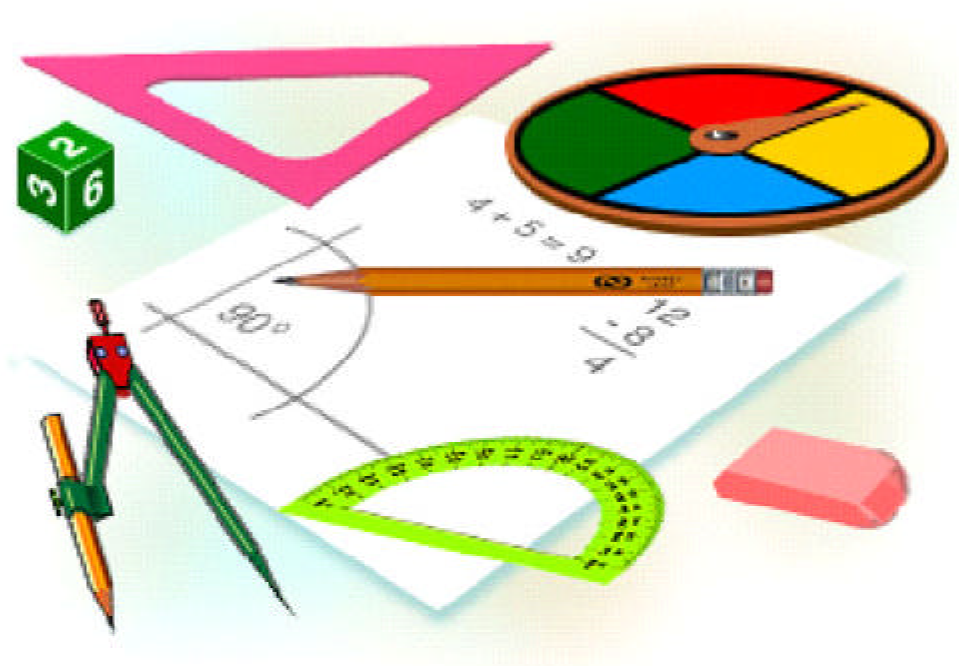
The Next Step

Mathematics Applications for Adults



Book 14016 – Whole Numbers

INTRODUCTION

Why Math?

The most important reason for learning math is that it teaches us how to think. Math is more than adding and subtracting, which can easily be done on a calculator; it teaches us how to organize thoughts, analyze information, and better understand the world around us.

Employers often have to re-educate their employees to meet the demands of our more complex technological society. For example, more and more, we must be able to enter data into computers, read computer displays, and interpret results. These demands require math skills beyond simple arithmetic.

Everyone Is Capable of Learning Math

There is no **type** of person for whom math comes easily. Even mathematicians and scientists spend a lot of time working on a single problem. Success in math is related to practice, patience, confidence in ability, and hard work.

It is true that some people can solve problems or compute more quickly, but speed is not always a measure of understanding. Being “faster” is related to **more practice or experience.**

For example, the reason why math teachers can work problems quickly is because they've done them so many times before, not because they have "mathematical minds".

Working with something that is familiar is natural and easy. For example, when cooking from a recipe we have used many times before or playing a familiar game, we feel confident. We automatically know what we need to do and what to expect. Sometimes, we don't even need to think. However, when using a recipe for the **first** time or playing a game for the **first** time, we must concentrate on each step. We double-check that we have done everything right, and even then we fret about the outcome. The same is true with math. When encountering problems for the very first time, **everyone must have patience** to understand the problem and work through it correctly.

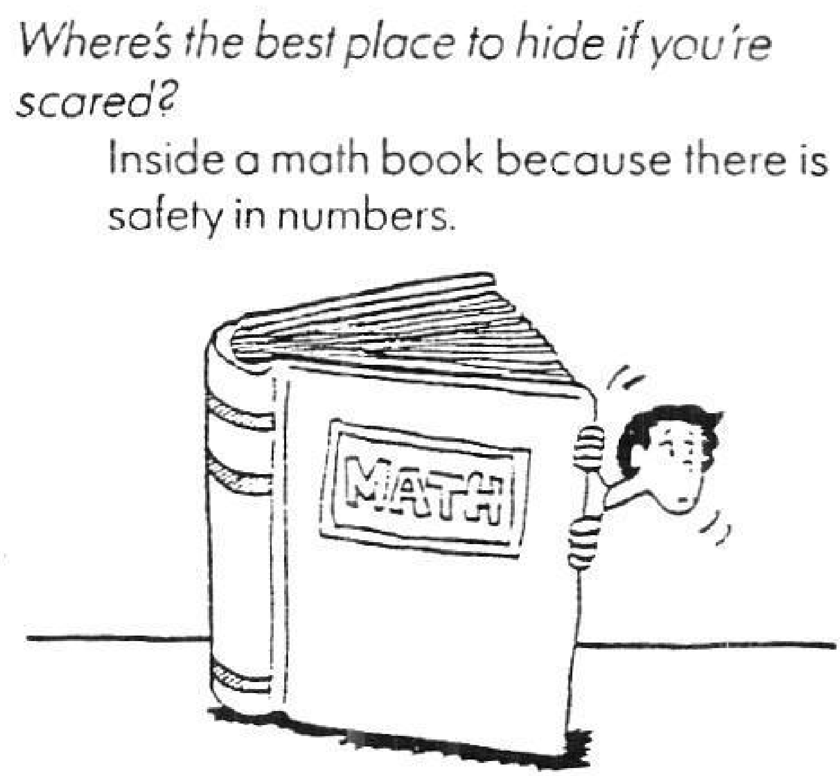
It's Never Too Late to Learn

One of the main reasons people don't succeed in math is that they don't start at the right place. **IMPORTANT! You must begin where *you* need to begin.** Could you hit a homerun if you hadn't figured out which end of the bat had to make contact with the ball? Why should learning math be any different?

If it has been a while since your last math class, **you must determine what level math you should take.** A teacher or trained tutor can help determine this with a few placement tests and questions.

Sometimes a few tutoring sessions can help you fill gaps in your knowledge or help you remember some of the things you have simply forgotten. It could also be the case where your foundations may be weak and it would be better for you to relearn the basics. **Get some help** to determine what is best for you.

Feeling good about ourselves is what all of us are ultimately striving for, and nothing feels better than conquering something that gives us difficulty. This takes a great deal of courage and the ability to rebound from many setbacks. This is a natural part of the learning process, and when the work is done and we can look back at our success, nothing feels better.



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OUTLINE

Mathematics - Book 14016

Whole Numbers
Number/Word Recognition
convert Arabic numbers to Roman numerals and vice versa (I – M…1 – 1,000).
read dates written in Roman numerals.
correctly write the number words for Arabic numbers (0 – 1,000,000).
correctly write the Arabic numerals for any number word (0 – 1,000,000).
Place Value
recognize the place value of each digit of a number to the million's place.
determine how many hundred thousands, thousands, hundreds, tens and ones in any number (0 – 1,000,000).
round off whole numbers to the nearest million, thousand, hundred, ten, and one.
Counting
count orally from 0 – 1,000,000 starting at any point in between those numbers.
count orally by 2's, 5's, and 10's to 100.
write all the even numbers from 2 - 100 and all the odd numbers from 1 - 99.
order numbers from greatest to least and least to greatest. (0 – 1,000,000).
Addition

find the sum of whole numbers up to 6 digits each.
use addition facts to compute sums up to and including 18.
Subtraction
subtract two whole numbers up to 6 digits (using borrowing/regrouping).
use subtraction facts to compute differences up to and including 18.
apply addition/subtraction skills by completing an incomplete equation (e.g. 14 + ? = 37).
Multiplication
multiply 3 digit factors by 3 digit factors.
write the times tables to 12 x 12 (within a specified time).
multiply by 1, 10, 100 quickly (within a specified time).
Division
explain factoring.
find the factors of a given list of products.
identify prime numbers from a given list.
how to calculate average.
when to use averages.
Word Problems with Whole Numbers
solve one/two step problems with addition, subtraction, multiplication or division of whole numbers.

THE NEXT STEP

Book 14016

Whole Numbers

Number Recognition

Digit is a counting word. A digit is any of the numerals from **1** to **9.** The word "digit" is also the name for a finger. So number digits can be counted on finger digits.

Our modern system of counting probably came from counting on fingers. Fingers and hands were among the earliest known calculators!

A CLOSER LOOK AT ROMAN NUMERALS

Roman numerals are still used today, more than 2000 years after their introduction. The history of Roman numerals is not well documented and written accounts are contradictory. Roman numerals are read from left to right.

It is likely that counting began on the fingers and that is why we count in tens.

A single stroke I represents one finger, five or a handful could possibly be represented by V and the X may have been used because if you stretch out two handfuls of fingers and place them close the two little fingers cross in an X. Alternatively, an X is like two Vs, one upside down.

The Roman numeral system uses seven letters to represent numbers. Combinations of these letters represent other numbers.

I = 1		**C = 100**	
V = 5		**D = 500**	
X = 10		**M = 1,000**	
L = 50			

Combining Roman Numerals

Here are the three rules for making numbers with Roman numerals:

1. If you put numbers of the same size together, then you add them.
   II=1+1=2
   XX=10+10=20
   XXX=10+10+10=30

2. If you put a small number to the right of a large number, then you add them, too.
   XV=10+5=15
   VIII=5+3=8
3. If you put a small number to the left of a large number, then you subtract the small one from the big one.

   IX=10-1=9
   CM=1000-100=900

Sample Numbers:

Letter	**Value**
I	1
II	2
III	3
IV	4
V	5
VI	6
VII	7
VIII	8
IX	9
X	10
XX	20
XXX	30
XL	40
L	50

LX	60
LXX	70
LXXX	80
XC	90
C	100
CC	200
CCC	300
CD	400
D	500
DC	600
DCC	700
DCCC	800
CM	900
M	1,000

How are larger numbers expressed using Roman numerals? A modern method has been developed.

Letter	**Value**
$\overline{V}$	5,000
$\overline{X}$	10,000
$\overline{L}$	50,000
$\overline{C}$	100,000
$\overline{D}$	500,000

$\overline{M}$	1,000,000

$\overline{M}$ represents 1,000,000—a small bar placed over the numeral multiplies the numeral by 1000. Thus, theoretically, it is possible, by using an infinite number of bars, to express the numbers from 1 to infinity. In practice, however, one bar is usually used; two are rarely used, and more than two are almost never used.

These are some examples of the use of Roman Numerals.

