

Example:

$$5 \times 7 = 7 \times 5$$
$$35 = 35$$

Associative Property of Multiplication

The property which states that when multiplying three or more factors , any two of the factors can be multiplied, and the remaining factors may then be multiplied without changing the total product

Example:

$$(3 \times 4) \times 5 = 3 \times (4 \times 5)$$
$$12 \times 5 = 3 \times 20$$
$$60 = 60$$

Practice Exercise

Solve for each of the given problems.

1. 673 × 46	**2.** 405 × 60	**3.** 215 × 10	**4.** 879 × 19	**5.** 713 × 74
6. 281 × 179	**7.** 633 × 260	**8.** 225 × 351	**9.** 831 × 142	**10.** 883 × 258

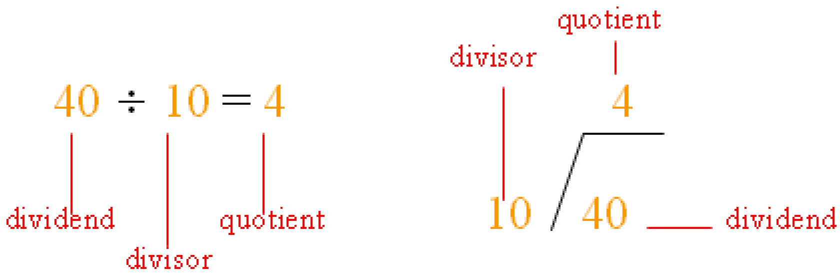
11. 223 × 755	12. 107 × 496	13. 345 × 587	14. 236 × 687	15. 530 × 631
16. 756 × 69	17. 879 × 78	18. 638 × 90	19. 306 × 62	20. 323 × 84
21. 327 × 8	22. 422 × 5	23. 804 × 9	24. 943 × 6	25. 697 × 3
26. 101 × 486	27. 539 × 84	28. 24 × 60	29. 51 × 8	30. 5 × 0

Division

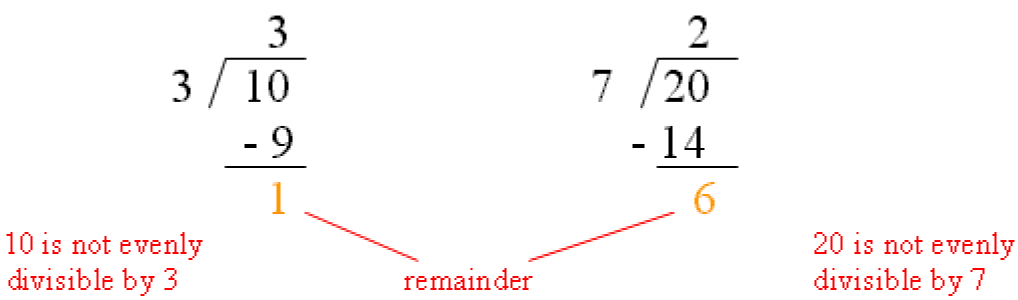
Division is the process of finding out how many times one number, the ***divisor***, will fit into another number, the ***dividend***. The division sentence results in a ***quotient***. The signs of division are ÷ and ⟌ , and mean ***divided by***. You can think of division as a series of repeated subtractions. For example, ***40*** ÷ ***10*** could also be solved by subtracting ***10*** from ***40*** four times:

40 – 10 – 10 – 10 – 10 = 0

Because ***10*** can be subtracted four times, you can say that ***40*** can be divided by ***10*** four times, or ***40*** ÷ ***10*** = ***4***.

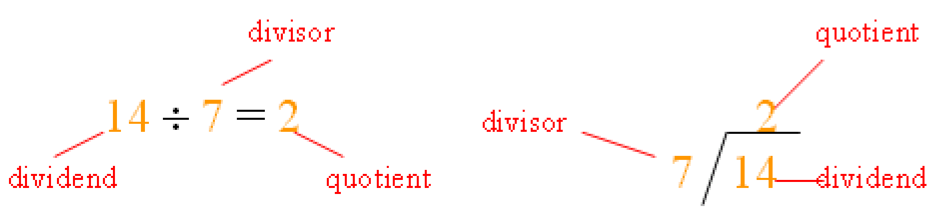


Many numbers do not fit evenly into other numbers. They are ***not evenly divisible by*** those numbers, and the number left over is called the ***remainder***.



We would record the answer for the first question as 3 r 1 and for the second question as 2 r 6. The “r” stands for remainder.

To divide whole numbers, reverse the process of multiplication. For example, if ***2 x 7 = 14*** in a multiplication equation, then in a division sentence, ***14*** is the ***dividend*** and ***7*** is the ***divisor*** with a ***quotient*** of ***2***.



A whole number divided by ***1*** will always equal itself.

1 ¸ 1 = 1

$$1\overline{)21}^{\;21}$$

36 ¸ 1 = 36

Zero divided by a whole number will always equal ***0***.

0 ¸ 12 = 0

$$3\overline{)0}^{\;0}$$

0/7 = 2

Division, Step-by-Step

Where the dividend and divisor are numbers with two or more digits, division becomes a step-by-step process.

```
     2
8 /208
  - 16
     4
```

First, round the divisor up - 8 rounds up to 10 - and estimate the number of 10s in 20. Answer: 2. Multiply the divisor – 8 x 2 – and subtract the product from the dividend.

```
    26
8/ 208
 -  16
    48
 -  48
     0
```

Next, pull down the next digit from the dividend – 8 – and repeat the estimation and subtraction process.

```
     26
8 / 208
 - 16
    48
 -  48
     0
```

Last, when you can subtract no more you've found the quotient.

No remainder

```
       1
23 / 276
  -  23
      4
```

First, round 23 to 25 and estimate the number of 25s in 27. Answer: 1. Multiply the divisor by 1 – 23 x 1 – and subtract.

```
      12
23 / 276
  -  23
     46
  -  46
```

Next, pull down the next digit from the dividend – 6 – and repeat the estimation and subtraction process.

```
      12
23 / 276
  -  23
      46
  -   46
       0
```

Then, pull down the next digit, estimate, and subtract, until you can subtract no more.

No remainder

Practice Exercise

Solve each problem.

1. $5\overline{)36}$

2. $8\overline{)461}$

3. $2\overline{)92}$

4. $4\overline{)45}$

5. $10\overline{)43}$

6. $112\overline{)213}$

7. $10\overline{)64}$

8. $8\overline{)794}$

9. $43\overline{)323}$

10. $2\overline{)13}$

11. $64\overline{)68}$

12. $5\overline{)535}$

13. $7\overline{)673}$

14. $111\overline{)260}$

15. $9\overline{)91}$

16. $5\overline{)456}$

17. $12\overline{)27}$

18. $66\overline{)487}$

19. $2\overline{)189}$

20. $6\overline{)34}$

21. $110\overline{)386}$

22. $5\overline{)443}$

23. $107\overline{)828}$

24. $49\overline{)153}$

Factors are numbers that, when multiplied together, form a new number called a ***product***. For example, ***1*** and ***2*** are factors of ***2***, and ***3*** and ***4*** are factors of ***12***.

Every number except ***1*** has at least two factors: ***1*** and itself.

Common Factor

A number that is a factor of two or more numbers

Example:

factors of 6: 1, 2, 3, 6

factors of 12: 1, 2, 3, 4, 6, 12

The common factors of 6 and 12 are
1, 2, 3, and 6.

Greatest Common Factor (GCF)

The greatest factor that two or more numbers have in common

Example:

18: 1, 2, 3, **6**, 9, 18

30: 1, 2, 3, 5, **6**, 10, 15, 30

5 is the GCF of 18 and 30.

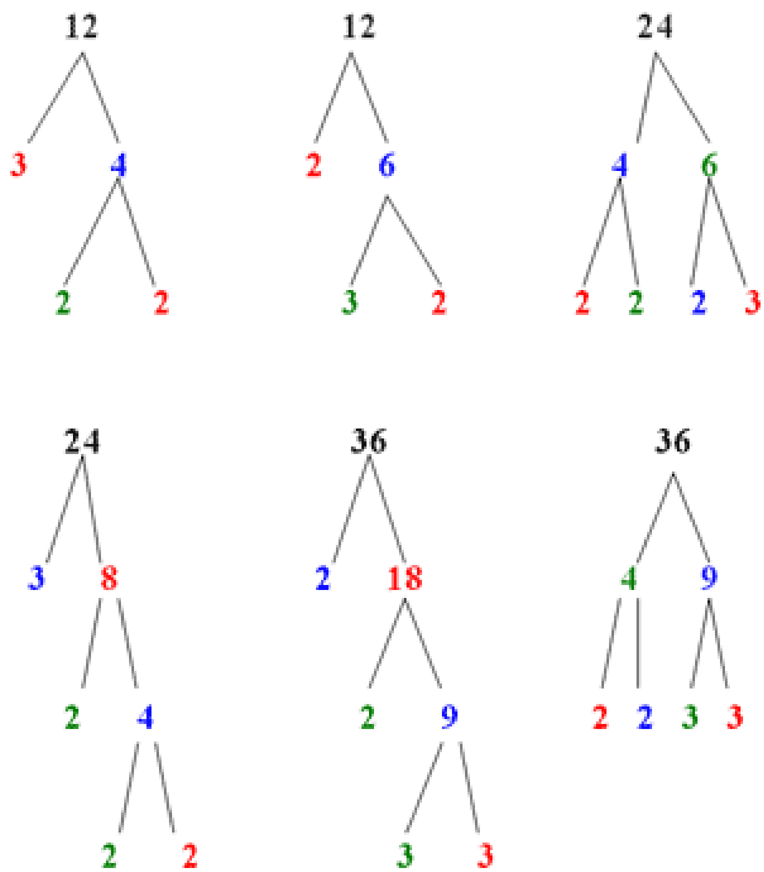
Practice Exercise

Find the greatest common factor (GCF) for the given numbers.

1. 3, 4 **1**
2. 4, 6
3. 3, 6
4. 2, 8
5. 11, 12
6. 5, 8
7. 8, 10

8. 5, 9
9. 5, 10
10. 6, 24
11. 4, 18
12. 20, 16
13. 15, 6
14. 9, 18
15. 12, 21
16. 16, 18
17. 4, 13
18. 8, 20
19. 18, 45
20. 16, 40
21. 24, 48
22. 10, 15
23. 35, 6
24. 45, 30
25. 66, 72
26. 64, 32
27. 40, 180
28. 12, 24
29. 30, 180
30. 80, 160

Composite numbers have more than two factors. In fact, every composite number can be written as the product of ***prime numbers***. You can see this on a ***factor tree***.



Prime numbers are counting numbers that can be divided by only two numbers---***1*** and themselves. A prime number can also be described as a counting number with only two factors, ***1*** and itself. The number ***1***, because it can be divided only by itself, is ***not*** a prime number.

Prime Numbers to 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,

53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Practice Exercise

Classify each number as prime or composite.

1. 87 ☐Prime ■Composite	2. 18 ☐Prime ☐Composite	3. 48 ☐Prime ☐Composite	4. 57 ☐Prime ☐Composite
5. 85 ☐Prime ☐Composite	6. 41 ☐Prime ☐Composite	7. 17 ☐Prime ☐Composite	8. 78 ☐Prime ☐Composite
9. 28 ☐Prime ☐Composite	10. 54 ☐Prime ☐Composite	11. 1 ☐Prime ☐Composite	12. 73 ☐Prime ☐Composite
13. 69 ☐Prime ☐Composite	14. 27 ☐Prime ☐Composite	15. 70 ☐Prime ☐Composite	16. 19 ☐Prime ☐Composite

17. 56 ☐Prime ☐Composite	18. 49 ☐Prime ☐Composite	19. 64 ☐Prime ☐Composite	20. 31 ☐Prime ☐Composite
21. 43 ☐Prime ☐Composite	22. 95 ☐Prime ☐Composite	23. 62 ☐Prime ☐Composite	24. 11 ☐Prime ☐Composite

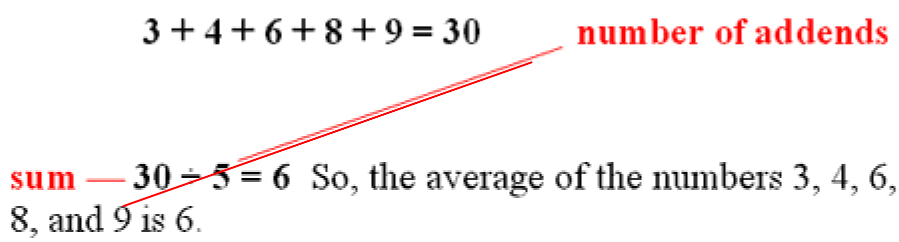
Find the prime factorization of each number.

1. 24 **2, 2, 2, 3**
2. 10
3. 14
4. 4
5. 12
6. 28
7. 42
8. 32
9. 78
10. 26
11. 58
12. 66
13. 20
14. 100
15. 36
16. 6

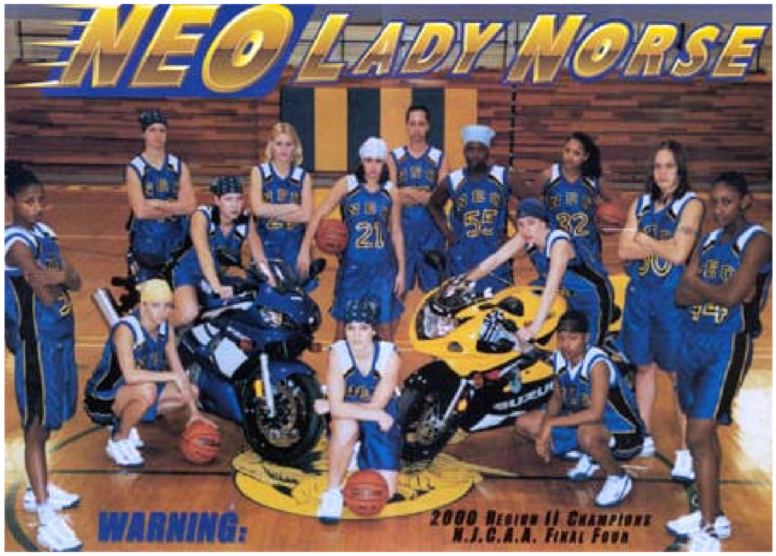
17. 64
18. 22

Averages

The most common way to find an ***average*** is to add up a list of numbers and divide the sum by the number of items on the list. Another word for average is ***mean***.



When do you need to calculate an average? Your grades may be based on the average of all your test scores. In sports, you might want to find out the average height of players on your favorite basketball team.



The height of the starters for this team is:

Anita 60”

Jane 58”

Cathy 57”

Joy 52”

Tanya 48”

The average height of these players is 55 inches.

Practice Exercise

Calculate the values to the nearest whole number.

1. 553, 680, 416, 416, 553, 554, 553, 416, 554, 416, and 553

   Write the mean (average): __________

2. 3, 7, 20, 17, 8, 3, 12, and 17

   Write the mean: __________

3. 14, 28, 5, 8, 5, 6, 5, 27, 7, 21, 28, 19, and 5

   Write the mean: __________

4. 15, 36, 36, 36, 15, 14, 15, 21, 24, 36, 21, 21, 8, and 29

   Write the mean: __________

5. 45, 45, 17, 22, 25, 17, 6, 6, 37, 45, 23, and 37

   Write the mean: __________

6. 507, 529, 9, 8, 8, 546, 1, 582, 8, 546, 573, 520, 545, 545, 512, 528, 520, and 582

   Write the mean: __________

7. 198, 194, 111, 4, 198, 108, 7, 150, 178, 195, 194, and 108

   Write the mean: __________

8. 728, 728, 728, 448, 929, 728, and 978

	Write the mean: __________
9.	853, 837, 812, 839, 853, 812, 856, 887, 812, 812, 812, and 812 Write the mean: __________
10.	127, 142, 188, 142, 142, 143, 107, 143, 107, 143, 127, 121, 195, 122, 142, 147, 190, and 190 Write the mean: __________

Word Problems with Whole Numbers

Within every story (word) problem are several ***clue words***. These words tell you the kind of math sentence (equation) to write to solve the problem.

Addition Clue Words

add
sum
total
plus
in all
both
together
increased by
all together
combined

Subtraction Clue Words

subtract
difference
take away
less than
are not
remain
decreased by
have or are left
change (money problems)
more

Multiplication Clue Words	Division Clue Words
times	quotient of
product of	divided by
multiplied by	half [or a fraction]
by (dimension)	split
	separated
	cut up
	parts
	shared equally

⇒ ***Division clue words are often the same as subtraction clue words. Divide when you know the total and are asked to find the size or number of "one part" or "each part."***

Following a system of steps can increase your ability to accurately solve problems. Use these steps to solve word problems.

1. Read the problem carefully. Look up the meanings of unfamiliar words.

2. Organize or restate the given information.

3. State what is to be found.

4. Select a strategy (such as making a chart of working backward) and plan the steps to solve the problem.

5. Decide on an approximate answer before solving the problem.

6. Work the steps to solve the problem.

7. Check the final result. Does your answer seem reasonable?

The Problem Solving System was used to solve the following problem:

Mary has ten marbles. Lennie has thirteen. How many marbles do they have in all?

**1. Mary has ten marbles. Lennie has thirteen.
How many marbles do they have in all?**

**2. Mary – 10 marbles
Lennie – 13 marbles**

3. How many marbles in all?

4. Add

5. A little over 20 marbles (10 + 10 = 20)

6.

$$\begin{array}{r} 10 \\ +13 \\ \hline 23 \end{array} \text{ marbles}$$

7. The final sum of 23 marbles is close to the estimated answer of 20 marbles. The final result is reasonable.

Þ ***Be sure to label answers whenever possible. For example: marbles, grams, pounds, feet, dogs, etc.***

Þ ***Some problems may require several steps to solve. Some may have more than one correct answer. And some problems may not have a solution.***

Have you ever tried to help someone else work out a word problem? Think about what you do. Often, you read the problem with the person, then discuss it or put it in your own words to help the person see what is happening. You can use this method---restating the problem---on your own as a form of "talking to yourself."

Restating a problem can be especially helpful when the word problem contains no key words. Look at the following example:

Example: Susan has already driven her car 2,700 miles since its last oil change. She still plans to drive 600 miles before changing the oil. How many miles does she plan to drive between oil changes?

Step 1: *question:* How many miles does she plan to drive between oil changes?

Step 2: *necessary information:* 2,700 miles, 600 miles

Step 3: *decide what arithmetic to use:* Restate the problem in your own words: "You are given the number of miles Susan has already driven and the number of miles more that she plans to drive. You need to add these together to find the total number of miles between oil changes."

Step 4: 2,700 miles + 600 miles = **3,300 miles** between oil changes.

Step 5: It makes sense that she will drive 3,300 miles between oil changes, since you are looking for a number larger than the 2,700 miles that she has already driven.

For some problems, you have to write two or three equations to solve the problem. For others, you may need to make charts or lists of information, draw pictures, find a pattern, or even guess and check. Sometimes you have to work backwards from a sum, product, difference, or quotient, or simply use your best logical thinking.

List/Chart

Marty's library book was six days overdue. The fine is $.05 the first day, $.10, the second, $.20 the third day, and so on. How much does Marty owe?

Marty's library book was six days overdue. The fine is $.05 the first day, $.10, the second, $.20 the third day, and so on. How much does Marty owe?

Days	1	2	3	4	5	6
Fine	$.05	$.10	$.20	$.40	$.80	$1.60

Answer: $1.60

Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?

Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?

Veronica	**Veronica**	**Archie**	**Archie**
Archie	**Betty**	**Veronica**	**Betty**
Betty	**Archie**	**Betty**	**Veronica**

Betty	**Betty**
Veronica	**Archie**
Archie	**Veronica**

Answer: 6 ways

Find a Pattern

Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X________. How did Jenny fill in the blanks?

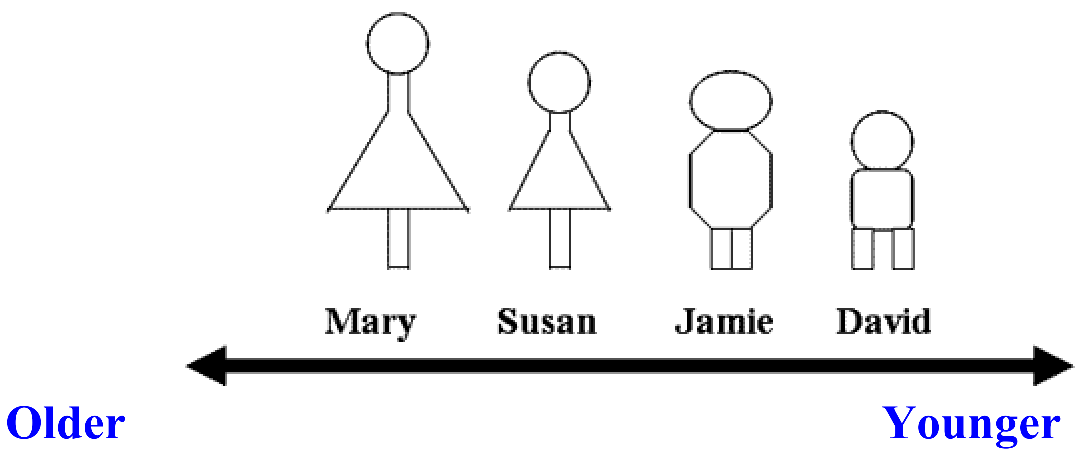
Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X_______. How did Jenny fill in the blanks?

Answer: 10, 11, 12 W

Draw a Picture

Mary is older than Jamie. Susan is older than Jamie, but younger than Mary. David is younger than Jamie. Who is oldest?

Mary is older than Jamie. Susan is older than Jamie, but younger than Mary. David is younger than Jamie. Who is oldest?



Answer: Mary is oldest.

Guess and Check

Farmer Joe keeps cows and chickens in the farmyard. All together, Joe can count 14 heads and 42 legs. How many cows and how many chickens does Joe have in the farmyard?

Farmer Joe keeps cows and chickens in the farmyard. All together, Joe can count 14 heads and 42 legs. How many cows and how many chickens does Joe have in the farmyard?

6 cows +8 chickens 14 heads	**Guess a number of cows. Then add the number of chickens to arrive at the sum of 14 heads. Then check the total legs.**	**6 cows = 24 legs +8 chickens = 16 legs 40 legs**
7 cows +7 chickens 14 heads	**Adjust your guesses. Then check again until you solve the problem.**	**7 cows = 28 legs +7 chickens = 14 legs 42 legs**

Answer: 7 cows and 7 chickens

Work Backwards

Marsha was banker for the school play. She took in $175 in ticket sales. She gave Wendy $75 for sets and costumes and Paul $17.75 for advertising and publicity. After paying for the props, Marsha had $32.25 left. How much did the props cost?

Marsha was banker for the school play. She took in $175 in ticket sales. She gave Wendy $75 for sets and costumes and Paul $17.75 for advertising and publicity. After paying for the props, Marsha had $32.25 left. How much did the props cost?

$ 175.00 tickets	**$ 82.25**
- 75.00 costumes	**- 32.25**
$ 100.00	**$ 50.00 cost of props**
- 17.75 advertising	
$ 82.25	

Logical Reasoning

Juan challenged Sheila to guess his grandmother's age in ten questions or less. It took her six. Here's what Sheila asked:

Juan challenged Sheila to guess his grandmother's age in ten questions or less. It took her six. Here's what Sheila asked:

"Is she less than fifty?" "No." **50+ years old**

"Less than seventy-five?" "Yes."	**50 to 74 years old**
"Is her age an odd or even number?" "Odd."	**ends in 1, 3, 5, 7 or 9**
"Is the last number less than or equal to five?" "No."	**ends in 7 or 9**
"Is it nine?" "No."	**ends in 7 – 57 or 67**
"Is she in her sixties?" "No."	**57 years old**

Not Enough Information

Now that you know how to decide whether to add, subtract, multiply, or divide to solve a word problem, you should be able to recognize a word problem that cannot be solved because not enough information is given.

