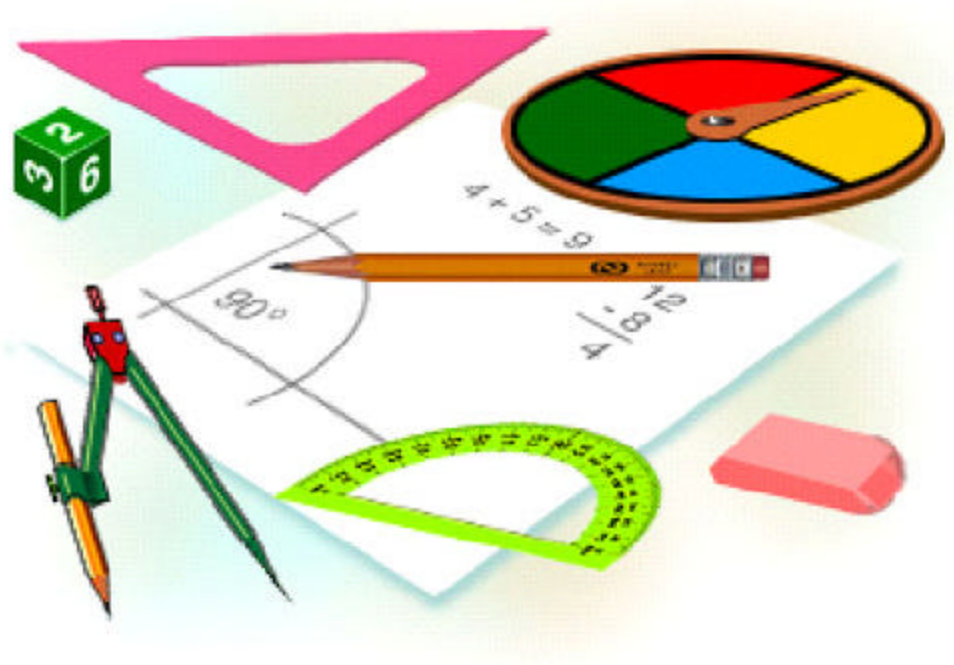
The Next Step

Mathematics Applications for Adults



Book 14016 – Fractions

OUTLINE

Mathematics - Book 14016

Fractions
Understanding and Comparing Fractions
explain proper and improper fractions.
explain equivalent fractions.
explain mixed number.
explain lowest common denominator (LCM).
demonstrate an understanding of fractions.
provide equivalent fractions.
recognize a fraction in its lowest terms.
demonstrate an understanding of reducing fractions to their lowest terms (e.g. 4/12=1/3).
convert improper fractions to mixed numbers and vice versa.
find lowest common denominator (LCM) given 2 or 3 fractions with unlike denominators.
Addition of Fractions
add fractions with like and unlike denominators.
Subtraction of Fractions
subtract fractions with like an unlike denominators.
Multiplication of Fractions
multiply common and improper fractions with a denominator up to and including 10.
find the greatest common factor (GCF).
find the lowest common denominator (LCM).
Division of Fractions
divide common and improper fractions with a

denominator up to and including10.
Word Problems with Fractions
solve one/two step problems with addition, subtraction, multiplication, and division of fractions.

THE NEXT STEP

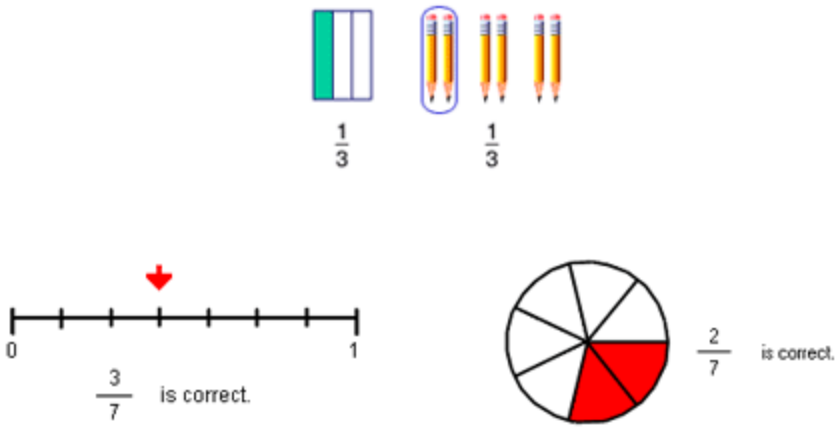
Book 14016

Fractions

Undertanding and Comparing Fractions

The word ***fraction*** means "part of a whole." The word comes from the Latin word ***fractio***, meaning "to break into pieces." In math, a fraction means one or more parts of a whole.

Example:



A fraction has two parts, a ***denominator*** and a ***numerator***. The denominator is the numeral written under the bar and tells the number of parts a whole is divided into. The numerator is the numeral written above the bar. The numerator tells the number of parts of the whole that are

being counted. A ***proper fraction*** has a numerator that is smaller than its denominator.

numerator	**number of parts counted**	**1**
denominator	**total parts of the whole**	**17**

Practice Exercise

Write the fraction of the coloured part.

1. 

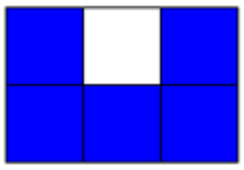
_____ of the circle is coloured.

2. 

_____ of the rectangle is coloured.

3. 

_____ of the triangle is coloured.

4. 

_____ of the rectangle is coloured.

Fill in the blanks.

1. ½ and _____ make 1 whole.
2. and _____ make 1 whole.
3. and _____ make 1 whole.