I. Chapters. Look at the chapter headings of any book you are using.

II. Buildings.

III. Movie copyright years.

IV. Tombstones

IV. Clocks and watches. Look at clocks and watches closely, especially how they represent the number **four**.

A NEW THEORY ABOUT IIII

A lot of people ask 'Why is the number 4 on a clock-face depicted as IIII and not as IV?' There is no certain answer to this question. One common suggestion is that around the circle the IIII balances the VIII which is in its mirror-symmetrical place –

that is if a mirror was placed vertically between the XII and VI, the VIII and IIII would reflect on to each other. There are problems with this theory – the V does not balance the VII, nor the I the XI. Another plausible explanation might be that IV has three strokes and is more likely to be confused with the neighbouring III, as both are at unfamiliar angles to the reader. But neither really offers an adequate explanation of why the normal rules of Roman numerals have been broken.

The oldest surviving clock-face in its original condition is on the clock inside Wells Cathedral in Somerset, England. It dates from before 1392 and the original mechanism – now in the Science Museum – has some claim to be the oldest surviving clock works in the world. The current mechanism that drives it is Victorian, but the face has not been changed for more than 600 years.

The outermost circle is more than six feet (1.93m) in diameter and around it, in Roman numerals, are the twenty four hours of the day with the 4 indicated by IIII.

Exceptions

The practice of using IIII rather than IV on clock-faces, although common, is not universal. The well-known clock, commonly called Big Ben, at the Palace of Westminster in London (where Parliament meets) has gothic style Roman numerals round its face and the 4 is depicted as iv. (Strictly speaking, the hour bell is Big Ben, the clock is the Great Clock, and the tower is the Clock Tower and although the building is correctly called the Palace of Westminster, most people refer to it as the Houses of Parliament). Other examples of an IV on a clock-face are rare in

England - but the clock in the South Transept of Norwich Cathedral is one example. Others are found in Spain. San Sebastian in northern Spain has at least two clocks – one on the cathedral and one on another church – which both have clear plain Roman numerals on the dial and which use IV for the 4.

Let's say that you wanted to read a date on a tombstone or building cornerstone. Most likely the date would have been etched into the stone as a Roman numeral.

What would the number MDCCXLVIII represent?

MDCCXLVIII =	M +	D +	C +	C +	XL +	V +	III
	1,000	500	100	100	(50-10)	5	3

MDCCXLVIII = 1748

Practice Exercise

What number does each of the following Roman numerals represent?

1. XXXVI
2. CLXV
3. MDCLIX
4. MCCLIV
5. MCMXCIII
6. MMXLVII

Write a Roman numeral for each of the following numbers:

7. 19
8. 299
9. 847
10. 1492
11. 1776
12. 2015
13. A cornerstone is marked MCMXIX. What date does it represent?

What number is represented by the Roman numeral in each of the following:

14. Page VII
15. Unit XLVI
16. Item CXC

Write the Roman numeral that comes next after each of the following:

17. XIII
18. XXIV
19. XCIX
20. XLVIII

Number/Word Recognition

Every number can be written two ways.
It can be written as a numeral.
Or it can be written as a word.
The numeral and word stand for the same thing.

Numeral	Word
0	zero
1	one
2	two
3	three
4	four
5	five
6	six
7	seven
8	eight
9	nine

Learn to say these 2-place numbers:

10	ten
11	eleven
12	twelve
13	thirteen
14	fourteen
15	fifteen
16	sixteen
17	seventeen
18	eighteen
19	nineteen

The 2-place numbers go from 10 (ten) to 99 (ninety-nine).
We have just learned about the 2-place numbers from 10 to 19.
Now learn these 2-place numbers:

20	twenty
21	twenty-one
22	twenty-two
23	twenty-three
24	twenty-four
25	twenty-five
26	twenty-six
27	twenty-seven
28	twenty-eight
29	twenty-nine
30	thirty
31	thirty-one
32	thirty-two
33	thirty-three

34	thirty-four
35	thirty-five
36	thirty-six
37	thirty-seven
38	thirty-eight
39	thirty-nine
40	forty
41	forty-one
42	forty-two
43	forty-three
44	forty-four
45	forty-five
46	forty-six
47	forty-seven
48	forty-eight
49	forty-nine
50	fifty
51	fifty-one
52	fifty-two
53	fifty-three
54	fifty-four
55	fifty-five
56	fifty-six
57	fifty-seven
58	fifty-eight
59	fifty-nine
60	sixty
61	sixty-one
62	sixty-two
63	sixty-three
64	sixty-four

65	sixty-five
66	sixty-six
67	sixty-seven
68	sixty-eight
69	sixty-nine
70	seventy
71	seventy-one
72	seventy-two
73	seventy-three
74	seventy-four
75	seventy-five
76	seventy-six
77	seventy-seven
78	seventy-eight
79	seventy-nine
80	eighty
81	eighty-one
82	eighty-two
83	eighty-three
84	eighty-four
85	eighty-five
86	eighty-six
87	eighty-seven
88	eighty-eight
89	eighty-nine
90	ninety
91	ninety-one
92	ninety-two
93	ninety-three
94	ninety-four
95	ninety-five

96	ninety-six
97	ninety-seven
98	ninety-eight
99	ninety-nine

The number 99 is the greatest 2-place number.
The next number in order is 100 (one hundred).

100 is one more than 99.
It is a 3-place number.
It has three numerals: 1, 0, and 0.
They stand for 1 hundred, 0 tens, and 0 ones

The greatest 3-place number is 999 (nine hundred ninety-nine).
It stands for 9 hundreds, 9 tens, and 9 ones.

Every 3-place number tells how many hundreds, tens, and ones the number stands for.

The number 999 is the greatest 3-place number.
The next number in order is 1,000 (one thousand).
It is one more than 999.
It is a 4-place number.
It has four numerals: 1, 0, 0, and 0.
They stand for 1 thousand, 0 hundreds, 0 tens, and 0 ones.

The greatest 4-place number is 9,999
(nine thousand, nine hundred ninety-nine).
The number after 9,999 is 10,000 (ten thousand).
Ten thousand is a 5-place number.

The greatest 5-place number is 99,999 (ninety-nine thousand, nine hundred ninety-nine).
The number after 99,999 is 100,000
(one hundred thousand).
One hundred thousand is a 6-place number.

The greatest 6-place number is 999,999 (nine hundred ninety-nine thousand, nine hundred ninety-nine).
The number after 999,999 is 1,000,000 (one million).
One million is a 7-place number

We use a comma after the number in the thousands' place.
The comma makes large numbers easier to read.

Practice Exercise

Write the numeral as a number word.
The first one has been done for you.

1. 37,215 **thirty-seven thousand, two hundred fifteen**

2. 42,499 ____

3. 45,291 ____

4. 10,148 ____

5. 9,030 ____

6. 33,000 ______________________________

7. 51,935 ______________________________

8. 1,099,868 ______________________________

9. 1,094,442 ______________________________

10. 1,570,000 ______________________________

11. 1,098,329 ______________________________

12. 1,964,835 ______________________________

13. 1,668,100 ______________________________

14. 1,010,521 ______________________________

15. 1,456,142 ______________________________

16. 1,942,818 ______________________________

17. 1,362,825 ______________________________

18. 1,011,431 ______________________________

Read the number word and write the number.

1. eighty thousand, thirty-seven

   80,037

2. four thousand, five hundred sixty-seven

   ____

3. ninety-two thousand, seven hundred thirty-four

   ____

4. two thousand, seventy-six

   ____

5. thirty-four thousand, six hundred seventy-six

   ____

6. twenty-three thousand

   ____

7. seventy-seven thousand, nine hundred

   ____

8. one million, sixty-three thousand, five hundred sixty-one

   ____

9. one million, sixty-six thousand, four hundred eleven

   ____

10. one million, nine hundred sixty thousand

   ____

11. one million, sixty thousand, seven hundred eight

   ____

12. one million, six hundred seventy-one thousand, eight hundred forty-three

____

13. one million, forty-eight thousand, five hundred thirty-four

____

14. one million, four hundred ninety-seven thousand, three hundred two

____

[Place Value]

To read the place value of numerals in a number, read from left to right.
Each column has a value 10 times greater than the column to its right.

Place Value

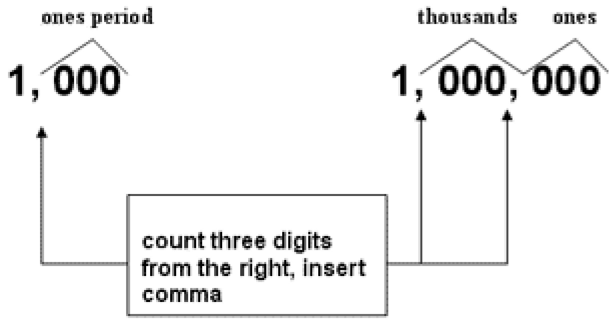
The value of a digit as determined by its position in a number

Example:

	PLACE VALUE									
	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
1, 623, 051→	1	6	2	3	0	5	1			
0.053→							0	0	5	3
32.4→						3	2	4		

Periods

Three places in the place value chart make up a ***period***. Periods are always counted from the right---from the "ones" column---of a number. Periods are separated in numerals by commas.



Millions Period			Thousands Period			Ones Period		
Hundred Millions	**Ten Millions**	**Millions**	**Hundred Thousands**	**Ten Thousands**	**Thousands**	**Hundreds**	**Tens**	**Ones**
100,000,000 **900,000,000**	**10,000,000** **90,000,000**	**1,000,000** **9,000,000**	**100,000** **900,000**	**10,000** **90,000**	**1,000** **9,000**	**100** **900**	**10** **90**	**1** **9**

Practice Exercise

Millions, Thousands, Hundreds, Tens, and Ones

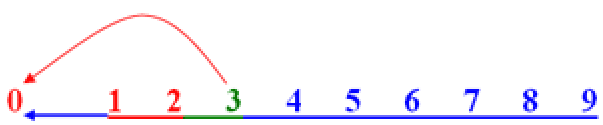
Write the place value of the bold number in each numeral.