Look at the following example:

Problem: At her waitress job, Sheila earns $4.50 an hour plus tips. Last week she got $65.40 in tips. How much did she earn last week?

Step 1: *question:* How much did she earn last week?

Step 2: *necessary information:* $4.50/hour, $65.40

Step 3: *decide what arithmetic to use:*

tips + (pay per hour **x** hours worked) = total earned *missing information:* hours worked

At first glance, you might think that you have enough information since there are 2 numbers. But when the solution is set up, you can see that you need to know the number of hours Sheila worked to find out what she earned. (**Be Careful!!!**)

Practice Exercise

Solve the following problems.

1. John bought a wallet and a book. The book cost $33. He gave the cashier $50 and received $6 for change. How much did the wallet cost?

2. Al bought 244 eggs for $24.40. He found that 16 eggs were rotten. He sold the rest at 12 eggs for $1.89. How much money did he make?

3. Jenny and Steven have $330 altogether. Jenny has $195. How much must Jenny take from Steven so that she has twice as much as Steven?

4. Nick weighs 52 kg. Peter is the same weight as Michael. If their total weight is 128 kg., what is each of their weights?

5. Michelle has 241 stamps. Janet has 142 more stamps than Michelle. Their friend Lucy also collects stamps. If they have 800 stamps altogether, how many stamps does Lucy have?

6. Mary needs 44 red beads, 39 purple beads and 63 yellow beads to make a necklace. Beads are sold 12 beads in a packet of the same colour? If each packet costs $2, how much does Mary need?

7. Gene bought $360 worth of sports equipment and $18 worth of office supplies for the boys' club. Since the boys' club is tax-exempt, he didn't have to pay the sales tax. If he had paid tax, how much would he have spent?

8. One computer costs $2430. One printer costs $630. What is the total cost for 14 computers and 8 printers?

9. A father has $240000 to be shared among his 3 children. Two of the children have equal amounts. The third child has $100000. How much does each child get?

10. John has 42 more stamps than Janet. Janet has 23 less stamps than Jack. If Janet has 324 stamps, how many stamps do they have altogether?

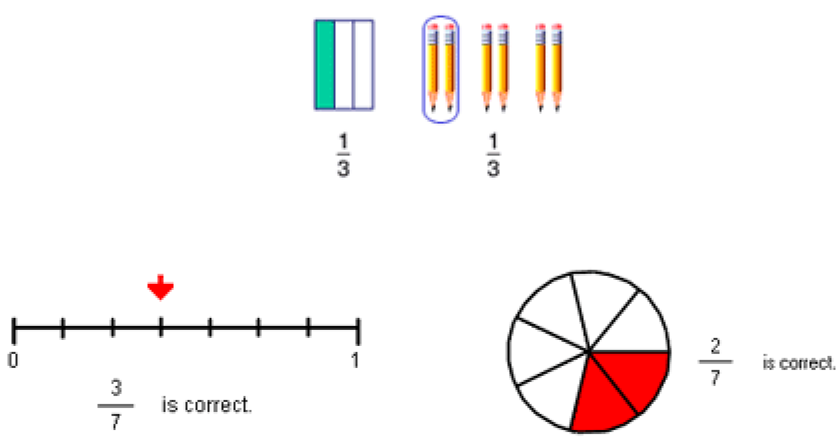
11. 1680 people attended a charity concert. 1320 adults paid $15 each. The rest were children who paid $4 each. The organizers had to pay $5300 for staging the concert. How much money went to charity?

Fractions

Undertanding and Comparing Fractions

The word ***fraction*** means “part of a whole.” The word comes from the Latin word ***fractio***, meaning “to break into pieces.” In math, a fraction means one or more parts of a whole.

Example:

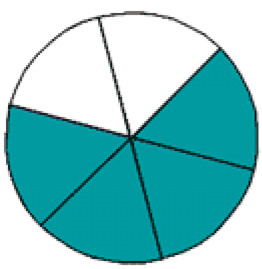


A fraction has two parts, a ***denominator*** and a ***numerator***. The denominator is the numeral written under the bar and tells the number of parts a whole is divided into. The numerator is the numeral written above the bar. The numerator tells the number of parts of the whole that are being counted. A ***proper fraction*** has a numerator that is smaller than its denominator.

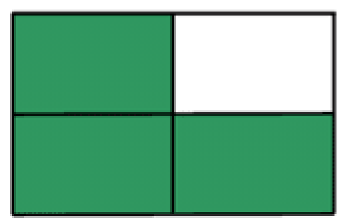
numerator	**number of parts counted**	**1**
----------------------	**----------------------------------**	**------**
denominator	**total parts of the whole**	**17**

Practice Exercise

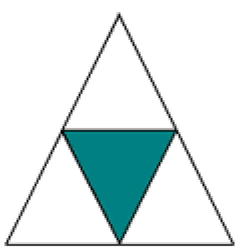
Write the fraction of the coloured part.

1\. 

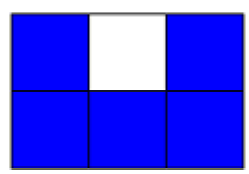
_______ of the circle is coloured.

2\. 

_______ of the rectangle is coloured.

3\. 

_______ of the triangle is coloured.

4\. 

_______ of the rectangle is coloured.

Fill in the blanks.

1. ½ and ________ make 1 whole.
2. and ________ make 1 whole.
3. and ________ make 1 whole.

Improper Fractions

When the numerator of a fraction is greater than or equal to the denominator, the fraction is called an ***improper fraction***.

$$\frac{3}{2} \quad \frac{4}{3} \quad \frac{5}{4} \quad \frac{6}{5} \quad \frac{7}{6} \quad \frac{8}{8}$$

Þ ***The value of an improper fraction is always greater than or equal to one.***

Mixed Numerals

Mixed numerals combine whole numbers and fractions. The values of mixed numerals can also be written as ***improper fractions***. To write a mixed numeral as an improper fraction, multiply the whole number by the denominator of the fraction, then add the numerator. Use your answer as the new numerator and eep the original enominator.

$$1\tfrac{1}{2} = \frac{(2 \times 1) + 1}{2} = \frac{3}{2} \qquad 2\tfrac{3}{4} = \frac{(2 \times 4) + 3}{4} = \frac{11}{4}$$

To change an improper fraction to a mixed numeral, divide the numerator by the denominator. Then place the remainder over the old denominator.

$$\frac{3}{2} = 2\overline{)3} \Rightarrow 1,\ 3 - 2 = 1 \quad = 1\tfrac{1}{2} \qquad \frac{11}{4} = 4\overline{)11} \Rightarrow 2,\ 11 - 8 = 3 \quad = 2\tfrac{3}{4}$$

Practice Exercise

Express each fraction as a whole number or as a mixed number.

1. $\frac{20}{7} =$
2. $\frac{23}{3} =$
3. $\frac{25}{2} =$
4. $\frac{33}{10} =$
5. $\frac{59}{6} =$
6. $\frac{85}{8} =$
7. $\frac{93}{12} =$
8. $\frac{39}{5} =$
9. $\frac{49}{4} =$
10. $\frac{45}{5} =$
11. $\frac{29}{6} =$
12. $\frac{24}{2} =$
13. $\frac{109}{10} =$
14. $\frac{53}{7} =$
15. $\frac{17}{4} =$
16. $\frac{7}{3} =$

17. $\frac{52}{8} =$ 18. $\frac{57}{9} =$ 19. $\frac{59}{11} =$ 20. $\frac{64}{7} =$

21. $\frac{114}{9} =$ 22. $\frac{26}{5} =$ 23. $\frac{41}{4} =$ 24. $\frac{36}{3} =$

25. $\frac{9}{2} =$ 26. $\frac{60}{5} =$ 27. $\frac{37}{4} =$ 28. $\frac{39}{6} =$

29. $\frac{20}{3} =$ 30. $\frac{151}{12} =$ 31. $\frac{64}{11} =$ 32. $\frac{52}{9} =$

Express each mixed numeral as an improper fraction.

a. 12 3/4 =
b. 99 1/2 =
c. 28 3/4 =
d. 2 1/4 =
e. 31 1/3 =
f. 75 2/3 =
g. 46 1/8 =
h. 64 3/8 =
i. 57 5/8 =
j. 7 1/2 =

Common Denominators

Many fractions have ***common denominators***. That means that the numbers in their denominators are the same.

$$\mathbf{\frac{1}{2} \quad \frac{3}{2} \quad \frac{5}{2}}$$

To find common denominators, (1) find the ***least common multiple*** for the denominators of the fractions you are comparing.

Find the ***multiples*** of a number by multiplying it by other whole numbers. The multiples of ***2***, for example, are:

0 x 2 = 0	**2 x 3 = 6**
1 x 2 = 2	**2 x 4 = 8**
2 x 2 = $\underline{4}$	**2 x 5 = $\underline{10}$**

… and so on.

As you can see, the multiples of ***2*** include ***0, 2, 4, 6, 8,*** and ***10***. The list continues into infinity!

Some numbers share the same multiples. Those multiples are known as ***common multiples***.

Number Multiples

0	**0**	**0**	**0**	**0**	**0**	**0**
1	**0**	**1**	**2**	**3**	**4**	**5**
2	**0**	**2**	**4**	**6**	**8**	**10**
3	**0**	**3**	**6**	**9**	**12**	**15**
4	**0**	**4**	**8**	**12**	**16**	**20**
5	**0**	**5**	**10**	**15**	**20**	**25**
	0	**1**	**2**	**3**	**4**	**5**

The least multiple of two or more numbers is the least common multiple. For example, the least common multiple of ***2*** and ***3*** is ***6***.

2 x 1 = 2　　**2 x 2 = 4**　　**2 x 3 = 6**
3 x 1 = 3　　**3 x 2 = 6**

Compare:

$\frac{1}{2}$ and $\frac{2}{3}$　　**Answer: least common multiple is 6**

(2) Divide the common multiple by the denominators.

$6 \div 2 = 3$　　$6 \div 3 = 2$

(3) Multiply the quotients by the old numerators to calculate the new numerators.

$$\begin{array}{r} 3 \\ \underline{\times\ 1} \\ 3 \end{array} \qquad\qquad \begin{array}{r} 2 \\ \underline{\times\ 2} \\ 4 \end{array}$$

(4) Place the new numerators over the common denominator.

$$\frac{3}{6} \qquad\qquad \frac{4}{6}$$

Þ ***To reduce a fraction to its lowest terms, divide both the numerator and the denominator by their greatest common denominator.***

$$\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$$

Practice Exercise

Rewrite each set of fractions using the least common denominator.

1. $\frac{5}{6}$, $\frac{1}{5}$	2. $\frac{3}{6}$, $\frac{1}{4}$
3. $\frac{5}{6}$, $\frac{1}{2}$	4. $\frac{1}{2}$, $\frac{2}{5}$

5. $\frac{2}{5}$, $\frac{9}{11}$	6. $\frac{2}{9}$, $\frac{5}{8}$
7. $\frac{5}{8}$, $\frac{8}{10}$	8. $\frac{1}{6}$, $\frac{3}{9}$
9. $\frac{4}{5}$, $\frac{1}{7}$	10. $\frac{6}{7}$, $\frac{1}{10}$
11. $\frac{2}{9}$, $\frac{2}{5}$	12. $\frac{4}{8}$, $\frac{3}{6}$
13. $\frac{3}{5}$, $\frac{1}{12}$	14. $\frac{7}{11}$, $\frac{3}{6}$
15. $\frac{4}{12}$, $\frac{4}{7}$	16. $\frac{6}{11}$, $\frac{5}{12}$
17. $\frac{6}{7}$, $\frac{3}{8}$, $\frac{4}{14}$	18. $\frac{6}{9}$, $\frac{3}{7}$, $\frac{11}{14}$

Reduce each fraction to lowest terms.
(Hint: Divide its numerator and denominator by their Greatest Common Factor)

1. $\frac{2}{12} =$	2. $\frac{24}{40} =$	3. $\frac{6}{60} =$	4. $\frac{20}{25} =$
5. $\frac{12}{24} =$	6. $\frac{24}{30} =$	7. $\frac{10}{90} =$	8. $\frac{30}{45} =$
9. $\frac{30}{60} =$	10. $\frac{22}{12} =$	11. $\frac{10}{20} =$	12. $\frac{38}{68} =$
13. $\frac{34}{43} =$	14. $\frac{12}{30} =$	15. $\frac{43}{50} =$	16. $\frac{7}{84} =$
17. $\frac{9}{46} =$	18. $\frac{2}{16} =$	19. $\frac{38}{56} =$	20. $\frac{24}{28} =$
21. $\frac{51}{28} =$	22. $\frac{15}{35} =$	23. $\frac{26}{30} =$	24. $\frac{20}{44} =$

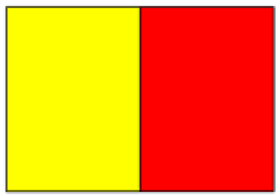
Equivalent Fractions

You know from experience that different fractions can have the same value.

> Since there are 100 pennies in a dollar, 25 pennies is equal to 25/100 of a dollar. The same amount also equals a quarter, or ¼ of a dollar.

On a measuring cup, ½ cup is the same amount as 2/4 cup.
On an odometer, 5/10 of a mile is the same as ½ mile.
Out of a dozen doughnuts, six doughnuts equal 6/12, or ½ dozen.

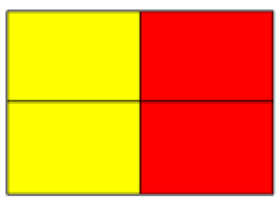
A napkin is folded into two parts. One part is yellow, the other red.



½ yellow

½ red

Then the napkin is folded again. Now there are two yellow parts and two red parts.



$\frac{2}{4}$ **yellow**

$\frac{2}{4}$ **red**

In this example, the red part of the napkin can be described as ½ red or 2/4 red. That makes ½ and 2/4 ***equivalent fractions***.

When solving math problems, reduce fractions to their lowest equivalent. Rather than describing the napkin as 2/4 yellow, call it ½ yellow.

Some Equivalent Fractions

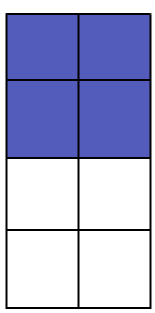
$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

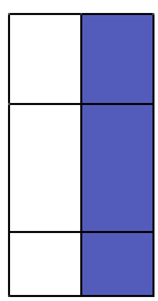
$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20}$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$$

You can tell if two fractions are equal by finding cross products.

Example Are 4/8 and 3/6 equal fractions?





Multiply diagonally as shown by the arrows below. If the cross products are equal, the fractions are equal.

$$\frac{4}{8} \times \frac{3}{6}$$

4 x 6 = 24
8 x 3 = 24

Since the cross products are equal, **4/8 = 3/6.**

Sometimes you need to find an equal fraction with higher terms. You raise a fraction to higher terms by multiplying both the numerator and the denominator by the same number (except 0).

5/8 and 20/32 are equal fractions because $\frac{5 \times 4}{8 \times 4} = \frac{20}{32}$

Often you will need to find an equal fraction with a specific denominator. To do this, think, "What number multiplied by the original denominator will result in the new denominator?" Then multiply the original numerator by the same number.

Example ¾ = ?/24

Since 4 x 6 = 24, multiply the numerator 3 by 6. $\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$

The fractions ¾ **and 18/24 are equal fractions.**

Comparing Fractions

When two fractions have the same number as the denominator, they are said to have a common denominator, and the fractions are called like fractions. When you compare like fractions, the fraction with the greater numerator is the greater fraction.

Example 1 Which fraction is greater, 3/5 or 4/5?

The fractions 3/5 and 4/5 are like fractions because they have a common denominator, 5. Compare the numerators.

Since 4 is greater than 3, **4/5 is greater than 3/5.**

Fractions with different denominators are called unlike fractions. To compare unlike fractions, you must change them to fractions with a common denominator.

The common denominator will always be a multiple of both of the original denominators. The multiples of a number are found by going through the times tables for the number. For instance, the multiples of 3 are 3, 6, 9, 12, 15, 18, and so on.

You can often find a common denominator by using mental math. If not, try these methods:

1. See whether the larger denominator could be a common denominator. In other words, if the smaller denominator can divide into the larger denominator evenly, use the larger denominator as the common denominator.

2. Go through the multiples of the larger denominator. The first one that can be divided evenly by the smaller denominator is the lowest common denominator.

Example 2 Which is greater, 5/6 or ¾?

Go through the multiples of the larger denominator: 6, 12, 18, 24, 30…. Since 12 can be divided evenly by both 4 and 6, 12 is the lowest common denominator.

Build equal fractions, each with the denominator 12:

$$\frac{5 \times 2}{6 \times 2} = \frac{10}{12} \qquad \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

Compare the like fractions. Since 10/12 > 9/12, the fraction **5/6 > ¾.**

Practice Exercise

Fill in the missing numerator or denominator for the following.

1.	$\frac{3}{5} = \frac{18}{}$	2.	$\frac{1}{8} = \frac{}{72}$
3.	$\frac{3}{6} = \frac{}{30}$	4.	$\frac{2}{7} = \frac{}{77}$
5.	$\frac{2}{3} = \frac{20}{}$	6.	$\frac{2}{9} = \frac{14}{}$
7.	$\frac{1}{4} = \frac{}{40}$	8.	$\frac{1}{2} = \frac{11}{}$
9.	$\frac{1}{3} = \frac{8}{}$	10.	$\frac{9}{10} = \frac{}{30}$
11.	$\frac{6}{7} = \frac{72}{}$	12.	$\frac{6}{9} = \frac{}{99}$
13.	$\frac{1}{6} = \frac{}{60}$	14.	$\frac{2}{4} = \frac{16}{}$
15.	$\frac{3}{8} = \frac{6}{}$	16.	$\frac{6}{12} = \frac{}{84}$

Fraction Comparison

1. $\frac{5}{8}$ > $\frac{2}{5}$	2. $\frac{2}{3}$ ___ $\frac{1}{11}$
3. $\frac{5}{9}$ ___ $\frac{1}{9}$	4. $\frac{9}{14}$ ___ $\frac{1}{2}$
5. $\frac{1}{6}$ ___ $\frac{1}{4}$	6. $\frac{3}{19}$ ___ $\frac{6}{11}$
7. $\frac{8}{16}$ ___ $\frac{2}{10}$	8. $\frac{3}{6}$ ___ $\frac{9}{18}$
9. $\frac{1}{3}$ ___ $\frac{20}{29}$	10. $\frac{5}{50}$ ___ $\frac{7}{35}$
11. $\frac{30}{60}$ ___ $\frac{2}{9}$	12. $\frac{45}{21}$ ___ $\frac{135}{63}$
13. $\frac{14}{15}$ ___ $\frac{1}{4}$	14. $\frac{3}{2}$ ___ $\frac{37}{20}$
15. $\frac{12}{36}$ ___ $\frac{9}{15}$	16. $\frac{5}{4}$ ___ $\frac{3}{4}$

Addition of Fractions

To add fractions, the fractions must have ***common denominators***. To add fractions with common denominators, simply add the numerators. The sum will become the numerator of your answer. The denominator will remain the same.

$$\frac{3}{8} + \frac{4}{8} = \frac{3+4}{8} = \frac{7}{8}$$

Unlike fractions have different denominators. Use these steps to add unlike fractions.

Step 1 Find a common denominator and change one or both of the fractions to make like fractions.

$$\tfrac{1}{2} + \tfrac{3}{4} = ?$$

$$\tfrac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Step 2 Add the like fractions

$$2/4 + \tfrac{3}{4} = 5/4$$

Step 3 Reduce the answer if necessary. If the answer is an improper fraction, rewrite it as a whole or mixed number.

$$5/4 = 1\,\tfrac{1}{4}$$

A mixed number is a whole number and a proper fraction. To add mixed numbers, work with each part separately and then combine the results.

Þ ***Adding fractions is impossible without first writing the fractions with common denominators.***

Step 1 Write the fractions with common denominators.

$$6\ 1/3 = 6\ \frac{1 \times 4}{3 \times 4} = 6\ \frac{4}{12}$$

$$+\ 4\ \tfrac{3}{4} = 4\ \frac{3 \times 3}{4 \times 3} = 4\ \frac{9}{12}$$

Step 2 Add the fractions first. Add the numerators and put the sum over the common denominator. Then add the whole numbers.

$$6\ \frac{4}{12}$$

$$+\ 4\ \frac{9}{12}$$

Step 3 Change the improper fraction to a mixed number. Add this to the whole number answer.

$$\frac{13}{12} = 1\ \frac{1}{12}$$

$$10 + 1\ \frac{1}{12} = 11\ \frac{1}{12}$$

Sometimes when you add the fraction parts, you get a whole number as an answer. If this happens, just add that whole number to the other one.

Example: $2\frac{3}{5} + 2\frac{2}{5}$

2 + 2 = 4

$\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$ **Remember that any number divided by itself is 1.**

4 + 1 = 5 **The answer is 5.**

Mixed numbers can be added to whole numbers by adding the whole numbers together and keeping the fraction. This makes sense because you are adding whole amounts plus another part of a whole.

Example: 3 + 2 ½ = 5 ½ 3 + 2 = 5, 5 + ½ = 5 ½

Practice Exercise

Solve for each of the given problems.

1. $\frac{4}{7} + 7\frac{2}{7}$	**2.** $\frac{1}{5} + \frac{3}{5}$	**3.** $4\frac{1}{2} + \frac{1}{2}$	**4.** $\frac{9}{10} + \frac{9}{10}$

5. $4\frac{3}{4} + \frac{3}{4}$	6. $10\frac{4}{6} + \frac{1}{6}$	7. $12\frac{5}{7} + 10\frac{1}{7}$	8. $\frac{8}{12} + \frac{5}{12}$
9. $3\frac{6}{8} + 11\frac{5}{8}$	10. $12\frac{3}{4} + \frac{6}{10}$	11. $\frac{5}{11} + \frac{6}{11}$	12. $\frac{7}{10} + \frac{6}{7}$
13. $5\frac{5}{6} + \frac{2}{8}$	14. $\frac{2}{6} + \frac{4}{5}$	15. $12\frac{2}{3} + 9\frac{2}{9}$	16. $9\frac{10}{12} + \frac{8}{9}$

Subtraction of Fractions

To subtract fractions, the fractions must have ***common denominators***. To subtract fractions with common denominators, simply subtract the numerators. The difference will become the numerator of your answer. The denominator will remain the same.

$$\frac{11}{12} - \frac{2}{12} = \frac{11 - 2}{12} = \frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

Unlike fractions have different denominators. Use these steps to subtract unlike fractions.

Step 1 Find a common denominator and change one or both of the fractions to make like fractions.

$$\frac{3}{4} - \frac{1}{2} = ?$$

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Step 2 Subtract the like fractions.

$$\frac{3}{4} - 2/4 = \frac{1}{4}$$

Step 3 Reduce the answer if necessary. If the answer is an improper fraction, rewrite it as a whole or mixed number.

A mixed number is a whole number and a proper fraction. To subtract mixed numbers, work with each part separately and then combine the results.

Þ ***Subtracting fractions is impossible without first writing the fractions with common denominators.***

Step 1 Write the fractions with common denominators.

$$6\,3/4 = 6\,\frac{3 \times 3}{4 \times 3} = 6\,\frac{9}{12}$$

$$-\,4\,1/3 = 4\,\frac{1 \times 4}{3 \times 4} = 4\,\frac{4}{12}$$

Step 2 Subtract the fractions first. Subtract the numerators and put the difference over the common denominator. Then subtract the whole numbers.

$$\begin{array}{r} 6\,\frac{9}{12} \\ -\,4\,\frac{4}{12} \\ \hline 2\,\frac{5}{12} \end{array}$$

Step 3 If necessary, reduce to lowest terms.