Improper Fractions

When the numerator of a fraction is greater than or equal to the denominator, the fraction is called an ***improper fraction***.

$$\frac{3}{2} \quad \frac{4}{3} \quad \frac{5}{4} \quad \frac{6}{5} \quad \frac{7}{6} \quad \frac{8}{8}$$

Þ ***The value of an improper fraction is always greater than or equal to one.***

Mixed Numerals

Mixed numerals combine whole numbers and fractions. The values of mixed numerals can also be written as ***improper fractions***. To write a mixed numeral as an improper fraction, multiply the whole number by the denominator of the fraction, then add the numerator. Use

your answer as the new numerator and eep the original enominator.

$$1\tfrac{1}{2} = \frac{(2 \times 1) + 1}{2} = \frac{3}{2} \qquad 2\tfrac{3}{4} = \frac{(2 \times 4) + 3}{4} = \frac{11}{4}$$

To change an improper fraction to a mixed numeral, divide the numerator by the denominator. Then place the remainder over the old denominator.

$$\frac{3}{2} = 2\overline{)3}\ \begin{matrix}1\\ -2\\ \hline 1\end{matrix} = 1\tfrac{1}{2} \qquad \frac{11}{4} = 4\overline{)11}\ \begin{matrix}2\\ -8\\ \hline 3\end{matrix} = 2\tfrac{3}{4}$$

Express each fraction as a whole number or as a mixed number.

1. $\frac{20}{7} =$
2. $\frac{23}{3} =$
3. $\frac{25}{2} =$
4. $\frac{33}{10} =$
5. $\frac{59}{6} =$
6. $\frac{85}{8} =$
7. $\frac{93}{12} =$
8. $\frac{39}{5} =$
9. $\frac{49}{4} =$
10. $\frac{45}{5} =$
11. $\frac{29}{6} =$
12. $\frac{24}{2} =$
13. $\frac{109}{10} =$
14. $\frac{53}{7} =$
15. $\frac{17}{4} =$
16. $\frac{7}{3} =$

17. $\frac{52}{8}$ =

18. $\frac{57}{9}$ =

19. $\frac{59}{11}$ =

20. $\frac{64}{7}$ =

21. $\frac{114}{9}$ =

22. $\frac{26}{5}$ =

23. $\frac{41}{4}$ =

24. $\frac{36}{3}$ =

25. $\frac{9}{2}$ =

26. $\frac{60}{5}$ =

27. $\frac{37}{4}$ =

28. $\frac{39}{6}$ =

29. $\frac{20}{3}$ =

30. $\frac{151}{12}$ =

31. $\frac{64}{11}$ =

32. $\frac{52}{9}$ =

Express each mixed numeral as an improper fraction.

a. 12 3/4 =
b. 99 1/2 =
c. 28 3/4 =
d. 2 1/4 =
e. 31 1/3 =
f. 75 2/3 =
g. 46 1/8 =
h. 64 3/8 =
i. 57 5/8 =
j. 7 1/2 =

Common Denominators

Many fractions have ***common denominators***. That means that the numbers in their denominators are the same.

$$\frac{1}{2} \quad \frac{3}{2} \quad \frac{5}{2}$$

To find common denominators,  find the ***least common multiple*** for the denominators of the fractions you are comparing.

Find the ***multiples*** of a number by multiplying it by other whole numbers. The multiples of ***2***, for example, are:

0 x 2 = 0
1 x 2 = 2
2 x 2 = 4

2 x 3 = 6
2 x 4 = 8
2 x 5 = 10

… and so on.

As you can see, the multiples of ***2*** include ***0, 2, 4, 6, 8,*** and ***10***. The list continues into infinity!

Some numbers share the same multiples. Those multiples are known as ***common multiples***.

Number	Multiples					
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25
	0	1	2	3	4	5

The least multiple of two or more numbers is the least common multiple. For example, the least common multiple of ***2*** and ***3*** is ***6***.

2 x 1 = 2 **2 x 2 = 4** **2 x 3 = 6**
3 x 1 = 3 **3 x 2 = 6**

Compare:

$\frac{1}{2}$ and $\frac{2}{3}$ **Answer: least common multiple is 6**

(2) Divide the common multiple by the denominators.

$2\overline{)6}$ $\quad 3$ $\qquad$ $3\overline{)6}$ $\quad 2$

(3) Multiply the quotients by the old numerators to calculate the new numerators.

$$\begin{array}{r} 3 \\ \times\ 1 \\ \hline 3 \end{array} \qquad \begin{array}{r} 2 \\ \times\ 2 \\ \hline 4 \end{array}$$

(4) Place the new numerators over the common denominator.

$$\frac{3}{6} \qquad \frac{4}{6}$$

Þ ***To reduce a fraction to its lowest terms, divide both the numerator and the denominator by their greatest common denominator.***

$$\frac{4}{8}, \frac{4}{4} = \frac{1}{2}$$

Rewrite each set of fractions using the least common denominator.

1. $\frac{5}{6}, \frac{1}{5}$	2. $\frac{3}{6}, \frac{1}{4}$
3. $\frac{5}{6}, \frac{1}{2}$	4. $\frac{1}{2}, \frac{2}{5}$

5. $\frac{2}{5}$, $\frac{9}{11}$	6. $\frac{2}{9}$, $\frac{5}{8}$
7. $\frac{5}{8}$, $\frac{8}{10}$	8. $\frac{1}{6}$, $\frac{3}{9}$
9. $\frac{4}{5}$, $\frac{1}{7}$	10. $\frac{6}{7}$, $\frac{1}{10}$
11. $\frac{2}{9}$, $\frac{2}{5}$	12. $\frac{4}{8}$, $\frac{3}{6}$
13. $\frac{3}{5}$, $\frac{1}{12}$	14. $\frac{7}{11}$, $\frac{3}{6}$
15. $\frac{4}{12}$, $\frac{4}{7}$	16. $\frac{6}{11}$, $\frac{5}{12}$
17. $\frac{6}{7}$, $\frac{3}{8}$, $\frac{4}{14}$	18. $\frac{6}{9}$, $\frac{3}{7}$, $\frac{11}{14}$

Reduce each fraction to lowest terms.
(Hint: Divide its numerator and denominator by their Greatest Common Factor)

1. $\frac{2}{12} =$	2. $\frac{24}{40} =$	3. $\frac{6}{60} =$	4. $\frac{20}{25} =$
5. $\frac{12}{24} =$	6. $\frac{24}{30} =$	7. $\frac{10}{90} =$	8. $\frac{30}{45} =$
9. $\frac{30}{60} =$	10. $\frac{22}{12} =$	11. $\frac{10}{20} =$	12. $\frac{38}{68} =$
13. $\frac{34}{43} =$	14. $\frac{12}{30} =$	15. $\frac{43}{50} =$	16. $\frac{7}{84} =$
17. $\frac{9}{46} =$	18. $\frac{2}{16} =$	19. $\frac{38}{56} =$	20. $\frac{24}{28} =$
21. $\frac{51}{28} =$	22. $\frac{15}{35} =$	23. $\frac{26}{30} =$	24. $\frac{20}{44} =$

Equivalent Fractions

You know from experience that different fractions can have the same value.