1. 11,513,5**1**2	Tens	2. 97**2**,329,932	________________
3. 2,388,7**1**8	________________	4. 4,57**3**	________________
5. 24,1**7**7	________________	6. **7**74,465	________________
7. 496,283,6**2**2	________________	8. 7,**4**78,314	________________
9. 33,74**6**	________________	10. 5**4**5,383	________________
11. 7,7**1**8	________________	12. 55,793,5**5**8	________________
13. **3**,211	________________	14. 20**3**,244,825	________________
15. 42,66**6**,497	________________	16. 2**5**9,154	________________
17. 1,2**1**7,992	________________	18. 86,85**9**	________________
19. 52,318,6**4**7	________________	20. 5,312,**1**48	________________
21. 3,98**5**	________________	22. 1**3**4,293	________________
23. **6**2,885	________________	24. 578,59**9**,999	________________
25. 9,19**4**	________________	26. 92,375,46**9**	________________
27. 637,5**5**2	________________	28. 92,**8**27	________________

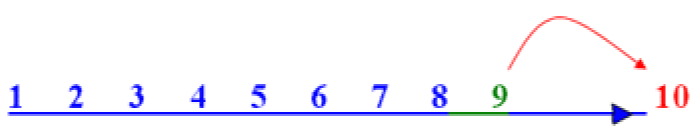
What's Rounding?

Rounding means to express a number to the nearest given place. The number in the given place is increased by one if the digit to its right is 5 or greater. The number in the given place remains the same if the digit to its right is less than 5. When rounding whole numbers, the digits to the right of the given place become zeros (digits to the left remain the same). When rounding decimal numbers, the digits to the right of the given place are dropped (digits to the left remain the same).

If you are rounding 3 to the nearest tens place, you would round down to 0, because 3 is closer to 0 than it is to 10.



If you were rounding 9, you would round up to 10.



General Rule for Rounding to the Nearest 10, 100, 1,000, and Higher!

Round down from numbers under 5 and round up from numbers 5 and greater.

The same holds true for multiples of 10. Round to the nearest 100 by rounding down from 49 or less and up from 50 or greater. Round to the nearest 1,000 by rounding down from 499 or less and up from 500 or greater.

Why Round?

Sometimes you have to figure out a math problem without using a pencil and paper or a calculator. Rounding numbers makes them easier to work with. Check out the shopping lists below. One list tells the prices of five items. The other list shows the same prices, rounded to the nearest ten cents. Which list is easier to add in your head?

Real Prices		**Prices Rounded**	
Pencil	**.29**	**Pencil**	**.30**
Eraser	**.19**	**Eraser**	**.20**
Stickers	**1.39**	**Stickers**	**1.40**
Notepad	**.89**	**Notepad**	**.90**
Pen	**.59**	**Pen**	**.60**
Total	**3.35**	**Total**	**3.40**

Practice Exercise

Round to the nearest <u>thousands place</u>.

1. 13,260	13,000	2. 9,459	________________
3. 3,113	________________	4. 23,175	________________
5. 95,276	________________	6. 13,705	________________
7. 81,724	________________	8. 7,848	________________
9. 7,590	________________	10. 7,094	________________
11. 9,072	________________	12. 13,893	________________
13. 2,942	________________	14. 16,168	________________
15. 1,093	________________	16. 2,295	________________

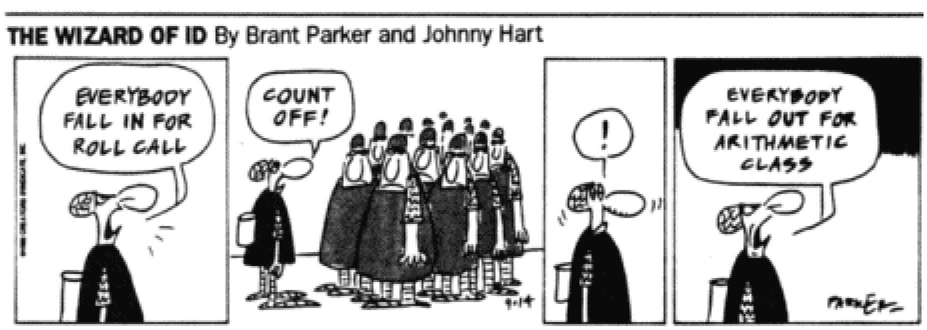
Round to the nearest <u>hundreds place</u>.

17. 361	400	18. 17,341	________________
19. 2,362	________________	20. 322	________________
21. 2,664	________________	22. 16,392	________________
23. 948	________________	24. 291	________________
25. 984	________________	26. 49,744	________________
27. 7,118	________________	28. 977	________________
29. 339	________________	30. 70,356	________________
31. 54,973	________________	32. 627	________________

Round to the nearest <u>tens place</u>.

33. 2,293	2,290	34. 669	________________
35. 38	________________	36. 845	________________
37. 58	________________	38. 26,322	________________
39. 47,958	________________	40. 95	________________
41. 79,313	________________	42. 712	________________
43. 66	________________	44. 24,092	________________
45. 94	________________	46. 50,398	________________
47. 67,284	________________	48. 523	________________

Counting



Wizard of ID by *Brant Parker & Johnny Hart*

The set of counting numbers has no end. It can go on forever. The idea that counting numbers can go on and on is called ***infinity***. Infinity has a special symbol:

$$\infty$$

There is no such thing as the "largest number." You can always add to or multiply a large number to make an even bigger number.

$$\infty + 3 = \infty$$

$$\infty \times 10 = \infty$$

If you began writing all the counting numbers today, you could continue writing every moment of every day for every day of the rest of your life and never be finished!

What's a googol?

A googol is a 1 with a hundred zeroes behind it. We can write a googol using exponents by saying a googol is 10^100 or 10 to the 100^{th} power.

The biggest named number that we know is googolplex, ten to the googol power, or (10)^(10^100). That's written as a one followed by googol zeroes.

It's funny that no one ever seems to ask, "What is the smallest number?" Again, there is really no such thing. You could always subtract from or divide a small number to make an even smaller number. As the number gets smaller and smaller, you would be approaching, but never reaching, negative infinity.

$$-\infty$$

The set of ***counting numbers***, or ***natural numbers***, begins with the number **1** and continues into infinity.

{1,2,3,4,5,6,7,8,9,10...}

The set of ***whole numbers*** is the same as the set of counting numbers, except that it begins with **0**.

{0,1,2,3,4,5,6,7,8,9,10...}

☞ ***All counting numbers are whole numbers. Zero is the only whole number that is not a counting number.***

Even numbers include the numbers ***0*** and ***2*** and all numbers that can be divided evenly by ***2***. ***Odd numbers*** are all numbers that cannot be divided evenly by ***2***.

Odd and **Even** Numbers to 100

1	3	5	7	9	11	13	15	17	19	21
0	2	4	6	8	10	12	14	16	18	20
23	25	27	29	31	33	35	37	39	41	
22	24	26	28	30	32	34	36	38	40	
43	45	47	49	51	53	55	57	59	61	
42	44	46	48	50	52	54	56	58	60	
63	65	67	69	71	73	75	77	79	81	
62	64	66	68	70	72	74	76	78	80	
83	85	87	89	91	93	95	97	99		
82	84	86	88	90	92	94	96	98	100	

Skip Counting

To count by 2's, simply count all the even numbers: 0, 2, 4, 6, 8, 10…and so on.

To count by 5's: 0, 5, 10, 15, 20…and so on.

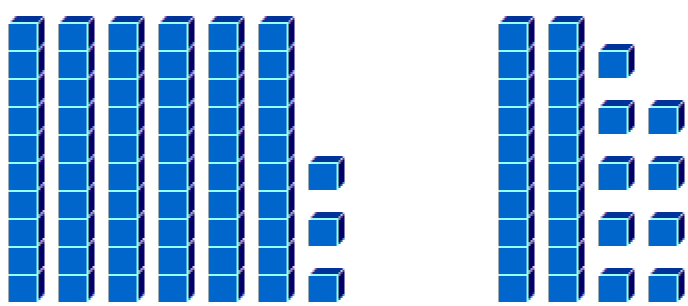
To count by 10's: 0, 10, 20, 30, 40…and so on.

To count by 100's: 0, 100, 200, 300, 400…and so on.

Ordering numbers means listing numbers from least to greatest, or from greatest to least. Two symbols are used in ordering.

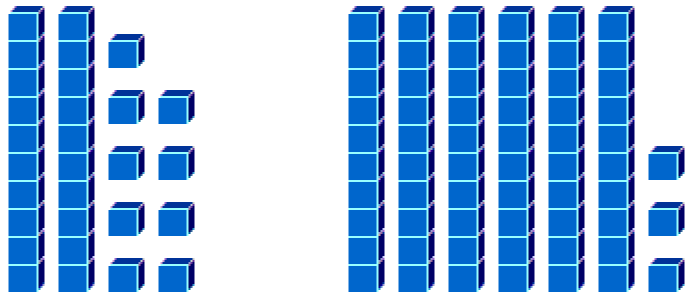
<	>
is less than	**is greater**
2 < 10	**10 > 2**

Greater Than >



63 is **greater than** 29.
63 > 29

<u>Less Than <</u>



29 is **less than** 63.
29 < 63

Practice Exercise

Fill in the blanks to complete the numerical sequence.
The first one has been done for you.

1.	8	9	10	11	**12**	**13**
2.	5	10	______	20	______	30
3.	18	20	______	24	26	30
	______					
4.	10	20	30	______	50	______
	70					
5.	106	107	______	109	______	111
	112					

6.	3,214	3,216	3,218	_______	3,222	_______
	3,226					
7.	45,625	45,630	45,635	45,640	_______	45,650
	_______					
8.	789,060	789,070	789,080	789,090	789,100	_______
	_______					
9.	95	96	97	98	_______	100
	_______					
10.	1,000	998	996	_______	992	_______
11.	210	205	_______	195	190	185
	_______	_______				

Compare the two numbers.
In the middle of the two numbers, write either **>** (greater than), **<** (less than), or = (equals) to complete the problem.

1.	604	>	367
2.	933	_____	356
3.	two hundred fifty-three	_____	eight hundred thirty-four
4.	510	_____	293
5.	299	_____	390

6.	673	_____	357
7.	two hundred ninety-nine	_____	eight hundred nineteen
8.	eight hundred fifty-nine	_____	652
9.	one hundred seventy-two	_____	699
10.	16	_____	903
11.	191	_____	470
12.	419	_____	580
13.	877	_____	286
14.	207	_____	454
15.	55	_____	83
16.	29	_____	702
17.	27	_____	65
18.	459	_____	895
19.	26	_____	39
20.	822	_____	330
21.	eight hundred twenty-five	_____	
22.	41	_____	224
23.	895	_____	28
24.	eight hundred eighty-three	_____	

Arrange the numbers from greatest to least.

1. 172 909 , 279 000 , 160 909 , 20 999
2. 135 620 , 513 527 , 315 427 , 315 727
3. 802 250 , 802 520 , 999 999 , 255 000

Arrange the numbers from least to greatest.

