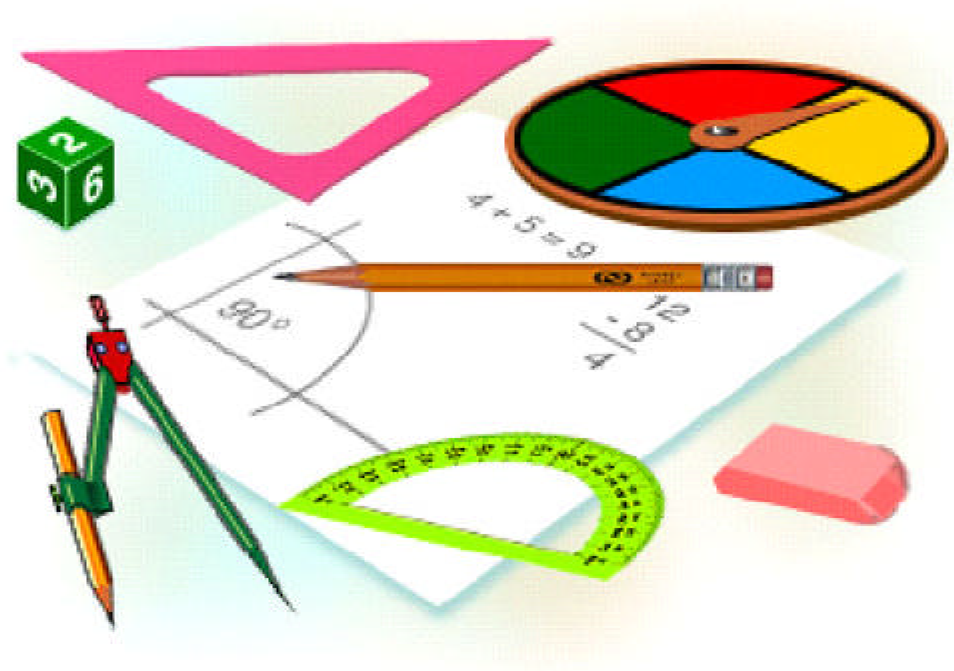
The Next Step

Mathematics Applications for Adults



Book 14018

INTRODUCTION

Why Math?

The most important reason for learning math is that it teaches us how to think. Math is more than adding and subtracting, which can easily be done on a calculator; it teaches us how to organize thoughts, analyze information, and better understand the world around us.

Employers often have to re-educate their employees to meet the demands of our more complex technological society. For example, more and more, we must be able to enter data into computers, read computer displays, and interpret results. These demands require math skills beyond simple arithmetic.

Everyone Is Capable of Learning Math

There is no **type** of person for whom math comes easily. Even mathematicians and scientists spend a lot of time working on a single problem. Success in math is related to practice, patience, confidence in ability, and hard work.

It is true that some people can solve problems or compute more quickly, but speed is not always a measure of understanding. Being “faster” is related to **more practice or experience.**

For example, the reason why math teachers can work problems quickly is because they’ve done them so many times before, not because they have “mathematical minds”.

Working with something that is familiar is natural and easy. For example, when cooking from a recipe we have used many times before or playing a familiar game, we feel confident. We automatically know what we need to do and what to expect. Sometimes, we don’t even need to think. However, when using a recipe for the **first** time or playing a game for the **first** time, we must concentrate on each step. We double-check that we have done everything right, and even then we fret about the outcome. The same is true with math. When encountering problems for the very first time, **everyone must have patience** to understand the problem and work through it correctly.

It’s Never Too Late to Learn

One of the main reasons people don’t succeed in math is that they don’t start at the right place. **IMPORTANT! You must begin where *you* need to begin.** Could you hit a homerun if you hadn’t figured out which end of the bat had to make contact with the ball? Why should learning math be any different?

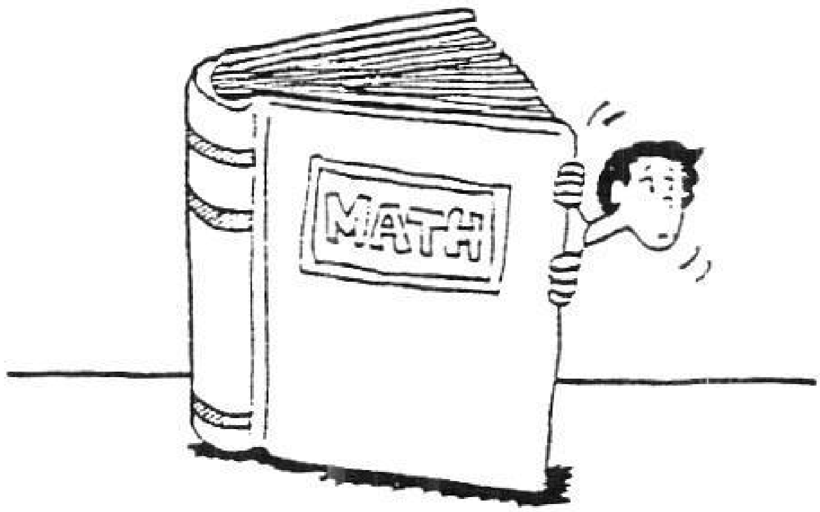
If it has been a while since your last math class, **you must determine what level math you should take.** A teacher or trained tutor can help determine this with a few placement tests and questions.

