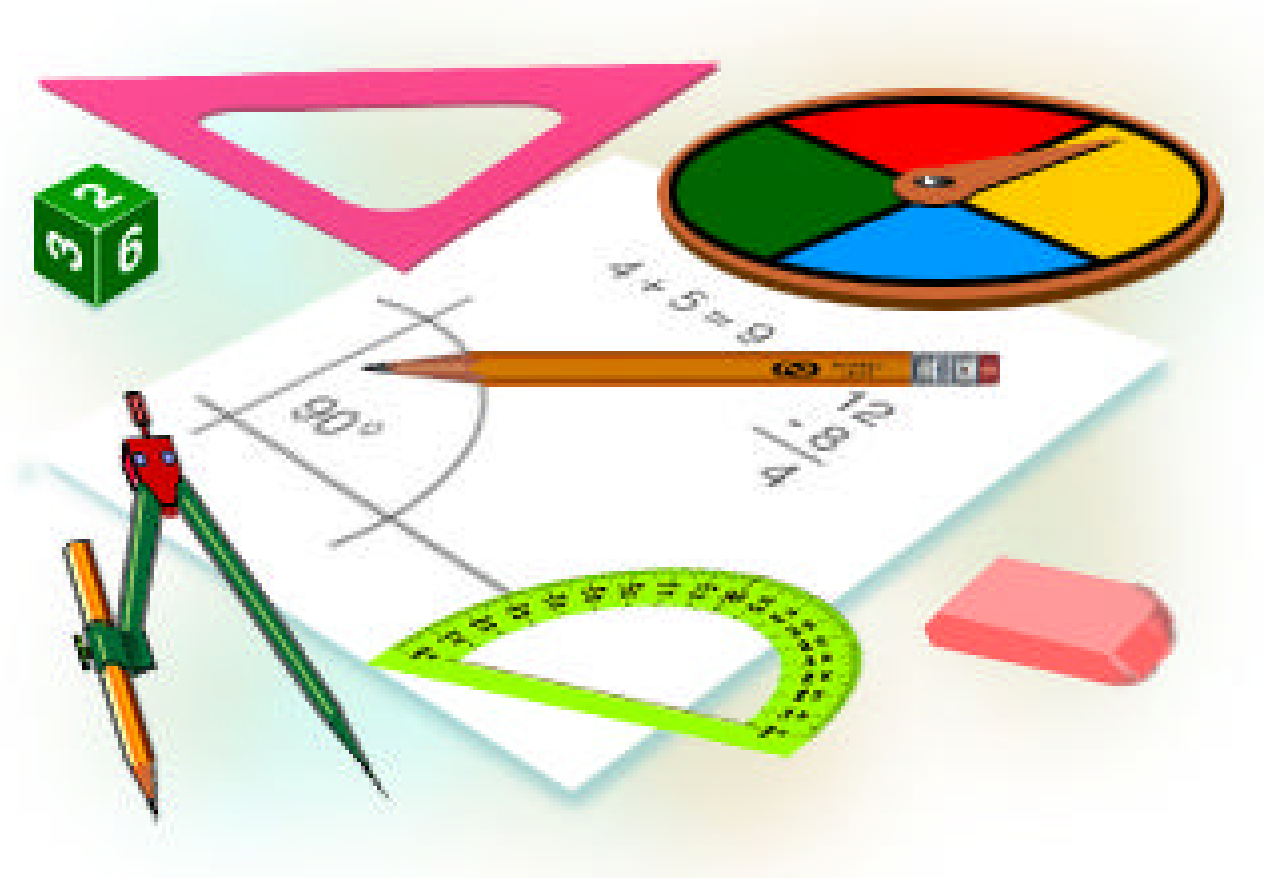
# The Next Step

## Mathematics Applications for Adults




**Book 14017 - Integers**

# OUTLINE

## Mathematics - Book 14017

| Integers |
| --- |
| **Introduction To Integers** |
| perform mathematical operations using integers. |
| explain the difference between signs of operations and signs of quantity. |
| **Problem Solving With Integers** |
| solve multi-step problems requiring the performance of any combination of mathematical operations involving integers, with or without a calculator. |

# THE NEXT STEP

# Book 14017

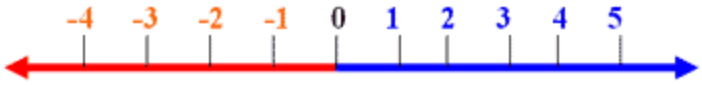
## Integers

## Introduction to Integers

The set of ***integers*** includes ***0***, all of the counting numbers (called ***positive*** whole numbers), and the whole numbers less than ***0*** (called ***negative*** numbers). Integers are shown below on a number line.

