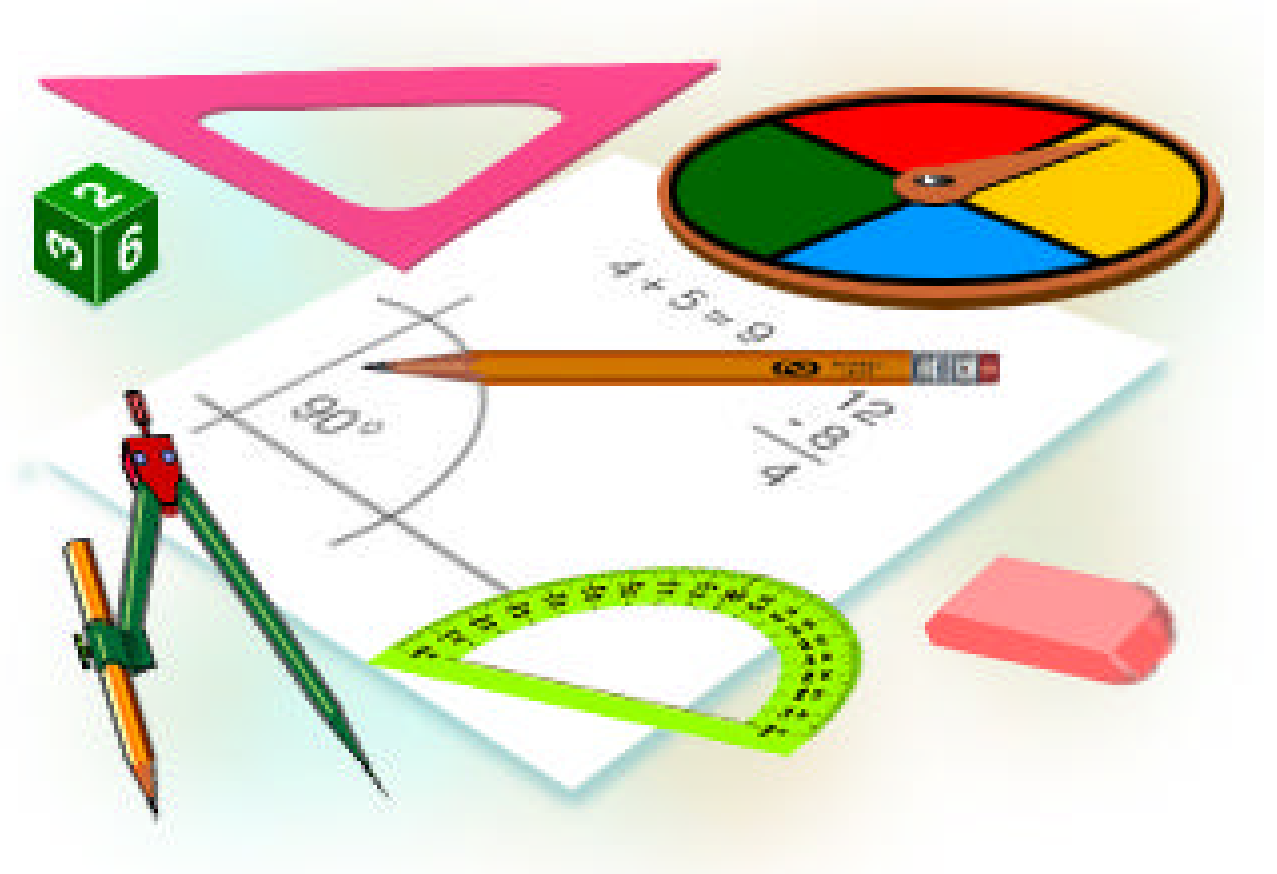


# The Next Step

## Mathematics Applications for Adults



**Book 14017 – Number Operations**

# INTRODUCTION

## Why Math?

The most important reason for learning math is that it teaches us how to think. Math is more than adding and subtracting, which can easily be done on a calculator; it teaches us how to organize thoughts, analyze information, and better understand the world around us.

Employers often have to re-educate their employees to meet the demands of our more complex technological society. For example, more and more, we must be able to enter data into computers, read computer displays, and interpret results. These demands require math skills beyond simple arithmetic.

### Everyone Is Capable of Learning Math

There is no **type** of person for whom math comes easily. Even mathematicians and scientists spend a lot of time working on a single problem. Success in math is related to practice, patience, confidence in ability, and hard work.

It is true that some people can solve problems or compute more quickly, but speed is not always a measure of understanding. Being “faster” is related to **more practice or experience**.

For example, the reason why math teachers can work problems quickly is because they've done them so many times before, not because they have "mathematical minds".

Working with something that is familiar is natural and easy. For example, when cooking from a recipe we have used many times before or playing a familiar game, we feel confident. We automatically know what we need to do and what to expect. Sometimes, we don't even need to think. However, when using a recipe for the **first** time or playing a game for the **first** time, we must concentrate on each step. We double-check that we have done everything right, and even then we fret about the outcome. The same is true with math. When encountering problems for the very first time, **everyone must have patience** to understand the problem and work through it correctly.

### It's Never Too Late to Learn

One of the main reasons people don't succeed in math is that they don't start at the right place. **IMPORTANT! You must begin where *you* need to begin.** Could you hit a homerun if you hadn't figured out which end of the bat had to make contact with the ball? Why should learning math be any different?

If it has been a while since your last math class, **you must determine what level math you should take.** A teacher or trained tutor can help determine this with a few placement tests and questions.

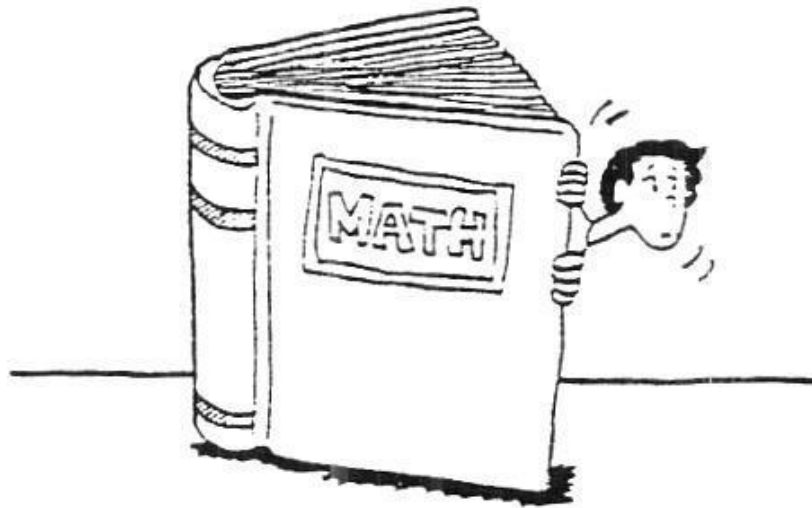
Sometimes a few tutoring sessions can help you fill gaps in your knowledge or help you remember some of the things you

have simply forgotten. It could also be the case where your foundations may be weak and it would be better for you to relearn the basics. **Get some help** to determine what is best for you.

Feeling good about ourselves is what all of us are ultimately striving for, and nothing feels better than conquering something that gives us difficulty. This takes a great deal of courage and the ability to rebound from many setbacks. This is a natural part of the learning process, and when the work is done and we can look back at our success, nothing feels better.

*Where's the best place to hide if you're scared?*

Inside a math book because there is safety in numbers.



*Artist Unknown*

## OUTLINE

### Mathematics - Book 14017

<b>Number Operations</b>
<b><u>Mathematical Operations, Average, Median, and Mode</u></b>
perform with accuracy and speed the four mathematical operations.
find average, median, and mode.
<b><u>Factors and Prime Numbers</u></b>
factor a given group of whole numbers.
determine which numbers are prime numbers.
find the Greatest Common Factor (GCM).
find the Least Common Multiple (LCM).
<b><u>Exponents</u></b>
express like factors using exponents.
express exponents using like factors.
<b><u>Problem Solving With Whole Numbers</u></b>
solve multi-step problems, with and without a calculator

# THE NEXT STEP

## Book 14017

### Number Operations

#### Mathematical Operations, Average, Median, and Mode



***Digit*** is a counting word. A digit is any of the numerals from **1** to **9**. The word “digit” is also the name for a finger. So number digits can be counted on finger digits.

Our modern system of counting or ***tallying*** probably came from counting on fingers. Fingers and hands were among the earliest known calculators!

The set of counting numbers has no end. It can go on forever. The idea that counting numbers can go on and on is called ***infinity***. Infinity has a special symbol:



There is no such thing as the “largest number.” You can always add to or multiply a large number to make an even bigger number.

$$\infty + 3 = \infty$$

$$\infty \times 10 = \infty$$

If you began writing all the counting numbers today, you could continue writing every moment of every day for every day of the rest of your life and never be finished!