Sometimes a few tutoring sessions can help you fill gaps in your knowledge or help you remember some of the things you have simply forgotten. It could also be the case where your foundations may be weak and it would be better for you to relearn the basics. **Get some help** to determine what is best for you.

Feeling good about ourselves is what all of us are ultimately striving for, and nothing feels better than conquering something that gives us difficulty. This takes a great deal of courage and the ability to rebound from many setbacks. This is a natural part of the learning process, and when the work is done and we can look back at our success, nothing feels better.

Where's the best place to hide if you're scared?

Inside a math book because there is safety in numbers.



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OUTLINE

Mathematics - Book 14018

Number Operations
Mathematical Operations, Average, Median, and Mode
perform with accuracy and speed the four mathematical operations.
find average, median, and mode.
Factors and Prime Numbers
factor a given group of whole numbers.
determine which numbers are prime numbers.
find the Greatest Common Factor (GCM).
find the Least Common Multiple (LCM).
Exponents
express like factors using exponents.
express exponents using like factors.
perform certain mathematical operations involving exponents.
Squares and Square Roots
find the square and square root of whole numbers which have perfect squares.
Problem Solving With Whole Numbers
solve multi-step problems, with and without a calculator.

Fractions, Decimals, and Percent
Fractions
perform the four mathematical operations with fractions.
write mixed fractions.
determine recipricol fractions.
write equivalent fractions.
determine the lowest term fraction.
Decimals
perform the four mathematical operations using decimals.
round off decimals to a given number of decimal points.
convert decimals to fractions.
convert fractions to decimals.
Percent
perform the four mathematical operations using percents.
write fractions or decimals as percents.
write percents as fractions or decimals.
Problem Solving With Fractions, Decimals, Percents
solve multi-step problems requiring any combination of mathematical operations with fractions, decimals, and percents, with or without the use of a calculator.
Percent, Ratio, and Proportion
Introduction To Ratio, Proportion, and Percent
find the percentage that one number is of another

number.
find the number when a percentage is given.
percent of a given number.
use the formula $r/100 = P/W$ and cross-multiplication.
determine which ratio in a given list is equal to given ratio.
determine which of a given list of compared ratios are proportions and which are false statements.
Problem Solving With Percent, Ratio, and Proportion
solve multi-step problems requiring the performance of any combination of mathematical operations involving ratio, proportion, and percent, with or without a calculator.
Geometry
Lines and Angles
identify parallel lines and perpendicular lines in a given selection of figures.
construct parallel lines.
determine angles when a transversal cuts parallel lines.
construct angles using a protractor, given a list of angle measurements.
identify types of angles: acute, right, obtuse, straight, complete, reflex.
illustrate with diagrams the following angle relations: complementary, adjacent, supplementary, exterior, and vertical or opposite.

Introduction to Geometric Figures
identify parts of a circle.
construct a circle and label its parts.
identify a variety of polygons.
identify a variety of polyhedrons.
use the Pythagorean Theorem to find length of one side of a triangle.
Measurement
The Metric System
use correct metric units to measure length, volume, capacity, mass, time, and temperature.
convert from any given metric unit to any stated metric unit.
Area, Perimeter, and Volume
find the perimeter of various regular and irregular geometric figures and shapes.
find the area of various regular and irregular geometric figures and shapes.
find the volume of various regular geometric figures.
Problem Solving Involving Measurement
solve multi-step problems requiring the performance of any combination of mathematical operations involving measurement, with or without a calculator.
Integers
Introduction To Integers
perform mathematical operations using integers.
explain the difference between signs of operations

and signs of quantity.
Equations: Equalities And Inequalities
Introduction To Equations: Equalities And Inequalities
rewrite English statements as math expressions.
solve equalities.
simplify an expression using correct order of operations.
Problem Solving With Equations and Equalities
solve multi-step problems requiring the performance of any combination of mathematical operations involving equalities, with or without a calculator.
Graphs
Introduction To Graphs
answer questions about information contained in graphs.
construct a variety of graphs, given the necessary information.
write a table of values for any relation.
Problem Solving Using Graphs
solve multi-step problems requiring the performance of any combination of mathematical operations involving graphs, with or without a calculator.