⇒ ***All counting numbers and whole numbers are integers.***

**Negative Numbers** **Positive Numbers**




**Numbers less than 0 are negative numbers. Numbers greater than 0 are positive.**

***Number lines*** show numbers in order. If you follow the number line to the right, the numbers get larger and larger. If you follow the number line to the left, the numbers get smaller and smaller.

⇒ ***To remember the order of negative and positive numbers on a number line, think of the alphabet (n,o,p = negative, zero, positive).***

It's important to understand the number line because it shows you that every number has an opposite. The famous German mathematician Leopold Kronecker once said: "God made the positive integers; everything else is the work of man." Why, then, did we confuse things with negative numbers? As it turns out, there are many, many everyday problems where negative numbers are useful. For example, we can both gain and lose weight.

The temperature can rise or fall. Locations on the earth can be above sea level, or below sea level.

Integers can be understood both as signs of operation and signs of quantity.

**Examples** +5 stands for 0 + 5 (operation), but it can also stand for a positive amount or a gain as in measurements such as temperature or weight (quantity).

-5 stands for 0 – 5 (operation), but it can also stand for a negative amount or a drop as in measurements such as temperature or weight (quantity).

# Practice Exercise

Write an integer for each description.
(Hint: include a negative sign for a description that is below zero)

1. **-9** 9 yards short for first down
2. ____ An altitude of 4000 ft
3. ____ A gain of 6 pounds
4. ____ A profit of $30 dollars
5. ____ 266 ft below sea level
6. ____ 15 units to the right of -6 on a number line
7. ____ A loss of 6 pounds
8. ____ The opposite of -10
9. ____ The temperature dropped 15 degrees overnight
10. ____ Withdraw $53 from an ATM machine
11. ____ 1 unit to the right of 7 on a number line
12. ____ A deposit of $158
13. ____ A gain of 23 yards
14. ____ 20 degrees below zero
15. ____ 12 units to the left of 5 on a number line
16. ____ 6 inches taller
17. ____ The opposite of 9
18. ____ The stock market went up 240 points today

# Integer Comparison

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | -4 | < | 0 | 2. | -7 | _____ | 4 |
| 3. | -5 | _____ | -4 | 4. | 8 | _____ | -7 |
| 5. | 9 | _____ | -2 | 6. | -49 | _____ | 34 |
| 7. | -18 | _____ | 18 | 8. | 87 | _____ | 29 |
| 9. | 12 | _____ | -14 | 10. | -11 | _____ | -48 |
| 11. | -75 | _____ | -75 | 12. | 14 | _____ | -16 |
| 13. | -3 | _____ | 9 | 14. | -58 | _____ | 84 |
| 15. | 31 | _____ | -64 | 16. | -12 | _____ | -45 |
| 17. | -52 | _____ | -52 | 18. | 18 | _____ | 19 |
| 19. | -65 | _____ | 65 | 20. | -12 | _____ | 0 |
| 21. | -38 | _____ | -68 | 22. | 11 | _____ | -2 |
| 23. | -15 | _____ | -10 | 24. | -56 | _____ | -56 |
| 25. | -19 | _____ | -31 | 26. | -37 | _____ | 21 |
| 27. | -69 | _____ | 69 | 28. | -23 | _____ | -19 |
| 29. | 34 | _____ | -51 | 30. | -9 | _____ | 13 |
| 31. | -87 | _____ | -87 | 32. | -4 | _____ | -14 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 33. | -66 | _____ | 66 | 34. | -10 | _____ | -34 |
| 35. | 19 | _____ | -92 | 36. | -2 | _____ | 15 |

## Adding and Subtracting Integers

Adding and subtracting positive integers works the same way as adding and subtracting whole numbers. Adding and subtracting negative numbers works differently.