When subtracting mixed numbers, sometimes the fraction you are subtracting from will be smaller than the fraction you are taking away. In this situation, you will need to regroup, or borrow, 1 from the whole number and rewrite it as a fraction. Remember, a fraction with the same numerator and denominator equals 1.

Example $5\frac{1}{8} - 3\frac{3}{4}$

$$\begin{array}{r} 5\frac{1}{8} \\ -3\frac{3}{4} \\ \hline \end{array}$$

Step 1 Write the fractions with common denominators. The lowest common denominator is 8.

$$5\frac{1}{8} = 5\frac{1 \times 1}{8 \times 1} = 5\frac{1}{8}$$

$$-3\frac{3}{4} = 3\frac{3 \times 2}{4 \times 2} = 3\frac{6}{8}$$

Step 2 Because 1/8 is less than 6/8, you need to regroup, or borrow. Borrow 1 from the whole number 5, rewriting 5 as 4 8/8. Then add the fractional parts 1/8 and 8/8.

$$5\frac{1}{8} = 4\frac{8}{8} + \frac{1}{8} = 4\frac{9}{8}$$

$$\begin{array}{r} 5\frac{1}{8} \\ -3\frac{6}{8} \\ \hline \end{array} \qquad \begin{array}{r} 4\frac{9}{8} \\ -3\frac{6}{8} \\ \hline 1\frac{3}{8} \end{array}$$

Step 3 Subtract. If necessary, reduce the fraction to lowest terms

Sometimes when you subtract the fraction parts, you get a whole number as an answer. If this happens, just subtract that whole number from the other one.

Example: $4\frac{7}{5} - 2\frac{2}{5}$

4 - 2 = 2

$\frac{7}{5} - \frac{2}{5} = \frac{5}{5} = 1$ **Remember that any number divided by itself is 1.**

2 - 1 = 1 **The answer is 1.**

Mixed numbers can be subtracted from whole numbers by subtracting the whole numbers and keeping the fraction.

Example: 3 - 2 ½ = 1 ½ 3 - 2 = 1, 5 + ½ = 1 ½

Practice Exercise

Solve for each of the given problems.

1. $2\frac{6}{10} - \frac{1}{10}$	**2.** $3\frac{5}{6} - \frac{4}{6}$	**3.** $\frac{6}{9} - \frac{3}{9}$	**4.** $12\frac{1}{2} - 9\frac{1}{2}$
5. $\frac{4}{7} - \frac{3}{7}$	**6.** $\frac{6}{12} - \frac{4}{12}$	**7.** $7\frac{7}{11} - 1\frac{10}{11}$	**8.** $2\frac{3}{5} - \frac{3}{5}$
9. $\frac{3}{4} - \frac{1}{4}$	**10.** $\frac{3}{8} - \frac{2}{9}$	**11.** $9\frac{5}{10} - \frac{5}{10}$	**12.** $7\frac{2}{7} - 3\frac{1}{5}$

13.	14.	15.	16.
$\begin{array}{r} 6\frac{4}{11} \\ -\ 5\frac{8}{9} \\ \hline \end{array}$	$\begin{array}{r} 11\frac{4}{5} \\ -\ 10\frac{6}{12} \\ \hline \end{array}$	$\begin{array}{r} 10\frac{5}{6} \\ -\ \frac{6}{7} \\ \hline \end{array}$	$\begin{array}{r} \frac{5}{8} \\ -\ \frac{5}{9} \\ \hline \end{array}$

Multiplication of Fractions

To multiply one fraction by another fraction, multiply the numerators. Their product will become the new numerator. Next, multiply the denominators. Their product will become the new denominator.

multiply the numerators

$$\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$$

multiply the denominators

$$\frac{7}{8} \times \frac{1}{3} = \frac{7}{24}$$

$$\frac{4}{3} \times \frac{1}{10} = \frac{4 \times 1}{3 \times 10} = \frac{4}{30}$$

To multiply a fraction by a whole number, change the whole number to a fraction by placing it over a denominator of one.

(This does not change the value of the whole number.) Multiply the numerators then multiply the denominators to get the product.

$$\frac{1}{2} \times 1 = \frac{1}{2} \times \frac{1}{1} = \frac{1 \times 1}{2 \times 1} = \frac{1}{2}$$

$$\frac{2}{7} \times 3 = \frac{2}{7} \times \frac{3}{1} = \frac{2 \times 3}{7 \times 1} = \frac{6}{7}$$

$$\frac{8}{9} \times 6 = \frac{8}{9} \times \frac{6}{1} = \frac{8 \times 6}{9 \times 1} = \frac{48}{9} = 5\frac{3}{9} = 5\frac{1}{3}$$

Þ ***Change improper fractions to mixed numerals. Be sure the fraction part of the mixed numeral is written in the lowest possible terms.***

To multiply mixed numerals by fractions, change the mixed numerals to improper fractions. Then multiply the fractions.

change the mixed numeral to an improper fraction

$$1\frac{6}{7} \times \frac{2}{3} = \frac{13}{7} \times \frac{2}{3} = \frac{13 \times 2}{7 \times 3} = \frac{26}{21} = 1\frac{5}{21}$$

$$2\frac{1}{8} \times 3\frac{1}{2} = \frac{17}{8} \times \frac{7}{2} = \frac{17 \times 7}{8 \times 2} = \frac{119}{16} = 7\frac{7}{16}$$

As you know, reducing a fraction means to divide the numerator and the denominator by the same number. You can use this principle to simplify before you work the problem. This process is called canceling.

Example Find 1/6 of 2/3.

Both the numerator of one fraction and the denominator of the other fraction can be divided by 2. Since 2 ÷ 2 = 1, draw a slash through the numerator 2 and write 1. Since 6 ÷ 2 = 3, draw a slash through the denominator 6 and write 3. Then multiply the simplified fractions.

$$\frac{1}{6} \times \frac{2}{3} = \frac{1 \times \overset{1}{\cancel{2}}}{\underset{3}{\cancel{6}} \quad 3} = \frac{1}{9}$$

Since you used canceling before multiplying, there is no need to reduce the answer: 1/6 of 2/3 is **1/9.**

When you cancel, make sure you divide a numerator and a denominator by the same number. The canceling shown in the following example is **incorrect**.

$$\frac{1}{6} \times \frac{2}{3} = \frac{1 \times 2}{\underset{2}{\cancel{6}} \quad \underset{1}{\cancel{3}}}$$

Although 6 and 3 can both be divided by 3, both numbers are in the denominator.

To multiply with mixed numbers, change the mixed numbers to improper fractions before you multiply.

Example Multiply 1 2/3 by 7 ½.

Step 1 Change to improper fractions.

$$1\frac{2}{3} \text{ x } 7\frac{1}{2} = \frac{5}{3} \text{ x } \frac{15}{2}$$

Step 2 Cancel and multiply.

$$\frac{5}{\cancel{3}_{1}} \text{ x } \frac{\cancel{15}^{5}}{2} =$$

Step 3 Write as a mixed number.

$$\frac{25}{2} = 12\frac{1}{2}$$

The product of 1 2/3 and 7 ½ is **12 ½.**

Practice Exercise

Solve for each of the given problems.
Write the answer in lowest terms.

1. $\frac{1}{2} \times \frac{2}{3}$	**2.** $\frac{2}{3} \times \frac{1}{2}$
3. $\frac{4}{7} \times \frac{7}{8}$	**4.** $\frac{1}{7} \times \frac{2}{3}$
5. $\frac{2}{7} \times \frac{3}{9}$	**6.** $\frac{2}{4} \times \frac{1}{3}$
7. $\frac{1}{3} \times \frac{2}{3}$	**8.** $\frac{4}{10} \times \frac{2}{7}$
9. $\frac{3}{4} \times \frac{6}{10}$	**10.** $\frac{2}{3} \times 9$
11. $12 \times \frac{1}{12}$	**12.** $12\frac{3}{6} \times 13$
13. $1 \times 15\frac{4}{11}$	**14.** $\frac{4}{9} \times 13\frac{2}{8}$
15. $10\frac{9}{14} \times \frac{7}{9}$	**16.** $14\frac{5}{7} \times 7\frac{2}{9}$
17. $8\frac{7}{15} \times 5\frac{4}{6}$	**18.** $6\frac{5}{11} \times 14\frac{1}{3}$

Factors are numbers that, when multiplied together, form a new number called a ***product***. For example, ***1*** and ***2*** are factors of ***2***, and ***3*** and ***4*** are factors of ***12***.

Every number except ***1*** has at least two factors: ***1*** and itself.

Common Factor

A number that is a factor of two or more numbers

Example:

factors of 6: 1, 2, 3, 6

factors of 12: 1, 2, 3, 4, 6, 12

The common factors of 6 and 12 are
1, 2, 3, and 6.

Greatest Common Factor (GCF)

The greatest factor that two or more numbers have in common

Example:

18: 1, 2, 3, **6**, 9, 18
30: 1, 2, 3, 5, **6**, 10, 15, 30
6 is the GCF of 18 and 30.

Practice Exercise

Find the greatest common factor (GCF) for the given numbers.

1. 2, 5 **1**
2. 10, 4
3. 8, 12
4. 3, 8
5. 6, 10

6. 12, 9
7. 12, 6
8. 2, 4
9. 2, 3
10. 18, 27
11. 12, 30
12. 12, 15
13. 5, 20
14. 8, 24
15. 12, 18
16. 5, 17
17. 20, 15
18. 3, 6
19. 16, 32
20. 12, 2
21. 48, 24
22. 24, 9
23. 12, 36
24. 10, 2
25. 16, 72
26. 112,160
27. 42, 63
28. 30,110
29. 42, 44
30. 80, 64

Find the least common multiple for the given numbers.

1. 6, 8 **48**
2. 5, 2
3. 4, 5
4. 5, 6
5. 6, 3
6. 9, 10
7. 12, 8
8. 6, 12
9. 11, 5
10. 15, 25
11. 4, 14
12. 28, 10
13. 18, 10
14. 27, 24
15. 6, 16
16. 22, 4
17. 5, 3
18. 12, 24
19. 23, 15
20. 7, 22
21. 16, 48
22. 2, 30
23. 12, 4
24. 13, 16
25. 25, 2

26. 150,180
27. 40, 120
28. 63, 48
29. 18, 5
30. 24, 42

Division of Fractions

To divide a fraction by a whole number, change the whole number to an improper fraction with a denominator of one. Invert the divisor fraction. Then multiply the fractions.

$$\frac{1}{2} \div 2 = \frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\frac{2}{7} \div 3 = \frac{2}{7} \div \frac{3}{1} = \frac{2}{7} \times \frac{1}{3} = \frac{2}{21}$$

To divide a whole number by a fraction or to divide a fraction by another fraction, ***invert*** the divisor fraction. Then multiply the fractions.

$$\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times \frac{3}{1} = \frac{1 \times 3}{2 \times 1} = \frac{3}{2} = 1\frac{1}{2}$$

Invert the divisor fraction and multiply

$$7 \div \frac{6}{8} = \frac{7}{1} \text{ x } \frac{8}{6} = \frac{7 \text{ x } 8}{1 \text{ x } 6} = \frac{56}{6} = 9\frac{2}{6} = 9\frac{1}{3}$$

To divide a mixed numeral by another mixed numeral, first change the mixed numerals to improper fractions. Then invert the divisor fraction and multiply.

$$4\frac{1}{2} \div 2\frac{1}{3} = \frac{9}{2} \div \frac{7}{3} = \frac{9}{2} \text{ x } \frac{3}{7} = \frac{27}{14} = 1\frac{13}{14}$$

$$7\frac{6}{8} \div 6\frac{1}{3} = \frac{62}{8} \div \frac{19}{3} = \frac{62}{8} \text{ x } \frac{3}{19} = \frac{186}{152} = 1\frac{34}{152} = 1\frac{17}{76}$$

Turn it Upside Down: Inverting

Inverting a fraction means turning it upside down, or reversing the numerator and the denominator.

$\frac{1}{3}$ **inverted is** $\frac{3}{1}$ $\frac{6}{8}$ **inverted is** $\frac{8}{6}$

Inverting a whole number means to make it the denominator of a fraction with 1 as the numerator. 3 inverted is 1/3, 7 inverted is 1/7.

So, to solve the problem 1/3 ÷ 3,

invert 3 or $\frac{3}{1}$ to $\frac{1}{3}$

then $\frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9}$

Practice Exercise

Solve for each of the given problems.
Write the answer in lowest terms.

1. $\frac{1}{2} \div \frac{2}{4}$	2. $\frac{1}{3} \div \frac{1}{2}$
3. $\frac{3}{8} \div \frac{2}{4}$	4. $\frac{4}{7} \div \frac{2}{3}$
5. $\frac{2}{7} \div \frac{1}{3}$	6. $\frac{2}{4} \div \frac{2}{7}$
7. $\frac{6}{9} \div \frac{1}{4}$	8. $\frac{2}{9} \div \frac{2}{7}$
9. $\frac{6}{7} \div \frac{7}{9}$	10. $\frac{4}{5} \div 8$
11. $4 \div \frac{9}{11}$	12. $11\frac{7}{9} \div 11$
13. $7 \div 10\frac{3}{10}$	14. $11\frac{7}{10} \div \frac{1}{3}$
15. $\frac{8}{11} \div 3\frac{1}{3}$	16. $6\frac{7}{8} \div 12\frac{3}{11}$

17. $2\frac{9}{10} \div 10\frac{4}{5}$	**18.** $2\frac{7}{15} \div 2\frac{4}{6}$

Word Problems with Fractions

Using the Substitution Method

So far, you have solved addition, subtraction, multiplication, and division word problems using whole numbers. Many students can do these word problems with ease, but they worry when they see word problems using large whole numbers, fractions, or decimals.

The difficulty has to do with "math intuition," or the feel that a person has for numbers. You have a very clear idea of the correct answer to 4 – 3. It is more difficult to picture 7,483,251 + 29,983 or 6.45 – 5.5. And for most of us, our intuition totally breaks down for 3/8 – 1/3.

Changing only the numbers in a word problem does not change what must be done to solve the problem. By substituting small whole numbers in a problem, you can understand the problem and how to solve it.

Look at the following example:

Example: A floor is to be covered with a layer of ¾-in. fiberboard and 7/16-in. plywood. By how much will the floor level be raised?

Fractions, especially those with different denominators, are especially hard to picture. You can make the problem easier to understand by substituting small whole numbers for the fractions. You can substitute any numbers, but try to use numbers under 10. These numbers do not have to look like the numbers they are replacing.

In the example, try substituting 3 for ¾ and 2 for 7/16. The problem now looks like this:

> A floor is to be covered by a layer of 3-in. fiberboard and 2-in. plywood. By how much will the floor level be raised?

You can now read this problem and know that you must add.

Once you make your decision about *how* to solve the problem, you can return the original numbers to the word problem and work out the solution. With the substituted numbers, you decided to *add* 3 and 2. Therefore, in the original, you must *add* ¾ and 7/16.

$$\frac{3}{4} = \frac{12}{16}$$

$$+\frac{7}{16} = \frac{7}{16}$$

$$\overline{\phantom{+\frac{7}{16} = \frac{7}{16}}}$$

$$\frac{19}{16} = \mathbf{1\frac{3}{16}}$$

Remember: Choosing 3 and 2 was completely up to you. You could have used any small whole numbers.

Practice Exercise

Solve for each of the given problems.

1.	Michael worked on the computer for 3 1/7 hours. Later, Michael talked to Paul on the phone for 1 1/2 hours. How many hours did Michael use the computer and talk on the phone altogether?
2.	Brad walked one-seventh of a mile from school before Mom arrived and drove Brad home. In all, Brad walked and rode in the car 6 miles. How many miles did Mom drive?
3.	Albert is baking cookies. The recipe calls for one-sixth cup of white flour per dozen. How much flour is needed to make 3 dozen cookies?
4.	Paul had 11/12 yard of string. One-fourth yard of string was used to tie newspapers. How much of the string is left (in yards)?

5.	What is seven-tenths of 24?
6.	The West Sussex book warehouse shipped a package of books that weighs 36 1/4 pounds. Each book in the package weighs 3 5/8 pounds each. How many books were shipped?
7.	Paul read 2/4 of a book. Of those pages, Paul read one-half of the book last week. How much of the book did Paul read last week?
8.	Jane had 7/8 yard of string. One-half of the string was used to tie newspapers. How much of the string is left (in yards)?
9.	A motorized scooter uses 8/24 gallon of gas each mile. How much gas will be used after traveling three miles?
10.	One-third of your grade is based on the final exam. One-sixth of your grade is based on homework. If the rest of your grade is based on participation, how much is participation worth?
11.	Month — Rainfall (inches) April — 4 1/4 May — 3 1/4 June — 4 1/4 July — 5 1/4 August — 3 5/8 September — 4 ¼ What is the total rainfall during these months?

12.	Amy lives 7/9 of a mile from the mall. Jill lives 8/9 of a mile from the mall. How much closer is Amy to the mall?
13.	Brad purchased a computer. Brad paid 2/4 of the $1500 price in cash and will pay the rest in five equal monthly payments. How much will Brad pay each month?
14.	Jane practices basketball 3 2/5 hours three times a week. How many hours will Jane practice each week?
15.	How much is four-fifths of one-third?
16.	How much is two-fourths of one-half?

Decimals

Understanding and Comparing Decimals

The numerals we use today are called ***decimal*** numerals. These numerals stand for the numbers in the decimal system. The decimal system is also known as the Arabic system. The decimal system was first created by Hindu astronomers in India over a thousand years ago. It spread into Europe around 700 years ago.

The ***decimal system*** uses ten symbols: ***0, 1, 2, 3, 4, 5, 6, 7, 8,*** and ***9***. The word "decimal" comes from the Latin root ***decem***, meaning "ten."

Comparing Decimals

Comparing decimals uses an important mathematical concept. You can add zeros to the right of the last decimal digit without changing the value of the number. Study these examples.

RULE When comparing decimals with the same number of decimal places, compare them as though they were whole numbers.

Example Which is greater, 0.364 or 0.329?
Both numbers have three decimal places. Since 364 is greater than 329, the decimal **0.364 > 0.329.**

The rule for comparing whole numbers in which the number with more digits is greater does not hold true for decimals. The decimal number with more decimal places is not necessarily the greater number.

RULE When decimals have a different number of digits, write zeros to the right of the decimal with fewer digits so the numbers have the same number of decimal places. Then compare.

Example Which is greater, 0.518 or 0.52?
Add a zero to 0.52.
Since 520 > 518, the decimal **0.52 > 0.518.**

RULE When numbers have both whole number and decimal parts, compare the whole numbers first.

Example 1 Compare 32.001 and 31.999.

Since 32 is greater than 31, the number **32.001 is greater than 31.999.** It does not matter that 0.999 is greater than 0.001.

Using the same rules, you can put several numbers in order according to value. When you have several numbers to compare, write the numbers in a column and line up the decimal points. Then add zeros to the right until all the decimals have the same number of decimal digits.

Example 2 A digital scale displays weight to thousandths of a pound.
Three packages weigh 0.094 pound, 0.91 pound, and 0.1 pound. Arrange the weights in order from greatest to least.

Step 1	Write the weights in a column, aligning the decimal point.	0.094 0.91**0**
Step 2	Add zeros to fill out the columns.	0.1**00**
Step 3	Compare as you would whole numbers.	

In order from greatest to least, the weights are **0.91, 0.1,** and **0.094 pound.**

Equivalent Decimals

Decimals that name the same number or amount

Example:

0.5 = 0.50 = 0.500

Practice Exercise

Compare the given decimals.

1.	0.78	<	0.95
2.	0.483	________	48.3
3.	0.68	________	0.86
4.	9926.493	________	9962.493
5.	0.48	________	0.055
6.	0.6	________	0.06
7.	0.165	________	16.5
8.	0.27	________	2.8
9.	0.968	________	0.008
10.	1252.479	________	1225.479
11.	1613.276	________	1631.276
12.	0.984	________	0.071
13.	0.34	________	0.34
14.	9.84	________	0.003
15.	0.905	________	0.01

16.	0.490	_______	0.94
17.	0.032	_______	0.032
18.	0.97	_______	9.7
19.	0.34	_______	0.09

Decimals and Place Value

Decimal

A number that uses place value and a decimal point to show values less than one, such as tenths and hundredths

Example:

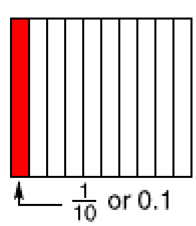
3.47

	hundreds	tens	ones	Decimal point	tenths	hundredths	thousandths
$10\frac{1}{10}$		1	0	.	1		
$205\frac{3}{100}$	2	0	5	.	0	3	
$4\frac{9}{1000}$			4	.	0	0	9

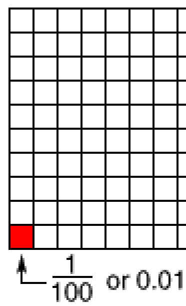
Tenth

One of ten equal parts

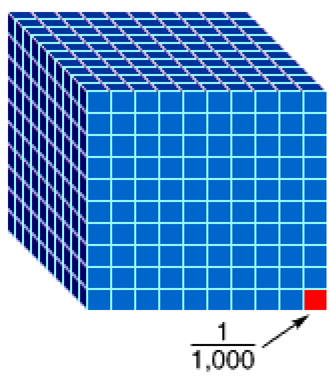
Example:



Hundredth
One of one hundred equal parts
Example:



Thousandth
One part of 1,000 equal parts
Example:



How do you write 16.034 in words?

Read the whole number part of the number. Say *and* to represent the decimal point. Read the digits to the right of the decimal point, and say the place name of the last digit on the right. Note that there are no commas setting off groups of three digits in the decimal part of the number to the right of the decimal point.

The number 16.034 is read ***sixteen and thirty-four thousandths.***

Þ Be careful!!! Although most Canadians and Americans recognize the "." as a decimal point, the decimal point is expressed as a comma in many countries. Most French Canadians use the comma to represent the decimal point.

Decimal Fractions and Decimal Numbers

Decimal fractions or ***decimals*** are fractions with denominators of ***10***, ***100***, ***1,000***, ***10,000***, and so on.

Decimal fractions are written using a decimal point:

$$\frac{1}{10} = .1 \qquad \frac{1}{100} = .01 \qquad \frac{1}{1000} = .001$$

Changing a Fraction to a Decimal

Any fraction can be written as a decimal by dividing the numerator by the denominator, and adding a decimal point in the correct place.

$$\frac{1}{10} = \begin{array}{r} .1 \\ 10\overline{\smash{)}1.0} \end{array} \qquad \frac{3}{5} = \begin{array}{r} .6 \\ 5\overline{\smash{)}3.0} \end{array} \qquad \frac{1}{4} = \begin{array}{r} .25 \\ 4\overline{\smash{)}1.00} \end{array}$$

Þ ***In decimal notation, a decimal point distinguishes whole numbers from decimal fractions:***

$$1 = 1.0$$
$$\frac{1}{10} = 0.1$$
$$1\frac{1}{10} = 1.1$$

Practice Exercise

Write each fraction in decimal format.

1. $\frac{2}{10}$ = **0.2**
2. $\frac{42}{100}$ = ______
3. $\frac{14}{20}$ = ______
4. $\frac{3}{4}$ = ______

5. $\frac{61}{50}$ = ______

6. $\frac{2}{5}$ = ______

7. $\frac{1}{5}$ = ______

8. $\frac{5}{20}$ = ______

9. $0\frac{316}{400}$ = ______

10. $9\frac{66}{100}$ = ______

11. $22\frac{52}{80}$ = ______

12. $7\frac{22}{50}$ = ______

13. $18\frac{4}{20}$ = ______

14. $\frac{388}{80}$ = ______

Changing Decimals to Fractions

Both decimals and fractions can be used to show part of a whole. Sometimes it is easier to calculate using fractions. At other times, decimals are more useful. If you know how to change from one form to the other, you can solve any problem using the form that is best for the situation.