THE NEXT STEP

Book 14018

Number Operations

Mathematical Operations, Average, Median, Mode

Digit is a counting word. A digit is any of the numerals from **1** to **9.** The word "digit" is also the name for a finger. So number digits can be counted on finger digits.

Our modern system of counting or ***tallying*** probably came from counting on fingers. Fingers and hands were among the earliest known calculators!

The set of counting numbers has no end. It can go on forever. The idea that counting numbers can go on and on is called ***infinity***. Infinity has a special symbol:

$$\infty$$

There is no such thing as the “largest number.” You can always add to or multiply a large number to make an even bigger number.

$$\infty + 3 = \infty$$

$$\infty \times 10 = \infty$$

If you began writing all the counting numbers today, you could continue writing every moment of every day for every day of the rest of your life and never be finished!

What's a googol?

A googol is a 1 with a hundred zeroes behind it. We can write a googol using exponents by saying a googol is 10^100 or 10 to the 100^{th} power.

The biggest named number that we know is googolplex, ten to the googol power, or (10)^(10^100). That's written as a one followed by googol zeroes.

It’s funny that no one ever seems to ask, “What is the smallest number?” Again, there is really no such thing. You could always subtract from or divide a small number to make an even smaller number. As the number gets smaller and smaller, you would be approaching, but never reaching, negative infinity.

$$-\infty$$

The set of ***counting numbers***, or ***natural numbers***, begins with the number **1** and continues into infinity.

{1,2,3,4,5,6,7,8,9,10...}

The set of ***whole numbers*** is the same as the set of counting numbers, except that it begins with **0**.

{0,1,2,3,4,5,6,7,8,9,10...}

☞ ***All counting numbers are whole numbers. Zero is the only whole number that is not a counting number.***

Even numbers include the numbers ***0*** and ***2*** and all numbers that can be divided evenly by ***2***. ***Odd numbers*** are all numbers that cannot be divided evenly by ***2***.

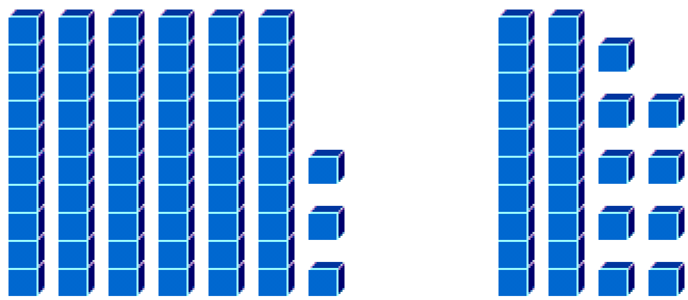
Odd and **Even** Numbers to 100

1 3 5 7 9 11 13 15 17 19 21
0 2 4 6 8 10 12 14 16 18 20
23 25 27 29 31 33 35 37 39 41
22 24 26 28 30 32 34 36 38 40
43 45 47 49 51 53 55 57 59 61
42 44 46 48 50 52 54 56 58 60
63 65 67 69 71 73 75 77 79 81
62 64 66 68 70 72 74 76 78 80
83 85 87 89 91 93 95 97 99
82 84 86 88 90 92 94 96 98 100

Ordering numbers means listing numbers from least to greatest, or from greatest to least. Two symbols are used in ordering.

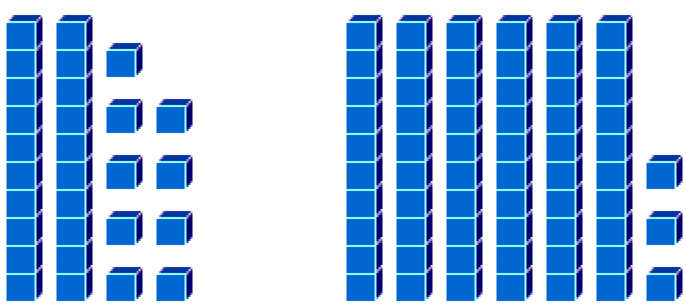
<	>
is less than	**is greater**
2 < 10	**10 > 2**

Greater Than >



63 is **greater than** 29.
63 > 29

Less Than <



29 is **less than** 63.
29 < 63

A ***mathematical operation*** is a process or action, such as addition, subtraction, multiplication, or division performed in a specified sequence and in accordance with specific rules.

Combining two or more numbers is called ***addition***. The term for addition is ***plus***, and the symbol for plus is +. The numbers that are combined in addition are called ***addends*** and together they form a new number called a ***sum***.