When you add a negative integer to a positive integer, you are actually subtracting the value of the negative integer from the positive integer.

**4 + -2 = 4 – 2 = 2**
**7 + 3 + -2 = 7 + 3 – 2 = 8**
**11 + -6 + 4 + -2 = 11 – 6 + 4 – 2 = 7**

When you add a negative integer to another negative integer, you add the values of the integers and then add a negative sign in front of them.

⇒ ***Adding a negative number to another negative number results in a sum less than either negative addend.***

**-4 + -2 = - (4 + 2) = -6**
**-7 + -3 = - (7 + 3) = -10**
**-11 + -6 + -4 + -2 = - (11 + 6 + 4 + 2) = -23**

⇒ ***Adding a positive number to a positive number always results in a sum greater than either addend.***

**2 + 3 = 5**

## Addition Property of Opposites

The property which states that the sum of a number and its opposite is zero

*Examples*:

5 + ⁻5 = 0  ⁻15 + 15 = 0

When you subtract a negative number from a negative integer, you are actually adding a positive integer to the negative integer.

**-4 - -2 = -4 + 2 = -2**
**-7 - -3 - -2 = -7 + 3 + 2 = -2**
**-11 - -6 - -4 - - 2 = -11 + 6 + 4 + 2 = 1**

When you subtract a positive integer of greater value from another positive integer, the difference will be a negative integer.

**2 – 4 = -2**
**3 – 7 – 2 = -6**
**11 – 6 – 4 – 2 = -1**

# Practice Exercise

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **1.** | **3 + 14** | **=** | **17** | **2.** | **-23 - 15** | **=** | **_____** |
| **3.** | **-8 - (-11)** | **=** | **_____** | **4.** | **-10 - (-11)** | **=** | **_____** |
| **5.** | **11 - (-12)** | **=** | **_____** | **6.** | **-18 + 12** | **=** | **_____** |
| **7.** | **6 - (-4)** | **=** | **_____** | **8.** | **-14 - (-5)** | **=** | **_____** |
| **9.** | **6 + -14** | **=** | **_____** | **10.** | **10 + 6** | **=** | **_____** |
| **11.** | **-5 + -12** | **=** | **_____** | **12.** | **9 - (-1)** | **=** | **_____** |
| **13.** | **-3 + -2** | **=** | **_____** | **14.** | **18 + -5** | **=** | **_____** |
| **15.** | **-7 - -0** | **=** | **_____** | **16.** | **7 + 5** | **=** | **_____** |
| **17.** | **17 - (-2)** | **=** | **_____** | **18.** | **14 + -13** | **=** | **_____** |
| **19.** | **0 - (-12)** | **=** | **_____** | **20.** | **-7 + -10** | **=** | **_____** |
| **21.** | **-5 + -13** | **=** | **_____** | **22.** | **-3 - (-13)** | **=** | **_____** |
| **23.** | **0 + 43** | **=** | **_____** | **24.** | **-6 - (-27)** | **=** | **_____** |
| **25.** | **-8 - (-8)** | **=** | **_____** | **26.** | **38 + 0** | **=** | **_____** |
| **27.** | **36 + 32** | **=** | **_____** | **28.** | **-7 + -11** | **=** | **_____** |
| **29.** | **15 - (-2)** | **=** | **_____** | **30.** | **-11 - 2** | **=** | **_____** |
| **31.** | **9 - (-6)** | **=** | **_____** | **32.** | **-22 - 12** | **=** | **_____** |
| **33.** | **-14 + 9** | **=** | **_____** | **34.** | **-161 - (-55)** | **=** | **_____** |
| **35.** | **-472 - (-124)** | **=** | **_____** | **36.** | **18 - (-7)** | **=** | **_____** |

## Multiplying and Dividing Integers

Multiplying and dividing integers works the same way as multiplying and dividing whole numbers, unless one or more of the integers is a negative number.

The product or quotient of a positive integer multiplied or divided by another positive integer will always be a positive integer. Positive integers may or may not be written with a positive sign: +8 = 8. Negative integers are ***always*** written with the minus sign.

**4 x 2 = 8**
**24 ¸ 6 = 4**

The product or quotient of a positive integer multiplied or divided by a negative integer will always be a negative integer.