### **What's a googol?**

A googol is a 1 with a hundred zeroes behind it. We can write a googol using exponents by saying a googol is  $10^{100}$  or 10 to the 100<sup>th</sup> power.

The biggest named number that we know is googolplex, ten to the googol power, or  $(10)^{(10^{100})}$ . That's written as a one followed by googol zeroes.

It's funny that no one ever seems to ask, “What is the smallest number?” Again, there is really no such thing. You could always subtract from or divide a small number to make an even smaller number. As the number gets smaller and smaller, you would be approaching, but never reaching, negative infinity.

$$-\infty$$

The set of *counting numbers*, or *natural numbers*, begins with the number 1 and continues into infinity.

$\{1,2,3,4,5,6,7,8,9,10...\}$

The set of *whole numbers* is the same as the set of counting numbers, except that it begins with 0.

$\{0,1,2,3,4,5,6,7,8,9,10...\}$

*☞ All counting numbers are whole numbers. Zero is the only whole number that is not a counting number.*

*Even numbers* include the numbers 0 and 2 and all numbers that can be divided evenly by 2. *Odd numbers* are all numbers that cannot be divided evenly by 2.

### Odd and Even Numbers to 100

1	3	5	7	9	11	13	15	17	19	21
0	2	4	6	8	10	12	14	16	18	20
23	25	27	29	31	33	35	37	39	41	
22	24	26	28	30	32	34	36	38	40	
43	45	47	49	51	53	55	57	59	61	
42	44	46	48	50	52	54	56	58	60	
63	65	67	69	71	73	75	77	79	81	
62	64	66	68	70	72	74	76	78	80	
83	85	87	89	91	93	95	97	99		
82	84	86	88	90	92	94	96	98	100	



**Ordering** numbers means listing numbers from least to greatest, or from greatest to least. Two symbols are used in ordering.

<

is less than

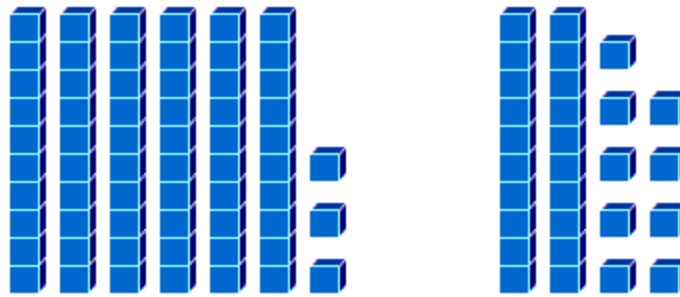
$$2 < 10$$

>

is greater

$$10 > 2$$

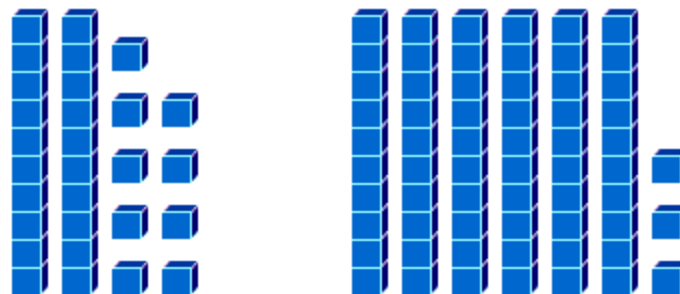
Greater Than >



63 is **greater than** 29.

$$63 > 29$$

Less Than <



29 is **less than** 63.

$$29 < 63$$



## Table of Addition Facts

<b>+</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
<b>1</b>	2	3	4	5	6	7	8	9	10	11
<b>2</b>	3	4	5	6	7	8	9	10	11	12
<b>3</b>	4	5	6	7	8	9	10	11	12	13
<b>4</b>	5	6	7	8	9	10	11	12	13	14
<b>5</b>	6	7	8	9	10	11	12	13	14	15
<b>6</b>	7	8	9	10	11	12	13	14	15	16
<b>7</b>	8	9	10	11	12	13	14	15	16	17
<b>8</b>	9	10	11	12	13	14	15	16	17	18
<b>9</b>	10	11	12	13	14	15	16	17	18	19
<b>10</b>	11	12	13	14	15	16	17	18	19	20

## Regrouping Numbers in Addition

Addition often produces sums with a value greater than **9** in a given place. The value of ten is then *regrouped* (or *carried*) to the next place.

tens	ones
+ 1	9
1	0

1

tens	ones
+ 1	3
2	2

1

hundreds	tens	ones
+ 4	1	3
4	2	1

1

hundreds	tens	ones
+ 4	9	6
5	0	1

1

thousands	hundreds	tens	ones
1,	3	4	3
+3,	7	9	8
5,	1	4	1

1

To explain addition another way, it can be done by adding the place value amounts separately.

e.g. 69

$$\begin{array}{r} + 8 \\ \hline 17 \end{array}$$

60 (the 6 in the tens place means 6 tens or “60”)

$$\underline{77}$$

⇒ If there are not enough digits in each number to make even columns under each place value, then zeros may be used **before** a given number to make adding easier. Do **not** add zeros **after** a number because it changes the value of the whole number.

e.g.  $6.9 + 8 + 125$  could be added as:

$$\begin{array}{r} 069 \\ 008 \\ +125 \\ \hline \end{array}$$

### Commutative Property of Addition

The property which states that two or more addends can be added in any order without changing the sum

$$a + b = b + a$$

*Examples:*

$$c + 4 = 4 + c$$

$$(2 + 5) + 4r = 4r + (2 + 5)$$

## Associative Property of Addition

The property which states that for all real numbers  $a$ ,  $b$ , and  $c$ , their sum is always the same, regardless of their grouping:

$$(a + b) + c = a + (b + c)$$

*Example:*

$$(2 + 3) + 4 = 2 + (3 + 4)$$

“Taking away” one or more numbers from another number is called **subtraction**. The term for subtraction is **minus**, and the symbol for minus is  $-$ . The number being subtracted is called a **subtrahend**. The number being subtracted from is called a **minuend**. The new number left after subtracting is called a **remainder** or **difference**.

$$\begin{array}{r} 4 \text{ ---- } \text{minuend} \text{ ---- } 4 \\ - 2 \text{ --subtrahend - - } 1 \\ \hline 2 \text{ - difference ---- } 3 \end{array}$$

The complete addition or subtraction “sentence” is called an **equation**. An equation has two parts. The two parts are separated by the **equal sign**,  $=$ . For example, **the minuend minus the subtrahend equals the difference**. An **addition fact** or a **subtraction fact** is the name given to specific addition or subtraction equations.

$$0 + 1 = 1$$

$$1 + 1 = 2$$

$$2 + 1 = 3$$

$$3 + 1 = 4$$

$$4 + 1 = 5$$

$$5 + 1 = 6$$

$$1 - 1 = 0$$

$$2 - 1 = 1$$

$$3 - 1 = 2$$

$$4 - 1 = 3$$

$$5 - 1 = 4$$

$$6 - 1 = 5$$

$6 + 1 = 7$

$7 + 1 = 8$

$8 + 1 = 9$

$7 - 1 = 6$

$8 - 1 = 7$

$9 - 1 = 8$

## Regrouping in Subtraction

**Regrouping**, sometimes called **borrowing**, is used when the subtrahend is greater than the minuend in a given place. Regrouping means to take a group of tens from the next greatest place to make a minuend great enough to complete the subtraction process.

	tens	ones
21	1 2	
- 3	-	3
-----	1	8

Diagram illustrating regrouping for  $21 - 3$ . A vertical dashed line separates the tens and ones places. A red arrow points from the 2 in the tens place to the 1 in the ones place, changing it to 11. The result is 18.

	tens	ones
46	3 4	
- 9	-	9
-----	3	7

Diagram illustrating regrouping for  $46 - 9$ . A vertical dashed line separates the tens and ones places. A red arrow points from the 4 in the tens place to the 6 in the ones place, changing it to 16. The result is 37.

	hundreds	tens	ones
343	3	3 4	
- 9	-	-	9
-----	3	3	4

Diagram illustrating regrouping for  $343 - 9$ . Vertical dashed lines separate the hundreds, tens, and ones places. A red arrow points from the 4 in the tens place to the 3 in the ones place, changing it to 13. The result is 334.

	hundreds	tens	ones
521	45	112	11
- 62	-	6	2
-----	-----	-----	-----
459	4	5	9

	hundreds	tens	ones
506	45	910	16
- 8	-		8
-----	-----	-----	-----
498	4	9	8

**Multiplication** is a quick form of addition. By multiplying numbers together, you are really adding a series of one number to itself. For example, you can add 2 plus 2. Both **2 plus 2** and **2 times 2** equal 4.