> Since there are 100 pennies in a dollar, 25 pennies is equal to 25/100 of a dollar. The same amount also equals a quarter, or ¼of a dollar.

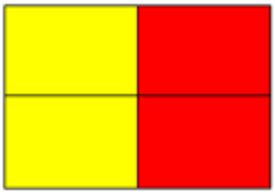
On a measuring cup, ½ cup is the same amount as 2/4 cup.
On an odometer, 5/10 of a mile is the same as ½ mile.
Out of a dozen doughnuts, six doughnuts equal 6/12, or ½ dozen.

A napkin is folded into two parts. One part is yellow, the other red.

½ yellow

½ red

Then the napkin is folded again. Now there are two yellow parts and two red parts.



$\frac{2}{4}$ yellow

$\frac{2}{4}$ red

In this example, the red part of the napkin can be described as ½ red or 2/4 red. That makes ½ and 2/4 ***equivalent fractions***.

When solving math problems, reduce fractions to their lowest equivalent. Rather than describing the napkin as 2/4 yellow, call it ½ yellow.

Some Equivalent Fractions

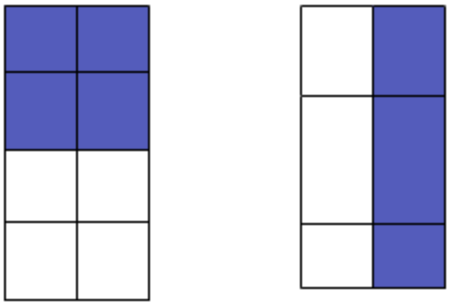
$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20}$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$$

You can tell if two fractions are equal by finding cross products.

Example Are 4/8 and 3/6 equal fractions?



Multiply diagonally as shown by the arrows below. If the cross products are equal, the fractions are equal.

$$\frac{4}{8} \quad \frac{3}{6}$$

4 x 6 = 24
8 x 3 = 24

Since the cross products are equal, **4/8 = 3/6.**

Sometimes you need to find an equal fraction with higher terms. You raise a fraction to higher terms by multiplying both

the numerator and the denominator by the same number (except 0).

5/8 and 20/32 are equal fractions because $\frac{5 \times 4}{8 \times 4} = \frac{20}{32}$

Often you will need to find an equal fraction with a specific denominator. To do this, think, "What number multiplied by the original denominator will result in the new denominator?" Then multiply the original numerator by the same number.

Example ¾ = ?/24

Since 4 x 6 = 24, multiply the numerator 3 by 6. $\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$

The fractions ¾ **and 18/24 are equal fractions.**

Comparing Fractions

When two fractions have the same number as the denominator, they are said to have a common denominator, and the fractions are called like fractions. When you compare like fractions, the fraction with the greater numerator is the greater fraction.

Example 1 Which fraction is greater, 3/5 or 4/5?

The fractions 3/5 and 4/5 are like fractions because they have a common denominator, 5. Compare the numerators.

Since 4 is greater than 3, **4/5 is greater than 3/5.**

Fractions with different denominators are called unlike fractions. To compare unlike fractions, you must change them to fractions with a common denominator.

The common denominator will always be a multiple of both of the original denominators. The multiples of a number are found by going through the times tables for the number. For instance, the multiples of 3 are 3, 6, 9, 12, 15, 18, and so on.

You can often find a common denominator by using mental math. If not, try these methods:

1. See whether the larger denominator could be a common denominator. In other words, if the smaller denominator can divide into the larger denominator evenly, use the larger denominator as the common denominator.

2. Go through the multiples of the larger denominator. The first one that can be divided evenly by the smaller denominator is the lowest common denominator.

Example 2 Which is greater, 5/6 or ¾?

Go through the multiples of the larger denominator: 6, 12, 18, 24, 30…. Since 12 can be divided evenly by both 4 and 6, 12 is the lowest common denominator.

Build equal fractions, each with the denominator 12:

$$\frac{5 \times 2}{6 \times 2} = \frac{10}{12} \qquad \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

Compare the like fractions. Since 10/12 > 9/12, the fraction **5/6 > ¾.**

Practice Exercise

Fill in the missing numerator or denominator for the following.

1.	$\frac{3}{5}$	=	$\frac{18}{\quad}$	2.	$\frac{1}{8}$	=	$\frac{\quad}{72}$
3.	$\frac{3}{6}$	=	$\frac{\quad}{30}$	4.	$\frac{2}{7}$	=	$\frac{\quad}{77}$
5.	$\frac{2}{3}$	=	$\frac{20}{\quad}$	6.	$\frac{2}{9}$	=	$\frac{14}{\quad}$
7.	$\frac{1}{4}$	=	$\frac{\quad}{40}$	8.	$\frac{1}{2}$	=	$\frac{11}{\quad}$
9.	$\frac{1}{3}$	=	$\frac{8}{\quad}$	10.	$\frac{9}{10}$	=	$\frac{\quad}{30}$
11.	$\frac{6}{7}$	=	$\frac{72}{\quad}$	12.	$\frac{6}{9}$	=	$\frac{\quad}{99}$
13.	$\frac{1}{6}$	=	$\frac{\quad}{60}$	14.	$\frac{2}{4}$	=	$\frac{16}{\quad}$
15.	$\frac{3}{8}$	=	$\frac{6}{\quad}$	16.	$\frac{6}{12}$	=	$\frac{\quad}{84}$