Example Change 0.375 to a fraction.

Step 1 Write the number without the decimal point as the numerator of the fraction.

$$0.375 = \frac{375}{?}$$

Step 2 Write the place value for the last decimal digit as the denominator.

$$0.375 = \frac{375}{1000}$$

Step 3 Reduce the fraction to lowest terms.

$$\frac{375 \div 125}{1000 \div 125} = \frac{3}{8}$$

The decimal 0.375 is equal to the fraction **3/8.**

Practice Exercise

Write each decimal as a fraction or mixed number in lowest terms.

1. 0.62 = __________ 2. 0.85 = __________

3. 0.41 = __________ 4. 0.44 = __________

5. 0.69 = __________ 6. 0.71 = __________

7. 0.16 = __________ 8. 0.75 = __________

9. 0.6 = __________ 10. 0.84 = __________

11. 64.5 = __________ 12. 40.5 = __________

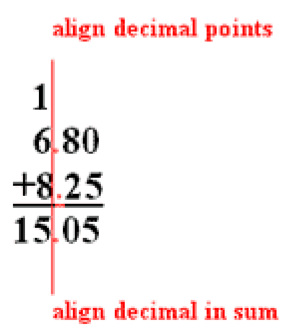
13. 41.9 = __________ 14. 40.55 = __________

15. 73.4 = __________ 16. 76.4 = __________

Addition of Decimals

Adding decimals is easy.

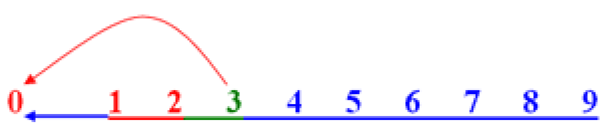
First, align the decimal points of the decimals. Then treat decimal fractions like whole numbers, aligning the decimal point in the sum. Adding decimals may look familiar---it's just like adding money.



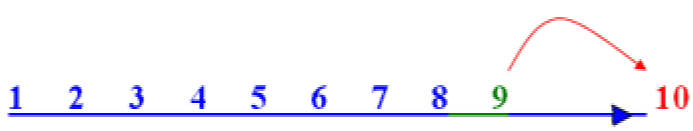
Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. You can estimate when you want to know if you have enough cash to pick up the three things you want at the grocery store or about how much each person should contribute to split the cost of lunch. In such cases, you can use amounts rounded to the nearest dollar (the ones place).

Rounding means to express a number to the nearest given place. The number in the given place is increased by one if the digit to its right is 5 or greater. The number in the given place remains the same if the digit to its right is less than 5. When rounding whole numbers, the digits to the right of the given place become zeros (digits to the left remain the same). When rounding decimal numbers, the digits to the right of the given place are dropped (digits to the left remain the same).

If you are rounding 3 to the nearest tens place, you would round down to 0, because 3 is closer to 0 than it is to 10.



If you were rounding 9, you would round up to 10.



General Rule for Rounding to the Nearest 10, 100, 1,000, and Higher!

Round down from numbers under 5 and round up from numbers 5 and greater.

The same holds true for multiples of 10. Round to the nearest 100 by rounding down from 49 or less and up from 50 or greater. Round to the nearest 1,000 by rounding down from 499 or less and up from 500 or greater.

Example   Using the following price list, about how much would Pat pay for a steering wheel cover, a wide-angle mirror, and an oil drip pan?

Auto Parts Price List	
Outside Wide-Angle Mirror	$13.45
Steering Wheel Cover	$15.95
Oil Drip Pan	$ 8.73
Windshield Washer Fluid	$ 2.85
Brake Fluid	$ 6.35

Round the cost of each item to the nearest dollar and find the total of the estimates.

Item	**Cost**	**Estimate**
Steering wheel cover	$15.95	$16
Wide-angle mirror	13.45	13
Oil drip pan	+ 8.73	+ 9
Total:	$38.13	$38

The best estimate is **$38** which is close to the actual cost of **$38.13.**

The steps for rounding decimals are similar to those you use for rounding whole numbers. The most important difference is that once you have rounded off your number, you must ***drop the remaining digits.***

Example Round 5.362 to the nearest tenth.

Step 1 Find the digit you want to round to.
It may help to circle, underline, or highlight it.

5.362

Step 2 Look at the digit immediately to the right of the highlighted digit.

5.3<u>6</u>2

Step 3 If the digit to the right is 5 or more, add 1 to the highlighted digit. If the digit to the right is less than 5, do not change the highlighted digit. ***Drop the remaining digits.***

5.4

Examples Round 1.832 to the nearest hundredth.

1.832 rounds to 1.83

Round 16.95 to the nearest tenth.

16.95 rounds to 17.0

Round 3.972 to the ones place.

3.972 rounds to 4

Practice Exercise

Solve each problem on the next page.

1. 1.8 + 72.4	**2.** 70.8 + 5.6	**3.** 9.19 + 6.8	**4.** 93.3 + 8.1
5. 15.6 + 44.9	**6.** 81.2 + 66.3	**7.** 87.9 + 7.1	**8.** 6.8 + 7.1
9. 88.6 + 49.4	**10.** 4.59 +78.09	**11.** 86.5 + 2.29	**12.** 74.9 + 92.3

13.	81.41 + 43.89	14.	9.4 + 8.3	15.	3.2 + 30.9	16.	94.86 +52.38
17.	4.76 + 80.83	18.	74.09 + 90.7	19.	5.9 + 1.4	20.	2.29 + 5.6
21.	58.2 + 94.6	22.	9.19 + 9.8	23.	54.47 + 73.14	24.	2.7 + 3.3
25.	32.94 + 96.33	26.	99.4 + 53.5	27.	44.5 + 21.4	28.	6.8 + 9.8

Round to tenths:

(1)	5.21	(2)	25.26	(3)	945.95
(4)	1.43	(5)	43.04	(6)	777.18

Round to hundredths:

(7)	0.638	(8)	4.08	(9)	43.698
(10)	0.413	(11)	0.178	(12)	92.415

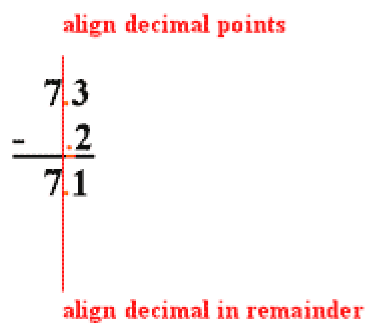
Round to the nearest dollar:

(13)	$19.02	(14)	$24.49	(15)	$19.30
(16)	$5.60	(17)	$22.74	(18)	$8.73

Subtraction of Decimals

Subtracting decimals is easy.

First, align the decimal points of the decimals. Then treat decimal fractions like whole numbers, aligning the decimal point in the remainder. Subtracting decimals may look familiar---it's just like subtracting money.



To subtract decimals, if necessary, use place-holding zeros. Note: Whole numbers are understood to have a decimal point to the right of the ones place.

$$12 - 4.08 = \begin{array}{r} 12.00 \\ \underline{-4.08} \\ 7.92 \end{array}$$

Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. In such cases, you can use amounts rounded to the nearest dollar (the ones place).

Example Susan has $213 in a checking account. If she writes a check for $32.60, about how much will be left in the account?

Round the amount of the check off to the nearest dollar and find the difference.

$213.00	$213
- $ 32.60	- $ 33
$180.40	$180

The best estimate is **$180** which is close to the actual amount of **$180.40.**

Practice Exercise

Solve each problem.

1. 83.2 − 3.4	**2.** 9.1 −8.19	**3.** 32.5 −17.6	**4.** 53.7 −22.2
5. 6.9 − 1.2	**6.** 48.4 − 1.7	**7.** 9.5 − 1.9	**8.** 97.5 −54.2
9. 59.7 − 2.1	**10.** 72.8 −61.3	**11.** 4.59 − 3.8	**12.** 71.59 − 7.3

13. 28.2 - 4.5	**14.** 81.14 -47.79	**15.** 8.5 - 1.4	**16.** 76.9 -18.3
17. 80.24 -42.66	**18.** 7.2 - 5.4	**19.** 77.7 -15.2	**20.** 68.97 -7.31
21. 71.9 - 5.4	**22.** 9.19 - 8.1	**23.** 90.2 -56.5	**24.** 4.4 -2.29
25. 4.4 - 1.9	**26.** 56.91 -46.54	**27.** 1.9 - 1.1	**28.** 59.9 - 2.1

Multiplication of Decimals

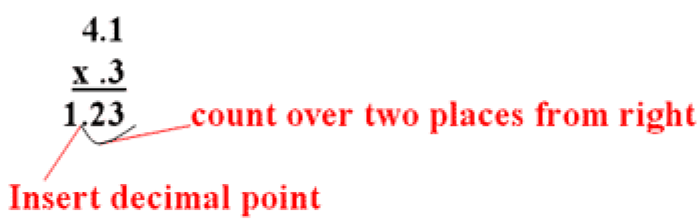
To multiply decimals, treat them as if they were whole numbers, at first ignoring the decimal point.

4.1
x .3
123

Next, count the number of places to the right of the decimal point in the multiplicand. Add this to the number of places to the right of the decimal point in the multiplier.

4.1 multiplicand ------------ one place
x .3 multiplier --------------- +one place
two places

Last, insert the decimal point in the product by counting over from the right the appropriate number of places.



Here are two other examples:

8.9	**65.003**
x 1.0	**x .025**
0 0	**325015**
8 9 0	**1300060**
8. 9 0	**1.625075**

Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. In such cases, round each factor to its greatest place. Then multiply.

Example Richard earns $7.90 per hour and works 38.5 hours each week. How much are his total earnings per week?

Round each factor to its greatest place and multiply.

38.5	40 hours
$7.90	$8 per hour
3950	$320 weekly wages, estimate
6320	
2370	
$304.150	

The best estimate is **$320** which is close to the actual solution of **$304.15.**

Multiplying Decimals by 10, 100, and 1,000

There are shortcuts you can use when multiplying decimals by 10, 100, and 1,000.

To multiply a decimal by 10, move the decimal point **one place to the right.**

Example .26 x 10

.26 x 10 = 2.6 = 2.6

To multiply a decimal by 100, move the decimal point **two places to the right.**

Example 3.7 x 100

3.7 x 100 = 3 70 = 370

To multiply a decimal by 1,000, move the decimal point **three places to the right.**

Example 1.4 x 1,000

1.4 x 1,000 = 1 400 = 1,400

Practice Exercise

Multiply each number by 10, 100, and 1,000.
Hint: You only need to move the decimal points and add 0 if needed.

		× 10	× 100	× 1,000
1.	910.86	**9,108.6**	**91,086**	**910,860**
2.	5.987537	______________	______________	______________
3.	31.83	______________	______________	______________
4.	76.5487	______________	______________	______________
5.	6,690.71	______________	______________	______________
6.	2.606	______________	______________	______________
7.	3.956666	______________	______________	______________
8.	577.35	______________	______________	______________
9.	46.541	______________	______________	______________
10.	626.317	______________	______________	______________
11.	2.163592	______________	______________	______________

12. 94.51 __________ __________ __________
13. 4,817.27 __________ __________ __________
14. 87.16543 __________ __________ __________
15. 751.72 __________ __________ __________
16. 26.95 __________ __________ __________
17. 6,523.16 __________ __________ __________
18. 12 __________ __________ __________
19. 261 __________ __________ __________
20. 39 __________ __________ __________

Solve each problem.

1. $\begin{array}{r} 2.8 \\ \times\ 0.9 \\ \hline \end{array}$ **2.** $\begin{array}{r} 0.4 \\ \times\ 3 \\ \hline \end{array}$ **3.** $\begin{array}{r} 0.4 \\ \times\ 5 \\ \hline \end{array}$ **4.** $\begin{array}{r} 7.3 \\ \times\ 0.5 \\ \hline \end{array}$

5. $\begin{array}{r} 7.7 \\ \times\ 0.88 \\ \hline \end{array}$ **6.** $\begin{array}{r} 0.1 \\ \times\ 0.89 \\ \hline \end{array}$ **7.** $\begin{array}{r} 4.8 \\ \times\ 0.43 \\ \hline \end{array}$ **8.** $\begin{array}{r} 7.7 \\ \times\ 0.92 \\ \hline \end{array}$

9. $\begin{array}{r} 8.1 \\ \times\ 0.62 \\ \hline \end{array}$ **10.** $\begin{array}{r} 7.1 \\ \times\ 0.82 \\ \hline \end{array}$ **11.** $\begin{array}{r} 1.4 \\ \times\ 0.8 \\ \hline \end{array}$ **12.** $\begin{array}{r} 5.8 \\ \times\ 0.9 \\ \hline \end{array}$

13. $\begin{array}{r} 42.7 \\ \times\ 30.08 \\ \hline \end{array}$ **14.** $\begin{array}{r} 25.18 \\ \times\ 44.927 \\ \hline \end{array}$ **15.** $\begin{array}{r} 40.551 \\ \times\ 35.054 \\ \hline \end{array}$ **16.** $\begin{array}{r} 8.97 \\ \times\ 32 \\ \hline \end{array}$

17.	**15.62**	**18.**	**5.5**	**19.**	**32.82**	**20.**	**38.18**
	× 4.006		**× 16.637**		**× 46.06**		**× 16.95**

Division of Decimals

Begin dividing decimals the same way you would divide whole numbers.

If the number in a division box (the dividend) has a decimal, but the number outside of the division box (the divisor) does not have a decimal, place the decimal point in the quotient (the answer) directly above the decimal point in the division box.

$$\begin{array}{r} 0.002 \\ 5\overline{)0.010} \end{array}$$

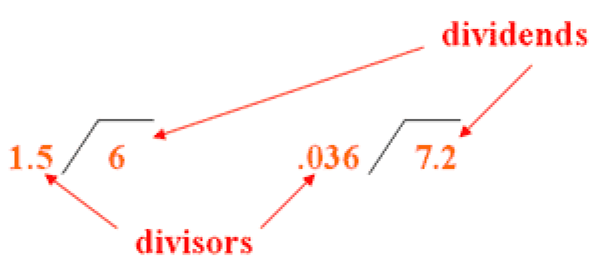
If both the numbers inside and outside of the division box have decimals, count how many places are needed to move the decimal point outside of the division box (the divisor) to make it a whole number. Move the decimal point in the number inside of the division box (the dividend) the same number of places.

Place the decimal point in the quotient (the answer) directly above the new decimal point.

$$0.05\overline{)0.01} = 5\overline{)1} = 5\overset{0.2}{\overline{)1.0}}$$

If the number outside of the division box has a decimal, but the number inside of the division box does not, move the decimal place on the outside number however many places needed to make it a whole number. Then to the right of the number in the division box (a whole number with an "understood decimal" at the end) add as many zeros to match the number of places the decimal was moved on the outside number. Place the decimal point in the quotient directly above the new decimal place in the division box.

$$0.05\overline{)1} = 0.05\overline{)1.00} = 5\overset{20}{\overline{)100}}$$



(Note that 6 = 6.0.)

Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. In such cases, round the divisor to its greatest place, and round the dividend so that it can be divided exactly by the rounded divisor. Then divide.

Example If a plane flew 2,419.2 miles in 6.3 hours, what was its average speed in miles per hour?

Round the divisor to its greatest place, round the dividend so that it can be divided exactly by the rounded divisor, and divide.

6.3	6 hours
2,419.2	2,400 miles

2,400 ÷ 6 = 400 miles per hour, estimate

2,419.2 ÷ 6.3 = 384 miles per hour

The best estimate is **400 miles per hour** which is close to the actual answer of **384 miles per hour.**

Dividing Decimals by 10, 100, and 1,000

There are shortcuts you can use when dividing decimals by 10, 100, and 1,000.

To divide a decimal by 10, move the decimal point **one place to the left.**

Example 7.2 ÷ 10

$$7.2 \div 10 = .7\,2 = .72$$

To divide a decimal by 100, move the decimal point **two places to the left.**

Example 364 ÷ 100

$$364 \div 100 = 3.64 = 3.64$$

To divide a decimal by 1,000, move the decimal point **three places to the left.**

Example 25.3 ÷ 1,000

$$25.3 \div 1{,}000 = .025\,3 = .0253$$

Practice Exercise

Divide each number by 10, 100, and 1,000.
Hint: You only need to move the decimal points and add 0 if needed.

	÷ 10	**÷ 100**	**÷ 1,000**
1. 84,644.8	**8,464.48**	**846.448**	**84.6448**
2. 51.97922	____________	____________	____________

3. 383.8 ________ ________ ________

4. 84.363 ________ ________ ________

5. 7,256.88 ________ ________ ________

6. 103,081.2 ________ ________ ________

7. 79.938 ________ ________ ________

8. 3.802 ________ ________ ________

9. 7,140.067 ________ ________ ________

10. 368.08 ________ ________ ________

11. 3,641.47 ________ ________ ________

12. 1.073 ________ ________ ________

13. 1.776 ________ ________ ________

14. 752.39 ________ ________ ________

15. 94,351.9 ________ ________ ________

16. 78.15737 ________ ________ ________

17. 79,119.9 ________ ________ ________

18. 66 ________ ________ ________

19. 343 ________ ________ ________

20. 40 ________ ________ ________

Solve each problem.

1. $705\overline{)169.905}$	**2.** $71\overline{)637.864}$	**3.** $91\overline{)69.6696}$
4. $0.3\overline{)2745}$	**5.** $6.5\overline{)456885}$	**6.** $9.9\overline{)298287}$
7. $0.67\overline{)369371}$	**8.** $3.53\overline{)228038}$	**9.** $0.45\overline{)112365}$
10. $2.5\overline{)11.745}$	**11.** $7.39\overline{)24.67521}$	**12.** $1.6\overline{)51.68}$
13. $4.96\overline{)4.37968}$	**14.** $5\overline{)7.2}$	**15.** $0.1\overline{)0.293}$

16.	$5.9\overline{)281.784}$	**17.**	$7.9\overline{)367.35}$	**18.**	$0.76\overline{)29.4576}$
19.	$4.2\overline{)37.212}$	**20.**	$1.2\overline{)0.040944}$	**21.**	$6.3\overline{)247.59}$
22.	$3.63\overline{)129.0828}$	**23.**	$5.2\overline{)16.4528}$	**24.**	$0.72\overline{)3.39408}$

Word Problems with Decimals

1. In 1991, 56.8 thousand people lived in Moncton, New Brunswick. By 1996, there were 2.5 thousand more people. How many people lived in Moncton in 1996?

2. Janet bought 2.6 pounds of beef, 1.3 pounds of cheese, 2.45 pounds of chicken, and 5 pounds of sugar. What was the total weight she had to carry?

3. In polishing a piece of pipe that was 2 inches thick, .016 inch of metal was worn away. What was the thickness of the pipe when it was polished?

4. In 1974, the government counted 23.37 million people as being poor. In 1975, the government said there were 25.88 million people who were poor. How many more people did the government count as being poor in 1975 than in 1974?

5. One inch is equal to 2.54 centimeters. How many centimeters are there in 6.5 inches?

6. At $1.80 a meter, how much does 3.75 meters of wood cost?

7. A train traveled 144.9 miles in 4.2 hours. What was its average speed in miles per hour?

8. Mary made $24.70 for 6.5 hours of work. How much did she make each hour?

Percents

Understanding and Comparing Percents

The term ***percent*** means ***parts per hundred***. Any fraction with a denominator of ***100*** can be written as a percentage, using a percent sign, ***%***. So, if you ate ½ of a pie, you ate **50/100** or ***.50*** or ***50%*** of the pie.

If you ate $\frac{1}{8}$ of the pie, you ate 12.5%

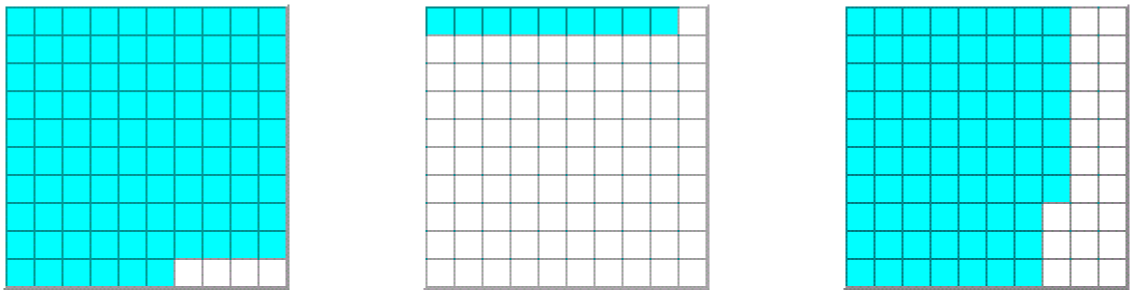
If you ate $\frac{3}{4}$ of the pie, you ate 75%

If you ate $\frac{1}{5}$ of the pie, you ate 20%

Þ If you ate 1 whole pie, you ate 1.00 or 100% of the pie. You ate the whole thing!!!

What fraction of each grid is shaded?

Grid 1 Grid 2 Grid 3



Each grid above has 100 boxes. For each grid, the ratio of the number of shaded boxes to the total number of boxes can be represented as a fraction.

Comparing Shaded Boxes to Total Boxes		
Grid	**Ratio**	**Fraction**
1	96 to 100	$\frac{96}{100}$
2	9 to 100	$\frac{9}{100}$
3	77 to 100	$\frac{77}{100}$

We can represent each of these fractions as a percent using the symbol %.

$\frac{96}{100}$ = 96% $\frac{9}{100}$ = 9% $\frac{77}{100}$ = 77%

Let's look at our comparison table again. This time the table includes percents.

Comparing Shaded Boxes to Total Boxes			
Grid	**Ratio**	**Fraction**	**Percent**
1	96 to 100	$\frac{96}{100}$	96%
2	9 to 100	$\frac{9}{100}$	9%
3	77 to 100	$\frac{77}{100}$	77%

It is easy to convert a fraction to a percent when its denominator is 100. If a fraction does not have a denominator of 100, you can convert it to an equivalent fraction with a denominator of 100, and then write the equivalent fraction as a percent.

Example 1: Write each fraction as a percent: $\frac{1}{2}, \frac{18}{20}, \frac{4}{5}$

Solution		
Fraction	**Equivalent Fraction**	**Percent**
$\frac{1}{2}$	$\frac{1 \times 50}{2 \times 50} = \frac{50}{100}$	50%
$\frac{18}{20}$	$\frac{18 \times 5}{20 \times 5} = \frac{90}{100}$	90%
$\frac{4}{5}$	$\frac{4 \times 20}{5 \times 20} = \frac{80}{100}$	80%

You may also change a fraction to a percentage by dividing the fraction.

$$\frac{2}{5} = 5\overline{)2.00} = .40$$

Then change the decimal to a fraction with ***100*** in the denominator.

$$.40 = \frac{40}{100} = 40\%$$

To change a percentage to a fraction, reverse the process. Be sure to write the fraction in its lowest possible terms.

$$4\% = \frac{4}{100} = \frac{1}{25} \qquad 13\% = \frac{13}{100}$$

Writing Decimals as Percents

Problem: What percent of a dollar is 76 cents?

76 cents = .76

.76 = 76%

Solution: 76 cents is 76% of a dollar.