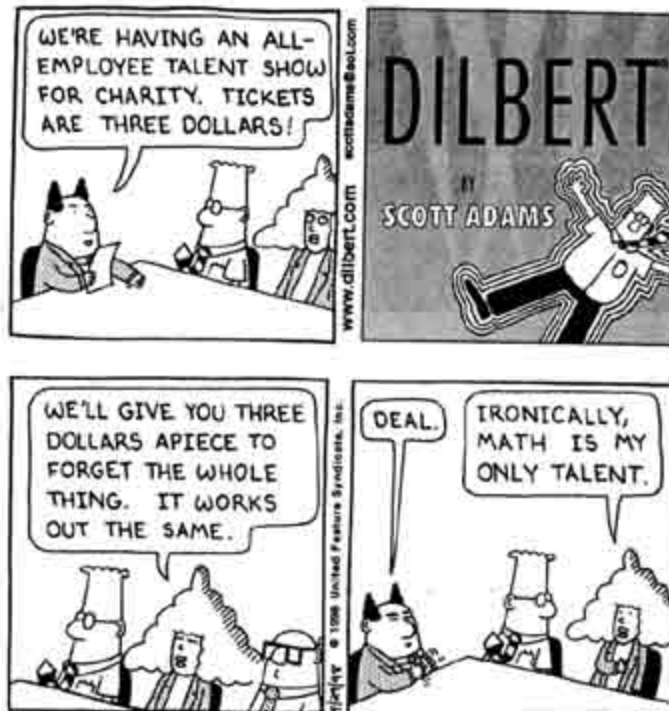
$2 + 2 = 4$	$2$	$2$
$2 \times 2 = 4$	$+ 2$	$\times 2$
	-----	-----
	4	4

But what if you wanted to calculate the number of days in five weeks? You could add 7 days + 7 days + 7 days + 7 days + 7



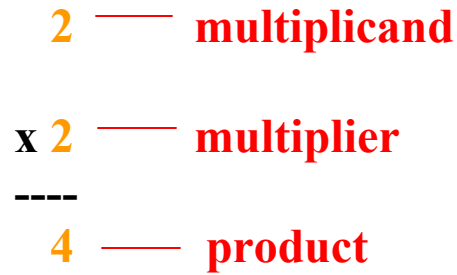
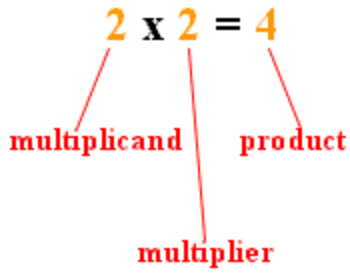
days or you could multiply 7 days times 5. Either way you arrive at 35, the number of days in five weeks.

$$\begin{array}{r} 7 + 7 + 7 + 7 + 7 = 35 \\ 5 \times 7 \qquad \qquad \qquad = 35 \end{array}$$



*Dilbert* by Scott Adams 9/29/98 Copyright 1998 by United Feature Syndicate, Inc.

Although multiplication is related to addition, the parts of multiplication are not known as addends. Instead, the parts are known as *multiplicands* and *multipliers*. A multiplication sentence, like an addition sentence, is called an *equation*. But a multiplication sentence results in a *product*, not a sum.



<b>X</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>12</b>
<b>1</b>	0	1	2	3	4	5	6	7	8	9	10	11	12
<b>2</b>	0	2	4	6	8	10	12	14	16	18	20	22	24
<b>3</b>	0	3	6	9	12	15	18	21	24	27	30	33	36
<b>4</b>	0	4	8	12	16	20	24	28	32	36	40	44	48
<del><b>5</b></del>	<del>0</del>	<del>5</del>	<del>10</del>	<del>15</del>	<del>20</del>	<del>25</del>	<del>30</del>	<del>35</del>	<b>40</b>	45	50	55	60
<b>6</b>	0	6	12	18	24	30	36	42	48	54	60	66	72
<b>7</b>	0	7	14	21	28	35	42	49	56	63	70	77	84
<b>8</b>	0	8	16	24	32	40	48	56	64	72	80	88	96
<b>9</b>	0	9	18	27	36	45	54	63	72	81	90	99	108
<b>10</b>	0	10	20	30	40	50	60	70	80	90	100	110	120
<b>11</b>	0	11	22	33	44	55	66	77	88	99	110	121	132
<b>12</b>	0	12	24	36	48	60	72	84	96	108	120	132	144

## Multiplication, Step-by-Step

When the multiplicand and the multiplier are numbers with two or more digits, multiplication becomes a step-by-step process.

Look at 15 x 13:

$$\begin{array}{r}
 15 \\
 \times 3 \\
 \hline
 15
 \end{array}$$

First, multiply the ones –  $3 \times 5$ . Write down the product so the ones columns line up.

$$\begin{array}{r}
 15 \\
 \times 3 \\
 \hline
 15
 \end{array}$$

Next, multiply the tens –  $3 \times 1$  ten. Line up the product with the tens column.

$$\begin{array}{r}
 30 \\
 \hline
 \end{array}$$

— Zero is the place holder.

$$\begin{array}{r}
 15 \\
 \times 3 \\
 \hline
 15
 \end{array}$$

Last, add the ones and tens to find the product of the equation.

$$\begin{array}{r}
 15 \\
 + 30 \\
 \hline
 45
 \end{array}$$

Here is a shorter way:

$$\begin{array}{r} 1 \\ 15 \\ \times 3 \\ \hline 45 \end{array}$$

1. Multiply the ones:  $3 \times 5 = 15$ .  
Put the 5 in the ones column and regroup the 1 to the tens column.

2. Multiply the tens:  $3 \times 1 = 3$ .

3. Add the 1 that you regrouped to the 3, put the sum in the tens column.

Look at  $265 \times 23$ :

$$\begin{array}{r} 265 \\ \times 23 \\ \hline 15 \\ 180 \\ 600 \end{array}$$

First, multiply the multiplicand by the ones in the multiplier –  $3 \times 5$ ,  $3 \times 6$ , and  $3 \times 2$ .  
Zero is the place holder.

$$\begin{array}{r} 265 \\ \times 23 \\ \hline 15 \\ 180 \\ 600 \\ \hline 100 \\ 1,200 \\ 4,000 \end{array}$$

Next, multiply by the tens –  $2 \times 5$ ,  $2 \times 6$ , and  $2 \times 2$ .  
Zero is the place holder.

	265	Last, add.
	x 23	
	-----	
	15	
+	180	
+	600	
	-----	
+	100	
+	1,200	
+	4,000	
	-----	
	6,095	

Here is a shorter way:

	11	
	11	
	265	1. Multiply the ones: $3 \times 265$
		$3 \times 5 = 15$ regroup the 1
x	23	$3 \times 6 = 18$ plus the regrouped 1 = 19;
	-----	regroup the 1
	795	$3 \times 2 = 6$ plus the regrouped 1 = 7
	5300	2. Multiply the tens: $2 \times 265$
	-----	0 is the place holder
	6,095	$2 \times 5 = 10$ regroup the 1
		$2 \times 6 = 12$ plus the regrouped 1 = 13;
		regroup the 1
		$2 \times 2 = 4$ plus the regrouped 1 = 5
		3. Add $795 + 5300 = 6,095$

## Partial Product

A method of multiplying where the ones, tens, hundreds, and so on are multiplied separately and then the products added together

*Example:*

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 12 \\ + 60 \\ \hline 72 \end{array}$$

← Multiply the ones:  $3 \times 4 = 12$   
← Multiply the tens:  $3 \times 20 = 60$

$$36 \times 17 = 42 + 210 + 60 + 300 = 612$$

When you multiply whole numbers, the *product* usually has a greater value than either the *multiplicand* or the *multiplier*.

But there are exceptions:

A number multiplied by *1* is always equal to itself.

$$\begin{array}{r} 1 \\ \times 1 \\ \hline 1 \end{array} \quad 21 \times 1 = 21 \quad \begin{array}{r} 36 \\ \times 1 \\ \hline 36 \end{array}$$

A number multiplied by *0* is always equal to *0*.

$$\begin{array}{r} 1 \\ \times 1 \\ \hline 0 \end{array} \quad 21 \times 0 = 0 \quad \begin{array}{r} 36 \\ \times 0 \\ \hline 0 \end{array}$$

To multiply a number by 10, add a 0 to the right of the number.

EXAMPLE

$$25 \times 10 = 250 \quad \text{or} \quad \begin{array}{r} 25 \\ \times 10 \\ \hline 250 \end{array}$$

To multiply a number by 100, add two 0's to the right of the number.