Fraction Comparison

1.	$\frac{5}{8}$	>	$\frac{2}{5}$	2.	$\frac{2}{3}$	—	$\frac{1}{11}$
3.	$\frac{5}{9}$	—	$\frac{1}{9}$	4.	$\frac{9}{14}$	—	$\frac{1}{2}$
5.	$\frac{1}{6}$	—	$\frac{1}{4}$	6.	$\frac{3}{19}$	—	$\frac{6}{11}$
7.	$\frac{8}{16}$	—	$\frac{2}{10}$	8.	$\frac{3}{6}$	—	$\frac{9}{18}$
9.	$\frac{1}{3}$	—	$\frac{20}{29}$	10.	$\frac{5}{50}$	—	$\frac{7}{35}$
11.	$\frac{30}{60}$	—	$\frac{2}{9}$	12.	$\frac{45}{21}$	—	$\frac{135}{63}$
13.	$\frac{14}{15}$	—	$\frac{1}{4}$	14.	$\frac{3}{2}$	—	$\frac{37}{20}$
15.	$\frac{12}{36}$	—	$\frac{9}{15}$	16.	$\frac{5}{4}$	—	$\frac{3}{4}$

Addition of Fractions

To add fractions, the fractions must have ***common denominators***. To add fractions with common denominators, simply add the numerators. The sum will become the numerator of your answer. The denominator will remain the same.

$$\frac{3}{8} + \frac{4}{8} = \frac{3+4}{8} = \frac{7}{8}$$

Unlike fractions have different denominators. Use these steps to add unlike fractions.

Step 1 Find a common denominator and change one or both of the fractions to make like fractions.

½ + ¾ = ?

$$\text{½} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Step 2 Add the like fractions

2/4 + ¾ = 5/4

Step 3 Reduce the answer if necessary. If the answer is an improper fraction, rewrite it as a whole or mixed number.

5/4 = 1 ¼

A mixed number is a whole number and a proper fraction. To add mixed numbers, work with each part separately and then combine the results.

Þ ***Adding fractions is impossible without first writing the fractions with common denominators.***

Step 1 Write the fractions with common denominators.

$$\begin{aligned} 6\ 1/3 &= 6\ \frac{1 \times 4}{3 \times 4} = 6\ \frac{4}{12} \\ +\ 4\ ¾ &= 4\ \frac{3 \times 3}{4 \times 3} = 4\ \frac{9}{12} \end{aligned}$$

Step 2 Add the fractions first. Add the numerators and put the sum over the common denominator. Then add the whole numbers.

$$\begin{array}{r} 6\ \frac{4}{12} \\ +\ 4\ \frac{9}{12} \\ \hline \end{array}$$

Step 3 Change the improper fraction to a mixed number. Add this to the whole number answer.

$$\frac{13}{12} = 1\ \frac{1}{12}$$

$$10 + 1\ \frac{1}{12} = 11\ \frac{1}{12}$$

Sometimes when you add the fraction parts, you get a whole number as an answer. If this happens, just add that whole number to the other one.

Example: $2\frac{3}{5} + 2\frac{2}{5}$

2 + 2 = 4

$\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$ **Remember that any number divided by itself is 1.**

4 + 1 = 5 **The answer is 5.**

Mixed numbers can be added to whole numbers by adding the whole numbers together and keeping the fraction. This makes sense because you are adding whole amounts plus another part of a whole.

Example: $3 + 2\frac{1}{2} = 5\frac{1}{2}$ $3 + 2 = 5, 5 + \frac{1}{2} = 5\frac{1}{2}$

Solve for each of the given problems.

1. $\begin{array}{r} \frac{4}{7} \\ 7\frac{2}{7} \\ + \\ \hline \end{array}$	**2.** $\begin{array}{r} \frac{1}{5} \\ \frac{3}{5} \\ + \\ \hline \end{array}$	**3.** $\begin{array}{r} 4\frac{1}{2} \\ \frac{1}{2} \\ + \\ \hline \end{array}$	**4.** $\begin{array}{r} \frac{9}{10} \\ \frac{9}{10} \\ + \\ \hline \end{array}$

5. $4\frac{3}{4} + \frac{3}{4}$	**6.** $10\frac{4}{6} + \frac{1}{6}$	**7.** $12\frac{5}{7} + 10\frac{1}{7}$	**8.** $\frac{8}{12} + \frac{5}{12}$
9. $3\frac{6}{8} + 11\frac{5}{8}$	**10.** $12\frac{3}{4} + \frac{6}{10}$	**11.** $\frac{5}{11} + \frac{6}{11}$	**12.** $\frac{7}{10} + \frac{6}{7}$
13. $5\frac{5}{6} + \frac{2}{8}$	**14.** $\frac{2}{6} + \frac{4}{5}$	**15.** $12\frac{2}{3} + 9\frac{2}{9}$	**16.** $9\frac{10}{12} + \frac{8}{9}$

Subtraction of Fractions

To subtract fractions, the fractions must have ***common denominators***. To subtract fractions with common denominators, simply subtract the numerators. The difference will become the numerator of your answer. The denominator will remain the same.

$$\frac{11}{12} - \frac{2}{12} = \frac{11-2}{12} = \frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

Unlike fractions have different denominators. Use these steps to subtract unlike fractions.

Step 1 Find a common denominator and change one or both of the fractions to make like fractions.

$$\tfrac{3}{4} - \tfrac{1}{2} = ?$$
$$\tfrac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Step 2 Subtract the like fractions.

$$\tfrac{3}{4} - 2/4 = \tfrac{1}{4}$$

Step 3 Reduce the answer if necessary. If the answer is an improper fraction, rewrite it as a whole or mixed number.

A mixed number is a whole number and a proper fraction. To subtract mixed numbers, work with each part separately and then combine the results.