2 ---- addends ---- 3
+ 2   + 1
4 ---- sum --------- 4

Adding whole numbers is as simple as 2 + 2! To add two whole numbers, you can simply follow the number line and complete the addition fact.

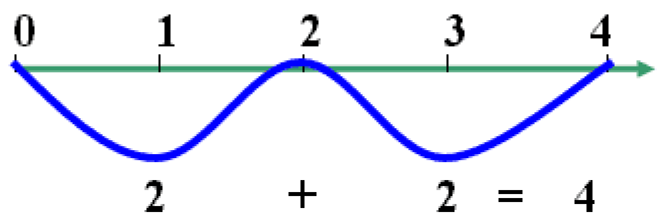
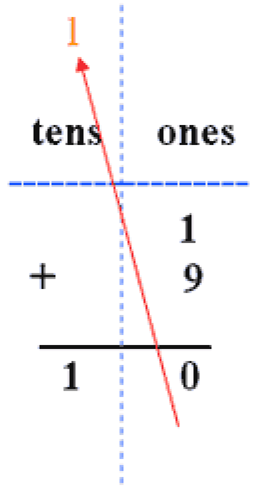


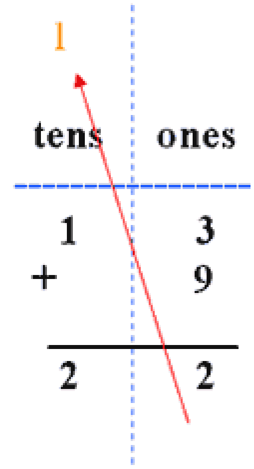
Table of Addition Facts

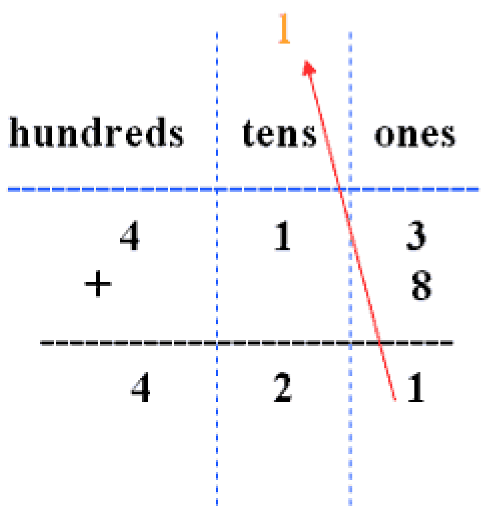
+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

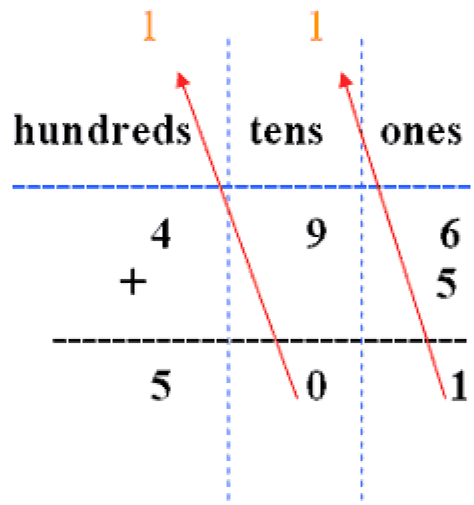
Regrouping Numbers in Addition

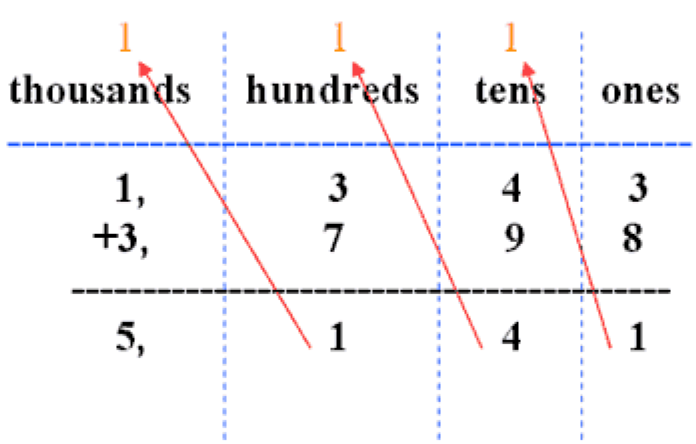
Addition often produces sums with a value greater than ***9*** in a given place. The value of ten is then ***regrouped*** (or ***carried***) to the next place.