#### EXAMPLE

$$36 \times 100 = 3,600 \quad \text{or} \quad \begin{array}{r} 36 \\ \times 100 \\ \hline 3,600 \end{array}$$

### Commutative Property of Multiplication

The property which states that two or more factors can be multiplied in any order without changing the product

$$a \cdot b = b \cdot a$$

*Examples:*

$$3 \cdot c = c \cdot 3$$

$$4 \cdot 5 \cdot y7 = 5 \cdot 4 \cdot y7$$

### Associative Property of Multiplication

The property which states that for all real numbers  $a$ ,  $b$ , and  $c$ , their product is always the same, regardless of their grouping:

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

*Example:*

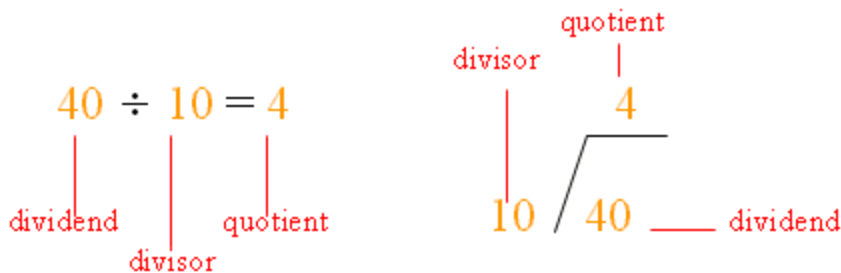
$$(5 \cdot 6) \cdot 7 = 5 \cdot (6 \cdot 7)$$

***Division*** is the process of finding out how many times one number, the ***divisor***, will fit into another number, the ***dividend***.

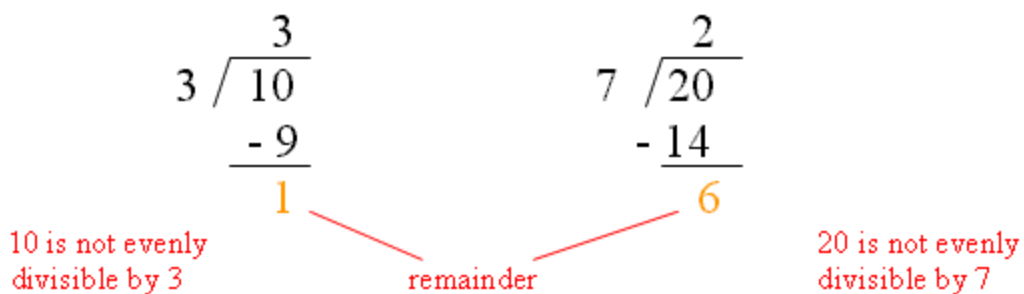
The division sentence results in a *quotient*. The signs of division are  $\div$  and  $\sqrt{\quad}$ , and mean *divided by*. You can think of division as a series of repeated subtractions. For example,  $40 \div 10$  could also be solved by subtracting  $10$  from  $40$  four times:

$$40 - 10 - 10 - 10 - 10 = 0$$

Because  $10$  can be subtracted four times, you can say that  $40$  can be divided by  $10$  four times, or  $40 \div 10 = 4$ .



Many numbers do not fit evenly into other numbers. They are *not evenly divisible by* those numbers, and the number left over is called the *remainder*.



To divide whole numbers, reverse the process of multiplication. For example, if  $2 \times 7 = 14$  in a multiplication equation, then in a division sentence,  $14$  is the *dividend* and  $7$  is the *divisor* with a *quotient* of  $2$ .



$$\begin{array}{ccc}
 & \text{divisor} & \\
 & \swarrow & \searrow \\
 14 \div 7 = 2 & & \\
 \swarrow & & \searrow \\
 \text{dividend} & & \text{quotient}
 \end{array}
 \qquad
 \begin{array}{ccc}
 & \text{quotient} & \\
 & \swarrow & \searrow \\
 7 \overline{)14} & & \\
 \swarrow & & \searrow \\
 \text{divisor} & & \text{dividend}
 \end{array}$$

A whole number divided by *1* will always equal itself.

$$1 \overline{)1} = 1 \qquad 1 \overline{)21} \qquad 36 \overline{)36}$$

Zero divided by a whole number will always equal *0*.

$$0 \div 12 = 0 \qquad 3 \overline{)0} \qquad 0/7 = 0$$

## Division, Step-by-Step

Where the dividend and divisor are numbers with two or more digits, division becomes a step-by-step process.

$$\begin{array}{r}
 2 \\
 8 \overline{)208} \\
 -16 \phantom{0} \\
 \hline
 4
 \end{array}$$

First, round the divisor up - 8 rounds up to 10 - and estimate the number of 10s in 20. Answer: 2. Multiply the divisor – 8 x 2 – and subtract the product from the dividend.

$$\begin{array}{r}
 26 \\
 8 \overline{) 208} \\
 - 16 \phantom{0} \\
 \hline
 48 \\
 - 48 \\
 \hline
 0
 \end{array}$$

Next, pull down the next digit from the dividend – 8 – and repeat the estimation and subtraction process.

$$\begin{array}{r}
 26 \\
 8 \overline{) 208} \\
 - 16 \phantom{0} \\
 \hline
 48 \\
 - 48 \\
 \hline
 0
 \end{array}$$

Last, when you can subtract no more you've found the quotient.

0 ——— No remainder

$$\begin{array}{r}
 1 \\
 23 \overline{) 276} \\
 - 23 \phantom{0} \\
 \hline
 4
 \end{array}$$

First, round 23 to 25 and estimate the number of 25s in 27. Answer: 1. Multiply the divisor by 1 – 23 x 1 – and subtract.

$$\begin{array}{r}
 12 \\
 23 \overline{) 276} \\
 - 23 \phantom{0} \\
 \hline
 46 \\
 - 46 \\
 \hline
 0
 \end{array}$$

Next, pull down the next digit from the dividend – 6 – and repeat the estimation and subtraction process.

$$\begin{array}{r}
 23 \overline{)276} \\
 \underline{-23} \phantom{0} \\
 46 \\
 \underline{-46} \\
 0
 \end{array}$$

Then, pull down the next digit, estimate, and subtract, until you can subtract no more.

0 ——— No remainder

Inverse (opposite) operations are used to simplify an equation for solving.

One operation is involved with the unknown and the inverse operation is used to solve the equation.

### **Addition and subtraction are inverse operations.**

Given an equation such as  $7 + x = 10$ , the unknown  $x$  and  $7$  are *added*. Use the inverse operation subtraction. To solve for  $x$ , subtract  $7$  from  $10$ . The unknown value is therefore  $3$ .

### Examples for addition and subtraction

Addition Problem

$$x + 15 = 20$$

Solution

$$x = 20 - 15 = 5$$

Subtraction Problem

$$x - 15 = 20$$

Solution

$$x = 20 + 15 = 35$$

### **Multiplication and division are inverse operations.**

Given an equation  $7x = 21$ .  $x$  and  $7$  are multiplied to create a value of  $21$ . To solve for  $x$ , divide  $21$  by  $7$  for an answer of  $3$ .

Examples for division and multiplication.

Multiplication Problem

$$3x = 21$$

Solution

$$x = 21 \div 3 = 7$$

Division Problem

$$x \div 12 = 3$$

Solution

$$y = 3 \times 12 = 36$$

# Practice Exercise

Solve each problem.

- |     |  |     |   |     |   |     |   |
|-----|--|-----|---|-----|---|-----|---|
| 1.  | $235 \overline{)9635}$                               | 2.  | $\begin{array}{r} 233 \\ \times 91 \\ \hline \end{array}$ | 3.  | $\begin{array}{r} 406 \\ + 37 \\ \hline \end{array}$      | 4.  | $707 \overline{)47369}$                                   |
| 5.  | $\begin{array}{r} 374 \\ + 44 \\ \hline \end{array}$ | 6.  | $\begin{array}{r} 585 \\ - 56 \\ \hline \end{array}$      | 7.  | $\begin{array}{r} 226 \\ - 22 \\ \hline \end{array}$      | 8.  | $\begin{array}{r} 602 \\ \times 16 \\ \hline \end{array}$ |
| 9.  | $580 \overline{)37700}$                              | 10. | $\begin{array}{r} 683 \\ + 89 \\ \hline \end{array}$      | 11. | $\begin{array}{r} 492 \\ \times 12 \\ \hline \end{array}$ | 12. | $\begin{array}{r} 828 \\ - 30 \\ \hline \end{array}$      |
| 13. | $\begin{array}{r} 165 \\ - 62 \\ \hline \end{array}$ | 14. | $\begin{array}{r} 692 \\ \times 30 \\ \hline \end{array}$ | 15. | $\begin{array}{r} 651 \\ + 71 \\ \hline \end{array}$      | 16. | $734 \overline{)58720}$                                   |