Þ ***Subtracting fractions is impossible without first writing the fractions with common denominators.***

Step 1 Write the fractions with common denominators.

$$6\ 3/4 = 6\ \frac{3 \times 3}{4 \times 3} = 6\ \frac{9}{12}$$

$$-\ 4\ 1/3 = 4\ \frac{1 \times 4}{3 \times 4} = 4\ \frac{4}{12}$$

Step 2 Subtract the fractions first. Subtract the numerators and put the difference over the common denominator. Then subtract the whole numbers.

$$\begin{array}{r} 6\ \frac{9}{12} \\ -\ 4\ \frac{4}{12} \\ \hline 2\ \frac{5}{12} \end{array}$$

Step 3 If necessary, reduce to lowest terms.

When subtracting mixed numbers, sometimes the fraction you are subtracting from will be smaller than the fraction you are taking away. In this situation, you will need to

regroup, or borrow, 1 from the whole number and rewrite it as a fraction. Remember, a fraction with the same numerator and denominator equals 1.

Example

$$\begin{array}{r} 5\,\frac{1}{8} \\ -3\,\frac{3}{4} \\ \hline \end{array}$$

Step 1 Write the fractions with common denominators. The lowest common denominator is 8.

$$5\,\frac{1}{8} = 5\,\frac{1 \times 1}{8 \times 1} = 5\,\frac{1}{8}$$

$$-3\,\frac{3}{4} = 3\,\frac{3 \times 2}{4 \times 2} = 3\,\frac{6}{8}$$

Step 2 Because 1/8 is less than 6/8, you need to regroup, or borrow. Borrow 1 from the whole number 5, rewriting 5 as 4 8/8. Then add the fractional parts 1/8 and 8/8.

$$5\,\frac{1}{8} = 4\,\frac{8}{8} + \frac{1}{8} = 4\,\frac{9}{8}$$

$$\begin{array}{r} -3\,\frac{6}{8} \\ \hline \end{array} \qquad \begin{array}{r} -3\,\frac{6}{8} \\ \hline \end{array}$$

$$1\,\frac{3}{8}$$

Step 3 Subtract. If necessary, reduce the fraction to lowest terms

Sometimes when you subtract the fraction parts, you get a whole number as an answer. If this happens, just subtract that whole number from the other one.

Example: $4\frac{7}{5} - 2\frac{2}{5}$

4 - 2 = 2

$\frac{7}{5} - \frac{2}{5} = \frac{5}{5} = 1$ **Remember that any number divided by itself is 1.**

2 - 1 = 1 **The answer is 1.**

Mixed numbers can be subtracted from whole numbers by subtracting the whole numbers and keeping the fraction.

Example: 3 - 2 ½ = 1 ½ 3 - 2 = 1, 5 + ½ = 1 ½

Practice Exercise

Solve for each of the given problems.

1. $2\frac{6}{10} - \frac{1}{10}$	**2.** $3\frac{5}{6} - \frac{4}{6}$	**3.** $\frac{6}{9} - \frac{3}{9}$	**4.** $12\frac{1}{2} - 9\frac{1}{2}$
5. $\frac{4}{7} - \frac{3}{7}$	**6.** $\frac{6}{12} - \frac{4}{12}$	**7.** $7\frac{7}{11} - 1\frac{10}{11}$	**8.** $2\frac{3}{5} - \frac{3}{5}$
9. $\frac{3}{4} - \frac{1}{4}$	**10.** $\frac{3}{8} - \frac{2}{9}$	**11.** $9\frac{5}{10} - \frac{5}{10}$	**12.** $7\frac{2}{7} - 3\frac{1}{5}$

13. $6\frac{4}{11} - 5\frac{8}{9}$	14. $11\frac{4}{5} - 10\frac{6}{12}$	15. $10\frac{5}{6} - \frac{6}{7}$	16. $\frac{5}{8} - \frac{5}{9}$

Multiplication of Fractions

To multiply one fraction by another fraction, multiply the numerators. Their product will become the new numerator. Next, multiply the denominators. Their product will become the new denominator.

multiply the numerators

$$\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$$

multiply the denominators

$$\frac{7}{8} \times \frac{1}{3} = \frac{7}{24}$$

$$\frac{4}{3} \times \frac{1}{10} = \frac{4 \times 1}{3 \times 10} = \frac{4}{30}$$

To multiply a fraction by a whole number, change the whole number to a fraction by placing it over a

denominator of one. (This does not change the value of the whole number.) Multiply the numerators then multiply the denominators to get the product.

$$\frac{1}{2} \times 1 = \frac{1}{2} \times \frac{1}{1} = \frac{1 \times 1}{2 \times 1} = \frac{1}{2}$$

$$\frac{2}{7} \times 3 = \frac{2}{7} \times \frac{3}{1} = \frac{2 \times 3}{7 \times 1} = \frac{6}{7}$$

$$\frac{8}{9} \times 6 = \frac{8}{9} \times \frac{6}{1} = \frac{8 \times 6}{9 \times 1} = \frac{48}{9} = 5\frac{3}{9} = 5\frac{1}{3}$$

Þ ***Change improper fractions to mixed numerals. Be sure the fraction part of the mixed numeral is written in the lowest possible terms.***

To multiply mixed numerals by fractions, change the mixed numerals to improper fractions. Then multiply the fractions.

change the mixed numeral to an improper fraction

$$1\frac{6}{7} \times \frac{2}{3} = \frac{13}{7} \times \frac{2}{3} = \frac{13 \times 2}{7 \times 3} = \frac{26}{21} = 1\frac{5}{21}$$

$$2\frac{1}{8} \times 3\frac{1}{2} = \frac{17}{8} \times \frac{7}{2} = \frac{17 \times 7}{8 \times 2} = \frac{119}{16} = 7\frac{7}{16}$$

As you know, reducing a fraction means to divide the numerator and the denominator by the same number. You can use this principle to simplify before you work the problem. This process is called canceling.