**4 x –2 = -8**
**24 ¸ -6 = -4**

The product or quotient of a negative integer multiplied or divided by a positive integer will always be a negative integer.

**-4 x 2 = -8**
**-24 ¸ 6 = -4**

The product or quotient of a negative integer multiplied or divided by a negative integer will always be a positive integer.

**-4 x –2 = 8**
**-24 ¸ -6 = 4**

***The rule for multiplying or dividing integers is if the signs are the same, the answer is positive. If the signs are different, the answer is negative.***

⇒ ***Remember:***

***positive x positive or positive , positive = positive***
***positive x negative or positive , negative = negative***
***negative x positive or negative , positive = negative***
***negative x negative or negative , negative = positive***

| | | | | | |
|---|---|---|---|---|---|
| 1. | -7 × -7 = | 49 | 2. | -7 × 4 = | _____ |
| 3. | 3 × -11 = | _____ | 4. | 3 × -6 = | _____ |
| 5. | 9 × 6 = | _____ | 6. | -2 × -12 = | _____ |
| 7. | -10 × -2 = | _____ | 8. | -6 × -9 = | _____ |
| 9. | 7 × -8 = | _____ | 10. | -10 × 10 = | _____ |
| 11. | -8 × -12 = | _____ | 12. | -10 × 2 = | _____ |
| 13. | -3 × -2 = | _____ | 14. | -8 × 8 = | _____ |
| 15. | -8 × -9 = | _____ | 16. | 5 × -6 = | _____ |
| 17. | -9 × -9 = | _____ | 18. | 4 × -8 = | _____ |
| 19. | 7 × -12 = | _____ | 20. | -10 × -10 = | _____ |
| 21. | 11 × -3 = | _____ | 22. | -9 × 11 = | _____ |
| 23. | -8 × -10 = | _____ | 24. | 6 × -11 = | _____ |
| 25. | -14 × -8 = | _____ | 26. | -13 × 24 = | _____ |
| 27. | -20 × -5 = | _____ | 28. | -41 × -33 = | _____ |
| 29. | -3 × -3 = | _____ | 30. | 18 × 10 = | _____ |
| 31. | -14 × 22 = | _____ | 32. | 6 × -4 = | _____ |
| 33. | -4 × 5 = | _____ | 34. | 33 × -19 = | _____ |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **35.** | **5 × -3 =** | _____ | | **36.** | **-10 × 23 =** | _____ | |
| **37.** | **99 ÷ -11 =** | **-9** | | **38.** | **120 ÷ -10 =** | _____ | |
| **39.** | **-30 ÷ -10 =** | _____ | | **40.** | **-84 ÷ 12 =** | _____ | |
| **41.** | **-24 ÷ 12 =** | _____ | | **42.** | **-36 ÷ -9 =** | _____ | |
| **43.** | **-30 ÷ 6 =** | _____ | | **44.** | **-120 ÷ 10 =** | _____ | |
| **45.** | **-21 ÷ 3 =** | _____ | | **46.** | **20 ÷ -5 =** | _____ | |
| **47.** | **-54 ÷ -9 =** | _____ | | **48.** | **-60 ÷ -5 =** | _____ | |
| **49.** | **27 ÷ -9 =** | _____ | | **50.** | **-120 ÷ 12 =** | _____ | |
| **51.** | **28 ÷ -4 =** | _____ | | **52.** | **25 ÷ -5 =** | _____ | |
| **53.** | **24 ÷ 3 =** | _____ | | **54.** | **-32 ÷ 8 =** | _____ | |
| **55.** | **345 ÷ 15 =** | _____ | | **56.** | **-63 ÷ -7 =** | _____ | |
| **57.** | **-56 ÷ 8 =** | _____ | | **58.** | **4 ÷ -2 =** | _____ | |
| **59.** | **12 ÷ -3 =** | _____ | | **60.** | **-18 ÷ -9 =** | _____ | |
| **61.** | **192 ÷ -24 =** | _____ | | **62.** | **273 ÷ 13 =** | _____ | |
| **63.** | **-54 ÷ -18 =** | _____ | | **64.** | **-240 ÷ 15 =** | _____ | |
| **65.** | **377 ÷ 13 =** | _____ | | **66.** | **88 ÷ -22 =** | _____ | |
| **67.** | **342 ÷ -18 =** | _____ | | **68.** | **-147 ÷ 21 =** | _____ | |
| **69.** | **171 ÷ -9 =** | _____ | | **70.** | **-30 ÷ -15 =** | _____ | |
| **71.** | **-36 ÷ 18 =** | _____ | | **72.** | **-322 ÷ -14 =** | _____ | |