17.	$\begin{array}{r} 567 \\ + 51 \\ \hline \end{array}$	18.	$\begin{array}{r} 642 \\ - 65 \\ \hline \end{array}$	19.	$470 \overline{)15510}$	20.	$\begin{array}{r} 570 \\ \times 70 \\ \hline \end{array}$
21.	$\begin{array}{r} 690 \\ \times 44 \\ \hline \end{array}$	22.	$196 \overline{)11956}$	23.	$\begin{array}{r} 412 \\ - 45 \\ \hline \end{array}$	24.	$\begin{array}{r} 511 \\ + 16 \\ \hline \end{array}$

## Order of Operations

Sometimes the order in which you add, subtract, multiply, and divide is very important. For example, how would you solve the following problem?

$$2 \times 3 + 6$$

Would you group

$$(2 \times 3) + 6 \text{ or } 2 \times (3 + 6) ?$$

Which comes first, addition or multiplication? Does it matter?  
Yes. Mathematicians have written two simple steps:

1. *All multiplication and division operations are carried out first, from left to right, in the order they occur.*

2. Then all addition and subtraction operations are carried out, from left to right, in the order they occur.

For example:

$$\begin{array}{ccccccc} 8 \div 2 + 2 \times 3 - 1 & = & 4 + 6 - 1 & = & 9 \\ \swarrow \quad \searrow & & \swarrow \quad \searrow & & \swarrow \quad \searrow \\ 4 & & 6 & & 10 \\ \text{step 1} & & & & \text{step 2} \end{array}$$

**P** Perform all operations with parentheses (brackets) and exponents before carrying out the remaining operations in an equation.

$$8 \div (2 + 2) \times 3 - 1 =$$

$$8 \div 4 \times 3 - 1 =$$

$$2 \times 3 - 1 =$$

$$6 - 1 = 5$$

To remember the order of operations, simply remember **BEDMAS**: **B**rackets, **E**xponents, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction.

*Example:*

$$\begin{array}{l} 10 \div (2 + 8) \times 2^3 - 4 \\ 10 \div 10 \times 2^3 - 4 \end{array}$$

*Add inside parentheses.  
Clear exponent.*

$$10 \div 10 \times (2 \times 2 \times 2) - 4$$

$$10 \div 10 \times 8 - 4$$

$$8 - 4$$

$$4$$

*Multiply and divide.*

*Subtract.*

# Practice Exercise

<b>1. <math>16 \div 8 + 19</math></b>	<b>2. <math>24 \times 18 - 14</math></b>
<b>3. <math>19 \times 19 + 4</math></b>	<b>4. <math>19 \times 13 - 20</math></b>
<b>5. <math>(20 \div 5) + 21 \times 11</math></b>	<b>6. <math>3 + 4</math></b>
<b>7. <math>5^3 + 4 + 11</math></b>	<b>8. <math>21 + 3^2 - 21</math></b>
<b>9. <math>25 \times 21</math></b>	<b>10. <math>(17 \times 19) \times 15 \times 5</math></b>
<b>11. <math>13 \times 12^2 - 17 + (12 + 12)</math></b>	<b>12. <math>6 \times 13</math></b>
<b>13. <math>(18 + 4) \times 19</math></b>	<b>14. <math>24 - 9</math></b>

15. $2 \times 15 \times (4 \times 17 \times 3)$	16. $18 \times 12 - 19$
17. $12 \times 5 + 23 \times 12$	18. $14 + 13 \times (23 - 3) + 9$
19. $13 \times (23 - 19) + 23$	20. $22 - 20$

## Averages

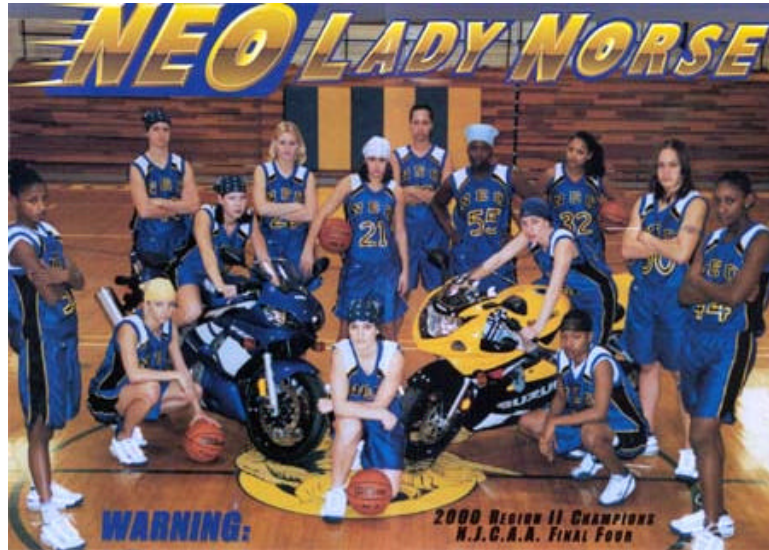
The most common way to find an *average* is to add up a list of numbers and divide the sum by the number of items on the list. Another word for average is *mean*.

$$3 + 4 + 6 + 8 + 9 = 30 \quad \text{number of addends}$$

**sum** —  $30 \div 5 = 6$  So, the average of the numbers 3, 4, 6, 8, and 9 is 6.

When do you need to calculate an average? Your grades may be based on the average of all your test scores. In sports, you might want to find out the average height of players on your favorite basketball team.





**The height of the starters for this team is:**

**Anita      60”**

**Jane        58”**

**Cathy      57”**

**Joy         52”**

**Tanya      48”**

**The average height of these players is 55 inches.**

## **Medians**

Average or mean is different from *median*. The median is the middle number in a series of numbers stated in order from least

to greatest. An average and a median can be the same number. The average of 3, 5, and 7 is 5:

$$3 + 5 + 7 = 15 \text{ and } 15 \div 3 = 5$$

and the median of 3, 5, and 7 is 5. But average and median are often different numbers.

**Anita**      **60"**

**Jane**      **58"**

**Cathy**     **57"**

**Joy**        **52"**

**Tanya**     **48"**

**The median height of these girls is 57 inches—Cathy's height-- because it is the middle number.**

If there is an even number of data items, the median is the average (mean) of the two middle numbers.

**Example** Amy's point totals for six games of basketball were 24, 16, 19, 22, 6, and 12 points. Find the median of her point totals.

**Step 1** Arrange the data in order.

24, 22, **19**, **16**, 12, 6

**Step 2** The two middle numbers are 19 and 16. Average these to find the median.

$$19 + 16 = 35$$

$$35 \div 2 = 17.5$$

Amy's median point total is **17.5 points**.

## Percentiles

Individual scores may be compared with all the other scores in a group by giving the score a positional standing or rank.

The **percentile rank** of a score indicates the percent of all the scores that are below this given score. If the rank of a particular score is the 60<sup>th</sup> percentile, it means that 60 % of all the scores are lower than this score.

**Examples** If Todd ranked fourth in a class of 16 students, there are 12 students of 16, or 75%, with a lower rank. He would have a percentile rank of 75 or a rank of the 75<sup>th</sup> percentile.

If Todd ranked fourth in a class of 40 students, there are 36 students of 40, or 90%, with a lower rank. He would have a percentile rank of 90 or a rank of the 90<sup>th</sup> percentile.

## Mode

The number or numbers that occur most often in a collection of data; there can be more than one mode or none at all.

*Examples:*

2, 3, 4, 5, 5, 6, 7, **8, 8, 8**, 9, 11

The mode is 8.

2, 3, 4, **5, 5, 5**, 7, **8, 8, 8**, 9, 11

The modes are 5 and 8.

2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 17

There is no mode.

# Practice Exercise

Calculate the values to the nearest tenth.