Example Find 1/6 of 2/3.

Both the numerator of one fraction and the denominator of the other fraction can be divided by 2. Since $2 \div 2 = 1$, draw a slash through the numerator 2 and write 1. Since $6 \div 2 = 3$, draw a slash through the denominator 6 and write 3. Then multiply the simplified fractions.

$$\frac{1}{6} \times \frac{2}{3} = \frac{1 \times \cancel{2}^{1}}{\cancel{6}_{3} \quad 3} = \frac{1}{9}$$

Since you used canceling before multiplying, there is no need to reduce the answer: 1/6 of 2/3 is **1/9.**

When you cancel, make sure you divide a numerator and a denominator by the same number. The canceling shown in the following example is **incorrect**.

$$\frac{1}{6} \times \frac{2}{3} = \frac{1 \times 2}{\cancel{6}_{2} \quad \cancel{3}_{1}}$$

Although 6 and 3 can both be divided by 3, both numbers are in the denominator.

To multiply with mixed numbers, change the mixed numbers to improper fractions before you multiply.

Example Multiply 1 2/3 by 7 ½.

Step 1 Change to improper fractions.

$$1\frac{2}{3} \times 7\frac{1}{2} = \frac{5}{3} \times \frac{15}{2}$$

Step 2 Cancel and multiply.

$$\frac{5}{\cancel{3}_{1}} \times \frac{\overset{5}{\cancel{15}}}{2} =$$

Step 3 Write as a mixed number.

$$\frac{25}{2} = 12\frac{1}{2}$$

The product of 1 2/3 and 7 ½ is **12 ½.**

Practice Exercise

Solve for each of the given problems.
Write the answer in lowest terms.

1. $\frac{1}{2} \times \frac{2}{3}$	**2.** $\frac{2}{3} \times \frac{1}{2}$
3. $\frac{4}{7} \times \frac{7}{8}$	**4.** $\frac{1}{7} \times \frac{2}{3}$
5. $\frac{2}{7} \times \frac{3}{9}$	**6.** $\frac{2}{4} \times \frac{1}{3}$
7. $\frac{1}{3} \times \frac{2}{3}$	**8.** $\frac{4}{10} \times \frac{2}{7}$
9. $\frac{3}{4} \times \frac{6}{10}$	**10.** $\frac{2}{3} \times 9$
11. $12 \times \frac{1}{12}$	**12.** $12\frac{3}{6} \times 13$
13. $1 \times 15\frac{4}{11}$	**14.** $\frac{4}{9} \times 13\frac{2}{8}$
15. $10\frac{9}{14} \times \frac{7}{9}$	**16.** $14\frac{5}{7} \times 7\frac{2}{9}$
17. $8\frac{7}{15} \times 5\frac{4}{6}$	**18.** $6\frac{5}{11} \times 14\frac{1}{3}$

Factors are numbers that, when multiplied together, form a new number called a ***product***. For example, ***1*** and ***2*** are factors of ***2***, and ***3*** and ***4*** are factors of ***12***.

Every number except ***1*** has at least two factors: ***1*** and itself.

Common Factor

A number that is a factor of two or more numbers

Example:

factors of 6: 1, 2, 3, 6

factors of 12: 1, 2, 3, 4, 6, 12

The common factors of 6 and 12 are 1, 2, 3, and 6.

Greatest Common Factor (GCF)

The greatest factor that two or more numbers have in common

Example:

18: 1, 2, 3, **6**, 9, 18

30: 1, 2, 3, 5, **6**, 10, 15, 30

4 is the GCF of 18 and 30.

Practice Exercise

Find the greatest common factor (GCF) for the given numbers.

1. 2, 5 **1**
2. 10, 4
3. 8, 12
4. 3, 8
5. 6, 10
6. 12, 9
7. 12, 6
8. 2, 4
9. 2, 3
10. 18, 27
11. 12, 30
12. 12, 15
13. 5, 20
14. 8, 24
15. 12, 18
16. 5, 17
17. 20, 15
18. 3, 6
19. 16, 32
20. 12, 2
21. 48, 24
22. 24, 9

23. 12, 36
24. 10, 2
25. 16, 72
26. 112,160
27. 42, 63
28. 30,110
29. 42, 44
30. 80, 64

Find the least common multiple for the given numbers.

1. 6, 8 **48**
2. 5, 2
3. 4, 5
4. 5, 6
5. 6, 3
6. 9, 10
7. 12, 8
8. 6, 12
9. 11, 5
10. 15, 25
11. 4, 14
12. 28, 10
13. 18, 10
14. 27, 24
15. 6, 16

16. 22, 4
17. 5, 3
18. 12, 24
19. 23, 15
20. 7, 22
21. 16, 48
22. 2, 30
23. 12, 4
24. 13, 16
25. 25, 2
26. 150,180
27. 40, 120
28. 63, 48
29. 18, 5
30. 24, 42

Division of Fractions

To divide a fraction by a whole number, change the whole number to an improper fraction with a denominator of one. Invert the divisor fraction. Then multiply the fractions.

$$\frac{1}{2} , 2 = \frac{1}{2} , \frac{2}{1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\frac{2}{7} \div 3 = \frac{2}{7} \div \frac{3}{1} = \frac{2}{7} \times \frac{1}{3} = \frac{2}{21}$$

To divide a whole number by a fraction or to divide a fraction by another fraction, ***invert*** the divisor fraction. Then multiply the fractions.

$$\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times \frac{3}{1} = \frac{1 \times 3}{2 \times 1} = \frac{3}{2} = 1\frac{1}{2}$$

Invert the divisor fraction and multiply

$$7 \div \frac{6}{8} = \frac{7}{1} \times \frac{8}{6} = \frac{7 \times 8}{1 \times 6} = \frac{56}{6} = 9\frac{2}{6} = 9\frac{1}{3}$$

To divide a mixed numeral by another mixed numeral, first change the mixed numerals to improper fractions. Then invert the divisor fraction and multiply.

$$4\frac{1}{2} \div 2\frac{1}{3} = \frac{9}{2} \div \frac{7}{3} = \frac{9}{2} \times \frac{3}{7} = \frac{27}{14} = 1\frac{13}{14}$$

$$7\frac{6}{8} \div 6\frac{1}{3} = \frac{62}{8} \div \frac{19}{3} = \frac{62}{8} \times \frac{3}{19} = \frac{186}{152} = 1\frac{34}{152} = 1\frac{17}{76}$$

Turn it Upside Down: Inverting

Inverting a fraction means turning it upside down, or reversing the numerator and the denominator.