The solution to the above problem should not be surprising, since both dollars and percents are based on the number 100. As a result, there is nothing complicated about converting a decimal to a percent. **To convert a decimal to a percent, move the decimal point two places to the right.** Look at the example below:

Example 1 Write each decimal as a percent: . 93, .08, .67, .41

Solution	
Decimal	**Percent**
.93	93%
.08	8%
.67	67%
.41	41%

Each of the decimals in Example 1 has two places to the right of the decimal point. However, a decimal can have any number of places to the right of the decimal point. Look at Example 2 and Example 3 below:

Example 2 Write each decimal as a percent: .786, .002, .059, .8719

Solution	
Decimal	**Percent**
.786	78.6%
.002	.2%
0.59	5.9%
.8719	87.19%

Example 3 Write each decimal as a percent: .1958, .007, .05623, .071362

Solution	
Decimal	**Percent**
.1958	19.58%
.007	.7%
.05623	5.623%
.071362	7.1362%

Writing Percents as Decimals

Problem: What is 35 percent of one dollar?

We know from the previous lesson that .35 = 35%. The word "of" means multiply. So we get the following:

35% x $1.00 = .35 x $1.00

.35 x $1.00 = .35 x 1 = .35

Solution: 35% of one dollar is $.35, or 35 cents.

The solution to the above problem should not be surprising, since percents, dollars and cents are all based on the number 100. **To convert a percent to a decimal, move the decimal point two places to the left.** Look at the example below:

Example 1 Write each percent as a decimal: 18%, 7%, 82%, 55%

Solution	
Percent	**Decimal**
18%	.18
7%	.07
82%	.82
55%	.55

In Example 1, note that for 7%, we needed to add in a zero. **To write a percent as a decimal, follow these steps:**

- Drop the percent symbol.
- Move the decimal point two places to the left, adding in zeros as needed.

Why do we move the decimal point 2 places to the left? Remember that percent means parts per hundred, so 18% equals $\frac{18}{100}$. From your knowledge of decimal place value, you know that $\frac{18}{100}$ equals eighteen hundredths (.18) . So 18% must also equal eighteen hundredths (.18). In Example 2 below, we take another look at Example 1, this time including the fractional equivalents.

Example 2 Write each percent as a decimal: 18%, 7%, 82%, 55%

Solution		
Percent	**Fraction**	**Decimal**
18%	$\frac{18}{100}$	.18
7%	$\frac{7}{100}$	.07
82%	$\frac{82}{100}$	.82
55%	$\frac{55}{100}$	.55

Let's look at some more examples of writing percents as decimals.

Example 3 Write each percent as a decimal:
12.5%, 89.19%, 39.2%, 71.935%

Solution	
Percent	**Decimal**
12.5%	.125
89.19%	.8919
39.2%	.392
71.935%	.71935

Þ **To remember which way to move the decimal point when changing from a decimal to a percent or vice versa, think of your alphabet. Think of the decimal as "d" and the percent as "p". To change from a decimal to a percent, move two places up your alphabet. Move two places down your alphabet to go from a percent to a decimal.**

Practice Exercise

Write each fraction as a percent.

1. $\frac{8}{10}$ = **80 %**
2. $\frac{44}{100}$ = ____
3. $\frac{7}{10}$ = ____
4. $\frac{1}{5}$ = ____
5. $\frac{3}{4}$ = ____
6. $\frac{33}{50}$ = ____
7. $\frac{294}{600}$ = ____
8. $\frac{6}{20}$ = ____
9. $\frac{27}{90}$ = ____
10. $\frac{2}{4}$ = ____
11. $\frac{9}{10}$ = ____
12. $\frac{27}{75}$ = ____
13. $\frac{56}{80}$ = ____
14. $\frac{45}{100}$ = ____
15. $\frac{17}{50}$ = ____
16. $\frac{7}{50}$ = ____
17. $\frac{21}{30}$ = ____
18. $\frac{3}{5}$ = ____
19. $\frac{24}{40}$ = ____
20. $\frac{4}{20}$ = ____
21. $\frac{6}{10}$ = ____

Write each decimal as a percent.

1. 0.76 = **76%**
2. 0.26 = ____
3. 0.31 = ____
4. 0.61 = ____
5. 0.63 = ____
6. 0.06 = ____

7.	0.47	=	_____	8.	0.5	=	_____	9.	0.22	=	_____
10.	0.9	=	_____	11.	0.73	=	_____	12.	0.55	=	_____
13.	0.77	=	_____	14.	0.08	=	_____	15.	0.25	=	_____
16.	0.71	=	_____	17.	0.69	=	_____	18.	0.17	=	_____
19.	0.01	=	_____	20.	0.09	=	_____	21.	0.72	=	_____
22.	0.38	=	_____	23.	0.97	=	_____	24.	0.39	=	_____

Express each of the following as a fraction in its simplest form.

1. 5%
2. 42%
3. 15%
4. 16%
5. 78%
6. 8%

Express each of the following as a decimal.

1. 13%
2. 72%
3. 27%
4. 85%

Using Percents

To find a percentage of a number, multiply the number by the percentage written in its decimal fraction form. Find 25% of 12.

.25 x 12 = 3

To find what percentage one number is of another, write the numbers as a fraction. Divide the fraction into its decimal form. Then change the decimal into its percentage form. ***12*** is what percent of ***48***?

$$\mathbf{\frac{12}{48}} \text{ or } 48\,\overline{)\,12.00} = .25 = 25\%$$

To find a number when a percentage of it is known, try this:

Nine is ***25%*** of what number?

$$\mathbf{\frac{25}{100} = \frac{9}{?}}$$

25 x ? = 100 x 9
25 x ? = 900
? = 900 ÷ 25
? = 36

Nine is ***25%*** of 36.

Some people like to use a formula to find the percent of a number, what percent one number is of another, or a number when a percent is given. The formula looks like this:

$$\frac{r}{100} = \frac{P}{W}$$

r **= percent rate**
P **= part of the number**
W **= the whole (entire) number**

So, to solve the problem, nine is ***25%*** of what number, we would follow these steps.

Step 1 Write down the formula.

$$\frac{r}{100} = \frac{P}{W}$$

Step 2 Insert the necessary information in the correct places.

$$\frac{25}{100} = \frac{9}{?}$$

Step 3 Cross multiply.

$$25 \times ? = 9 \times 100$$
$$25 \times ? = 900$$

Step 4 Divide and solve.

$$? = 900 \div 25$$
$$? = 36$$

Therefore, nine is ***25%*** of 36.

Practice Exercise

1. 15 is what % of 60?
2. 27 is what % of 81?
3. 9 is what % of 90?
4. 12 is what % of 72?
5. 8 is what % of 16?
6. 40 is what % of 320?
7. 16 is what % of 20?
8. 14 is what % of 35?
9. 32 is what % of 48?
10. 75 is what % of 90?
11. 33 is what % of 44?
12. 56 is what % of 64?
13. 75% of 68
14. 65% of 80
15. 78% of 100
16. 50% of 90
17. 36% of 62
18. 6% of 68
19. 39% of 76
20. 58% of 41
21. 72% of 79
22. 40% of 50
23. 54% of 48
24. 13% of 85
25. 25% of what number is 8?
26. 50% of what number is 45?
27. 75% of what number is 48?

28. 60% of what number is 75?
29. 40% of what number is 60?
30. 15% of what number is 12?
31. 10% of what number is 6.3?
32. 35% of what number is 8.4?

Simple Interest

Simple interest is the amount obtained by multiplying the principal by the rate by the time; $I = prt$.

The principal is the amount of money borrowed or saved.

Example:

Carol invested $150 at a simple interest rate of 4%. Find the interest she will earn in 1 year.

$I = prt$

$I = 150 \times 4\% \times 1$ $p = \$150$, $r = 4\%$, $t = 1$ year

$I = 150 \times 0.04 \times 1$ *Multiply.*

$I = 6$

So, the interest earned in 1 year is $6.

Watch Out!!! We know that there are 365 days in a year but with interest you calculate with 360 days (a business year).

$$\textbf{30 days} = \frac{30}{360} = \frac{1}{12}$$

$$120 \text{ days} = \frac{120}{360} = \frac{1}{2}$$

1 year = 1 1 ½ years = 1.5 2 ¾ years = 2.75

Example What would the interest be on a 90 day loan of $500.00, if the rate was 15%?

$$15\% = .15$$
$$\frac{90}{360} = \frac{1}{4}$$

$$I = PRT$$
$$I = 500 \times .15 \times \tfrac{1}{4}$$
$$I = \$18.75$$

To find the rate, principal, or time, you may rewrite the interest formula as follows:

$$\text{Rate} = \frac{\text{interest}}{\text{principal x time}}$$
$$\text{Principal} = \frac{\text{interest}}{\text{rate x time}}$$
$$\text{Time} = \frac{\text{interest}}{\text{principal x rate}}$$

To figure out the total amount owed or total payment due to the lender (bank, credit card company, etc.) add the accumulated interest to the original principal.

Example Mr. Jones borrowed $1600.00 for a period of 2 years. He is paying a rate of 12% a year. How much interest will he have to pay? What is the total amount that he will owe to the lender?

12% = .12

I = PRT
I = 1600 x .12 x 2
I = $384.00

Total Amount = P + T
Total Amount = 1600 + 384
Total Amount = $1984

Mr. Jones will have to pay $384 in interest. At the end of the lending period (2 years), he will owe the lender $1984.

Practice Exercise

Complete the following.

	Principal	**rate**	**time**	**interest**
1)	$400.00	7%	1 year	
2)	$800.00		60 days	$7.00
3)	$1550.00	6%		$232.50
4)	$880.00		2 years	$149.60
5)	$525.00	5%	2 years	
6)	$400.00	8%		$16.00

Calculate the interest and total payment assuming this is a loan.

	principal	*rate*	*time*	*interest*	*total payments*
1.	$200	11%	30 days	**$1.83**	**$201.83**
2.	$300	5%	2 years	__________	__________
3.	$200	7%	1 year	__________	__________
4.	$710	5%	1 year	__________	__________
5.	$570	8%	$4\frac{3}{4}$ years	__________	__________
6.	$390	12%	60 days	__________	__________
7.	$380	7%	210 days	__________	__________
8.	$730	9%	1 year	__________	__________
9.	$1,880	9%	1 year	__________	__________
10.	$680	16.2%	150 days	__________	__________
11.	$2,040	9%	1 year	__________	__________
12.	$1,675	7.4%	51 years	__________	__________
13.	$1,920	11.34%	120 days	__________	__________

14.	$14,410	13.11%	$1\frac{3}{4}$ years	__________	__________
15.	$17,020	5%	1 year	__________	__________

Word Problems with Percent

Identifying the Parts of and Solving a Percent Word Problem

Read the statement below:

The 8-ounce glass is 50% full. It contains 4 ounces.

This statement contains three facts:

the whole: the 8-ounce glass
the part: 4 ounces
the percent: 50%

A percent word problem would be missing one of these facts. When you are solving a percent word problem, the first step is to identify what you are looking for. As shown above, you have three possible choices: *the part*, *the whole*, or *the percent*.

It is usually easiest to figure out that you are being asked to find the percent. Word problems asking for the percent usually ask for it directly, with a question such as “What is the percent?” or “Find the percent” or “3 is what percent?” Occasionally, other percent-type words are used, such as “What is the *interest rate*?”

Example 114 city employees were absent yesterday. This was 4% of the city work force. How many people work for the city?

Step 1: *question:* How many people work for the city?
Step 2: *necessary information:* 114 city employees, 4%
Step 3: You are given the number of city employees who were absent (114) and the percent of the work force that this represents (4%). You are looking for the total number of people who work for the city, the whole.

Once you identify what you are looking for in a percent word problem, set up the problem and solve it.

Percent word problems can be set up in the following form:

$$\frac{P}{W} = \frac{r}{100}$$

$$\frac{114}{?} = \frac{4}{100}$$

$$114 \times 100 = 4 \times ?$$

$$11400 = 4 \times ?$$

$$11400 \div 4 = ?$$

$$2850 = ?$$

2850 people work for the city.

Practice Exercise

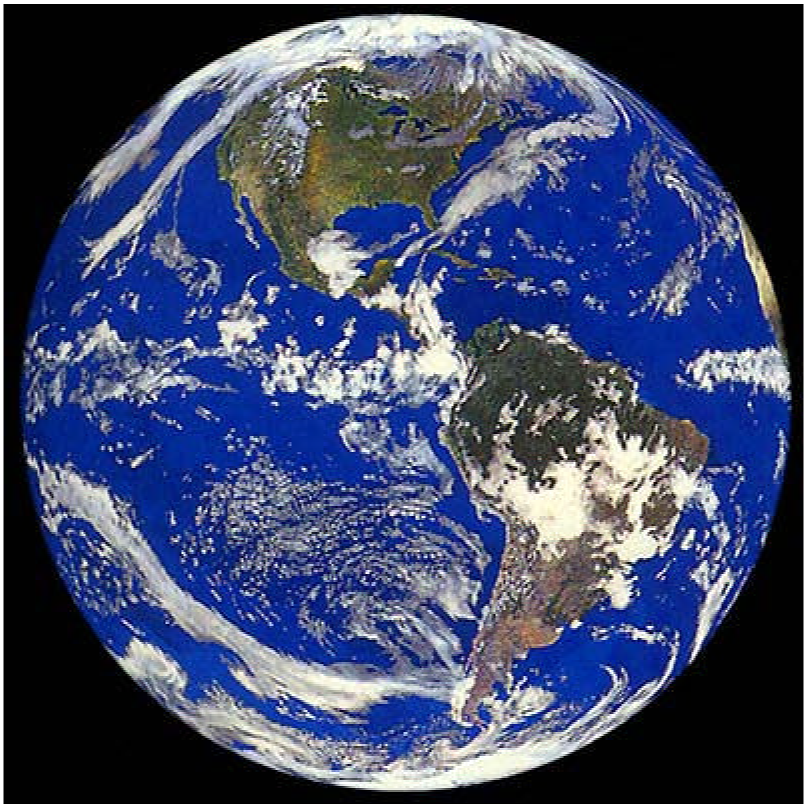
Solve for each of the given problems.

1.	A survey of 1,760 people was done by a newspaper. 40% of people did not know the name of their representative in Parliament. How many of the 1,760 people knew the answer?
2.	A class of thirty voted for class president. 30% voted for Brad and 70% voted for Amy. How many votes did the winner receive?
3.	Jane is in a class of 15 boys and 10 girls. 28% of the students in the class take the bus to school. How many students do not take the bus to school?
4	The movie theater has 250 seats. 150 seats were sold for the current showing. What percentage of seats are empty?
5.	3 out of 4 dentists recommend a fluoride toothpaste. What percent of all dentists recommend a fluoride toothpaste?
6.	112,492 voted for mayor in the city. This was 40% of the registered voters. How many registered voters are there in the city?
7.	Jane purchased a house for $82,000. To pay for the house, Jane took out a 30 year mortgage and pays the bank a yearly interest fee of 8.9%. In eight years, how

	much in interest fees was paid to the bank?
8.	Brad deposited $40,000 at a bank that pays 11% interest. Amy deposited $32,000 at a bank that pays 14% interest. Who will receive more interest in a year, and by how much more?

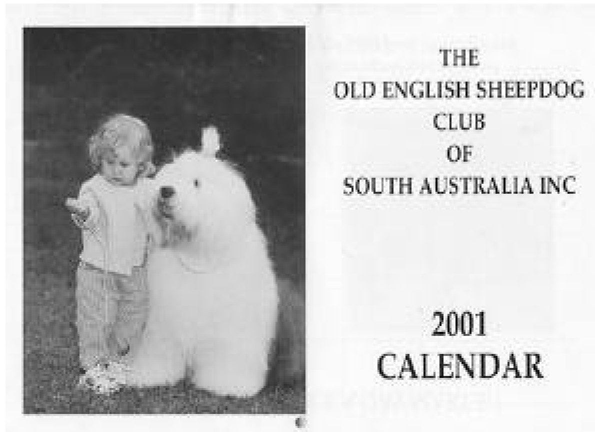
Measurement

Time



A ***day*** is the time it takes earth to spin around once on its *axis*, or twenty-four hours. (The axis is an imaginary pole that runs through the middle of the planet from the North Pole to the South Pole.) Seven days make up one *week*. Twenty-eight to thirty-one days make up one ***month***. A month is the approximate time needed for the moon to revolve once around earth. The lunar month actually takes twenty-nine days, twelve hours, forty-four minutes, and three seconds.

Twelve months make up one ***year***. A year is the time it takes earth to revolve once around the sun, or 365 days, five hours, forty-eight minutes, and forty-six seconds.

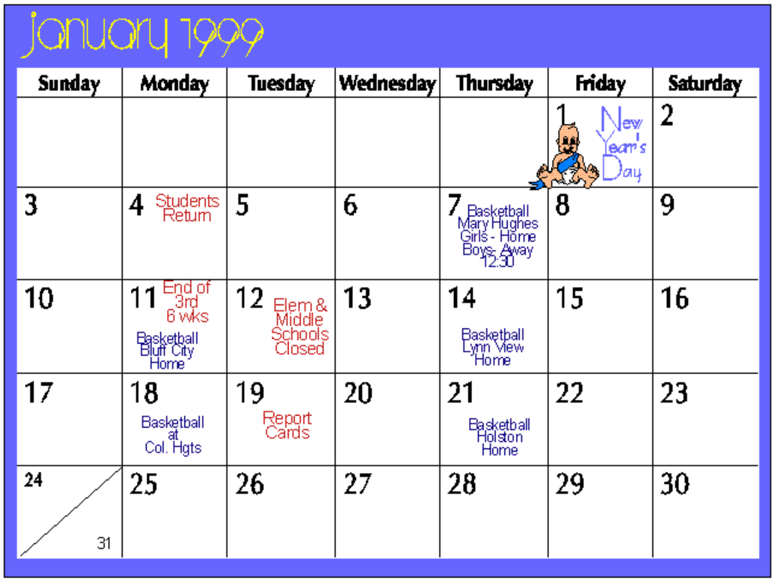


Calendars are tools that help us group days into weeks, months, and years. The calendar used throughout the world today is called the ***Gregorian*** calendar.

The astronomer Sosigenes was asked by Julius Caesar to created a calendar for the Roman Empire. The calendar was based on the solar year of 365 days. The year was divided into twelve months. Each month lasted thirty or thirty-one days, with the exception of February, which lasted either twenty-eight or twenty-nine days. The Julian calendar is the basis for the Gregorian calendar that was introduced by Pope Gregory VIII in 1582. The names used for the months in the Roman calendar were used in the Julian calendar. These names are also used today.

Roman	**Gregorian**	**Roman**	**Gregorian**
Januarius	**January**	**Quintilis**	**July**
Februarius	**February**	**Sextilis**	**August**
Martius	**March**	**September**	**September**

Aprilis	**April**	**October**	**October**
Maius	**May**	**November**	**November**
Junius	**June**	**December**	**December**



The names we use for weekdays come from the Saxons of England. The Saxons named the days for the planets and their gods.

SUN'S day ....Sunday
MOON'S day ....Monday
TIW'S day ....Tuesday
WODEN'S day ....Wednesday
THOR'S day................Thursday
FRIGG'S day ....Friday
SATURN'S day ....Saturday

Sosigenes made a mistake in the Julian calendar, but nobody found the mistake for hundreds of years. He made every fourth year a leap year, but these leap years made the calendar too long to measure the cycle of the sun. By the 1500s, the Julian calendar was almost two weeks ahead of the actual solar year.

Pope Gregory VIII fixed the mistake in 1582. Leap years were now added to the calendar every four years except for the years that begin new centuries, unless the number of the new century can be divided evenly by 400.

The century date 1900 was not a leap year (1900 ¸ 400 = 4 3/4), but the year 2000 was a leap year (2000 ¸ 400 = 5).

Pope Gregory VIII's calendar is accurate to within sixteen seconds per year. That's the reason we still use it today.

⇒ **Remember: 30 days has September**
April, June, and November,
All the rest have 31,
Except February which has 28 days clear
And 29 each leap year.

Numeric Dating

Numeric dating is the way of recording the date with 8 digits.
year: the last two digits 1977 = 77
month: 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 11, 12
day: number of the day

The three styles are: d/m/y 01/11/95

y/m/d	95/11/01
m/d/y	11/01/95

These are all different ways of writing November 1, 1995.

Numeric dating is usually used when filling in forms.

Remember all the concerns that we had around the year 2000? This was all due to the fact that we were using numeric dating. As the year 2000 was approaching, we had a problem with computers that were reading only the last 2 digits of the year. If the computers were not 2000 compatible, they were reading 2001 as 1901 or 2021 as 1921.

Practice Exercise

Express the following with numeric dating using m/d/y.

1) January 17, 2002
2) July 2, 1994
3) May 14, 1965
4) September 17, 1889
5) Today's date

Schedule

A table that lists activities and the times they happen

Example:

FLIGHTS FROM MIAMI TO NEW YORK CITY	
Each flight lasts about 2 hours and 45 minutes.	
Airline	**Departure Time**
Airline A	9:10 A.M.
Airline B	10:15 A.M.
Airline C	12:50 P.M.
Airline D	1:20 P.M.

We divide ***days*** into 24 ***hours***, but hours are divided into ***60*** parts. Roman astronomers called each division a ***par minuta*** or "small part of an hour." From the Latin name comes our word ***minute***. These early astronomers also divided minutes into *60* equal parts. They called each division ***par seconda***, or ***second***.

Measures of Time

60 seconds (sec) = 1 minute (min)
60 minutes = 1 hour (hr)
24 hours = 1 day
7 days = 1 week (wk)
12 months (mo), or 52 weeks, or 365 days = 1 year (yr)
366 days = 1 leap year

Practice Exercise

Fill in the answer

1. 52 mins = ________ secs
2. 51 secs 57 mins = _______ secs
3. 9 mins = ________ secs
4. 56 secs 32 mins = _______ secs
5. 51 mins = ________ secs
6. 26 mins 57 secs = _______ secs
7. 2,700 secs = ________ mins
8. 2,580 secs 14 hours = _______ mins
9. 2,520 secs = ________ mins
10. 6 hours 1,560 secs = ________ mins
11. 2 days = ________ hours
12. 1,920 mins 43,200 secs = ________ hours
13. 840 mins = ________ hours
14. 5 hours 1,200 mins = ________ hours
15. 17,280 mins = _______ days
16. 953 days 5,760 mins = ________ days
17. 15,840 mins = _______ days
18. 264 hours 15,840 mins = ________ days
19. 1,531 mins = _______ hours ________ mins
20. 2,250 mins 3,600 secs = ________ hours ________ mins
21. 765 mins = ________ hours ________ mins
22. 2,112 mins 10 days = ________ hours ________ mins

Adding and Subtracting Measurements of Time

When you add or subtract time measurements, you may have to carry or borrow between the different units. Here are two examples.

Example Barb worked 6 hours and 50 minutes on Tuesday and 5 hours 30 minutes on Thursday. How much total time did she work on the two days?

Step 1 Line up the measurements, putting like units under like units.

$$\begin{array}{r} \textbf{6 hr 50 min} \\ \underline{+\ \textbf{5 hr 30 min}} \end{array}$$

Step 2 Add the minutes and add the hours.

$$\begin{array}{r} \textbf{6 hr 50 min} \\ \underline{+\ \textbf{5 hr 30 min}} \\ \textbf{11 hr 80 min} \end{array}$$

Step 3 80 minutes is 1 hour and 20 minutes. Rewrite the sum.

11 hr + 80 min = 11 hr + 1 hr + 20 min
= 12 hr + 20 min

Answer: Barb worked 12 hours and 20 minutes on the two days.

Example Subtract 6 minutes 45 seconds from 12 minutes 20 seconds.

Step 1 Line up the measurements, putting the like units under like units.

12 min 20 sec
- 6 min 45 sec

Step 2 One minute is the same as 60 seconds. Rewrite the top number as 11 minutes + 60 seconds + 20 seconds, or 11 minutes 80 seconds.

11 min 80 sec
- 6 min 45 sec

Step 3 Subtract the seconds and subtract the minutes.