1. 13, 10, 18, 13, and 18

Write the median:

Write the mean:

Write the mode:

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

2. 3, 24, 11, 3, and 11

Write the median:

Write the mean:

Write the mode:

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

3. 10, 14, 16, 14, and 20

Write the median:

Write the mean:

Write the mode:

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

4. 5, 13, 5, 5, and 17

Write the median:

Write the mean:

Write the mode:

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

5. Given the following annual mutual fund returns:

12.33	69.31	41.8	9.84
24.78	16.5	23.29	23.51
27.28	11.41	14.36	7.39
12.69	45.52	27.28	14.14
62.58	14.14	40.08	44.73

Write the median:

Write the mean:

Write the mode:

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

6. Students with the following GPAs applied for a job: 2.8, 2.8, 2.8, 2.8, 2.4, 3.7, 2.7, 3.2, 3, and 2.6

Write the median:

Write the mean:

Write the mode:

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

7. The following grades were posted on the latest exam: 69, 66, 95, 89, 77, 87, 59, 95, 95, and 89

Write the median:

Write the mean:

Write the mode:

\_\_\_\_\_

\_\_\_\_\_

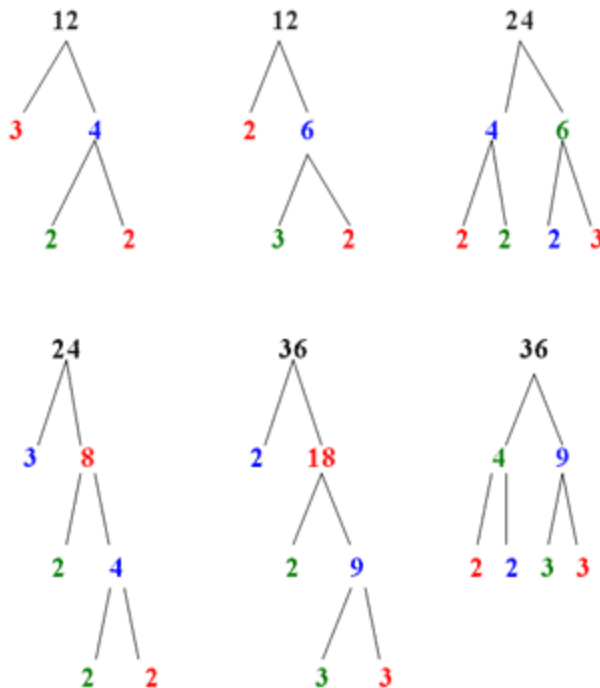
\_\_\_\_\_

## Factors and Prime Numbers

**Factors** are numbers that, when multiplied together, form a new number called a **product**. For example, **1** and **2** are factors of **2**, and **3** and **4** are factors of **12**.

Every number except **1** has at least two factors: **1** and itself.

**Composite numbers** have more than two factors. In fact, every composite number can be written as the product of **prime numbers**. You can see this on a **factor tree**.



**Prime numbers** are counting numbers that can be divided by only two numbers---**1** and themselves. A prime number can also

be described as a counting number with only two factors, *1* and itself. The number *1*, because it can be divided only by itself, is *not* a prime number.

### Prime Numbers to 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,  
53, 59, 61, 67, 71, 73, 79, 83, 89, 97

# Practice Exercise

Classify each number as prime or composite.

1. 58 <input type="checkbox"/> Prime <input checked="" type="checkbox"/> Composite	2. 82 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	3. 5 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	4. 3 <input type="checkbox"/> Prime <input type="checkbox"/> Composite
5. 14 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	6. 40 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	7. 15 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	8. 6 <input type="checkbox"/> Prime <input type="checkbox"/> Composite
9. 27 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	10. 68 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	11. 8 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	12. 37 <input type="checkbox"/> Prime <input type="checkbox"/> Composite

13. 44 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	14. 51 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	15. 12 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	16. 42 <input type="checkbox"/> Prime <input type="checkbox"/> Composite
17. 90 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	18. 95 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	19. 16 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	20. 20 <input type="checkbox"/> Prime <input type="checkbox"/> Composite
21. 70 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	22. 9 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	23. 57 <input type="checkbox"/> Prime <input type="checkbox"/> Composite	24. 65 <input type="checkbox"/> Prime <input type="checkbox"/> Composite

Find the prime factorization of each number.

1. 12      **2, 2, 3**
2. 30
3. 28
4. 14
5. 10
6. 8
7. 22
8. 44
9. 64
10. 86
11. 66
12. 100



- 13. 58
- 14. 88
- 15. 18
- 16. 144
- 17. 42
- 18. 60

### **The Greatest Common Factor**

*Common factors* are numbers that are factors of two or more numbers. For example, **2** is a factor of **12** and **36**, which makes **2** a common factor of **12** and **36**. The common factor of two numbers with the greatest value is called the ***greatest common factor***. For example, **2**, **3**, **4**, **6**, and **12** are common factors of **12** and **36**, but **12** is the greatest common factor.

### **Multiples**

Find the ***multiples*** of a number by multiplying it by other whole numbers. The multiples of **2**, for example, are:

$$0 \times 2 = \underline{0}$$

$$2 \times 3 = \underline{6}$$

$$1 \times 2 = \underline{2}$$

$$2 \times 4 = \underline{8}$$

$$2 \times 2 = \underline{4}$$

$$2 \times 5 = \underline{10}$$

...and so on.

As you can see, the multiples of 2 include 0, 2, 4, 6, 8, and 10. The list continues into infinity!

Some numbers share the same multiples. Those multiples are known as *common multiples*.

## Number Multiples

0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25
	0	1	2	3	4	5

The least multiple of two or more numbers is the least common multiple. For example, the least common multiple of 2 and 3 is 6.

$$2 \times 1 = 2$$

$$2 \times 2 = 4$$

$$2 \times 3 = 6$$

$$3 \times 1 = 3$$

$$3 \times 2 = 6$$

# Practice Exercise

Find the greatest common factor (GCF) for the given numbers.

1. 12, 9    **3**
2. 9, 11
3. 6, 12
4. 2, 12
5. 5, 8
6. 6, 10
7. 5, 10
8. 8, 12
9. 3, 4
10. 12, 30
11. 21, 24
12. 6, 4
13. 12, 4
14. 16, 24
15. 15, 20
16. 17, 27
17. 6, 30
18. 20, 18
19. 12, 40
20. 30, 45
21. 12, 24
22. 35, 42

23. 12, 11
24. 8, 48
25. 162, 6
26. 42, 126
27. 84, 168
28. 60, 120
29. 56, 70
30. 30, 120

Find the least common multiple for the given numbers.

1. 9, 8      **72**
2. 4, 12
3. 10, 8
4. 9, 6
5. 8, 12
6. 6, 3
7. 5, 4
8. 5, 2
9. 12, 6
10. 14, 17
11. 30, 5
12. 12, 18
13. 6, 20
14. 8, 7
15. 2, 30
16. 29, 18

17. 20, 10
18. 6, 16
19. 20, 22
20. 22, 5
21. 20, 12
22. 24, 12
23. 8, 4
24. 24, 36
25. 54, 108
26. 18, 28
27. 64, 160
28. 2, 48
29. 165, 18
30. 100, 90

## Exponents

### Powers and Exponents

To find the *powers* of a number, multiply the number over and over by itself. The *first power* is the number. The *second power* is the product of the number multiplied once by itself or *squared*. The *third power* is the number multiplied twice by itself or *cubed*, and so on. For example:

$$2^1 = 2 \times 1 \quad 2^2 = 2 \times 2 \quad 2^3 = 2 \times 2 \times 2$$

$$10^1 = 10 \times 1 \quad 10^2 = 10 \times 10 \quad 10^3 = 10 \times 10 \times 10$$

The numbers above are written in expanded form.

$5^2$  can be read as “five to the second power” or “five squared”.

$5^3$  can be read as “five to the third power” or “five cubed”.

$5^4$  can be read as “five to the fourth power”.

**P** *There is a special way of writing the power of a number called an **exponent**. It's the tiny number written above and to the right of the number.*

**P** *Sometimes you may see an exponent expressed like this,  $2^5$ . This would be the same as  $2^5$ .*

### Base

A number used as a repeated factor

*Example:*

$$8^3 = 8 \times 8 \times 8$$

The base is 8. It is used as a factor three times.

The exponent is 3.

## Converting Numbers to Scientific (Exponential) Notation

Scientific notation is used to express very large or very small numbers. A number in scientific notation is written as the product of a number (integer or decimal) and a power of 10. This number is always 1 or more and less than 10.

For example, there are approximately 6,000,000,000 humans on earth. This number could be written in scientific notation as  $6 \times 10^9$ . The number 6,000,000,000 is equivalent to  $6 \times 1,000,000,000$ . The number 1,000,000,000 is equivalent to  $10^9$  or  $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ .

A number can be converted to scientific notation by increasing the power of ten by one for each place the decimal point is moved to the left. In the example above, the decimal point was moved 9 places to the left to form a number more than 1 and less than 10.

Number	Scientific Notation?	Product of	Places after 1st Digit
1	$1.0 \times 10^0$	1	0 places
10	$1.0 \times 10^1$	$1 \times 10$	1 places
100	$1.0 \times 10^2$	$1 \times 10 \times 10$	2 places

1,000	$1.0 \times 10^3$	$1 \times 10 \times 10 \times 10$	3 places
10,000	$1.0 \times 10^4$	$1 \times 10 \times 10 \times 10 \times 10$	4 places
100,000	$1.0 \times 10^5$	$1 \times 10 \times 10 \times 10 \times 10 \times 10$	5 places
1,000,000	$1.0 \times 10^6$	$1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	6 places

**Scientific notation numbers may be written in different forms. The number  $6 \times 10^9$  could also be written as  $6e+9$ . The +9 indicates that the decimal point would be moved 9 places to the right to write the number in standard form.**

Negative powers of 10 are useful for writing very small numbers. Any number to a negative power represents a fraction or decimal.

$$10^{-1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10} \times \frac{1}{10} = 0.01$$

$$10^{-3} = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.001$$

**Example** In a scientific experiment, the mass of a sample is  $2 \times 10^{-5}$  kilogram. Write the mass in standard notation.