$\frac{1}{3}$ inverted is $\frac{3}{1}$ $\frac{6}{8}$ inverted is $\frac{8}{6}$

Inverting a whole number means to make it the denominator of a fraction with 1 as the numerator. 3 inverted is 1/3, 7 inverted is 1/7.
So, to solve the problem 1/3 ÷ 3,

invert 3 or $\frac{3}{1}$ to $\frac{1}{3}$

then $\frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9}$

Practice Exercise

Solve for each of the given problems.
Write the answer in lowest terms.

1. $\frac{1}{2} \div \frac{2}{4}$	**2.** $\frac{1}{3} \div \frac{1}{2}$
3. $\frac{3}{8} \div \frac{2}{4}$	**4.** $\frac{4}{7} \div \frac{2}{3}$
5. $\frac{2}{7} \div \frac{1}{3}$	**6.** $\frac{2}{4} \div \frac{2}{7}$
7. $\frac{6}{9} \div \frac{1}{4}$	**8.** $\frac{2}{9} \div \frac{2}{7}$
9. $\frac{6}{7} \div \frac{7}{9}$	**10.** $\frac{4}{5} \div 8$
11. $4 \div \frac{9}{11}$	**12.** $11\frac{7}{9} \div 11$
13. $7 \div 10\frac{3}{10}$	**14.** $11\frac{7}{10} \div \frac{1}{3}$
15. $\frac{8}{11} \div 3\frac{1}{3}$	**16.** $6\frac{7}{8} \div 12\frac{3}{11}$
17. $2\frac{9}{10} \div 10\frac{4}{5}$	**18.** $2\frac{7}{15} \div 2\frac{4}{6}$

Word Problems with Fractions

Using the Substitution Method

So far, you have solved addition, subtraction, multiplication, and division word problems using whole numbers. Many students can do these word problems with ease, but they worry when they see word problems using large whole numbers, fractions, or decimals.

The difficulty has to do with "math intuition," or the feel that a person has for numbers. You have a very clear idea of the correct answer to 4 – 3. It is more difficult to picture 7,483,251 + 29,983 or 6.45 – 5.5. And for most of us, our intuition totally breaks down for 3/8 – 1/3.

Changing only the numbers in a word problem does not change what must be done to solve the problem. By substituting small whole numbers in a problem, you can understand the problem and how to solve it.

Look at the following example:

Example: A floor is to be covered with a layer of ¾-in. fiberboard and 7/16-in. plywood. By how much will the floor level be raised?

Fractions, especially those with different denominators, are especially hard to picture. You can make the problem easier to understand by substituting

small whole numbers for the fractions. You can substitute any numbers, but try to use numbers under 10. These numbers do not have to look like the numbers they are replacing.

In the example, try substituting 3 for ¾ and 2 for 7/16. The problem now looks like this:

> A floor is to be covered by a layer of 3-in. fiberboard and 2-in. plywood. By how much will the floor level be raised?

You can now read this problem and know that you must add.

Once you make your decision about *how* to solve the problem, you can return the original numbers to the word problem and work out the solution. With the substituted numbers, you decided to *add* 3 and 2. Therefore, in the original, you must *add* ¾ and 7/16.

$$\frac{3}{4} = \frac{12}{16}$$

$$+\ \frac{7}{16} = \frac{7}{16}$$

$$\frac{19}{16} = \mathbf{1\ \frac{3}{16}}$$

Remember: Choosing 3 and 2 was completely up to you. You could have used any small whole numbers.

Practice Exercise

Solve for each of the given problems.

1.	Michael worked on the computer for 3 1/7 hours. Later, Michael talked to Paul on the phone for 1 1/2 hours. How many hours did Michael use the computer and talk on the phone altogether?
2.	Brad walked one-seventh of a mile from school before Mom arrived and drove Brad home. In all, Brad walked and rode in the car 6 miles. How many miles did Mom drive?
3.	Albert is baking cookies. The recipe calls for one-sixth cup of white flour per dozen. How much flour is needed to make 3 dozen cookies?
4.	Paul had 11/12 yard of string. One-fourth yard of string was used to tie newspapers. How much of the string is left (in yards)?
5.	What is seven-tenths of 24?
6.	The West Sussex book warehouse shipped a package of books that weighs 36 1/4 pounds. Each

	book in the package weighs 3 5/8 pounds each. How many books were shipped?
7.	Paul read 2/4 of a book. Of those pages, Paul read one-half of the book last week. How much of the book did Paul read last week?
8.	Jane had 7/8 yard of string. One-half of the string was used to tie newspapers. How much of the string is left (in yards)?
9.	A motorized scooter uses 8/24 gallon of gas each mile. How much gas will be used after traveling three miles?
10.	One-third of your grade is based on the final exam. One-sixth of your grade is based on homework. If the rest of your grade is based on participation, how much is participation worth?
11.	Month — Rainfall (inches) April — 4 1/4 May — 3 1/4 June — 4 1/4 July — 5 1/4 August — 3 5/8 September — 4 ¼ What is the total rainfall during these months?
12.	Amy lives 7/9 of a mile from the mall. Jill lives 8/9 of a mile from the mall. How much closer is Amy to the mall?

13.	Brad purchased a computer. Brad paid 2/4 of the $1500 price in cash and will pay the rest in five equal monthly payments. How much will Brad pay each month?
14.	Jane practices basketball 3 2/5 hours three times a week. How many hours will Jane practice each week?
15.	How much is four-fifths of one-third?
16.	How much is two-fourths of one-half?