1. 266 225 , 662 226 , 226 216 , 626 225
2. 90 395 , 890 359 , 92 953 , 190 595
3. 822 320 , 882 220 , 228 302 , 882 202

Addition

Combining two or more numbers is called ***addition***. The term for addition is ***plus***, and the symbol for plus is +. The numbers that are combined in addition are called ***addends*** and together they form a new number called a ***sum***.

2 ---- addends ---- 3
+ 2 + 1
4 ---- sum --------- 4

Adding whole numbers is as simple as 2 + 2! To add two whole numbers, you can simply follow the number line and complete the addition fact.

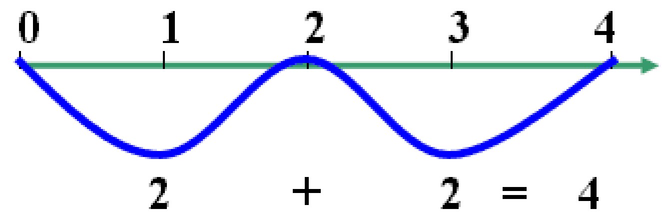
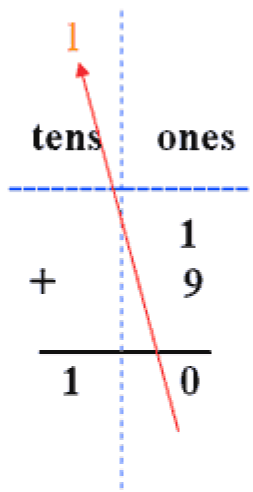


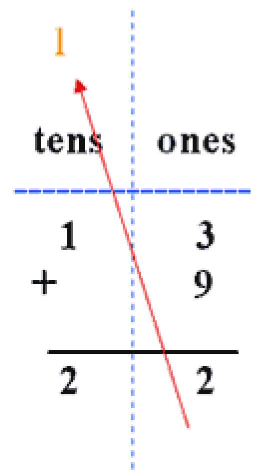
Table of Addition Facts

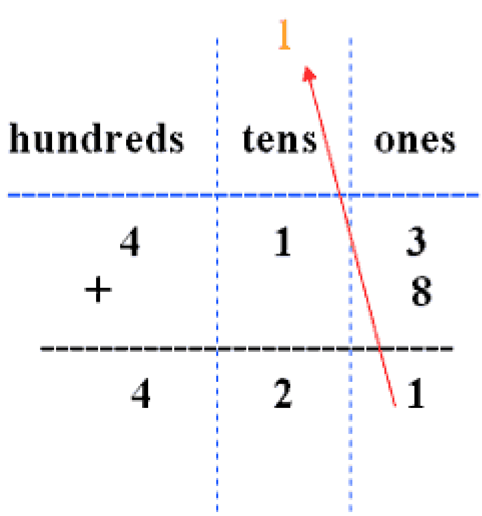
+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

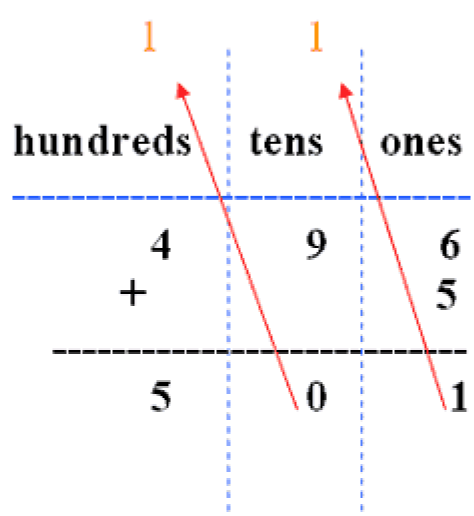
Regrouping Numbers in Addition

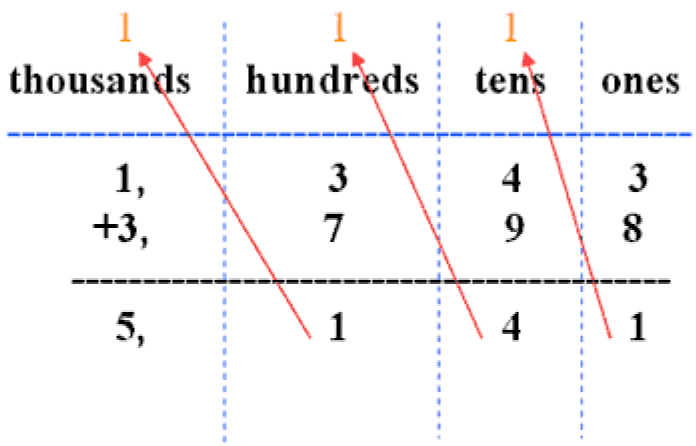
Addition often produces sums with a value greater than ***9*** in a given place. The value of ten is then ***regrouped*** (or ***carried***) to the next place.











To explain addition another way, it can be done by adding the place value amounts separately.

e.g. 69
+ 8
17
60 (the 6 in the tens place means 6 tens or "60")
77

⇒ If there are not enough digits in each number to make even columns under each place value, then zeros may be used **before** a given number to make adding easier. Do **not** add zeros **after** a number because it changes the value of the whole number.

e.g. 69 + 8 + 125 could be added as:

069
008
+125

Commutative Property of Addition

The property which states that addends can be added in any order. The sum is always the same

Example:

2.67 + 1.32 = 1.32 + 2.67

3.99 = 3.99

Associative Property

Addends can be grouped differently; the sum is always the same.

Example:

(8 + 7) + 4 = 8 + (7 + 4)

Practice Exercise

1. 26 +12	**2.** 481 +23	**3.** 4321 +103	**4.** 32452 +2667
5. 33 +32	**6.** 49 + 9	**7.** 3283 + 31	**8.** 3694 +270
9. 23 +15	**10.** 221 +13	**11.** 4625 +403	**12.** 176 + 9
13. 21 +18	**14.** 354 +19	**15.** 4757 +226	**16.** 38788 +1290

17. 389 + 13 + 33

18. 80 + 12 + 18

19. 1011 + 23 + 18

20. 1022 + 123 + 15

17.	**389**	**18.**	**80**	**19.**	**1011**	**20.**	**1022**
	13		**12**		**23**		**123**
	+33		**+18**		**+18**		**+15**
21.	**3643**	**22.**	**394**	**23.**	**205**	**24.**	**276**
	115		**5**		**30**		**19**
	+386		**+ 9**		**+ 8**		**+18**

25.	**397877**	**26.**	**349080**
	368901		**331234**
	234567		**123456**
	+118901		**+956789**

27. 99 + 2 + 24 + 16 = ______

28. 270 + 22 + 12 + 14 = _____

29. 131 + 0 + 20 = ______

30. 192 + 4 + 13 + 0 = ______

31. 18 + 834 + 2256 + 478 = ___

32. 3143 + 20 + 20 = ______

33. 179054 + 1712 + 3534 = ____

34. 3365 + 13 + 11 + 9 = _____

35. 1378 + 1490 + 6123 = ______

36. 34 + 165 + 26 + 297 = ____

37. 246 + 18 + 2 = ______

38. 8 + 269 + 350 + 221 = ____

39. 27 + 23 + 2 + 19 = ______

40. 1128 + 193 + 114 = ______

41. 319 + 11 + 9 + 28 = ______

42. 1 + 236 + 277 + 128 = ____

Subtraction

"Taking away" one or more numbers from another number is called ***subtraction***. The term for subtraction is ***minus***, and the symbol for minus is **-**. The number being subtracted is called a ***subtrahend***. The number being subtracted from is called a

minuend. The new number left after subtracting is called a ***remainder*** or ***difference***.

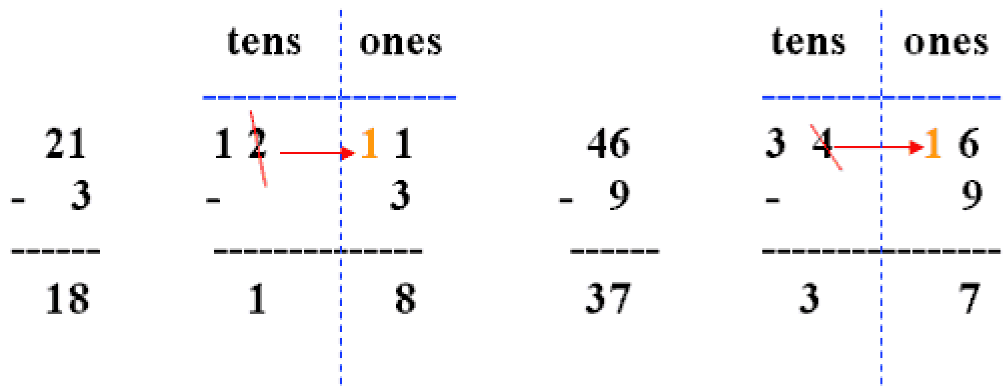
4	---- **minuend** ----	4
- 2	--**subtrahend** -	- 1
2	– **difference** ----	3

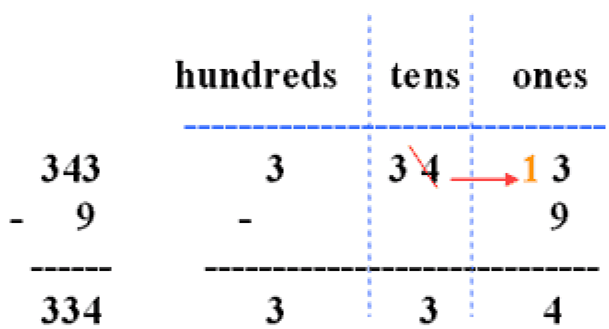
The complete addition or subtraction "sentence" is called an ***equation***. An equation has two parts. The two parts are separated by the ***equal sign***, =. For example, ***the minuend minus the subtrahend equals the difference***. An ***addition fact*** or a ***subtraction fact*** is the name given to specific addition or subtraction equations.