**Step 1** Write the given number with a string of zeros in front of it. You haven't changed the value.



0000002.

**Step 2** Move the decimal point to the left by the number of places shown in the exponent. Discard extra zeros.

00.00002.

$$2 \times 10^{-5} = 0.00002$$

## Practice Exercise

Solve each problem and write the answer in expanded form. The first one has been done for you.

1. $3^2$ $3 \times 3 = 9$	2. $6^3$
3. $10^3$	4. $4^2$
5. $9^2$	6. $3^3$
7. $10^4$	8. $4^5$
9. $8^4$	10. $3^6$

<b>11. <math>8^5</math></b>	<b>12. <math>12^2</math></b>
<b>13. <math>9^4</math></b>	<b>14. <math>3^4</math></b>
<b>15. <math>4^3</math></b>	<b>16. <math>9^3</math></b>
<b>17. <math>11^3</math></b>	<b>18. <math>4^4</math></b>
<b>19. <math>3^5</math></b>	<b>20. <math>5^3</math></b>
<b>21. <math>8^3</math></b>	<b>22. <math>5^4</math></b>
<b>23. <math>11^2</math></b>	<b>24. <math>5^2</math></b>
<b>25. <math>9^5</math></b>	<b>26. <math>6^4</math></b>

## Word Problems with Whole Numbers

Within every story (word) problem are several *clue words*. These words tell you the kind of math sentence (equation) to write to solve the problem.

### **Addition Clue Words**

add  
sum  
total  
plus  
in all  
both  
together  
increased by  
all together  
combined

### **Subtraction Clue Words**

subtract  
difference  
take away  
less than  
are not  
remain  
decreased by  
have or are left  
change (money problems)  
more  
fewer

### **Multiplication Clue Words**

times  
product of  
multiplied by  
by (dimension)

### **Division Clue Words**

quotient of  
divided by  
half [or a fraction]  
split  
separated  
cut up  
parts  
shared equally

⇒ *Division clue words are often the same as subtraction clue words. Divide when you know the total and are asked to find the size or number of “one part” or “each part.”*

Following a system of steps can increase your ability to accurately solve problems. Use these steps to solve word problems.

1. Read the problem carefully. Look up the meanings of unfamiliar words.
2. Organize or restate the given information.
3. State what is to be found.
4. Select a strategy (such as making a chart of working backward) and plan the steps to solve the problem.
5. Decide on an approximate answer before solving the problem.
6. Work the steps to solve the problem.
7. Check the final result. Does your answer seem reasonable?

The Problem Solving System was used to solve the following problem:

**Mary has ten marbles. Lennie has thirteen. How many marbles do they have in all?**

**1. Mary has ten marbles. Lennie has thirteen.**

**How many marbles do they have in all?**

**2. Mary – 10 marbles**

**Lennie – 13 marbles**

**3. How many marbles in all?**

**4. Add**

**5. A little over 20 marbles ( $10 + 10 = 20$ )**

**6. 10**

**+13**

**23 marbles**

**7. The final sum of 23 marbles is close to the estimated answer of 20 marbles. The final result is reasonable.**

**P**

*Be sure to label answers whenever possible. For example: marbles, grams, pounds, feet, dogs, etc.*

**P** *Some problems may require several steps to solve. Some may have more than one correct answer. And some problems may not have a solution.*

Have you ever tried to help someone else work out a word problem? Think about what you do. Often, you read the problem with the person, then discuss it or put it in your own words to help the person see what is happening. You can use this method---restating the problem---on your own as a form of “talking to yourself.”

Restating a problem can be especially helpful when the word problem contains no key words. Look at the following example:

**Example:** Susan has already driven her car 2,700 miles since its last oil change. She still plans to drive 600 miles before changing the oil. How many miles does she plan to drive between oil changes?

**Step 1:** *question:* How many miles does she plan to drive between oil changes?

**Step 2:** *necessary information:* 2,700 miles, 600 miles

**Step 3:** *decide what arithmetic to use:* Restate the problem in your own words: “You are given the number of miles Susan has already driven and the number of miles more that she plans to drive. You need to add these together to find the total number of miles between oil changes.”

**Step 4:** 2,700 miles + 600 miles = **3,300 miles** between oil changes.

**Step 5:** It makes sense that she will drive 3,300 miles between oil changes, since you are looking for a number larger than the 2,700 miles that she has already driven.

For some problems, you have to write two or three equations to solve the problem. For others, you may need to make charts or lists of information, draw pictures, find a pattern, or even guess and check. Sometimes you have to work backwards from a sum, product, difference, or quotient, or simply use your best logical thinking.

### List/Chart

**Marty's library book was six days overdue. The fine is \$.05 the first day, \$.10, the second, \$.20 the third day, and so on. How much does Marty owe?**

**Marty's library book was six days overdue. The fine is \$.05 the first day, \$.10, the second, \$.20 the third day, and so on. How much does Marty owe?**

<b>Days</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>Fine</b>	<b>\$.05</b>	<b>\$.10</b>	<b>\$.20</b>	<b>\$.40</b>	<b>\$.80</b>	<b>\$1.60</b>

**Answer: \$1.60**

Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?

Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?

Veronica	Veronica	Archie	Archie
Archie	Betty	Veronica	Betty
Betty	Archie	Betty	Veronica
Betty	Betty		
Veronica	Archie		
Archie	Veronica		

**Answer: 6 ways**

### Find a Pattern

Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X\_\_\_\_\_. How did Jenny fill in the blanks?

Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X\_\_\_\_\_. How did Jenny fill in the blanks?

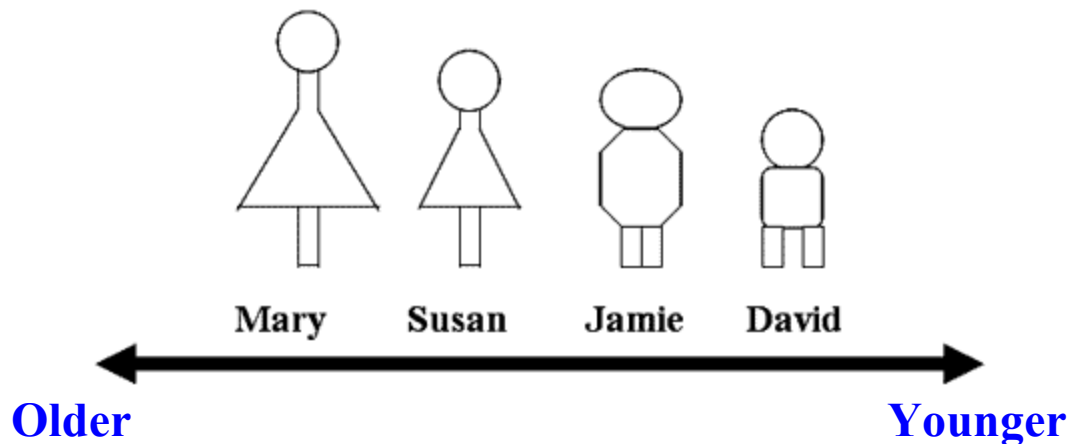
**Answer: 10, 11, 12 W**



## Draw a Picture

Mary is older than Jamie. Susan is older than Jamie, but younger than Mary. David is younger than Jamie. Who is oldest?

Mary is older than Jamie. Susan is older than Jamie, but younger than Mary. David is younger than Jamie. Who is oldest?



**Answer: Mary is oldest.**

## Guess and Check

Farmer Joe keeps cows and chickens in the farmyard. All together, Joe can count 14 heads and 42 legs. How many cows and how many chickens does Joe have in the farmyard?

Farmer Joe keeps **cows and chickens** in the farmyard. **All together**, Joe can count **14 heads** and **42 legs**. **How many cows and how many chickens** does Joe have in the farmyard?

$\begin{array}{r} 6 \text{ cows} \\ +8 \text{ chickens} \\ \hline 14 \text{ heads} \end{array}$	<p>Guess a number of cows. Then add the number of chickens to arrive at the sum of 14 heads. Then check the total legs.</p>	$\begin{array}{r} 6 \text{ cows} = 24 \text{ legs} \\ +8 \text{ chickens} = 16 \text{ legs} \\ \hline 40 \text{ legs} \end{array}$
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$\begin{array}{r} 7 \text{ cows} \\ +7 \text{ chickens} \\ \hline 14 \text{ heads} \end{array}$	<p>Adjust your guesses. Then check again until you solve the problem.</p>	$\begin{array}{r} 7 \text{ cows} = 28 \text{ legs} \\ +7 \text{ chickens} = 14 \text{ legs} \\ \hline 42 \text{ legs} \end{array}$
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**Answer: 7 cows and 7 chickens**

### Work Backwards

Marsha was banker for the school play. She took in \$175 in ticket sales. She gave Wendy \$75 for sets and costumes and Paul \$17.75 for advertising and publicity. After paying for the props, Marsha had \$32.25 left. How much did the props cost?