1
tens
ones
1
+ 9
1
0


1
tens
ones
1 3
+ 9
2 2


1
hundreds
tens
ones
4 1 3
+ 8
4 2 1


1 1
hundreds
tens
ones
4 9 6
+ 5
5 0 1


1 1 1
thousands
hundreds
tens
ones
1, 3 4 3
+3, 7 9 8
5, 1 4 1

To explain addition another way, it can be done by adding the place value amounts separately.

e.g.
```
  69
+  8
  17
  60 (the 6 in the tens place means 6 tens or "60")
  77
```

⇒ If there are not enough digits in each number to make even columns under each place value, then zeros may be used **before** a given number to make adding easier. Do **not** add zeros **after** a number because it changes the value of the whole number.

e.g. 69 + 8 + 125 could be added as:

```
 069
 008
+125
```

Commutative Property of Addition

The property which states that two or more addends can be added in any order without changing the sum

$a + b = b + a$

Examples:

$c + 4 = 4 + c$

$(2 + 5) + 4r = 4r + (2 + 5)$

Associative Property of Addition

The property which states that for all real numbers *a*, *b*, and *c*, their sum is always the same, regardless of their grouping:

$$(a + b) + c = a + (b + c)$$

Example:

$$(2 + 3) + 4 = 2 + (3 + 4)$$

"Taking away" one or more numbers from another number is called ***subtraction***. The term for subtraction is ***minus***, and the symbol for minus is **-**. The number being subtracted is called a ***subtrahend***. The number being subtracted from is called a ***minuend***. The new number left after subtracting is called a ***remainder*** or ***difference***.

4 ---- minuend ---- 4
- 2 --subtrahend - - 1
2 – difference ---- 3

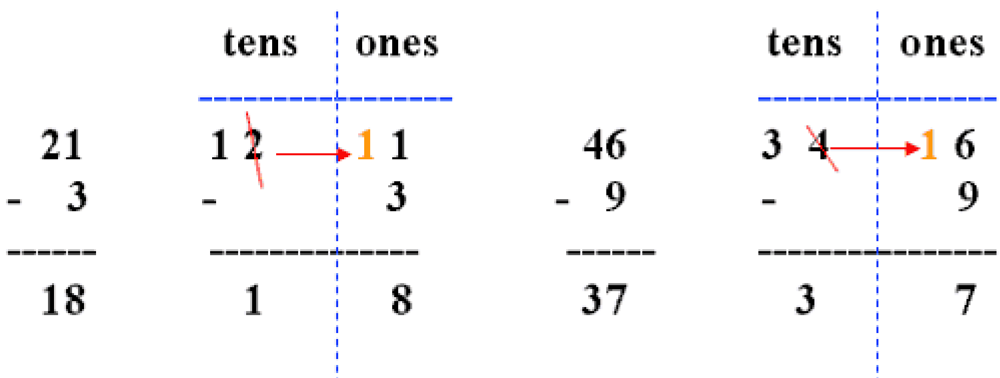
The complete addition or subtraction "sentence" is called an ***equation***. An equation has two parts. The two parts are separated by the ***equal sign***, **=**. For example, ***the minuend minus the subtrahend equals the difference***. An ***addition fact*** or a ***subtraction fact*** is the name given to specific addition or subtraction equations.

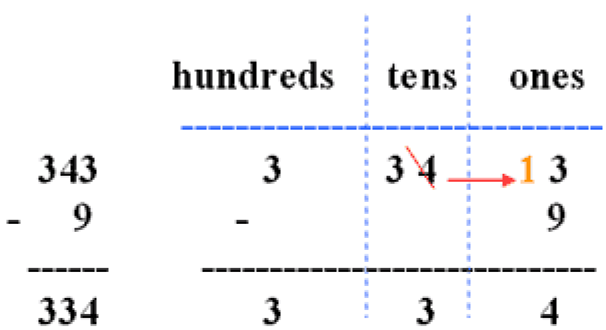
0 + 1 = 1	**1 – 1 = 0**
1 + 1 = 2	**2 – 1 = 1**
2 + 1 = 3	**3 – 1 = 2**
3 + 1 = 4	**4 – 1 = 3**

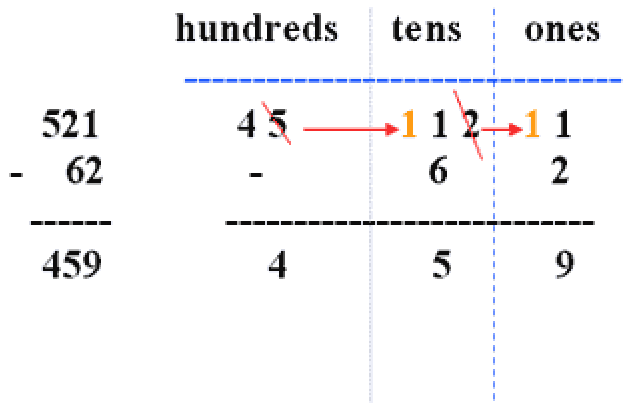
4 + 1 = 5 **5 – 1 = 4**
5 + 1 = 6 **6 – 1 = 5**
6 + 1 = 7 **7 – 1 = 6**
7 + 1 = 8 **8 – 1 = 7**
8 + 1 = 9 **9 – 1 = 8**