11 min 80 sec
- 6 min 45 sec
5 min 35 sec

Answer: The result of the subtraction is 5 minutes 35 seconds.

Practice Exercise

Fill in the answer.
(Hint: remember to reduce to lowest terms. 2 hr 69 min = 3 hr 9 min)
seconds (sec) - minutes (min) - hours - days

1. 17hours 59mins +8hours 28mins	**2.** 3mins 20secs -3mins 17secs	**3.** 10hours 32mins +13hours 55mins
4. 11hours 41mins -3hours 56mins	**5.** 3mins 20secs +3mins 17secs	**6.** 9 mins 39 secs - 2mins 13 secs

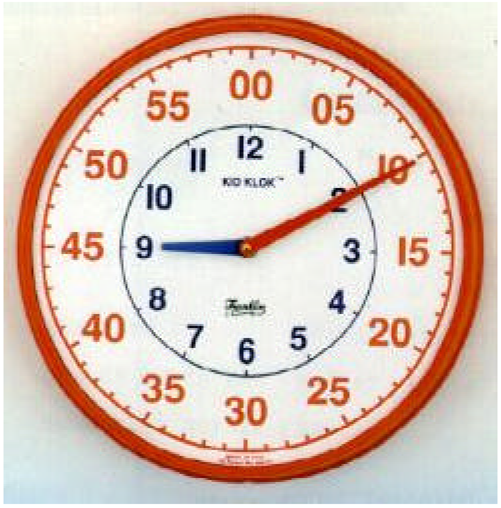
Standard time means the measurement of the day in two blocks of twelve hours each. The twelve hours from midnight to just before noon are ***a.m.*** hours. The twelve hours from noon until just before midnight are ***p.m.*** hours. The abbreviations "a.m." and "p.m." come from the Latin for ***ante meridiem*** and ***post meridiem***, meaning ***before*** (ante) and ***after*** (post) midday or noon (***meridiem***).

Today many clocks and watches use the battery-powered vibrations of a quartz crystal to keep time. The natural vibration of a quartz crystal is 100,000 times per second. Modern clocks and watches show the time in digital as well as analog displays.

Digital

Analog

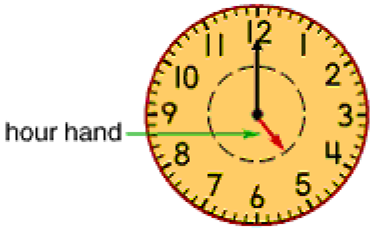
How to Tell Time



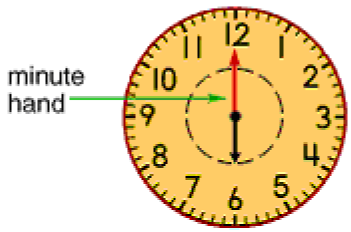
This clock demonstrates how minutes are to be read on an analog clock face. We know that there are 60 minutes in one hour, so the minute hand indicates the number of minutes that we are to read. In the picture on page 113, the minute hand (the longer red hand) is pointing at the 2 which stands for 10 minutes. The hour hand (the shorter blue hand) is pointing at the 9. We can read the time as "10 minutes after 9", "10 minutes past 9", or "9:10". You could even say that it is "50 minutes before 10", because it will take another 50 minutes before the hour hand points at the 10.

To figure out the minutes on a clock face, you must skip count by fives. For example, the 1 represents 5 minutes, the 2 represents 10 minutes, the 3 represents 15 minutes…and so on. Each mark between the large numbers represents 1 minute.

Hour Hand



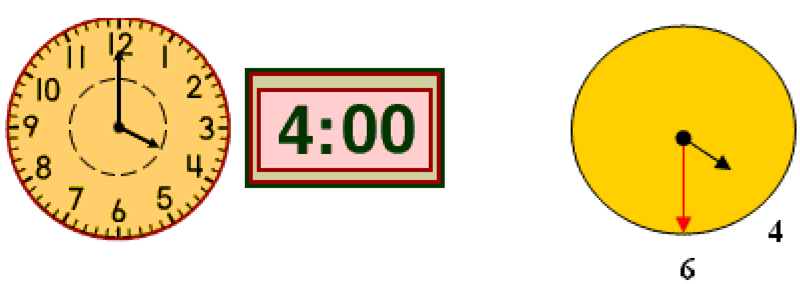
Minute Hand



O'clock

The clock shows 1 **o'clock**

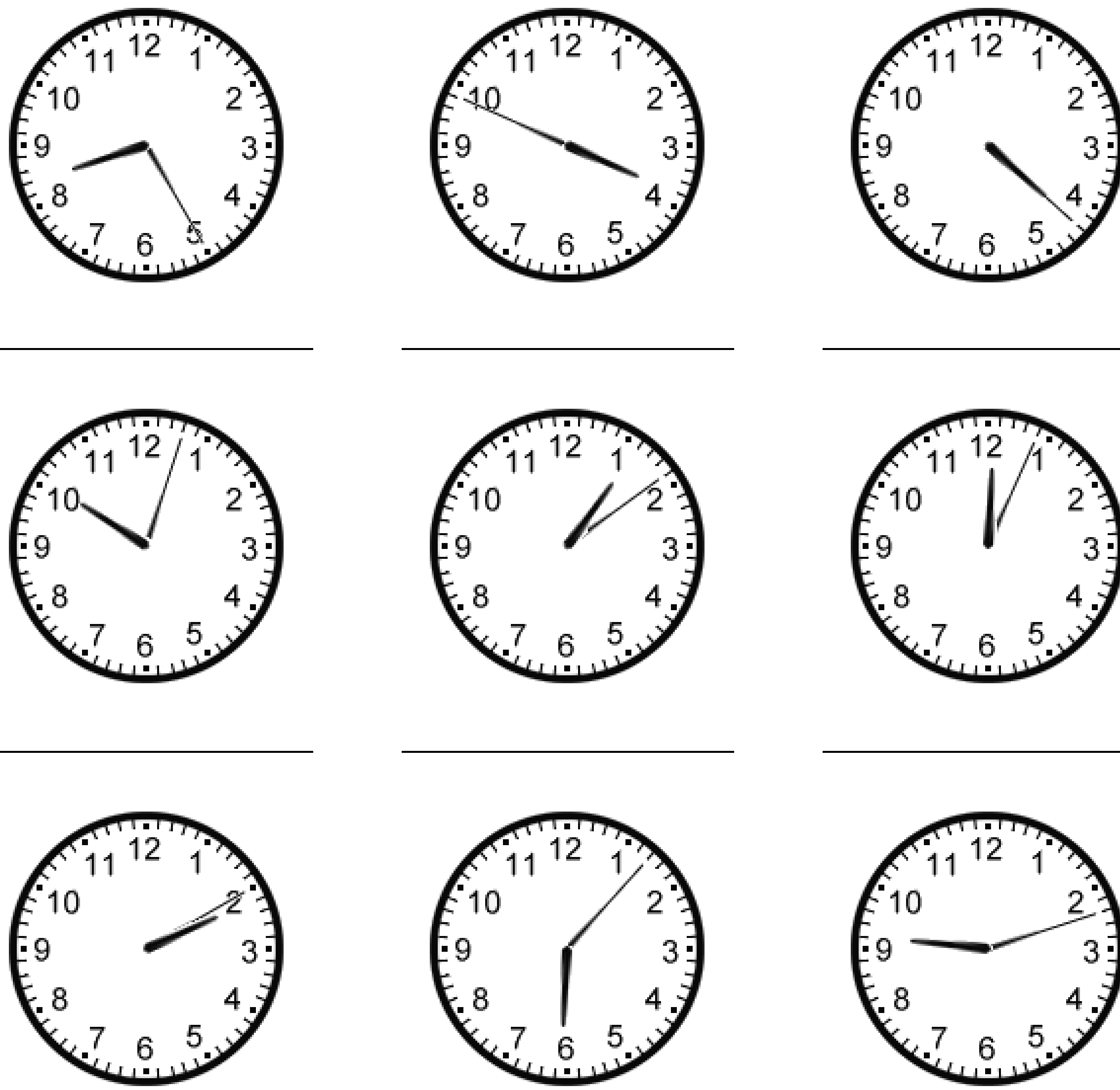
Half Hour



A **half hour** is 30 minutes, so when the **minute hand** reaches the six and the hour hand remains on four, the new time will be **4:30.**

Practice Exercise

What time is it?



Digital time is read from left to right. The first number stands for hours and the second number, after the colon, stands for minutes.

The clock above reads “10:20”. That means 10 hours and 20 minutes. You will also notice that the numbers are proceeded by the letters “P.M.” which tells us that this clock is reading “10:20 in the evening”, “20 minutes after 10”, “20 minutes past 10”, “40 minutes before 11”, or “40 minutes to 1”.

Military Time

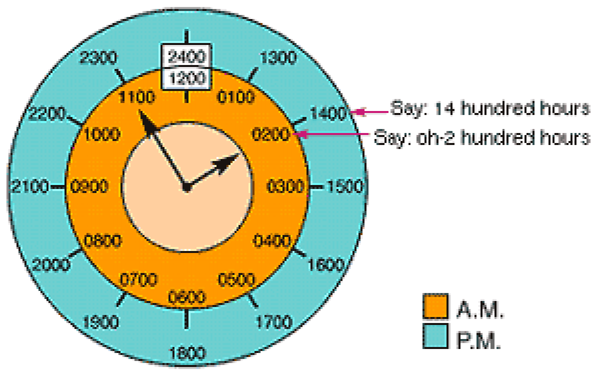
Standard time can be confusing. For example, eight o’clock can mean eight in the morning or eight in the evening. To avoid confusion, scientists created a 24-hour clock. The hours are numbered ***1*** through ***24***, beginning at midnight. This way of counting the hours in a day is called ***military time***. People who use military time say the time in a special way. For example, 11:00 is not called “eleven o’clock,” but “eleven hundred hours.”

Standard Time	24-Hour Time	Military Time
12:01 midnight	00:00	0001 hours
1:00 am	01:00	0100 hours
2:00 am	02:00	0200 hours
3:00 am	03:00	0300 hours
4:00 am	04:00	0400 hours
5:00 am	05:00	0500 hours
6:00 am	06:00	0600 hours
7:00 am	07:00	0700 hours
8:00 am	08:00	0800 hours
9:00 am	09:00	0900 hours
10:00 am	10:00	1000 hours
11:00 am	11:00	1100 hours
12:00 noon	12:00	1200 hours
1:00 pm	13:00	1300 hours
2:00 pm	14:00	1400 hours
3:00 pm	15:00	1500 hours
4:00 pm	16:00	1600 hours
5:00 pm	17:00	1700 hours
6:00 pm	18:00	1800 hours
7:00 pm	19:00	1900 hours
8:00 pm	20:00	2000 hours
9:00 pm	21:00	2100 hours
10:00 pm	22:00	2200 hours
11:00 pm	23:00	2300 hours
12:00 midnight	24:00	2400 hours

24-Hour Clock

A clock that does not use A.M. or P.M.

Example:



Money

The word ***dollar*** comes from the German word for a large silver coin, the ***Thaler***. In 1781, ***cent*** was suggested as a name for the smallest division of the dollar. Thomas Jefferson, third President of the United States and an amateur scientist, thought that the dollar should be divided into 100 parts. The word ***cent*** comes from the Latin ***centum***, which means one hundred.

Canadian currency was first proposed in 1850, but the first coins were not released for circulation until December 12, 1858.

1 penny = 1 cent (¢)

1 nickel = 5 cents

1 dime = 10 cents
1 quarter = 25 cents
1 dollar ($) = 100 cents

Penny (Cent)

Nickel

Dime

Quarter

Dollar (Loonie)

Toonie

Canadian money is created in decimal-based currency. That means we can add, subtract, divide, and multiply money the same way we do any decimal numbers.

The basic unit of Canadian currency is the "loonie" or dollar. The dollar has the value of one on a place value chart. The decimal point separates dollars from cents, which are counted as tenths and hundredths in a place value chart.

$1.11	**ones = dollars**	**.**	**tenths = dimes**	**hundredths = pennies**
one cent				**1**
ten cents		**.**	**1**	**0**
one dollar	**1**	**.**	**0**	**0**

$4.63	**ones = dollars**	**.**	**tenths = dimes**	**hundredths = pennies**
three cents				**3**
sixty cents		**.**	**6**	**0**
four dollars	**4**	**.**	**0**	**0**

$1.11 = $1.00 + 10Ë + 1Ë is read as 1 dollar and 11 cents
$4.63 = $4.00 + 60Ë + 3Ë is read as 4 dollars and 63 cents

When you write down amounts of money using the dollar sign, ***$***, you write the amounts the same way as you write decimal numbers—in decimal notation. There is a separate cents sign, ***Ë***. The cents sign does not use decimal notation. So if you have to add cents to dollars, you have to change cents to dollar notation.

8Ë = $.08

36Ë = $.36

100Ë = $1.00

Practice Exercise

Fill in the blank.

1. 8 quarters equals ___________________ cents.
2. 5 dimes, 5 quarters equals __________________ cents.
3. **457** cents equals __________ dimes, __________ pennies, __________ quarter, __________ dollars.
4. **221** cents equals __________ nickels, __________ dollars, __________ penny.
5. 5 quarters, 1 nickel, 2 pennies, 5 dollars, 5 dimes

equals __________________ cents.

6. 3 dollars equals __________________ cents.

7. **186** cents equals __________ dollar, __________ penny, __________ nickels, __________ dimes, __________ quarters.

8. 4 dimes, 4 dollars, 1 penny equals ______________ cents.

9. 7 quarters equals __________________ cents.

10. 9 pennies equals __________________ cents.

11. 3 dimes, 4 nickels, 2 pennies equals _____________ cents.

12. **43** cents equals __________ pennies, __________ quarter, __________ nickels.

13. 7 dollars equals __________________ cents.

14. **68** cents equals __________ pennies, __________ nickels, __________ dimes.

15. 1 dime, 1 penny, 1 dollar, 2 nickels equals __________________ cents.

16. **275** cents equals __________ dollars, __________ quarters.

17. 5 dimes, 4 dollars, 4 quarters, 1 penny equals __________________ cents.

18. 6 pennies equals __________________ cents.

19. 3 quarters, 1 nickel, 4 dimes equals ____________ cents.

20. **181** cents equals _________ dollar, _________ quarter, _________ penny, _________ nickel, _________ dimes.

21. 2 quarters equals _________________ cents.

22. 2 dollars, 3 nickels, 2 pennies, 4 quarters equals _________________ cents.

Complete the following.
Remember to include a $ in your answer.

1. $80.99 × 4	**2.** $687.43 +501.96	**3.** $658.74 -176.31	**4.** $454.35 -263.51
5. $973.94 -851.67	**6.** $165.01 -154.54	**7.** $55.99 +26.81	**8.** 4 ⟌ $52
9. 7 ⟌ $9.03	**10.** 10 ⟌ $89.30	**11.** $427.79 +117.45	**12.** $92.69 × 3

13. $44.44 +87.67	**14.** $134.02 -121.76	**15.** $73.42 × 8	**16.** $553.98 +368.18
17. $30.68 +94.91	**18.** $89.92 +16.40	**19.** $39.24 -36.22	**20.** $575.17 +515.15
21. $19.17 -14.49	**22.** $543.43 -533.37	**23.** 4 ⟌ $25.60	

Unit Pricing

Family members are consumers as well as workers. They spend a considerable amount of money to purchase food and other items that they need or desire. To obtain the maximum value for their money it is important to shop wisely. One way to stretch a dollar in the supermarket is to compare *unit prices* of items. A unit price is the amount charged for a single unit of measure such as one ounce or one pound. The unit price of an item is frequently printed on a price label along with the total cost of the item. If two items are of the same quality, it is worthwhile to buy the item that is a cent or two less per unit. Small savings repeated many times add up to big savings. The following formula may be used to compute the unit price of an item:

Unit Price = (Price of Item) ÷ (Weight of Item)

Example 1: If a ten pound bag of potatoes costs $1.25, what is the price per pound of the potatoes?

Solution: Price per pound $1.25 ÷ 10 = $.125

The unit price is approximately 13 cents per lb.

Example 2: Is it better to buy a 2 pound jar of jelly for $1.18 or a 3 pound jar of the same jelly for $1.68?

Solution:

$1.18 ÷ 2 = $.59 per pound

$1.68 ÷ 3 = $.56 per pound

The 3 pound jar for $1.68 is the better buy.

Practice Exercise

Find the unit price for each item.

1) 12 oranges $6.00
2) 1 dozen eggs $2.00
3) 5 pencils $1.25
4) 2 books $7.40
6) 2 bags of flour $12.50
7) 2 kg. of potatoes $1.99
8) 7 bubble gum 50¢
9) 2 l of milk $5.40

5) 3 cans of peas $1.30 10) 6 socks $8.40

What is GST/HST?

GST is a 7% tax on the sale of most goods and services in Canada. Three participating provinces- Nova Scotia, New Brunswick, and Newfoundland- harmonized their provincial sales tax with GST to create the harmonized sales tax (HST). HST applies to the same base of goods and services as GST but at a rate of 15%. Of this, 7% is the federal portion and 8% is the provincial portion.

When you calculate tax, you are working with percent and decimals. First you find out how much something is and then multiply by the percent. Almost every time you buy something you will have to include the tax into the total cost.

Example You buy a coat for $120.00 with a tax of 15%. What would the total cost be?

15% = .15

$120.00 x .15 = $18.00 is the tax

$120.00 + $18.00 = $138.00 is the total

Discount

The amount by which the original price is reduced is called a discount and is usually received in a sale.

Example:

To find the discount, multiply the cost of the item by the rate of discount. Subtract the discount from the original price to find the new cost.

Example There is a discount of 35% on a $130.00 dress. What is the discount or saving to you?

35% = .35

$130.00 x .35 = $45.50

The new cost is $130.00 - $45.50 = $84.50

Practice Exercise

Store Discounts and Taxes

Sparky's Electronics Price List

DVD Player	$90	Digital Camera	$303
VCR	$97	101-disc CD Changer	$168
13 inch television	$70	50 inch television	$825
Laptop Computer	$2,088	Portable CD Player	$116
2-Way Radio	$41	Cordless Phone	$39
Answering Machine	$93	Wireless Phone	$85

Using the price list, calculate each question to the nearest cent.

1.	8% sales tax on one 50 inch television. What is the sales tax?
2.	You want to buy the Answering Machine and also the 101-disc CD Changer. If the sales tax is 6.5%, what is your after-tax total?
3.	28% discount on one 2-Way Radio. Sales tax is 9%. How much is the after-tax total?

4.	4% sales tax on one Wireless Phone. What is the sales tax?
5.	You ordered two 13 inch televisions on-line. Sparky's offers a 15% discount off the price of the 13 inch television. You pay no tax, but the total shipping charge for the order is $5.49. What is the total to pay?
6.	4.4% sales tax on one Laptop Computer. What is the sales tax?
7.	10% discount on one Digital Camera. What is the discount?
8.	You want to buy the VCR and also the 50 inch television. If the sales tax is 8.5%, what is your after-tax total?
9.	9.5% sales tax on one Cordless Phone. What is the sales tax?
10.	70% discount on one Portable CD Player. What is the discount?

What is Credit?

Credit is *money* that you borrow. You use the money now. You promise to pay back the money later.

Credit is also *time.* You get time to pay for goods or services. You use the goods or services now. You promise to pay for them later.

"Buy now, pay later" is the common saying.

Credit is good for an emergency, and you might save money if you buy when the price is right.

Most credit is not free. Say you borrow some money. In some cases, you must pay back more money than you borrowed. The extra money you pay back is called interest.

Credit ties up future income. It is a promise to pay. Don't buy on credit unless you know you *can* pay. Pay off your debt as soon as you can.

Resist the temptation to buy too much or to pay more than you should.

Remember that if you can't pay, you may lose goods or income.

Developing a Budget

In order to be able to spend money wisely, an individual or family should devise a spending plan. Some people appear to be afraid of the idea of planning how to spend their money. They may fear that such planning will prevent them from using their money as they wish. Having a spending plan is a way of using available money to your goals. The spending plan should agree with the actual income. If the plan does not agree an adjustment should be made.

A realistic spending plan for managing an income will begin with a list of available resources. The list should include the sources of income, the amount of money from each source, and the times when each amount can be expected.

A spending plan is known as a budget and a budget should give a clear picture of where you stand financially. The basic budget is a four-point plan for spending.

1. Spending for comfortable daily living. This includes having enough money on hand to pay for basic items that keep you going from day to day.
2. Spending for major purchases.
   Major purchases includes household appliances, a house, car, or special vacation.
3. Spending for financial security.
   Savings accounts, insurance, and investment are a form of spending and one of the most

rewarding. You are buying peace of mind, and the ability to borrow money inexpensively. It is getting extra value for every dollar spent. You can receive interest from the bank, insurance companies and corporations for placing your dollars with them.

4. Splurge spending.

There is an occasional “throw caution to the winds” buy in each of us. With splurge spending you can dine at a superb restaurant; go to an unplanned baseball game. Keep splurge spending in proportion to the overall budget.

There is a checklist to begin a budget.

1. Open a checking account.
2. Start a savings account.
3. Total net income (income after taxes)
5. List all expenses (those that are constant, those that can change).

If the money going out does not match or is less than what is coming in, then you must cut some of your expenses so that you can ***balance*** your budget. A balance is achieved when your income matches your expenses. The goal is to achieve a ***surplus*** where you will have more money coming in than going out. You never know when you might need some extra money for an emergency.

Charts and Graphs

Charts/Tables

Charts or tables use lines or columns of numbers or words to provide information. The information could relate to distances on a map, weight or height charts, nutritional value information on a box of food, etc.

Example Nutritive value chart

The nutrition information is given for the consumer who wants to see:

1) what is in the product
2) how it compares to another brand

	BRAND A	BRAND B
per 30 g serving		
Energy	75 cal	115 cal
Protein	3.6 g	3.1 g
Fat	1.0 g	2.2 g
Carbohydrate	23 g	23 g
Sugar	5.5 g	5.0 g
Starch	7.2 g	15 g
Fibre	10 g	2.9 g
Sodium	300 mg	145 mg
Potassium	350 mg	120 mg

From the example above of the two boxes of cereal, you could answer the following questions:

1) Which brand has more calories per serving? (Brand B)
2) Which brand provides the higher fibre content? (Brand A)
3) If you were on a low sodium diet, which brand would be better for you? (Brand B)
4) Which brand has the higher starch? (Brand B)
5) Which brand has the higher potassium? (Brand A)

Example Height/Weight Chart

The information given on height/weight charts is for the person who may have health concerns or concerns about body image.

The Metropolitan Life Insurance Company revised their height/weight charts in 1983. New statistics showed that despite a few more pounds, their subscribers were not dying earlier. The new charts reflect the weights of people that lived the longest in each height category; they are not ideal weight tables. Some health educators recommend lower weight ranges than those specified here. But, if you have chased a few pounds for years that put you over "ideal body weight", relax. The new charts show that those few pounds will probably not increase your mortality.

Frame Size

If you have always wondered what size frame you are, here is the method the insurance company used. This will be easier with the help of a friend.

1. Extend your arm in front of your body bending your elbow at a ninety degree angle to your body. (your forearm is parallel to your body).
2. Keep your fingers straight and turn the inside of your wrist to your body.
3. Place your thumb and index finger on the two prominent bones on either side of your elbow, measure the distance between the bones with a tape measure or calipers.

Compare to the medium-framed chart below. Select your height based on what you are barefoot. If you are below the listed inches, your frame is small. If you are above, your frame is large.