Marsha was banker for the school play. She **took in \$175** in ticket sales. She **gave Wendy \$75** for sets and costumes **and Paul \$17.75** for advertising and publicity. **After paying for the props**, Marsha had **\$32.25 left**. **How much did the props cost?**

\$ 175.00 tickets	\$ 82.25
- 75.00 costumes	- 32.25
<hr style="width: 100%; border: 0.5px solid black;"/>	<hr style="width: 100%; border: 0.5px solid black;"/>
\$ 100.00	\$ 50.00 cost of props
- 17.75 advertising	
<hr style="width: 100%; border: 0.5px solid black;"/>	
\$ 82.25	

## Logical Reasoning

Juan challenged Sheila to guess his grandmother’s age in ten questions or less. It took her six. Here’s what Sheila asked:

Juan challenged Sheila to **guess his grandmother’s age** in ten questions or less. It took her six. Here’s what Sheila asked:

“Is she less than fifty?” “No.”

**50+ years old**

“Less than seventy-five?” “Yes.”

**50 to 74 years old**

“Is her age an odd or even number?”

“Odd.”

**ends in 1, 3, 5, 7 or 9**

“Is the last number less than or equal to five?” “No.”

**ends in 7 or 9**

“Is it nine?” “No.”

**ends in 7 – 57 or 67**

“Is she in her sixties?” “No.”

57 years old

## Not Enough Information

Now that you know how to decide whether to add, subtract, multiply, or divide to solve a word problem, you should be able to recognize a word problem that cannot be solved because not enough information is given.

Look at the following example:

**Problem:** At her waitress job, Sheila earns \$4.50 an hour plus tips. Last week she got \$65.40 in tips. How much did she earn last week?

**Step 1:** *question:* How much did she earn last week?

**Step 2:** *necessary information:* \$4.50/hour, \$65.40

**Step 3:** *decide what arithmetic to use:*

$\text{tips} + (\text{pay per hour} \times \text{hours worked}) = \text{total earned}$

*missing information:* hours worked

At first glance, you might think that you have enough information since there are 2 numbers. But when the solution is set up, you can see that you need to know the number of hours Sheila worked to find out what she earned. **(Be Careful!!!)**

# Practice Exercise

1. The New Software Company received a shipment of 200,000 foam pellets to be used in packing boxes. If New Software uses on the average 400 pellets for each box, how many boxes can be packed using the shipment of pellets?
2. Jill's allergy pills come in a bottle containing 250 tablets. She takes 4 tablets a day. How many tablets did she have left after taking the tablets for 30 days?
3. An oil truck carried 9,008 gallons of oil. After making 7 deliveries averaging 364 gallons each, how much oil was left in the truck?
4. Memorial Stadium has 72,070 seats. 58,862 people had seats at the football game. How many seats were empty?
5. In one garden bed, a gardener grew spinach, and when the spinach was harvested, he grew green beans. The spinach was harvested after 49 days. The green beans were harvested after 56 days. For how many days were vegetables growing in the garden bed?
6. A cafeteria serves 3,820 people a day, with each person being served an 8-ounce portion of soup. How many ounces of soup must be made in 1 day?

7. During an average 12-hour workday, the fast-food restaurant sold 3,852 hamburgers. On the average, how many hamburgers were sold per hour?
8. Every day, Kevin has to drive 7 miles each way to work and back. At work, he has to drive his truck on a 296-mile delivery route. How many miles does he drive during a 5-day workweek?
9. Jerry's restaurant had 4 small dining rooms with a capacity of 28 people each and a main dining room with a capacity of 94 people. What was the total capacity of the restaurant?
10. For the past five days the high temperatures in San Francisco were 64.4 degrees, 59.3 degrees, 68.0 degrees, 48.8 degrees, and 53.6 degrees. What was the average (mean) high temperature for those days, rounded to the nearest tenth of a degree?

## Answer Key

### Book 14017 – Number Operations

- Page 28**    1. 41    2. 21203    3. 443    4. 67    5. 418  
6. 529    7. 204    8. 9632    9. 65    10. 772  
11. 5904    12. 798    13. 103    14. 20760  
15. 722    16. 80    17. 618    18. 577  
19. 33    20. 39900    21. 30360    22. 61  
23. 367    24. 527

- Page 31**    1. 21    2. 418    3. 365    4. 227    5. 235  
6. 7    7. 140    8. 9    9. 525    10. 24225  
11. 1879    12. 78    13. 418    14. 15  
15. 6120    16. 197    17. 336    18. 283  
19. 75    20. 2

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	<b>Median</b>	<b>Mean</b>	<b>Mode</b>
<b>1.</b>	13	14.4	13, 18
<b>2.</b>	11	10.4	3, 11
<b>3.</b>	14	14.8	14
<b>4.</b>	5	9	5
<b>5.</b>	23.4	27.1	14.14
<b>6.</b>	2.8	2.9	2.8
<b>7.</b>	88	82.1	95

- Page 39**    1. Composite    2. Composite    3. Prime  
4. Prime    5. Composite    6. Composite

7. Composite    8. Composite    9. Composite  
 10. Composite    11. Composite    12. Prime  
 13. Composite    14. Composite  
 15. Composite    16. Composite  
 17. Composite    18. Composite  
 19. Composite    20. Composite  
 21. Composite    22. Composite  
 23. Composite    24. Composite

**Page 40**

2. 2, 3, 5    3. 2, 2, 7    4. 2, 7    5. 2, 5  
 6. 2, 2, 2    7. 2, 11    8. 2, 2, 11  
 9. 2, 2, 2, 2, 2, 2    10. 2, 43    11. 2, 3, 11  
 12. 2, 2, 5, 5    13. 2, 29    14. 2, 2, 2, 11  
 15. 2, 3, 3    16. 2, 2, 2, 2, 3, 3    17. 2, 3, 7  
 18. 2, 2, 3, 5

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2. 1    3. 6    4. 2    5. 1    6. 2    7. 5  
 8. 4    9. 1    10. 6    11. 3    12. 2    13. 4  
 14. 8    15. 5    16. 1    17. 6    18. 2    19. 4  
 20. 15    21. 12    22. 7    23. 1    24. 8  
 25. 6    26. 42    27. 84    28. 60    29. 14  
 30. 30

**Page 44**

2. 12    3. 40    4. 18    5. 24    6. 6    7. 20  
 8. 10    9. 12    10. 238    11. 30    12. 36  
 13. 60    14. 56    15. 30    16. 522    17. 20  
 18. 48    19. 220    20. 110    21. 60    22. 24  
 23. 8    24. 72    25. 108    26. 252    27. 320  
 28. 48    29. 990    30. 900

**Page 49**

2.  $6 \times 6 \times 6 = 216$     3.  $10 \times 10 \times 10 = 1000$



4.  $4 \times 4 = 16$     5.  $9 \times 9 = 81$   
2.  $3 \times 3 \times 3 = 27$   
3.  $10 \times 10 \times 10 \times 10 = 10000$   
4.  $4 \times 4 \times 4 \times 4 \times 4 = 1024$   
5.  $8 \times 8 \times 8 \times 8 = 4096$   
10.  $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$   
11.  $8 \times 8 \times 8 \times 8 \times 8 = 32768$   
12.  $12 \times 12 = 144$     13.  $9 \times 9 \times 9 \times 9 = 6561$   
14.  $3 \times 3 \times 3 \times 3 = 81$     15.  $4 \times 4 \times 4 = 64$   
16.  $9 \times 9 \times 9 = 729$     17.  $11 \times 11 \times 11 = 1331$   
18.  $4 \times 4 \times 4 \times 4 = 256$   
19.  $3 \times 3 \times 3 \times 3 \times 3 = 243$     20.  $5 \times 5 \times 5 = 125$   
21.  $8 \times 8 \times 8 = 512$     22.  $5 \times 5 \times 5 \times 5 = 625$   
23.  $11 \times 11 = 121$     24.  $5 \times 5 = 25$   
25.  $9 \times 9 \times 9 \times 9 \times 9 = 59049$   
26.  $6 \times 6 \times 6 \times 6 = 1296$

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1. 500 boxes    2. 130 tablets  
3. 6460 gallons    4. 13208 seats  
5. 105 days    6. 30560 ounces  
7. 321 hamburgers    8. 1550 miles  
9. 206 people    10. 58.8 degrees