## Problem Solving With Integers

1. During the day, the temperature rose from -5℃ to +17℃. How many degrees did the temperature rise?

2. Mount McKinley, the highest mountain in North America, is 20,320 feet above sea level. Death Valley, the lowest point in North America, is 280 feet below

sea level. Find the difference in height between Mount McKinley and Death Valley.

3. The penalty points for riders in a show jumping competition are shown in the table below.

| Rider | Penalty points |
|---|---|
| Dale | -3, -3, -4, -8 |
| Toni | -3, –4, -4, -5 |
| Tracey | -3, -3, -3, -8 |

a. Find the total penalty points for each rider.
b. A penalty point total closest to 0 wins. Who placed first? second? third?

4. A submarine descends at the rate of 50 meters per minute. How many minutes will it take to descend from sea level to –1500 meters?

5. A meteorologist measured the air pressure every hour. Each hour the change in pressure was –5 millibars. What was the total change after 3 hours?

# Answer Key

## Book 14017 - Integers

**Page 5**

**2.** +4000 **3.** +6 **4.** +30 **5.** –266
**6.** +9 **7.** –6 **8.** +10 **9.** –15 **10.** –53
**11.** +8 **12.** +158 **13.** +23 **14.** –20
**15.** –7 **16.** +6 **17.** –9 **18.** +240

**Page 6**

**2.** < **3.** < **4.** > **5.** > **6.** < **7.** <
**8.** > **9.** > **10.** > **11.** = **12.** >
**13.** < **14.** < **15.** > **16.** > **17.** =
**18.** < **19.** < **20.** < **21.** > **22.** >
**23.** < **24.** = **25.** > **26.** < **27.** <
**28.** < **29.** > **30.** < **31.** = **32.** >
**33.** < **34.** > **35.** > **36.** <

**Page 9**

**2.** –38 **3.** 3 **4.** 1 **5.** 23 **6.** –6
**7.** 10 **8.** –9 **9.** –8 **10.** 16 **11.** –17
**12.** 10 **13.** –5 **14.** 13 **15.** –7 **16.** 12
**17.** 19 **18.** 1 **19.** 12 **20.** –17 **21.** –18
**22.** 10 **23.** 43 **24.** 21 **25.** 0 **26.** 38
**27.** 68 **28.** –18 **29.** 17 **30.** –13
**31.** 15 **32.** –34 **33.** –5 **34.** –106
**35.** –348 **36.** 25

**Page 11**

**2.** –28 **3.** –33 **4.** –18 **5.** 54 **6.** 24
**7.** 20 **8.** 54 **9.** –56 **10.** –100 **11.** 96
**12.** –20 **13.** 6 **14.** –64 **15.** 72
**16.** –30 **17.** 81 **18.** –32 **19.** –84

**20.** 100 **21.** –33 **22.** –99 **23.** 80
**24.** –66 **25.** 112 **26.** –312 **27.** 100
**28.** 1353 **29.** 9 **30.** 180 **31.** –308
**32.** –24 **33.** –20 **34.** –627 **35.** –15
**36.** –230

**38.** –12 **39.** 3 **40.** –7 **41.** –2 **42.** 4
**43.** –5 **44.** –12 **45.** –7 **46.** –4 **47.** 6
**48.** 12 **49.** –3 **50.** –10 **51.** –7 **52.** –5
**53.** 8 **54.** –4 **55.** 23 **56.** 9 **57.** –7
**58.** –2 **59.** –4 **60.** 2 **61.** –8 **62.** 21
**63.** 3 **64.** –16 **65.** 29 **66.** –4 **67.** –19
**68.** –7 **69.** –19 **70.** 2 **71.** –2 **72.** 23

**Page 12**

**1.** 22 degrees **2.** 20600 feet
**3.** **a.** Dale, -8; Toni, -16; Tracey; -17
**b.** Dale; Toni; Tracey
**4.** 30 minutes **5.** –15 millibars