ELBOW MEASUREMENTS FOR MEDIUM FRAME			
Height in 1" heels	Elbow	Height in 1" heels	Elbow
Men	Breadth	**Women**	Breadth
5'2"-5'3"	2 1/2"-2 7/8"	4'10"-4'11"	2 1/4"-2 1/2"
5'4"-5'7"	2 5/8"-2 7/8"	5'0"-5'3"	2 1/4"-2 1/2"
5'8"-5'11"	2 3/4"-3"	5'4"-5'7"	2 3/8"-2 5/8"
6'0"-6'3"	2 3/4"-3 1/8"	5'8"-5'11"	2 3/8"-2 5/8"
6'4"	2 7/8"-3 1/4"	6'0"	2 1/2"-2 3/4"

HEIGHT & WEIGHT TABLE FOR WOMEN

Weights at ages 25-59 based on lowest mortality.
Weight in pounds according to frame
(in indoor clothing weighing 3 lbs.; shoes with 1" heels).

Height Feet Inches	Small Frame	Medium Frame	Large Frame
4' 10"	102-111	109-121	118-131
4' 11"	103-113	111-123	120-134
5' 0"	104-115	113-126	122-137
5' 1"	106-118	115-129	125-140
5' 2"	108-121	118-132	128-143
5' 3"	111-124	121-135	131-147
5' 4"	114-127	124-138	134-151
5' 5"	117-130	127-141	137-155
5' 6"	120-133	130-144	140-159
5' 7"	123-136	133-147	143-163
5' 8"	126-139	136-150	146-167
5' 9"	129-142	139-153	149-170
5' 10"	132-145	142-156	152-173
5' 11"	135-148	145-159	155-176
6' 0"	138-151	148-162	158-179

HEIGHT & WEIGHT TABLE FOR MEN

Weights at ages 25-59 based on lowest mortality.
Weight in pounds according to frame
(in indoor clothing weighing 5 lbs.; shoes with 1" heels).

Height Feet Inches	Small Frame	Medium Frame	Large Frame
5' 2"	128-134	131-141	138-150
5' 3"	130-136	133-143	140-153
5" 4"	132-138	135-145	142-156
5' 5"	134-140	137-148	144-160
5' 6"	136-142	139-151	146-164
5' 7"	138-145	142-154	149-168
5' 8"	140-148	145-157	152-172
5' 9"	142-151	148-160	155-176
5' 10"	144-154	151-163	158-180
5' 11"	146-157	154-166	161-184
6' 0"	149-160	157-170	164-188
6' 1"	152-164	160-174	168-192
6' 2"	155-168	164-178	172-197

6' 3"	158-172	167-182	176-202
6' 4"	162-176	171-187	181-207

Example Distance tables (map)

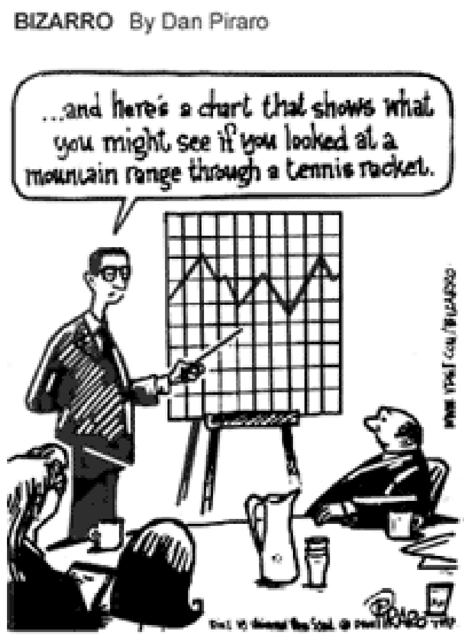
The information contained on these distance charts or tables is ideal for a tourist who is trying to plan a day-trip and is not familiar with the area. People who do a lot of traveling for business or pleasure find this type of information quite useful.

All distances on this map and along the New Brunswick highways are stated in kilometres.
To convert to miles multiply by 5/8. (Example 50x5=250 ÷8=31.2miles)
The chart below gives approximate distances between principal communities in both kilometres and miles.
In all cases the distance given is of the shortest route between the two points.

Kilometres ☐ Miles ☐	Bathurst		Campbellton		Edmundston		Fredericton		Miramichi		Moncton		Sackville		Saint John		Woodstock	
Bathurst			114	71	300	186	252	157	79	49	222	138	249	155	355	221	353	219
Campbellton	114	71			201	125	362	225	190	118	332	206	359	223	466	290	295	183
Caraquet	66	41	180	112	380	236	290	180	118	73	260	162	288	179	413	257	390	242
Charlo	85	53	24	15	224	139	338	201	166	103	307	191	335	208	417	261	321	199
Chatham	72	45	182	113	275	170	182	113	8	5	149	93	177	110	285	177	283	176
Dalhousie	94	58	23	14	224	139	340	211	171	106	310	193	341	212	440	275	320	199
Edmundston	300	186	201	125			275	171	266	165	460	286	505	314	381	237	177	110
Fredericton	252	157	362	225	275	171			172	107	182	113	228	142	103	64	104	65
Miramichi	79	49	190	118	266	165	172	107			158	98	185	115	275	171	275	171
Moncton	222	138	332	206	460	286	182	113	158	98			53	33	152	94	285	177
Sackville	249	155	359	223	505	314	228	142	185	115	53	33			204	127	330	205
Saint John	355	221	466	290	381	237	103	64	275	171	152	94	204	127			206	128
St. Andrews	387	240	460	286	355	220	135	84	307	191	248	154	300	186	96	60	178	111
Saint-Léonard	273	167	159	99	42	26	236	147	224	139	422	264	464	288	339	211	135	84
St. Stephen	375	233	460	286	342	213	125	78	296	184	255	158	312	194	107	66	170	106
Sussex	279	173	390	242	399	248	121	75	199	124	75	47	131	81	73	45	224	139
Woodstock	353	219	295	183	177	110	104	65	275	171	285	177	330	205	206	128		

You could use the information in the chart/table on page 210 to find the distance in kilometres between the following places:

1) Saint John to Moncton (152 km)
2) Woodstock to Campbellton (295 km)
3) Sackville to Bathurst (249 km)
4) Edmundston to Fredericton (275 km)
5) Caraquet to Miramichi (118 km)



Bizarro by *Dan Piraro*

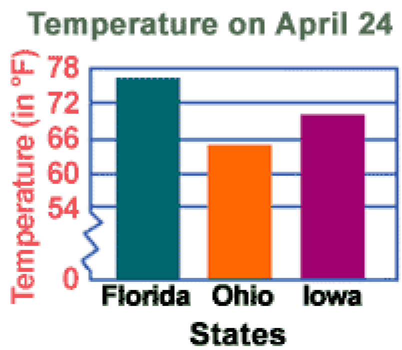
Graphs

A ***graph*** is a kind of drawing or diagram that shows ***data***, or information, usually in numbers. In order to make a graph, you must first have data.

Bar Graph

A graph that uses separate bars (rectangles) of different heights (lengths) to show and compare data

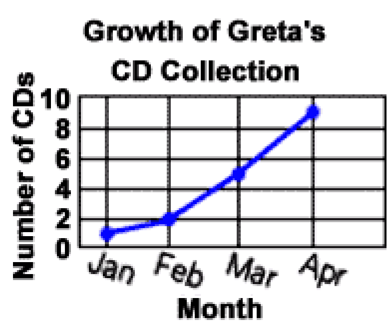
Example:



Line Graph

A graph in which line segments are used to show changes over time

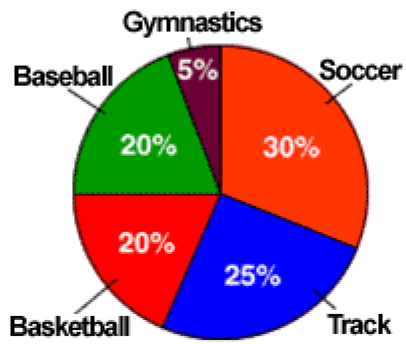
Example:



Circle Graph

A graph using a circle that is divided into pie-shaped sections showing percents or parts of the whole

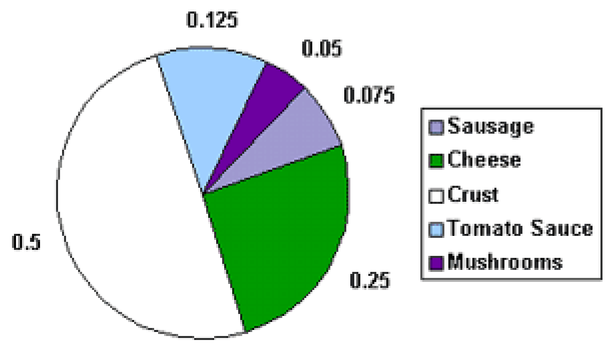
Example:



Pie Charts

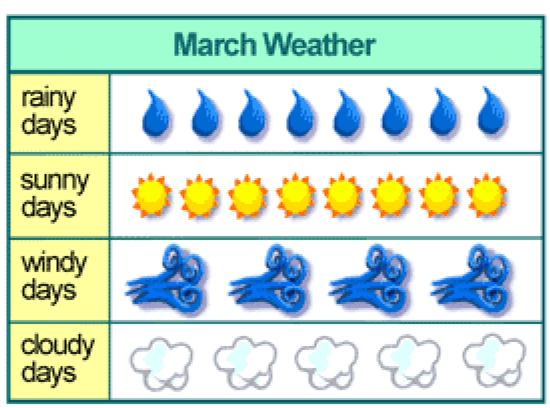
A pie chart is a circle graph divided into pieces, each displaying the size of some related piece of information. Pie charts are used to display the sizes of parts that make up some whole.

The pie chart below shows the ingredients used to make a sausage and mushroom pizza. The fraction of each ingredient by weight is shown in the pie chart below.



We see that half of the pizza's weight on the previous page comes from the crust. The mushrooms make up the smallest amount of the pizza by weight, since the slice corresponding to the mushrooms is smallest. Note that the sum of the decimal sizes of each slice is equal to 1 (the "whole" pizza").

Pictographs (picture graphs) are graphs that use pictures called ***icons*** to display data. Pictographs are used to show data in a small space. Pictographs, like bar graphs, compare data. Because pictographs use icons, however, they also include keys, or definitions of the icons.



Practice Exercise

1. This table shows the distance ran by 6 children.

Name	Distance
Andrew	2500 m
Nick	3800 m
Ken	2050 m
Kimberly	3300 m
James	2800 m
Jeremy	4025 m

(a) What distance did Nick run? How much further did he run than James?

(b) Who ran more than 3 km?

(c) What is the total distance of the longest and shortest runs?

(d) Who ran the third longest distance?

Complete the table below.

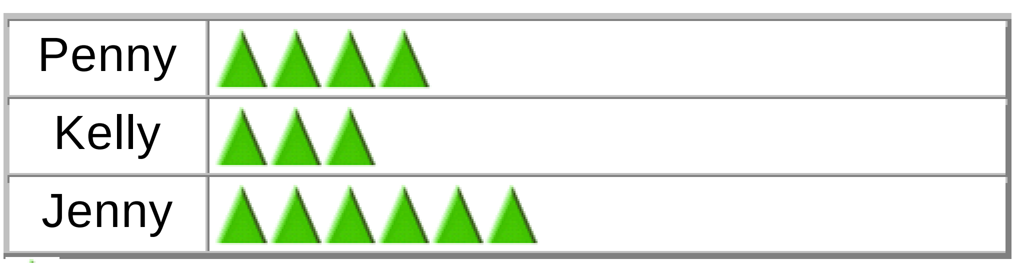
	Number of boys	Number of girls	Total
Eat noodles		**6**	**12**
Eat hamburgers	**7**		**12**
Eat chicken rice	**9**		
Total		**21**	**43**

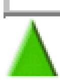
(a) How many children ate chicken rice?

(b) How many boys ate noodles and hamburgers?

(c) Which food was most popular?

This picture graph shows the number of stamps kept by each child.



 stands for 3 stamps.

(a) Kelly has ________ stamps.

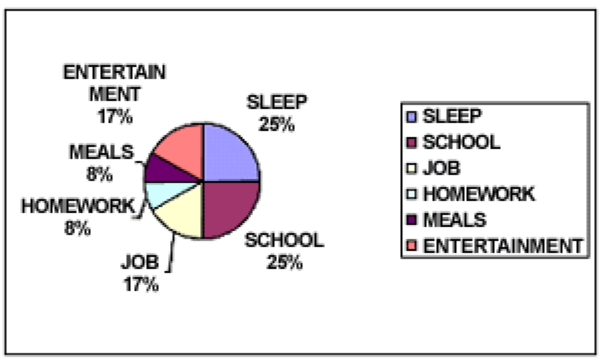
(b) Penny has ________ less stamps than Jenny.

(c) Jenny has twice as many stamps as ______.

(d) Penny has ________ stamps.

(e) The 3 girls have ________ stamps altogether.

Percent of Hours of a Day Spent on Activities



1. Which two activities took up half of the time of the day?

2. Which two activities took up the least amount of time?

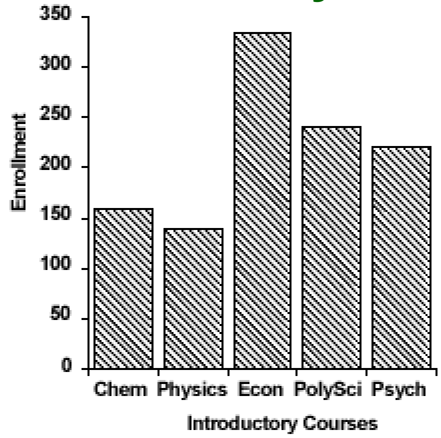
3. Which activity took up one fourth of the day?

4. What percent of the day does homework take up?

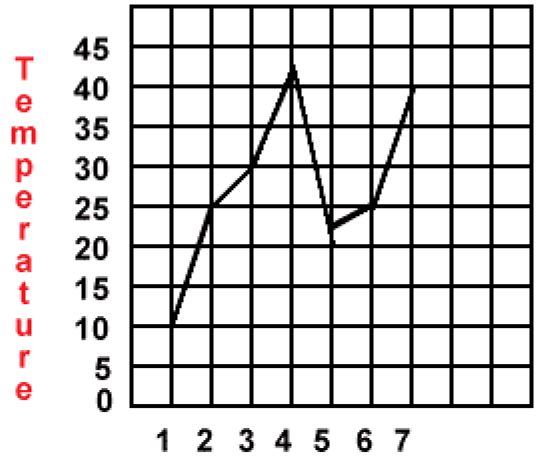
5. Which activity takes up the same amount of time as meals and entertainment together?

Given the graph below, answer the following questions.

Enrollment in Introductory Courses at Union University



1. Which course has the most students enrolled in it?
2. Order the courses by enrollment from lowest to highest.
3. The enrollment in Econ (Economics) is approximately how many times bigger than the enrollment in Chem (Chemistry)?
4. Approximately how many students were enrolled in the course with the most students?
5. Approximately how many more students are in Econ than in Physics?



Average Daily Temperature for
January 1-7 in Degrees Fahrenheit

1. This graph shows the temperatures during the period of a week, month, or year?

2. The temperatures in the beginning of the week were rising or falling?

3. Between what days did the least amount of change take place?

4. If freezing is 32 degrees, which day was above freezing?

5. Between what days was the greatest drop in temperature?

Metric Measurement

In the 1790s, French scientists worked out a system of measurement based on the ***meter***. The meter is one ten-millionth of the distance between the North Pole and the Equator. The French scientists made a metal rod equal to the length of the standard meter.

By the 1980s, the French metal bar was no longer a precise measure for the meter. Scientists figured out a new standard for the meter. They made it equal to 1/299,792,548 of the distance light travels in a vacuum in one second. Since the speed of light in a vacuum never changes, the distance of the meter will not change.

The French scientists developed the ***metric*** system to cover measurement of length, area, volume, and weight.

Metric Length Equivalents

Metric Unit	Abbreviation	Metric Equivalent
millimeter	mm	.1 centimeter
centimeter	cm	10 millimeters
decimeter	dm	10 centimeters
meter	m	100 centimeters
decameter	dam	10 meters
hectometer	hm	100 meters
kilometer	km	1000 meters

Metric Weight Equivalents

Metric Unit	Abbreviation	Metric Equivalent
milligram	mg	.001 gram
centigram	cg	10 milligrams
decigram	dg	10 centigrams
gram	g	1,000 milligrams
decagram	dag	10 grams
hectogram	hg	100 grams
kilogram	kg	1,000 grams

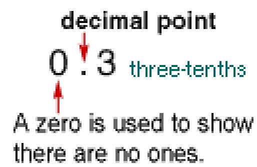
Metric Volume Measures

Metric Unit	Abbreviation	Metric Equivalent
milliliter	ml	.001 liter
centiliter	cl	10 milliliters
deciliter	dl	10 centiliters
liter	l	1,000 milliliters
decaliter	dal	10 liters
hectoliter	hl	100 liters
kiloliter	kl	1,000 liters

Decimal Point

A period that separates the whole numbers from the fractional part of a number; or that separates dollars from cents

Example:



Kilometers	**Hectometers**	**Decameters**	**Meters**	**Decimeters**	**Centimeters**	**Millimeters**
Kilograms	**Hectograms**	**Decagrams**	**Grams**	**Decigrams**	**Centigrams**	**Milligrams**
Kiloliters	**Hectoliters**	**Decaliters**	**Liters**	**Deciliters**	**Centiliters**	**Milliliters**

To use this chart, if a question asks you how many grams that you can get from 200 centigrams, for example, try this:

Start by putting down the number:

200

If we don't see a decimal point, the number is a whole number; and therefore, a decimal point may be inserted to the right of the last digit:

200.

Now, using your chart, start at centigrams and count back to grams (two spaces to the left).

Move the decimal point in your number the same amount of spaces in the same direction:

2.00

The answer to the question is that 200 centigrams is equal to 2 grams.

If a question asks you to tell how many millimeters are is 8.3 decimeters, try this:
Write down the number:

8.3

We already see a decimal point, so there is no need to guess where to place it:

8.3

Now, using your chart, start at decimeters and count forward to millimeters (two spaces to the right).

Move the decimal point in your number the same amount of spaces in the same direction:

830.

The answer to the question is that 830 millimeters is equal to 8.3 decimeters.

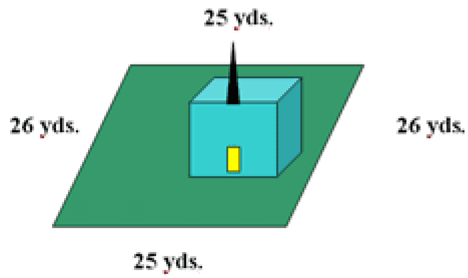
Practice Exercise

Fill in the answer.

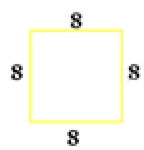
1. 1370 g = ________ kg
2. 36.61 mm = ________ cm
3. 1158 cg = ________ g
4. 105.39 mg = ________ cg
5. 10 L = ________ cl
6. 10.91 cl = ________ ml
7. 8000 L = ________ kl
8. 7.2 cm = ________ mm
9. 2.79 g = ________ cg
10. 12.23 cl = ________ ml
11. 3000 L = ________ kl
12. 11.5 cm = ________ mm
13. 9000 L = ________ kl
14. 909.7 cm = ________ m
15. 4 L = ________ ml
16. 9.75 m = ________ cm
17. 90 mg = ________ cg
18. 10.23 kl = ________ L
19. 11 L = ________ ml
20. 7.51 m = ________ cm
21. 1000 mg = ________ g
22. 10471 m = ________ km
23. 100 ml = ________ cl
24. 12.876 m = ________ cm
25. 8.1 km = ________ m
26. 9520 mm = ________ m
27. 2.32 L = ________ cl
28. 1500 mg = ________ g
29. 11 cl = ________ ml
30. 9000 ml = ________ L
31. 11.59 g = ________ cg
32. 11.62 kg = ________ g
33. 29 ml = ________ cl

Calculating Perimeter

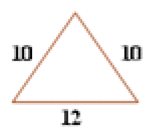
Perimeter is calculated in different ways, depending upon the shape of the surface. The perimeter of a surface outlined by straight lines is calculated by adding together the lengths of its sides.



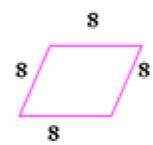
25 + 26 + 25 + 26 = 102 yds. perimeter of the rectangular lot



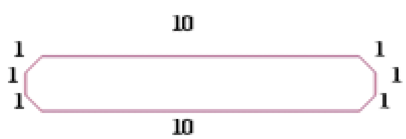
8 + 8 + 8 + 8 = 4 x 8 = 32
4s (4 sides) = perimeter of a square



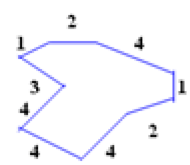
10 + 10 + 12 = 32
3s (3 sides) = perimeter of a triangle



8 + 8 + 8 + 8 = 4 x 8 = 32
4s = perimeter of a rhombus



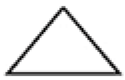
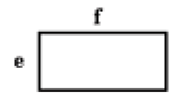
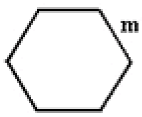
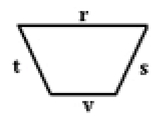
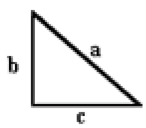
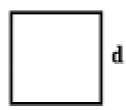
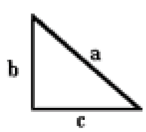
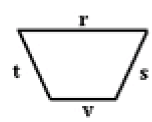
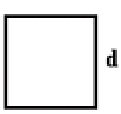
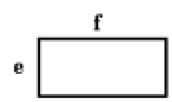
1 + 1 + 10 + 1 + 1 + 1 + 10 + 1 = 26
8s = perimeter of an irregular octagon



4 + 1 + 2 + 4 + 4 + 4 + 3 + 1 + 2 = 25
all sides = perimeter of an irregular polygon

Practice Exercise

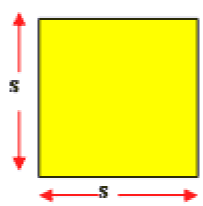
Find the perimeter.

1.  All sides equal 8 m **24 m**	2.  e = 9 cm f = 17 cm __________
3.  m = 18 cm All sides are equal __________	4.  v = 3 m t = 7 m r = 10 m s = t __________
5.  a = 6 m c = 2 m b = c __________	6.  The side d of this square is 32 m __________
7.  a = 9 m b = 2 m c = b __________	8.  v = 5 yd t = 8 yd r = 14 yd s = t __________
9.  The side d of this square is 38 m __________	10.  e = 7 yd f = 12 yd __________

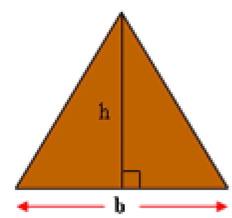
Calculating Area

Area is calculated in different ways, depending on the shape of the surface. Area is expressed in squares: square inches, square meters, etc.

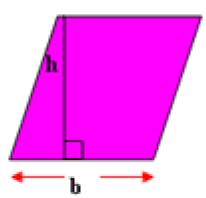
An area with a perimeter made up of straight lines is calculated in different ways for different shapes.