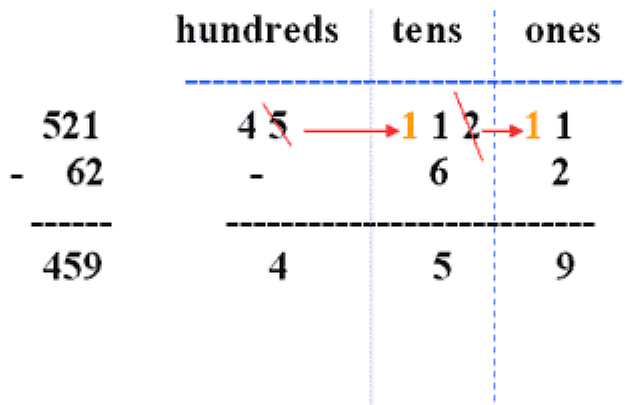
0 + 1 = 1	**1 – 1 = 0**
1 + 1 = 2	**2 – 1 = 1**
2 + 1 = 3	**3 – 1 = 2**
3 + 1 = 4	**4 – 1 = 3**
4 + 1 = 5	**5 – 1 = 4**
5 + 1 = 6	**6 – 1 = 5**
6 + 1 = 7	**7 – 1 = 6**
7 + 1 = 8	**8 – 1 = 7**
8 + 1 = 9	**9 – 1 = 8**

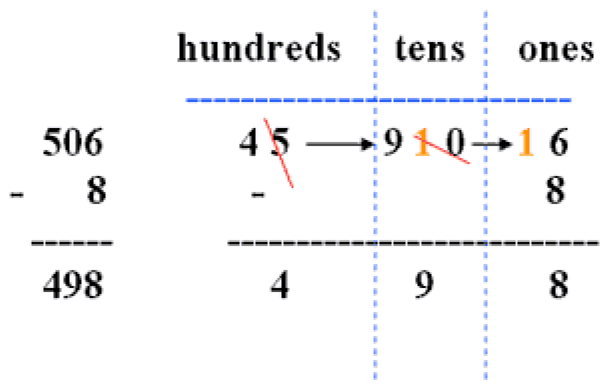
Regrouping in Subtraction

Regrouping, sometimes called ***borrowing***, is used when the subtrahend is greater than the minuend in a given place. Regrouping means to take a group of tens from the next greatest place to make a minuend great enough to complete the subtraction process.









Practice Exercise

Solve for each of the given problems.

1. 291 - 64	2. 756 – 220	3. 889 - 380	4. 842 - 529
5. 7,110 - 5,431	6. 12,733 -5,812	7. 40,782 - 38,073	8. 22,235 - 12,757

9. 41,969 **- 19,756**	**10.** 68,932 **- 63,034**	**11.** 46,775 **- 42,402**	**12.** 17,882 **- 2,416**
13. 767,851 **- 649,634**	**14.** 834,101 **- 648,103**	**15.** 430,492 **- 272,645**	**16.** 774,056 **- 160,412**
17. 535,823 **- 232,711**	**18.** 649,153 **- 149,334**	**19.** 629,063 **- 511,007**	**20.** 878,709 **- 400,376**

Solving Incomplete Addition or Subtraction Equations

Inverse (opposite) operations are used to simplify an equation for solving.
One operation is involved with the unknown and the inverse operation is used to solve the equation.
Addition and subtraction are inverse operations.
Given an equation such as 7 + x = 10, the unknown x and 7 are *added*. Use the inverse operation subtraction. To solve for n, subtract 7 from 10. The unknown value is therefore 3.

Examples for addition and subtraction

Addition Problem	Solution
x + 15 = 20	x = 20 - 15 = 5

Subtraction Problem	Solution
x - 15 = 20	x = 20 + 15 = 35

Practice Exercise

Solve each equation.
(Hint: Use inverse operation rules to solve)

1. 41 = a - 16 57
2. x + 31 = 51 ______
3. 78 = a - 22 ______
4. x - 67 = 18 ______
5. x - 27 = 59 ______
6. 60 + y = 141 ______
7. 19 = a - 57 ______
8. x - 59 = 38 ______
9. 57 + y = 119 ______
10. 3 = a - 36 ______
11. x + 80 = 124 ______
12. 92 + y = 171 ______
13. x + 81 = 102 ______
14. x + 83 = 100 ______
15. 83 = a - 3 ______
16. 31 = a - 26 ______
17. x + 67 = 108 ______
18. x - 64 = 28 ______

Multiplication

Multiplication is a quick form of addition. By multiplying numbers together, you are really adding a series of one number to itself. For example, you can add ***2*** plus ***2***. Both ***2 plus 2*** and ***2 times 2*** equal ***4***.

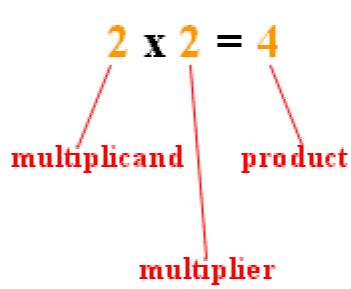
2 + 2 = 4
2 x 2 = 4

```
   2      2
 + 2    x 2
 ----   ----
   4      4
```

But what if you wanted to calculate the number of days in five weeks? You could add *7* days + *7* days + *7* days + *7* days + *7* days or you could multiply *7* days times ***5***. Either way you arrive at ***35***, the number of days in five weeks.

7 + 7 + 7 + 7 + 7 = 35
5 x 7 = 35

Although multiplication is related to addition, the parts of multiplication are not known as addends. Instead, the parts are known as ***multiplicands*** and ***multipliers***. A multiplication sentence, like an addition sentence, is called an ***equation***. But a multiplication sentence results in a ***product***, not a sum.



2 —— multiplicand
x 2 —— multiplier
----
4 —— product

X	0	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Multiplication, Step-by-Step

When the multiplicand and the multiplier are numbers with two or more digits, multiplication becomes a step-by-step process.

Look at 15 x 13:

```
   1 5
x    3
------
   1 5
```

First, multiply the ones – 3 x 5. Write down the product so the ones columns line up.

```
  1   5        Next, multiply the
               tens – 3 x 1 ten.
 x    3        Line up the product
----------     with the tens
  1   5        column.

  3   0   —    Zero is the place
----------     holder.
```

```
  1   5        Last, add the ones
               and tens to find the
 x    3        product of the
----------     equation.
  1   5

+ 3   0
----------
  4   5
```

Here is a shorter way:

```
  1            1.  Multiply the ones:  3 x 5 = 15.
  1   5        Put the 5 in the ones column and
               regroup the 1 to the tens column.
 x    3
-----------    2.  Multiply the tens:  3 x 1 = 3.
  4   5
               3.  Add the 1 that you regrouped to the 3,
               put the sum in the tens column.
```

Look at 265 x 23:

```
   265        First, multiply the
              multiplicand by the
x   23        ones in the
---------     multiplier – 3 x 5,
    15        3 x 6, and 3 x 2.
   180        Zero is the place
   600        holder.
```

```
   265        Next, multiply
              by the tens –
x   23        2 x 5, 2 x 6,
--------      and 2 x 2.
    15        Zero is the
   180        place holder.
   600
--------
   100
 1,200
 4,000
```

```
      265     Last, add.

x      23
---------
       15
+     180
+     600
---------
+     100
+   1,200
+   4,000
---------
    6,095
```

Here is a shorter way:

```
   11
   11
   265
x   23
---------
  795
 5300
---------
6,095
```

1. Multiply the ones: 3x265
   3x5=15 regroup the 1
   3x6=18 plus the regrouped 1=19; regroup the 1
   3x2=6 plus the regrouped 1=7

2. Multiply the tens: 2 x 265
   0 is the place holder
   2x5=10 regroup the 1
   2x6=12 plus the regrouped 1=13; regroup the 1
   2 x 2 – 4 plus the regrouped 1 = 5

3. Add 795 + 5300 = 6,095

Partial Product

A method of multiplying where the ones, tens, hundreds, and so on are multiplied separately and then the products added together

Example:

```
   24
 ×  3
 ----
   12  ←  Multiply the ones: 3 × 4 = 12
 + 60  ←  Multiply the tens: 3 × 20 = 60
 ----
   72
```

36 x **17 = 42 + 210 + 60 + 300 = 612**

When you multiply whole numbers, the ***product*** usually has a greater value than either the ***multiplicand*** or the ***multiplier***.

But there are exceptions:

A number multiplied by ***1*** is always equal to itself.

$$\begin{array}{r} \mathbf{1} \\ \underline{\mathbf{x\ 1}} \\ \mathbf{1} \end{array} \qquad \mathbf{21 \text{ x } 1 = 21} \qquad \begin{array}{r} \mathbf{36} \\ \underline{\mathbf{x\ 1}} \\ \mathbf{36} \end{array}$$

A number multiplied by ***0*** is always equal to ***0***.

$$\begin{array}{r} \mathbf{1} \\ \underline{\mathbf{x\ 0}} \\ \mathbf{0} \end{array} \qquad \mathbf{21 \text{ x } 0 = 0} \qquad \begin{array}{r} \mathbf{36} \\ \underline{\mathbf{x\ 0}} \\ \mathbf{0} \end{array}$$

To multiply a number by 10, add a 0 to the right of the number.

EXAMPLE

25 X 10 = 250 or

$$\begin{array}{r} 25 \\ \underline{\text{x}10} \\ 250 \end{array}$$

To multiply a number by 100, add two 0's to the right of the number.

EXAMPLE

36 X 100 = 3,600 or

$$\begin{array}{r} 36 \\ \underline{\text{x}100} \\ 3{,}600 \end{array}$$

Commutative Property of Multiplication

The property which states that factors can be multiplied in any order. The product is always the same.

Example:

$5 \times 7 = 7 \times 5$

$35 = 35$

Associative Property of Multiplication

The property which states that when multiplying three or more factors , any two of the factors can be multiplied, and the remaining factors may then be multiplied without changing the total product

Example:

(3 X 4) X 5 = 3 X (4 X 5)

12 X 5 = 3 X 20

60 = 60

Practice Exercise

Solve for each of the given problems.