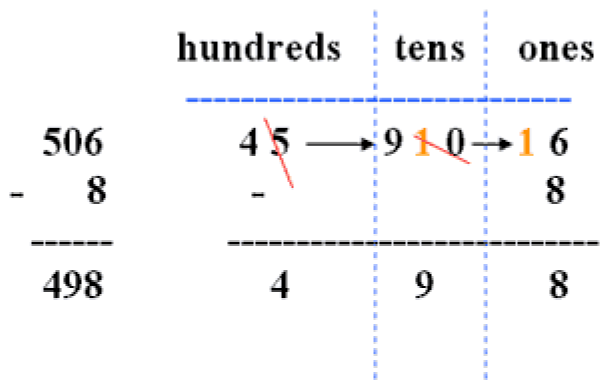
Regrouping in Subtraction

Regrouping, sometimes called ***borrowing***, is used when the subtrahend is greater than the minuend in a given place. Regrouping means to take a group of tens from the next greatest place to make a minuend great enough to complete the subtraction process.









Multiplication is a quick form of addition. By multiplying numbers together, you are really adding a series of one number to itself. For example, you can add ***2*** plus ***2***. Both ***2 plus 2*** and ***2 times 2*** equal ***4***.

2 + 2 = 4
2 x 2 = 4

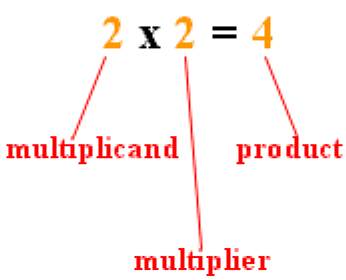
```
  2      2
+ 2    x 2
----   ----
  4      4
```

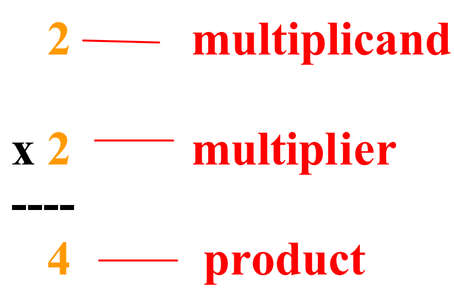
But what if you wanted to calculate the number of days in five weeks? You could add *7* days + *7* days + *7* days + *7* days + *7* days or you could multiply *7* days times ***5***. Either way you arrive at ***35***, the number of days in five weeks.

7 + 7 + 7 + 7 + 7 = 35

5 x 7 = 35

Although multiplication is related to addition, the parts of multiplication are not known as addends. Instead, the parts are known as ***multiplicands*** and ***multipliers***. A multiplication sentence, like an addition sentence, is called an ***equation***. But a multiplication sentence results in a ***product***, not a sum.





X	0	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Multiplication, Step-by-Step

When the multiplicand and the multiplier are numbers with two or more digits, multiplication becomes a step-by-step process.

Look at 15 x 13:

```
   1  5        First, multiply the
               ones –- 3 x 5.  Write
 x    3        down the product
 ________      so the ones columns
   1  5        line up.


   1  5        Next, multiply the
               tens – 3 x 1 ten.
  x   3        Line up the product
----------     with the tens
   1  5        column.

   3  0   —    Zero is the place
----------     holder.
```

```
   1   5        Last, add the ones
                and tens to find the
 x     3        product of the
---------       equation.
   1   5

 + 3   0
---------
   4   5
```

Here is a shorter way:

```
  1
   1   5
 x     3
----------
   4   5
```

1. Multiply the ones: 3 x 5 = 15. Put the 5 in the ones column and regroup the 1 to the tens column.

2. Multiply the tens: 3 x 1 = 3.

3. Add the 1 that you regrouped to the 3, put the sum in the tens column.

Look at 265 x 23:

```
   265        First, multiply the       265      Next, multiply
              multiplicand by the                by the tens –
x   23        ones in the              x  23     2 x 5, 2 x 6,
---------     multiplier – 3 x 5,     --------   and 2 x 2.
    15        3 x 6, and 3 x 2.          15      Zero is the
   180        Zero is the place         180      place holder.
   600        holder.                   600
                                      --------
                                           100
                                         1,200
                                         4,000
```

```
      265        Last, add.

x     23
---------
       15
+     180
+     600
---------
+     100
+   1,200
+   4,000
---------
    6,095
```