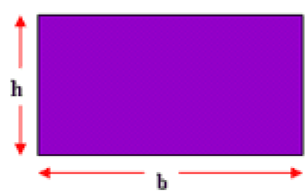
S^2 = area of a square



$\frac{\text{base x height}}{2}$ = area of a triangle



base x height = area of a rhombus



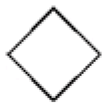
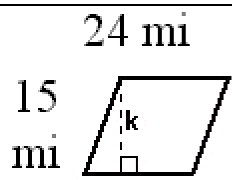
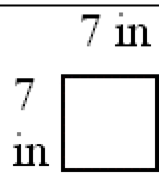
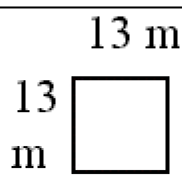
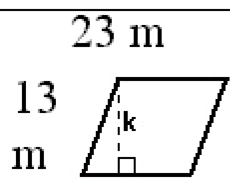
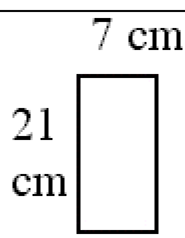
b x h = area of a rectangle

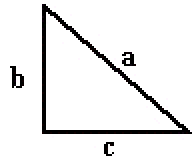
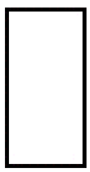
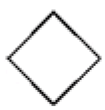
Þ ***The area of a rectangle, square, or rhombus is sometimes referred to as length x width (l x w) instead of base x height.***

Þ ***The area of a triangle is sometimes expressed as ½ the base x height (1/2 b x h).***

Practice Exercise

Find the area for each.

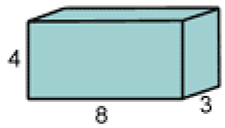
1.		All sides are 9 ft **81 square ft**
2.		k = 21 mi ________________________
3.		________________________
4.		________________________
5.		k = 18 m ________________________
6.		________________________

7.	c = 26 ft b = 26 ft 	______________________________
8.	23 in 16 in 	______________________________
9.	11 cm 55 cm 	______________________________
10.		All sides are 17 cm ______________________________

Calculating Volume

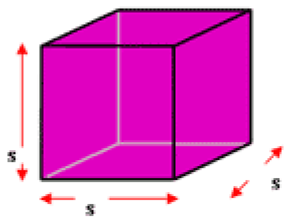
Volume is the amount of space contained in a three-dimensional shape. Area is a measurement of only ***two*** dimensions, usually length and width. Volume is a measurement of ***three*** dimensions, usually ***length***, ***width***, and ***height***, and is measured in cubic units.

To find the volume of a ***cube*** or a ***rectangular prism***, multiply length by width by height.



l x w x h = volume of a rectangular prism
8 x 3 x 4 = 96

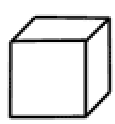
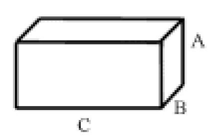
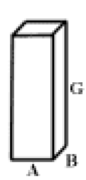
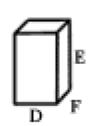
Since a cube has sides of equal length, multiply the length of one side by itself three times, S^3:



S^3 = volume of a cube

Practice Exercise

Find the volume.

1.		All sides are 5 cm ____________	2.		A = 9 ft B = 5 ft C = 20 ft ____________
3.		A = 15 cm B = 3 cm G = 28 cm ____________	4.		D = 26 cm E = 32 cm F = 5 cm ____________

Fill in the missing spaces and complete the table.
Round to the nearest hundredth.

	length	*width*	*height*	*volume*
5.	7 ft	9 ft	12 ft	___ cubic feet
6.	4 cm	13 cm	6 cm	___ cubic centimeters
7.	20 mm	12 mm	56 mm	___ cubic millimeters
8.	50 mm	75 mm	80 mm	___ cubic millimeters
9.	___ m	11 m	6 m	528 cubic meters
10.	10 mm	___ mm	15 mm	900 cubic millimeters
11.	7 m	5 m	___ m	455 cubic meters
12.	12 mm	10 mm	19.3 mm	___ cubic millimeters
13.	5 mm	11 mm	12.9 mm	___ cubic millimeters
14.	15.64 cm	4.97 cm	4 cm	___ cubic centimeters
15.	5.32 m	4.46 m	6 m	___ cubic meters

Perimeter	
Polygon	P = sum of the lengths of the sides
Rectangle	$P = 2(l + w)$
Square	$P = 4s$
Area	
Parallelogram	$A = bh$
Rectangle	$A = lw$
Square	$A = s^2$
Triangle	$A = \frac{1}{2}bh$
Volume	
Rectangular Prism	$V = lwh$

Word Problems with Measurement

Converting Measurements

When solving problems you must always keep in mind what units are being used. Converting measurements involves using ratios and rates correctly to change from one unit to another.

Example What is the volume of a rectangular solid with a length of 3 m, a width of 2 m, and a height of 90 cm?

The formula you will need to solve the problem is $V = lwh.$ But before you multiply you will need to convert the measurements given in the problem to the same unit of measurement.

Here the problem is solved by converting to metres.

Convert 90 cm to metres: **90 cm = 0.90 m.**

Volume = length x width x height
= 3 m x 2 m x 0.9 m
= 5.4 cubic metres

This example can also be solved in terms of centimetres.

Convert to centimetres:

Volume = length x width x height
= 3m x 2 m x 90 cm
= 300 cm x 200 cm x 90 cm
= 5 400 000 cubic centimetres

Each answer represents the same quantity:

5 400 000 cubic centimetres = 5.4 cubic metres

How do you decide which unit to choose? Generally, it is easiest to convert to the smallest unit of measure so that you will not have to work with fractions. Another factor to consider is the answer options in multiple choice questions. Always convert to the unit of measure given in the answer options.

Practice Exercise

1. Kevin has 1200 long-distance minutes per month on his phone-calling plan. How many hours of calls does this represent?
2. A piece of wire is 150 centimeters long. How many meters in length is the wire?
3. Monica needs to glue yarn around the perimeter of a rectangular piece of poster board that is 36 centimeters long and 15 centimeters wide. How much yarn does she need?

4. Ken's laundry room floor has the shape of a square. He wants to tile the room. If one side of the room measures 15 meters, what is the area of the floor in square meters (sq m)?
5. A shipping crate has the shape of a cube that is 8 meters long on each edge. What is its volume in cubic meters?
6. Mary worked from 7:15 A.M. to 12:30 P.M. on Monday and from 7:30 A.M. to 1:45 P.M. on Tuesday. If she earns $7.20 an hour, how much did she earn for her work on Monday and Tuesday?
7. What is the volume of a cube with sides 10 feet long?
8. Troy used 22 ½ centimeters of copper wire in each appliance he repaired. If he fixed eight appliances, how many meters of copper wire did he use?
9. Joe has received an antibiotic for his bronchitis. The instructions say to take four capsules, three times a day. If he takes his first set of capsules at 6:45 A.M., what time should he take his next set of capsules?
10. At Jill's work site, she is supposed to have a 15-minute break every 3 hours. She has worked 200 minutes since her last break. Has she worked long enough to earn the 15-minute break?

CURRICULUM OBJECTIVES

NUMBER RECOGNITION			
Arabic Numbers	1	understand and use correctly the word “digit”	
	2	recognize Arabic numbers: 0 – 1,000	
	3	recognize Arabic numbers: 1,000 +……..	
Roman Numerals	4	recognize Roman numerals: I – XXXIX (1 – 39)	
	5	recognize Roman numerals: XL – M (40 – 1,000)	
	6	read dates in Roman numerals (i.e. MCMLXXI – 1971)	
NUMBER/WORD RECOGNITION			
Number Words	1	write the number words for 0 -10	
	2	write the number words for 10 – 1,000	
	3	write the number words for 1,000 – 1,000,000	
Conventions	4	use of comma to separate thousands, (i.e. 1,000, etc.)	
	5	use of hyphen to separate number words (e.g. forty-one)	
	6	use of zero as a place holder	
PLACE VALUE			
Place Value	1	identify place value in numbers 0 – 1,000	
	2	demonstrate an understanding of “place value”	
	3	identify place value in numbers 1,000 – 1,000,000	
	4	place value in whole numbers is found from right to left	
	5	explain expanded notation	
	6	value of expanded notation for identifying numbers	
Rounding Off	7	round off whole numbers (to the nearest one, ten, hundred, thousand, million)	
COUNTING			
Counting	1	orally from 0 – 1,000, starting any place in between	
	2	drill and practice counting by 1’s, 2’s, 5’s, and 10’s (0 – 100)	
	3	orally from 1,000 – 1,000,000, starting any place between	
	4	drill and practice counting by 1’s, 2’s, 5’s, 10’s (1,000 – 1,000,000)	
Other	5	recognize “<” and “>” signs	
	6	compare numbers with , and . signs	
	7	explain even and odd numbers	
	8	order numbers from least to greatest and greatest to least	
ADDITION			
Terms	1	use the terms “addend” and “sum”	
	2	explain relationship between adding and counting	
	3	recognize and use “+” sign and the “=” sign	

	4	explain "whole number"	
Addition	5	demonstrate an understanding of addition	
	6	master addition facts up to and including 18	
	7	find sum of four whole numbers up to 3 digits	
	8	find sum of four whole numbers up to 6 digits	
	9	add numbers in columns	
	10	add numbers written in equation form	
	11	regroup ones, tens, hundreds, thousands	
	12	insert zero in blank spaces to make addition easier	
	13	the order in which numbers are added doesn't change the sum	
SUBTRACTION			
Terms	1	use "find the difference between" to signify subtraction	
	2	know the meaning of the subtraction sign "-"	
Subtraction	3	demonstrate an understanding of subtraction	
	4	master subtraction facts up to and including 18	
	5	find the difference in 2 whole numbers up to 3 digits	
	6	find the difference in 2 whole numbers up to 6 digits	
	7	subtract numbers written in columns	
	8	borrow numbers	
	9	regroup ones, tens, hundreds, thousands, etc.	
	10	subtract numbers written in equation format	
	11	insert zeros in blank spaces to make subtraction easier	
	12	explain the relation between addition and subtraction	
	13	solve addition/subtraction equations (37 - ? = 14)	
MULTIPLICATION			
Terms	1	understand and use the term "factor"	
	2	understand and use the term "multiple"	
	3	understand and use the term "product"	
	4	recognize and use the multiplication sign "x"	
Multiplication	5	demonstrate an understanding of multiplication	
	6	relation between addition and multiplication	
	7	memorize times table to 12 x 12; use a chart showing relation between numbers	
	8	carry numbers	
	9	regroup ones, tens, hundreds, and thousands	
	10	multiply by zero	
	11	multiply numbers in columns	
	12	multiply numbers written in equation format	
	13	importance of accuracy	
	14	double checking for computational errors	
	15	neatness in recording columns	
	16	printing legibly	
	17	order in which numbers are multiplied doesn't affect the answer	

DIVISION			
Terms	1	understand and use the term dividend	
	2	understand and use the term divisor	
	3	understand and use the term quotient	
	4	understand and use the term remainder	
	5	understand and use the term prime number	
	6	understand and use the term average (mean)	
	7	use the division signs "÷", "-", and "/"	
	8	divide using horizontal format "⟌ "	
Division	9	demonstrate an understanding of division	
	10	carrying numbers	
	11	dividing numbers in horizontal format	
	12	dividing numbers in equation format	
	13	dividing with zero	
	14	expressing remainders using "r"	
	15	explain relation between multiplication and division	
Factoring	16	explain factoring	
	17	find the factors of a given list of products	
	18	identify prime numbers from a given list	
Average	19	how to calculate average	
	20	when to use averages	
WORD PROBLEMS WITH WHOLE NUMBERS			
Problems	1	demonstrate ability to solve word problems with whole numbers	
Strategies	2	develop good work habits	
	3	read all parts of question carefully	
	4	determine what is asked for or required	
	5	separate information given from question being asked	
	6	record information given and solution required separately	
	7	decide what arithmetic process will solve the problem	
	8	work neatly and arrange work in rows where possible	
	9	label the answer in terms of values given in question	
	10	estimate an answer	
	11	check every step	
	12	compare estimated answer with answer found	
	13	use clue words to solve word problems (e.g. total, sum, how much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at)	
UNDERSTANDING AND COMPARING FRACTIONS			
Terms	1	explain fraction	

	2	explain numerator and denominator	
	3	explain proper and improper fraction	
	4	explain equivalent fractions	
	5	explain mixed number	
	6	lowest common denominator (LCM)	
Fractions	7	demonstrate an understanding of fractions	
	8	visualize fractions: divide circle or line into correct number of segments to represent a given fraction	
	9	compare fractions: order list of fractions from greatest to least	
	10	compare fractions: order list of fractions from least to greatest	
	11	compare 2 fractions using "<" and ">" (like denominators)	
	12	read "<" and ">" left to right and right to left	
	13	provide equivalent fractions for each fraction in a given list	
	14	identify fractions in their lowest terms	
	15	reduce fractions to their lowest terms	
	16	express improper fractions as mixed numbers	
	17	express mixed numbers as improper fractions	
	18	find lowest common denominator for group of 2 or 3 fractions	
	19	express these fractions using lowest common denominator	
ADDITION OF FRACTIONS			
Adding Fractions	1	add fractions with like denominators	
	2	reduce fractions to lowest terms	
	3	change improper fractions to mixed numbers	
	4	report answers as mixed numbers	
	5	add fractions with unlike denominators	
	6	find common denominators	
	7	fractions that equal 1	
	8	add mixed numbers	
	9	change mixed numbers to improper fractions	
SUBTRACTION OF FRACTIONS			
Subtracting Fractions	1	subtract fractions with like denominators	
	2	reduce fractions to lowest terms	
	3	change improper fractions to mixed numbers	
	4	report answers as mixed numbers	
	5	subtract fractions with unlike denominators	
	6	find common denominators	
	7	fractions that equal one	
	8	subtract mixed numbers	
	9	borrowing	

	10	change mixed numbers to improper fractions	
	11	subtract fractions from whole numbers	
	12	change whole numbers to fractions	
MULTIPLICATION OF FRACTIONS			
Terms	1	explain greatest common factor (GCF)	
	2	explain lowest common multiple (LCM)	
	3	explain cancelling	
Multiplying Fractions	4	demonstrate an understanding of cancelling	
	5	find GCF and LCM	
	6	find factors for each number	
	7	find greatest factor common to both	
	8	find multiples of two numbers	
	9	find LCM common to both	
	10	reduce fractions and/or cancel before multiplying	
	11	multiplication of numerator and denominator by same number	
	12	multiply fractions by whole numbers	
	13	multiply mixed numbers	
	14	reduce answers to lowest common denominator	
	15	change improper fractions to mixed numbers	
	16	report answers as mixed numbers	
DIVISION OF FRACTIONS			
Dividing Fractions	1	review cancelling	
	2	finding GCF and LCM	
	3	find factors for each number	
	4	find greatest factor common to both	
	5	find multiples	
	6	find the LCM	
	7	division of numerator and denominator by the same number	
	8	divide with fractions and whole numbers	
	9	division rule: cancel; invert 2nd fraction; multiply	
	10	divide mixed numbers	
	11	express remainders as fractions	
	12	reduce fractions to lowest terms	
	13	change improper fractions to mixed numbers	
	14	report answers as mixed numbers	
WORD PROBLEMS WITH FRACTIONS			
Problems	1	demonstrate ability to solve word problems with fractions	
Strategies	2	develop good work habits	
	3	read all parts of question carefully	

	4	determine what is asked for or required	
	5	separate information given from question being asked	
	6	record information given and solution required separately	
	7	decide what arithmetic process will solve the problem	
	8	work neatly and arrange work in rows where possible	
	9	label the answer in terms of values given in question	
	10	estimate an answer	
	11	check every step	
	12	compare estimated answer with answer found	
	13	use clue words to solve word problems (e.g. total, sum, how much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at, divide , and each)	
UNDERSTANDING AND COMPARING DECIMALS			
Decimal System	1	demonstrate and understanding of the decimal system	
	2	explain the decimal system	
	3	use of decimal point	
	4	compare: decimals	
	5	compare: mixed decimals	
	6	compare: decimals and fractions	
	7	read decimals from left to right	
	8	translate decimal as "and" in English	
	9	decimal point is expressed as a comma in many countries	
	10	convert mixed numbers to mixed decimals	
	11	compare decimal values by adding zeros to right of decimal	
	12	place value of each digit in a decimal through thousandths	
	13	convert fractions to decimals	
	14	convert decimals to fractions (excluding repeating decimals)	
ADDITION OF DECIMALS			
Adding Decimals	1	review addition number facts to 18	
	2	estimate answer before adding	
	3	add decimals and mixed decimals	
	4	placement of decimal points under one another	
	5	use of zero as a place holder	
	6	carrying numbers	
	7	round off decimals to the hundredths place	
SUBTRACTION OF DECIMALS			
Subtracting Decimals	1	review subtraction number facts to 18	
	2	estimate answer before subtracting	
	3	subtract decimals and mixed decimals	
	4	placement of decimal points under one another	

	5	use of zero as a place holder	
	6	borrowing numbers	
	7	round off decimals to the hundredths place	
MULTIPLICATION OF DECIMALS			
Multiplying Decimals	1	review multiplication facts	
	2	estimate answer before multiplying	
	3	multiply decimals and mixed decimals	
	4	placement of decimal point in final answer	
	5	no need to line up decimal points in question	
	6	use of zero as a place holder	
	7	carry numbers	
	8	round off decimals to the hundredths place	
	9	multiply decimals by 10, 100, 1,000 (decimal to right)	
DIVISION OF DECIMALS			
Dividing Decimals	1	review division number facts	
	2	estimate answer before dividing	
	3	divide decimals and mixed decimals	
	4	use zero as a place holder in dividing	
	5	place decimal point in answer directly above that in problem	
	6	zero as a place holder in expressing answer (i.e. 0.98)	
	7	express remainder as decimals	
	8	divide whole numbers and decimals by decimals moving the decimal point in divisor and dividend	
	9	add zeros to divide	
	10	round off decimals	
WORD PROBLEMS WITH DECIMALS			
Problems	1	demonstrate an understanding of problem solving with decimals	
Strategies	2	develop good work habits	
	3	read all parts of question carefully	
	4	determine what is asked for or required	
	5	separate information given from question being asked	
	6	record information given and solution required separately	
	7	decide what arithmetic process will solve the problem	
	8	work neatly and arrange work in rows where possible	
	9	label the answer in terms of values given in question	
	10	estimate an answer	
	11	check every step	
	12	compare estimated answer with answer found	
	13	use clue words to solve word problems (e.g. total, sum, how	

		much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at, divide, and each)	
UNDERSTANDING AND COMPARING PERCENTS			
Terms	1	explain percent	
	2	explain use of the "%"	
Percents	3	demonstrate an understanding of percent	
	4	introduce percent through use of grids or pie charts	
	5	shade in portions of figure to represent percentage of that figure	
	6	100% of a number = all of that number	
	7	0% of a number = none of that number	
	8	convert fractions to percent (multiply by 100)	
	9	convert decimals to percent (multiply by 100 or move decimals 2 places to right)	
	10	convert percent to fractions (divide percent over 100 and reduce fraction)	
	11	convert percent to decimals (divide percent by 100 or move decimal 2 places to left)	
USING PERCENTS			
Using Percents	1	use $r/100 = P/W$ to find percent of a number, what percent one number is of another, and a number when a percent is given	
SIMPLE INTEREST			
Terms	1	explain simple interest	
	2	explain the interest formula I = Prt (where P = principal, r = rate, t = time)	
Simple Interest	3	demonstrate an understanding of simple interest	
	4	discuss when simple interest is charged	
	5	relationship between percentage and interest	
	6	gather information needed to use I = Prt (implied multiplication between P and r and t)	
	7	calculate simple interest	
WORD PROBLEMS WITH PERCENT			
Problems	1	demonstrate an understanding of problem solving with percent	
Strategies	2	develop good work habits	
	3	read all parts of question carefully	
	4	determine what is asked for or required	
	5	separate information given from question being asked	
	6	record information given and solution required separately	

	7	decide what arithmetic process will solve the problem	
	8	work neatly and arrange work in rows where possible	
	9	label the answer in terms of values given in question	
	10	estimate an answer	
	11	check every step	
	12	compare estimated answer with answer found	
	13	use clue words to solve word problems (e.g. total, sum, how much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at, divide, and each)	
TIME			
Using Time Divisions	1	60 seconds in a minute	
	2	60 minutes in an hour	
	3	24 hours in a day	
	4	7 days in a week	
	5	approximately four weeks in a month	
	6	name the days of the week	
	7	name months of the year	
	8	correctly use a.m. and p.m.	
	9	calculate the number of seconds in a given number of minutes	
	10	calculate the number of minutes in a given number of hours	
	11	calculate the number of hours in a given number of days	
	12	state the number of days in a given month	
	13	explain the term "leap year"	
	14	calculate the leap years between two given dates	
	15	express time in words	
	16	use timetables (e.g. bus, train, school)	
Clocks	17	tell time with an analog clock to the nearest minute	
	18	recognize times (e.g. quarter past four, ten to six)	
	19	tell time with a digital clock (e.g. 1:50 is ten to one)	
	20	briefly describe the "twenty-four hour" clock	
	21	discuss where the twenty-four hour clock might be found	
	22	recognize written time (e.g. half hour, quarter hour)	
	23	use of the colon in writing time (e.g. 2:53)	
Calendars	24	numeric dating: various styles (d/m/y; y/m/d; m/d/y)	
Calculating with Time	25	add time	
	26	subtract time	
MONEY			
Coin Values	1	identify value of coins: penny (cent), nickel, dime, quarter, loonie, and toonie	
	2	use of the "$" and "¢" signs	
	3	use of decimal point to write dollar/cent amounts	
	4	convert cents to dollars and dollars to cents	
Calculating with	5	use knowledge of decimals to add and subtract money	

Money			
	6	use knowledge of decimals to multiply and divide money	
	7	practice counting money and making change	
	8	find cost of several items, given the unit cost	
	9	find unit cost, given number of items and total cost	
	10	calculate tax payable (HST)	
	11	calculate discount, given regular price and % discount	
	12	buying on credit	
	13	round off to the nearest cent	
	14	procedure for developing a budget	
CHARTS AND GRAPHS			
Chart	1	explain chart/table	
	2	interpret information in distance tables (map)	
	3	interpret information in weight/height charts	
	4	interpret information in nutritive value charts	
Graphs	5	explain graph	
	6	interpret information in circle graphs	
	7	interpret information in bar graphs	
	8	interpret information in pictographs	
	9	interpret information in line graphs	
METRIC MEASUREMENT			
Terms	1	linear, volume, mass measurement	
	2	gram (mass), litre (volume), metre (linear)	
	3	prefixes: milli, centi, deci, (metre, gram, litre), deka, hecto, kilo	
	4	abbreviations linear measure: mm, dm, m, dam, km, cm, hm	
	5	abbreviations volume measure: ml, dl, L, dal, hl, kl, cl	
	6	abbreviations mass measure: mg, cg, g, dag, hg, kg,dg	
	7	faces, edges, vertex, vertices	
	8	square, rectangle, triangle	
	9	perimeter	
	10	area	
Using Metric Measurement	11	estimate and measure accurately: linear measurement	
	12	estimate and measure accurately: volume measurement	
	13	estimate and measure accurately: mass measurement	
	14	identify geometric figures by counting faces, edges, and vertices	
	15	formula: calculate perimeter of rectangle (2l + 2w) and use the correct units: (mm, cm, dm, m, km)	
	16	formula: calculate area of rectangle (l x w)	
	17	use correct area units: (e.g. square centimetres, square kilometres, etc.)	

	18	formula: calculate volume of rectangular prism (l x w x h)	
	19	use correct volume units: (e.g. cubic centimetres, cubic metres, etc.)	
WORD PROBLEMS WITH MEASUREMENT			
Problems	1	demonstrate ability to solve word problems with addition, subtraction, multiplication, and division of whole numbers, fractions, decimals, percentages, time, money, temperature, and metric measurement	
Strategies	2	develop good work habits	
	3	read all parts of question carefully	
	4	determine what is asked for or required	
	5	separate information given from question being asked	
	6	record information given and solution required separately	
	7	decide what arithmetic process will solve the problem	
	8	work neatly and arrange work in rows where possible	
	9	label the answer in terms of values given in question	
	10	estimate an answer	
	11	check every step	
	12	compare estimated answer with answer found	
	13	use clue words to solve word problems (e.g. total, sum, how much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at)	