1. 673 × 46	**2.** 405 × 60	**3.** 215 × 10	**4.** 879 × 19	**5.** 713 × 74
6. 281 × 179	**7.** 633 × 260	**8.** 225 × 351	**9.** 831 × 142	**10.** 883 × 258

11. 223 × 755
12. 107 × 496
13. 345 × 587
14. 236 × 687
15. 530 × 631

16. 756 × 69
17. 879 × 78
18. 638 × 90
19. 306 × 62
20. 323 × 84

21. 327 × 8
22. 422 × 5
23. 804 × 9
24. 943 × 6
25. 697 × 3

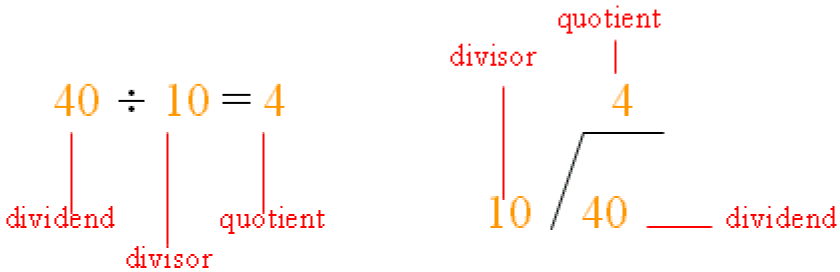
26. 101 × 486
27. 539 × 84
28. 24 × 60
29. 51 × 8
30. 5 × 0

Division

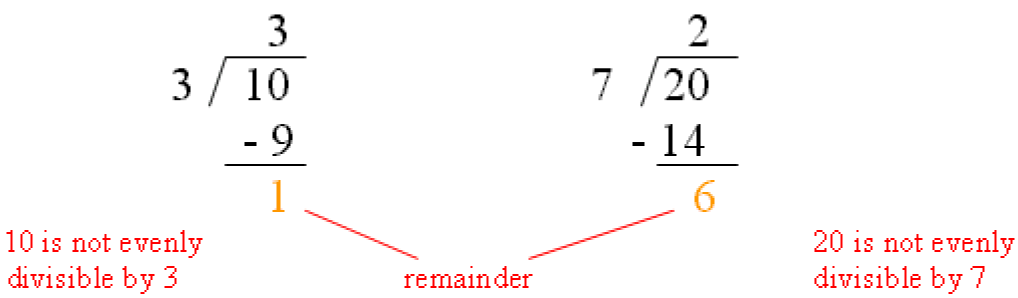
Division is the process of finding out how many times one number, the ***divisor***, will fit into another number, the ***dividend***. The division sentence results in a ***quotient***. The signs of division are ÷ and $\overline{)\quad}$, and mean ***divided by***. You can think of division as a series of repeated subtractions. For example, ***40*** **÷** ***10*** could also be solved by subtracting ***10*** from ***40*** four times:

40 – 10 – 10 – 10 – 10 = 0

Because ***10*** can be subtracted four times, you can say that ***40*** can be divided by ***10*** four times, or ***40 ¸ 10 = 4***.

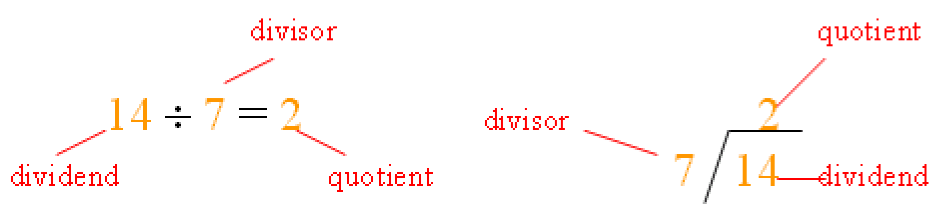


Many numbers do not fit evenly into other numbers. They are ***not evenly divisible by*** those numbers, and the number left over is called the ***remainder***.



We would record the answer for the first question as 3 r 1 and for the second question as 2 r 6. The "r" stands for remainder.

To divide whole numbers, reverse the process of multiplication. For example, if ***2 x 7 = 14*** in a multiplication equation, then in a division sentence, ***14*** is the ***dividend*** and ***7*** is the ***divisor*** with a ***quotient*** of ***2***.



A whole number divided by ***1*** will always equal itself.

1 ¸ 1 = 1 $\quad$ $1\overline{)21}$ with quotient **21** $\quad$ **36 ¸ 1 = 36**

Zero divided by a whole number will always equal ***0***.

0 ¸ 12 = 0 $\quad$ $3\overline{)0}$ with quotient **0** $\quad$ **0/7 = 2**

Division, Step-by-Step

Where the dividend and divisor are numbers with two or more digits, division becomes a step-by-step process.

```
     2
8 /208
  - 16
     4
```

First, round the divisor up - 8 rounds up to 10 - and estimate the number of 10s in 20. Answer: 2. Multiply the divisor – 8 x 2 – and subtract the product from the dividend.

```
    26
8/ 208
 -  16
    48
 -  48
     0
```

Next, pull down the next digit from the dividend – 8 – and repeat the estimation and subtraction process.

26
8/ 208
- 16
48
- 48
0 —— No remainder

Last, when you can subtract no more you've found the quotient.

1
23 /276
- 23
4

First, round 23 to 25 and estimate the number of 25s in 27. Answer: 1. Multiply the divisor by 1 – 23 x 1 – and subtract.

12
23 /276
- 23
46
- 46

Next, pull down the next digit from the dividend – 6 – and repeat the estimation and subtraction process.

12
23 /276
- 23
46
- 46
0 —— No remainder

Then, pull down the next digit, estimate, and subtract, until you can subtract no more.

Practice Exercise

Solve each problem.

1. $5\overline{)36}$	**2.** $8\overline{)461}$	**3.** $2\overline{)92}$
4. $4\overline{)45}$	**5.** $10\overline{)43}$	**6.** $112\overline{)213}$
7. $10\overline{)64}$	**8.** $8\overline{)794}$	**9.** $43\overline{)323}$
10. $2\overline{)13}$	**11.** $64\overline{)68}$	**12.** $5\overline{)535}$
13. $7\overline{)673}$	**14.** $111\overline{)260}$	**15.** $9\overline{)91}$
16. $5\overline{)456}$	**17.** $12\overline{)27}$	**18.** $66\overline{)487}$
19. $2\overline{)189}$	**20.** $6\overline{)34}$	**21.** $110\overline{)386}$
22. $5\overline{)443}$	**23.** $107\overline{)828}$	**24.** $49\overline{)153}$

Factors are numbers that, when multiplied together, form a new number called a ***product***. For example, ***1*** and ***2*** are factors of ***2***, and ***3*** and ***4*** are factors of ***12***.

> Every number except ***1*** has at least two factors: ***1*** and itself.

Common Factor

A number that is a **factor** of two or more numbers

Example:

factors of 6: 1, 2, 3, 6

factors of 12: 1, 2, 3, 4, 6, 12

The common factors of 6 and 12 are
1, 2, 3, and 6.

Greatest Common Factor (GCF)

The greatest factor that two or more numbers have in common

Example:

18: 1, 2, 3, **6**, 9, 18

30: 1, 2, 3, 5, **6**, 10, 15, 30

5 is the GCF of 18 and 30.

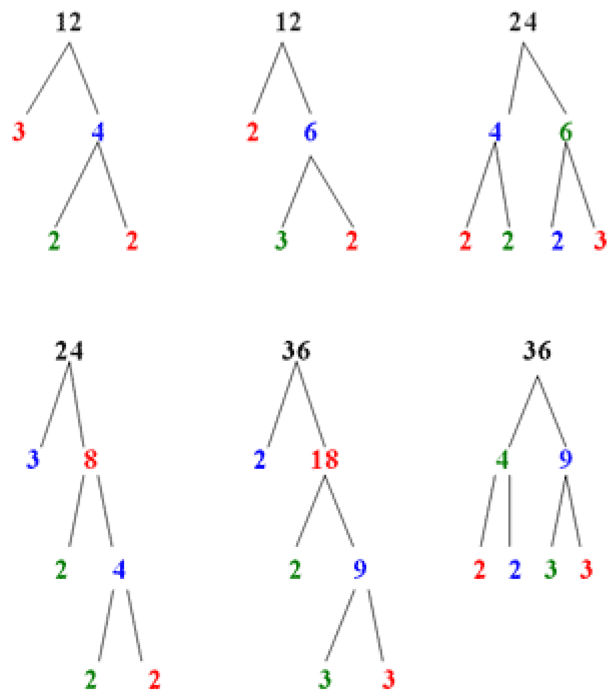
Practice Exercise

Find the greatest common factor (GCF) for the given numbers.

1. 3, 4 **1**
2. 4, 6
3. 3, 6
4. 2, 8
5. 11, 12
6. 5, 8

7. 8, 10
8. 5, 9
9. 5, 10
10. 6, 24
11. 4, 18
12. 20, 16
13. 15, 6
14. 9, 18
15. 12, 21
16. 16, 18
17. 4, 13
18. 8, 20
19. 18, 45
20. 16, 40
21. 24, 48
22. 10, 15
23. 35, 6
24. 45, 30
25. 66, 72
26. 64, 32
27. 40, 180
28. 12, 24
29. 30, 180
30. 80, 160

Composite numbers have more than two factors. In fact, every composite number can be written as the product of ***prime numbers***. You can see this on a ***factor tree***.



Prime numbers are counting numbers that can be divided by only two numbers**---*1*** and themselves. A prime number can also be described as a counting number with only two factors, ***1*** and itself. The number ***1***, because it can be divided only by itself, is ***not*** a prime number.

Prime Numbers to 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,

53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Practice Exercise

Classify each number as prime or composite.

1. 87 ☐Prime ■Composite	2. 18 ☐Prime ☐Composite	3. 48 ☐Prime ☐Composite	4. 57 ☐Prime ☐Composite
5. 85 ☐Prime ☐Composite	6. 41 ☐Prime ☐Composite	7. 17 ☐Prime ☐Composite	8. 78 ☐Prime ☐Composite
9. 28 ☐Prime ☐Composite	10. 54 ☐Prime ☐Composite	11. 1 ☐Prime ☐Composite	12. 73 ☐Prime ☐Composite
13. 69 ☐Prime ☐Composite	14. 27 ☐Prime ☐Composite	15. 70 ☐Prime ☐Composite	16. 19 ☐Prime ☐Composite
17. 56 ☐Prime ☐Composite	18. 49 ☐Prime ☐Composite	19. 64 ☐Prime ☐Composite	20. 31 ☐Prime ☐Composite
21. 43 ☐Prime ☐Composite	22. 95 ☐Prime ☐Composite	23. 62 ☐Prime ☐Composite	24. 11 ☐Prime ☐Composite

Find the prime factorization of each number.

1. 24 **2, 2, 2, 3**
2. 10
3. 14
4. 4
5. 12
6. 28
7. 42
8. 32
9. 78
10. 26
11. 58
12. 66
13. 20
14. 100
15. 36
16. 6
17. 64
18. 22

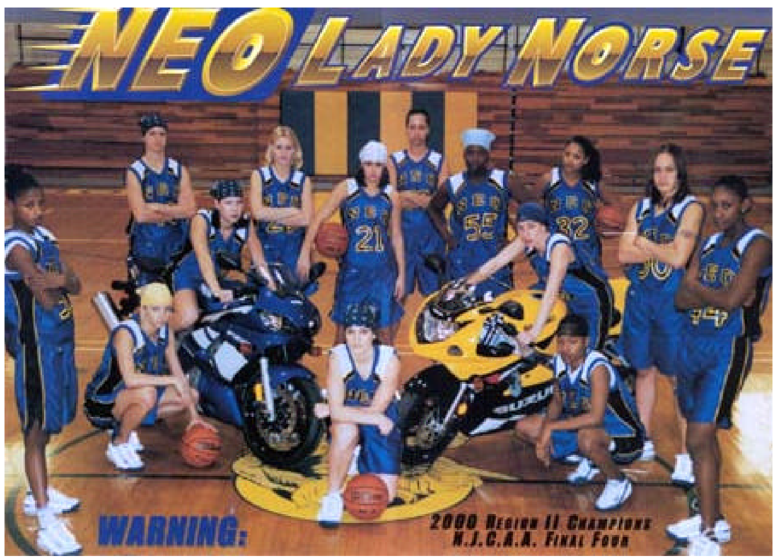
Averages

The most common way to find an ***average*** is to add up a list of numbers and divide the sum by the number of items on the list. Another word for average is ***mean***.