Answer Key

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Page 5 **1.** 4/6 = 2/3 **2.** ¾ **3.** ¼ **4.** 5/6

Page 6 **1.** ½ **2.** 5/8 **3.** 2/3

Page 7 **1.** 2 6/7 **2.** 7 2/3 **3.** 12 ½ **4.** 3 3/10
5. 9 5/6 **6.** 10 5/8 **7.** 7 ¾ **8.** 7 4/5
9. 12 ¼ **10.** 9 **11.** 4 5/6 **12.** 12
13. 10 9/10 **14.** 7 4/7 **15.** 4 ¼ **16.** 2 1/3
17. 6 ½ **18.** 6 1/3 **19.** 5 4/11 **20.** 9 1/7
21. 12 2/3 **22.** 5 1/5 **23.** 10 ¼ **24.** 12
25. 4 ½ **26.** 12 **27.** 9 ¼ **28.** 6 ½
29. 6 2/3 **30.** 12 7/12 **31.** 5 9/11
32. 5 7/9

Page 8 **a.** 51/4 **b.** 199/2 **c.** 115/4 **d.** 9/4
e. 94/3 **f.** 227/3 **g.** 369/8 **h.** 515/8
i. 461/8 **j.** 15/2

Page 11 **1.** 25/30, 6/30 **2.** 6/12, 3/12 **3.** 5/6, 3/6
4. 5/10, 4/10 **5.** 22/55, 45/55
6. 16/72, 45/72 **7.** 25/40, 32/40
8. 3/18, 6/18 **9.** 28/35, 5/35
10. 60/70, 7/70 **11.** 10/45, 18/45
12. 12/24, 12/24 **13.** 36/60, 5/60
14. 42/66, 33/66 **15.** 28/84, 48/84
16. 72/132, 55/132

17. 672/784, 294/784, 224/784
18. 588/882, 378/882, 693/882

Page 13 **1.** 1/6 **2.** 3/5 **3.** 1/10 **4.** 4/5 **5.** ½
6. 4/5 **7.** 1/9 **8.** 2/3 **9.** ½ **10.** 1 5/6
11. ½ **12.** 19/34 **13.** 34/43 **14.** 2/5
15. 43/50 **16.** 1/12 **17.** 9/46 **18.** 1/8
19. 19/28 **20.** 6/7 **21.** 1 23/28 **22.** 3/7
23. 13/15 **24.** 5/11

Page 18 **1.** 30 **2.** 9 **3.** 15 **4.** 22 **5.** 30 **6.** 63
7. 10 **8.** 22 **9.** 24 **10.** 27 **11.** 84
12. 66 **13.** 10 **14.** 32 **15.** 16 **16.** 42

Page 19 **2.** > **3.** > **4.** > **5.** < **6.** < **7.** >
8. = **9.** < **10.** < **11.** > **12.** = **13.** >
14. < **15.** < **16.** >

Page 22 **1.** 7 6/7 **2.** 4/5 **3.** 5 **4.** 1 4/5 **5.** 5 ½
6. 10 5/6 **7.** 22 6/7 **8.** 1 1/12 **9.** 15 3/8
10. 13 7/20 **11.** 1 **12.** 1 39/70 **13.** 6 1/12
14. 1 2/15 **15.** 21 8/9 **16.** 10 13/18

Page 28 **1.** 2 ½ **2.** 3 1/6 **3.** 1/3 **4.** 3 **5.** 1/7
6. 1/6 **7.** 5 8/11 **8.** 2 **9.** ½
10. 11/72 **11.** 9 **12.** 4 3/35 **13.** 47/99
14. 1 3/10 **15.** 9 41/42 **16.** 5/72

Page 33 **1.** 1/3 **2.** 1/3 **3.** ½ **4.** 2/21 **5.** 2/21
6. 1/6 **7.** 2/9 **8.** 4/35 **9.** 9/20 **10.** 6
11. 1 **12.** 162 ½ **13.** 15 4/11 **14.** 5 8/9

15. 8 5/18 **16.** 106 17/63 **17.** 47 44/45
18. 92 17/33

Page 35 **2.** 2 **3.** 4 **4.** 1 **5.** 2 **6.** 3 **7.** 6
8. 2 **9.** 1 **10.** 9 **11.** 6 **12.** 3 **13.** 5
14. 8 **15.** 6 **16.** 1 **17.** 5 **18.** 3
19. 16 **20.** 2 **21.** 24 **22.** 3 **23.** 12
24. 2 **25.** 8 **26.** 16 **27.** 21 **28.** 10
29. 2 **30.** 16

Page 36 **2.** 10 **3.** 20 **4.** 30 **5.** 6 **6.** 90
7. 24 **8.** 12 **9.** 55 **10.** 75 **11.** 28
12. 140 **13.** 90 **14.** 216 **15.** 48
16. 44 **17.** 15 **18.** 24 **19.** 345
20. 154 **21.** 48 **22.** 30 **23.** 12
24. 208 **25.** 50 **26.** 900 **27.** 120
28. 1008 **29.** 90 **30.** 168

Page 40 **1.** 1 **2.** 2/3 **3.** ¾ **4.** 6/7 **5.** 6/7
6. 1 ¾ **7.** 2 2/3 **8.** 2/5 **9.** 1 5/49
10. 1/10 **11.** 4 8/9 **12.** 1 7/99
13. 70/103 **14.** 35 1/10 **15.** 12/55
16. 121/216 **17.** 29/108 **18.** 37/40

Page 43 **1.** 4 9/14 hours **2.** 11 5/7 miles **3.** ½cup
4. 2/3 of a yard **5.** 16 4/5 **6.** 10 books
7. ¼ of a book **8.** 7/16 of a yard
9. 1 gallon of gas **10.** ½of your grade
11. 24 7/8 inches **12.** 1/9 of a mile
13. $150 each month **14.** 10 1/5 hours
15. 4/15 **16.** ¼