$3 + 4 + 6 + 8 + 9 = 30$ number of addends

sum — $30 \div 5 = 6$ So, the average of the numbers 3, 4, 6, 8, and 9 is 6.

When do you need to calculate an average? Your grades may be based on the average of all your test scores. In sports, you might want to find out the average height of players on your favorite basketball team.



The height of the starters for this team is:

Anita 60”

Jane 58”

Cathy 57”

Joy 52”

Tanya 48”

The average height of these players is 55 inches.

Practice Exercise

Calculate the values to the nearest whole number.

1. 553, 680, 416, 416, 553, 554, 553, 416, 554, 416, and 553 Write the mean (average): __________
2. 3, 7, 20, 17, 8, 3, 12, and 17 Write the mean: __________
3. 14, 28, 5, 8, 5, 6, 5, 27, 7, 21, 28, 19, and 5 Write the mean: __________
4. 15, 36, 36, 36, 15, 14, 15, 21, 24, 36, 21, 21, 8, and 29 Write the mean: __________
5. 45, 45, 17, 22, 25, 17, 6, 6, 37, 45, 23, and 37 Write the mean: __________
6. 507, 529, 9, 8, 8, 546, 1, 582, 8, 546, 573, 520, 545, 545, 512, 528, 520, and 582 Write the mean: __________

7.	198, 194, 111, 4, 198, 108, 7, 150, 178, 195, 194, and 108 Write the mean: __________
8.	728, 728, 728, 448, 929, 728, and 978 Write the mean: __________
9.	853, 837, 812, 839, 853, 812, 856, 887, 812, 812, 812, and 812 Write the mean: __________
10.	127, 142, 188, 142, 142, 143, 107, 143, 107, 143, 127, 121, 195, 122, 142, 147, 190, and 190 Write the mean: __________

Word Problems with Whole Numbers

Within every story (word) problem are several ***clue words***. These words tell you the kind of math sentence (equation) to write to solve the problem.

Addition Clue Words

add
sum
total
plus
in all

Subtraction Clue Words

subtract
difference
take away
less than
are not

both
together
increased by
all together
combined

remain
decreased by
have or are left
change (money problems)
more
fewer

Multiplication Clue Words

times
product of
multiplied by
by (dimension)

Division Clue Words

quotient of
divided by
half [or a fraction]
split
separated
cut up
parts
shared equally

⇒ ***Division clue words are often the same as subtraction clue words. Divide when you know the total and are asked to find the size or number of "one part" or "each part."***

Following a system of steps can increase your ability to accurately solve problems. Use these steps to solve word problems.

1. Read the problem carefully. Look up the meanings of unfamiliar words.

2. Organize or restate the given information.

3. State what is to be found.

4. Select a strategy (such as making a chart of working backward) and plan the steps to solve the problem.

5. Decide on an approximate answer before solving the problem.

6. Work the steps to solve the problem.

7. Check the final result. Does your answer seem reasonable?

The Problem Solving System was used to solve the following problem:

Mary has ten marbles. Lennie has thirteen. How many marbles do they have in all?

1. Mary has ten marbles. Lennie has thirteen.
How many marbles do they have in all?

2. Mary – 10 marbles
Lennie – 13 marbles

3. How many marbles in all?

4. Add

5. A little over 20 marbles (10 + 10 = 20)

6.   10
+13
23 marbles

7. The final sum of 23 marbles is close to the estimated answer of 20 marbles. The final result is reasonable.

Þ *Be sure to label answers whenever possible. For example: marbles, grams, pounds, feet, dogs, etc.*

Þ *Some problems may require several steps to solve. Some may have more than one correct answer. And some problems may not have a solution.*

Have you ever tried to help someone else work out a word problem? Think about what you do. Often, you read the problem with the person, then discuss it or put it in your own words to help the person see what is happening. You can use this method---restating the problem---on your own as a form of "talking to yourself."

Restating a problem can be especially helpful when the word problem contains no key words. Look at the following example:

Example: Susan has already driven her car 2,700 miles since its last oil change. She still plans to drive 600 miles before changing the oil. How many miles does she plan to drive between oil changes?

Step 1: *question:* How many miles does she plan to drive between oil changes?

Step 2: *necessary information:* 2,700 miles, 600 miles

Step 3: *decide what arithmetic to use:* Restate the problem in your own words: "You are given the number of miles Susan has already driven and the number of miles more that she plans to drive. You need to add these together to find the total number of miles between oil changes."

Step 4: 2,700 miles + 600 miles = **3,300 miles** between oil changes.

Step 5: It makes sense that she will drive 3,300 miles between oil changes, since you are looking for a number larger than the 2,700 miles that she has already driven.

For some problems, you have to write two or three equations to solve the problem. For others, you may need to make charts or lists of information, draw pictures, find a pattern, or even guess and check. Sometimes you have to work backwards from a sum, product, difference, or quotient, or simply use your best logical thinking.

List/Chart

Marty's library book was six days overdue. The fine is $.05 the first day, $.10, the second, $.20 the third day, and so on. How much does Marty owe?

Marty's library book was six days overdue. The fine is $.05 the first day, $.10, the second, $.20 the third day, and so on. How much does Marty owe?

Days	1	2	3	4	5	6
Fine	$.05	$.10	$.20	$.40	$.80	$1.60

Answer: $1.60

Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?

Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?

Veronica	Veronica	Archie	Archie
Archie	Betty	Veronica	Betty
Betty	Archie	Betty	Veronica

Betty	Betty
Veronica	Archie
Archie	Veronica

Answer: 6 ways

Find a Pattern

Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X_______. How did Jenny fill in the blanks?

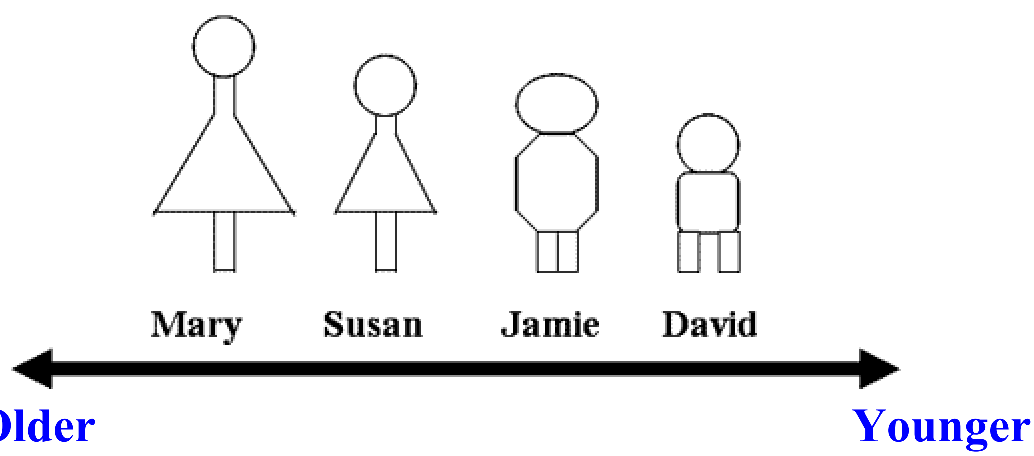
Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X_______. How did Jenny fill in the blanks?

Answer: 10, 11, 12 W

Draw a Picture

Mary is older than Jamie. Susan is older than Jamie, but younger than Mary. David is younger than Jamie. Who is oldest?

Mary is older than Jamie. Susan is older than Jamie, but younger than Mary. David is younger than Jamie. Who is oldest?



Answer: Mary is oldest.

Guess and Check

Farmer Joe keeps cows and chickens in the farmyard. All together, Joe can count 14 heads and 42 legs. How many cows and how many chickens does Joe have in the farmyard?

Farmer Joe keeps cows and chickens in the farmyard. All together, Joe can count 14 heads and 42 legs. How many cows and how many chickens does Joe have in the farmyard?

6 cows +8 chickens 14 heads	Guess a number of cows. Then add the number of chickens to arrive at the sum of 14 heads. Then check the total legs.	6 cows = 24 legs +8 chickens = 16 legs 40 legs
7 cows +7 chickens 14 heads	Adjust your guesses. Then check again until you solve the problem.	7 cows = 28 legs +7 chickens = 14 legs 42 legs

Answer: 7 cows and 7 chickens

Work Backwards

Marsha was banker for the school play. She took in $175 in ticket sales. She gave Wendy $75 for sets and costumes and Paul $17.75 for advertising and publicity. After paying for the props, Marsha had $32.25 left. How much did the props cost?

Marsha was banker for the school play. She took in $175 in ticket sales. She gave Wendy $75 for sets and costumes and Paul $17.75 for advertising and publicity. After paying for the props, Marsha had $32.25 left. How much did the props cost?

$ 175.00 tickets
- 75.00 costumes
$ 100.00
- 17.75 advertising
$ 82.25

$ 82.25
- 32.25
$ 50.00 cost of props

Logical Reasoning

Juan challenged Sheila to guess his grandmother's age in ten questions or less. It took her six. Here's what Sheila asked:

Juan challenged Sheila to guess his grandmother's age in ten questions or less. It took her six. Here's what Sheila asked:

"Is she less than fifty?" "No." **50+ years old**

"Less than seventy-five?" "Yes."	**50 to 74 years old**
"Is her age an odd or even number?" "Odd."	**ends in 1, 3, 5, 7 or 9**
"Is the last number less than or equal to five?" "No."	**ends in 7 or 9**
"Is it nine?" "No."	**ends in 7 – 57 or 67**
"Is she in her sixties?" "No."	**57 years old**

Not Enough Information

Now that you know how to decide whether to add, subtract, multiply, or divide to solve a word problem, you should be able to recognize a word problem that cannot be solved because not enough information is given.

Look at the following example:

Problem: At her waitress job, Sheila earns $4.50 an hour plus tips. Last week she got $65.40 in tips. How much did she earn last week?

Step 1: *question:* How much did she earn last week?

Step 2: *necessary information:* $4.50/hour, $65.40

Step 3: *decide what arithmetic to use:*

tips + (pay per hour **x** hours worked) = total earned

missing information: hours worked

At first glance, you might think that you have enough information since there are 2 numbers. But when the solution is set up, you can see that you need to know the number of hours Sheila worked to find out what she earned. (**Be Careful!!!**)

Practice Exercise

Solve the following problems.

1. John bought a wallet and a book. The book cost $33. He gave the cashier $50 and received $6 for change. How much did the wallet cost?

2. Al bought 244 eggs for $24.40. He found that 16 eggs were rotten. He sold the rest at 12 eggs for $1.89. How much money did he make?

3. Jenny and Steven have $330 altogether. Jenny has $195. How much must Jenny take from Steven so that she has twice as much as Steven?

4. Nick weighs 52 kg. Peter is the same weight as Michael. If their total weight is 128 kg., what is each of their weights?

5. Michelle has 241 stamps. Janet has 142 more stamps than Michelle. Their friend Lucy also collects stamps. If they have 800 stamps altogether, how many stamps

does Lucy have?

6. Mary needs 44 red beads, 39 purple beads and 63 yellow beads to make a necklace. Beads are sold 12 beads in a packet of the same colour? If each packet costs $2, how much does Mary need?

7. Gene bought $360 worth of sports equipment and $18 worth of office supplies for the boys' club. Since the boys' club is tax-exempt, he didn't have to pay the sales tax. If he had paid tax, how much would he have spent?

8. One computer costs $2430. One printer costs $630. What is the total cost for 14 computers and 8 printers?

9. A father has $240000 to be shared among his 3 children. Two of the children have equal amounts. The third child has $100000. How much does each child get?

10. John has 42 more stamps than Janet. Janet has 23 less stamps than Jack. If Janet has 324 stamps, how many stamps do they have altogether?

11. 1680 people attended a charity concert. 1320 adults paid $15 each. The rest were children who paid $4 each. The organizers had to pay $5300 for staging the concert. How much money went to charity?

Answer Key

Book 14016 – Whole Numbers

Page 14 **1.** 36 **2.** 165 **3.** 1659 **4.** 1254 **5.** 1993
6. 2047 **7.** XIX **8.** CCXCIX
9. DCCCXLVII **10.** MCDXCII
11. MDCCLXXVI **12.** MMXV **13.** 1919
14. 7 **15.** 46 **16.** 190 **17.** XIV
18. XXV **19.** C **20.** XLIX

Page 20 **2.** forty-two thousand, four hundred ninety-nine **3.** forty-five thousand, two hundred ninety-one **4.** ten thousand, one hundred forty-eight **5.** nine thousand, thirty **6.** thirty-three thousand **7.** fifty-one thousand, nine hundred thirty-five **8.** one million, ninety-nine thousand, eight hundred sixty-eight **9.** one million, ninety-four thousand, four hundred forty-two **10.** one million, five hundred seventy thousand **11.** one million, ninety-eight thousand, three hundred twenty-nine **12.** one million, nine hundred sixty-four thousand, eight hundred thirty-five **13.** one million, six hundred sixty-eight thousand, one hundred **14.** one million, ten thousand, five hundred twenty-one **15.** one million, four hundred fifty-six thousand, one hundred forty-two **16.** one million, nine hundred forty-two thousand, eight hundred eighteen **17.** one million, three hundred sixty-two thousand, eight hundred twenty-five **18.** one million, eleven thousand, four hundred thirty-one

Page 22 **2.** 4567 **3.** 92734 **4.** 2076 **5.** 34676
6. 23000 **7.** 77900 **8.** 1063561
9. 1066411 **10.** 1960000 **11.** 1060708
12. 1671843 **13.** 1048534 **14.** 1497302

Page 25 **2.** Millions **3.** Tens **4.** Ones **5.** Tens
6. Hundred Thousands **7.** Tens
8. Hundred Thousands **9.** Ones
10. Ten Thousands **11.** Tens **12.** Tens
13. Thousands **14.** Millions **15.** Thousands
16. Ten Thousands **17.** Ten Thousands
18. Ones **19.** Tens **20.** Hundreds
21. Ones **22.** Ten Thousands
23. Ten Thousands **24.** Thousands
25. Ones **26.** Ones **27.** Tens
28. Hundreds

Page 28 **2.** 9000 **3.** 3000 **4.** 23000 **5.** 95000
6. 14000 **7.** 82000 **8.** 8000 **9.** 8000
10. 7000 **11.** 9000 **12.** 14000 **13.** 3000
14. 16000 **15.** 1000 **16.** 2000

Page 28 **18.** 17300 **19.** 2400 **20.** 300 **21.** 2700
22. 16400 **23.** 900 **24.** 300 **25.** 1000
26. 49700 **27.** 7100 **28.** 1000 **29.** 300
30. 70400 **31.** 55000 **32.** 600

Page 29 **34.** 670 **35.** 40 **36.** 850 **37.** 60
38. 26320 **39.** 47960 **40.** 100 **41.** 79310
42. 710 **43.** 70 **44.** 24090 **45.** 90
46. 50400 **47.** 67280 **48.** 520

Page 33 **2.** 15, 25 **3.** 22, 32 **4.** 40, 60
5. 108, 110 **6.** 3220, 3224
7. 45645, 45655 **8.** 789110, 789120
9. 99, 101, 102 **10.** 994, 990
11. 200, 180, 175

Page 34 **2.** > **3.** < **4.** > **5.** < **6.** > **7.** <
8. > **9.** < **10.** < **11.** < **12.** < **13.** >
14. < **15.** < **16.** < **17.** < **18.** <
19. < **20.** > **21.** > **22.** < **23.** >
24. >

Page 36 **1.** 279000, 172909, 160909, 20999
2. 513527, 315727, 315427, 135620
3. 999999, 802520, 802250, 255000

Page 36 **1.** 226216, 266225, 626225, 662226
2. 90395, 92953, 190595, 890359
3. 228302, 822320, 882202, 882220

Page 40 **1.** 38 **2.** 504 **3.** 4424 **4.** 35119 **5.** 65
6. 58 **7.** 3314 **8.** 3964 **9.** 38 **10.** 234
11. 5028 **12.** 185 **13.** 39 **14.** 373
15. 4983 **16.** 40078 **17.** 435 **18.** 110
19. 1052 **20.** 1160 **21.** 4144 **22.** 408
23. 243 **24.** 313 **25.** 1120246

26. 1760559 **27.** 141 **28.** 318 **29.** 151
30. 209 **31.** 3586 **32.** 3183 **33.** 184300
34. 3398 **35.** 8991 **36.** 522 **37.** 266
38. 848 **39.** 71 **40.** 1435 **41.** 367
42. 642

Page 44 **1.** 227 **2.** 536 **3.** 509 **4.** 313
5. 1679 **6.** 6921 **7.** 2709 **8.** 9478
9. 22213 **10.** 5898 **11.** 4373 **12.** 15466
13. 118217 **14.** 185998 **15.** 157847
16. 613644 **17.** 303112 **18.** 499819
19. 118056 **20.** 478333

Page 46 **2.** 20 **3.** 100 **4.** 85 **5.** 86 **6.** 81
7. 76 **8.** 97 **9.** 62 **10.** 39 **11.** 44
12. 79 **13.** 21 **14.** 17 **15.** 86 **16.** 57
17. 41 **18.** 92

Page 53 **1.** 30958 **2.** 24300 **3.** 2150 **4.** 16701
5. 52762 **6.** 50299 **7.** 164580 **8.** 78975
9. 118002 **10.** 227814 **11.** 168365
12. 53072 **13.** 202515 **14.** 162132
15. 334430 **16.** 52164 **17.** 68562
18. 57420 **19.** 18972 **20.** 27132
21. 2616 **22.** 2110 **23.** 7236 **24.** 5658
25. 2091 **26.** 49086 **27.** 45276 **28.** 1440
29. 408 **30.** 0

Page 58 **1.** 7 r 1 **2.** 57 r 5 **3.** 46 **4.** 11 r 1
5. 4 r 3 **6.** 1 r 101 **7.** 6 r 4 **8.** 99 r 2
9. 7 r 22 **10.** 6 r 1 **11.** 1 r 4 **12.** 107

13. 96 r 1 **14.** 2 r 38 **15.** 10 r 1
16. 91 r 1 **17.** 2 r 3 **18.** 7 r 25 **19.** 94 r 1
20. 5 r 4 **21.** 3 r 56 **22.** 88 r 3 **23.** 7 r 79
24. 3 r 6

Page 59 **2.** 2 **3.** 3 **4.** 2 **5.** 1 **6.** 1 **7.** 2
8. 1 **9.** 5 **10.** 6 **11.** 2 **12.** 4 **13.** 3
14. 9 **15.** 3 **16.** 2 **17.** 1 **18.** 4 **19.** 9
20. 8 **21.** 24 **22.** 5 **23.** 1 **24.** 5
25. 6 **26.** 32 **27.** 20 **28.** 12 **29.** 30
30. 80

Page 62 **2.** Composite **3.** Composite **4.** Composite
5. Composite **6.** Prime **7.** Prime
8. Composite **9.** Composite
10. Composite **11.** Prime **12.** Prime
13. Composite **14.** Composite
15. Composite **16.** Prime **17.** Composite
18. Composite **19.** Composite **20.** Prime
21. Prime **22.** Composite **23.** Composite
24. Prime

Page 63 **2.** 2, 5 **3.** 2, 7 **4.** 2, 2 **5.** 2, 2, 3
6. 2, 2, 7 **7.** 2, 3, 7 **8.** 2, 2, 2, 2, 2
9. 2, 3, 13 **10.** 2, 13 **11.** 2, 29
12. 2, 3, 11 **13.** 2, 2, 5 **14.** 2, 2, 5, 5
15. 2, 2, 3, 3 **16.** 2, 3 **17.** 2, 2, 2, 2, 2, 2
17. 2, 11

Page 65 **1.** 515 **2.** 11 **3.** 14 **4.** 43 **5.** 27
6. 393 **7.** 137 **8.** 752 **9.** 833 **10.** 145

Page 76 **1.** $11 **2.** $11.51 **3.** $25
4. Nick: 52 kg, Peter: 38 kg, Michael: 38 kg
5. 176 stamps **6.** She will need 14 packets of beads altogether, so she will need $28 **7.** Not enough information **8.** $39060 **9.** Two children get $70000 each. The third child already has $100000 **10.** $1037
11. $15940