Here is a shorter way:

```
   11
   11
  265
x  23
---------
  795
 5300
---------
6,095
```

1. Multiply the ones: 3 x 265
   3 x 5 = 15 regroup the 1
   3 x 6 = 18 plus the regrouped 1 = 19; regroup the 1
   3 x 2 = 6 plus the regrouped 1 = 7

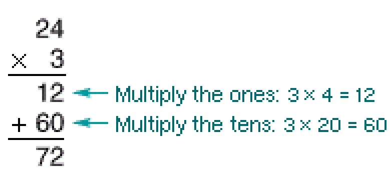
2. Multiply the tens: 2 x 265
   0 is the place holder
   2 x 5 = 10 regroup the 1
   2 x 6 = 12 plus the regrouped 1 = 13; regroup the 1
   2 x 2 – 4 plus the regrouped 1 = 5

3. Add 795 + 5300 = 6,095

Partial Product

A method of multiplying where the ones, tens, hundreds, and so on are multiplied separately and then the products added together

Example:



36 x **17 = 42 + 210 + 60 + 300 = 612**

When you multiply whole numbers, the ***product*** usually has a greater value than either the ***multiplicand*** or the ***multiplier***.

But there are exceptions:

A number multiplied by ***1*** is always equal to itself.

$$\begin{array}{r} 1 \\ \times\, 1 \\ \hline 1 \end{array} \qquad 21 \times 1 = 21 \qquad \begin{array}{r} 36 \\ \times\, 1 \\ \hline 36 \end{array}$$

A number multiplied by ***0*** is always equal to ***0***.

$$\begin{array}{r} 1 \\ \times\, 0 \\ \hline 0 \end{array} \qquad 21 \times 0 = 0 \qquad \begin{array}{r} 36 \\ \times\, 0 \\ \hline 0 \end{array}$$

To multiply a number by 10, add a 0 to the right of the number.

EXAMPLE

$25 \text{ X } 10 = 250$ or

$$\begin{array}{r} 25 \\ \times 10 \\ \hline 250 \end{array}$$

To multiply a number by 100, add two 0's to the right of the number.

EXAMPLE

36 X 100 = 3,600 or

$$\begin{array}{r} 36 \\ \underline{x100} \\ 3{,}600 \end{array}$$

Commutative Property of Multiplication

The property which states that two or more factors can be multiplied in any order without changing the product

$a \cdot b = b \cdot a$

Examples:

$3 \cdot c = c \cdot 3$

$4 \cdot 5 \cdot y7 = 5 \cdot 4 \cdot y7$

Associative Property of Multiplication

The property which states that for all real numbers *a*, *b*, and *c*, their product is always the same, regardless of their grouping:

$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

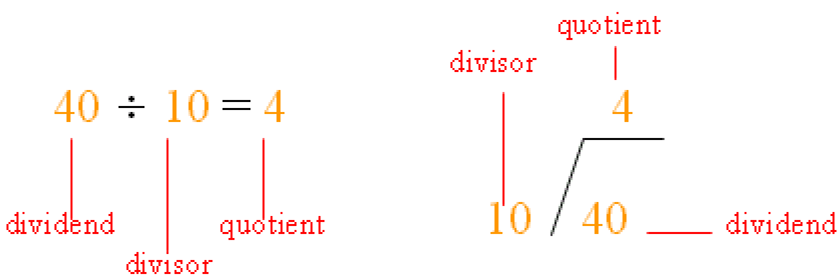
Example:

$(5 \cdot 6) \cdot 7 = 5 \cdot (6 \cdot 7)$

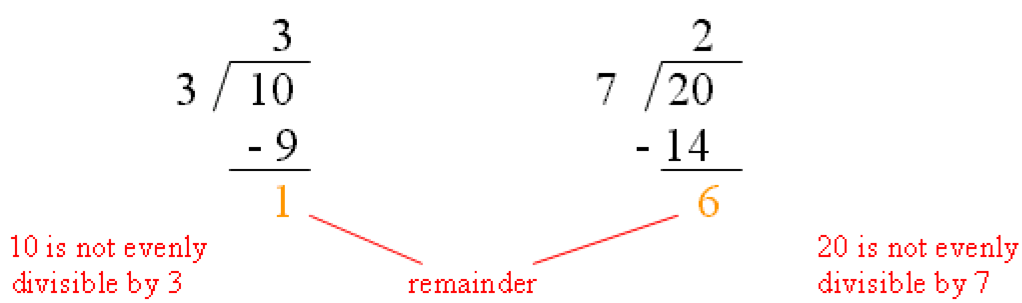
Division is the process of finding out how many times one number, the ***divisor***, will fit into another number, the ***dividend***. The division sentence results in a ***quotient***. The signs of division are ÷ and /‾‾‾, and mean ***divided by***. You can think of division as a series of repeated subtractions. For example, ***40*** ÷ ***10*** could also be solved by subtracting ***10*** from ***40*** four times:

40 – 10 – 10 – 10 – 10 = 0

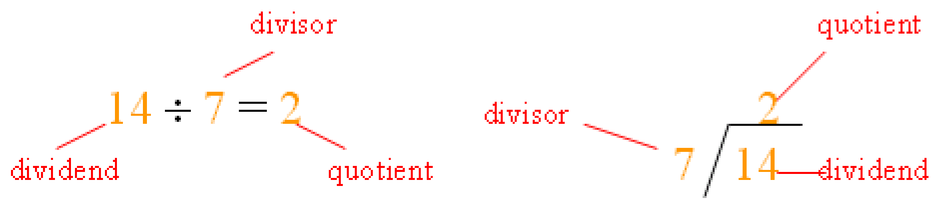
Because ***10*** can be subtracted four times, you can say that ***40*** can be divided by ***10*** four times, or ***40*** ÷ ***10 = 4***.



Many numbers do not fit evenly into other numbers. They are ***not evenly divisible by*** those numbers, and the number left over is called the ***remainder***.



To divide whole numbers, reverse the process of multiplication. For example, if ***2 x 7 = 14*** in a multiplication equation, then in a division sentence, ***14*** is the ***dividend*** and ***7*** is the ***divisor*** with a ***quotient*** of ***2***.



A whole number divided by ***1*** will always equal itself.

1 ¸ 1 = 1　　**1 / 21 = 21**　　**36 ¸ 1 = 36**

Zero divided by a whole number will always equal ***0***.

0 ÷ 12 = 0　　**12 / 0 = 0**　　**3　0**　　**0/7 = 2**

Division, Step-by-Step

Where the dividend and divisor are numbers with two or more digits, division becomes a step-by-step process.

```
      2
8  /208
   - 16
      4
```

First, round the divisor up - 8 rounds up to 10 - and estimate the number of 10s in 20. Answer: 2. Multiply the divisor – 8 x 2 – and subtract the product from the dividend.

```
     26
8 / 208
 -  16
    48
 -  48
     0
```

Next, pull down the next digit from the dividend – 8 – and repeat the estimation and subtraction process.

```
     26
8 / 208
 -  16
    48
 -  48
     0  ———— No remainder
```

Last, when you can subtract no more you've found the quotient.

No remainder

```
       1
23 / 276
 -   23
      4
```

First, round 23 to 25 and estimate the number of 25s in 27. Answer: 1. Multiply the divisor by 1 – 23 x 1 – and subtract.

```
      12
23 / 276
 -   23
     46
 -   46
```

Next, pull down the next digit from the dividend – 6 – and repeat the estimation and subtraction process.

```
        12
23  /276
  -  23
      46
  -   46
       0 ——— No remainder
```

Then, pull down the next digit, estimate, and subtract, until you can subtract no more.

Inverse (opposite) operations are used to simplify an equation for solving.
One operation is involved with the unknown and the inverse operation is used to solve the equation.

Addition and subtraction are inverse operations.
Given an equation such as 7 + x = 10, the unknown x and 7 are *added*. Use the inverse operation subtraction. To solve for n, subtract 7 from 10. The unknown value is therefore 3.

<u>Examples for addition and subtraction</u>

Addition Problem	Solution
x + 15 = 20	x = 20 - 15 = 5
Subtraction Problem	Solution
x - 15 = 20	x = 20 + 15 = 35

Multiplication and division are inverse operations.
Given an equation 7x = 21. x and 7 are multiplied to create a value of 21. To solve for x, divide 21 by 7 for an answer of 3.

<u>Examples for division and multiplication.</u>

Multiplication Problem	Solution
$3x = 21$	$x = 21 \div 3 = 7$

Division Problem	Solution
$x \div 12 = 3$	$y = 3 \text{ x } 12 = 36$

Practice Exercise

Mixed Problems

Solve each problem.

1.	$\begin{array}{r} 287 \\ \underline{+\ 34} \end{array}$	**2.**	$506 \overline{)48576}$	**3.**	$\begin{array}{r} 532 \\ \underline{-\ \ 98} \end{array}$	**4.**	$\begin{array}{r} 330 \\ \underline{\times 23} \end{array}$
5.	$\begin{array}{r} 722 \\ \underline{\times\ 78} \end{array}$	**6.**	$\begin{array}{r} 873 \\ \underline{-\ \ 33} \end{array}$	**7.**	$\begin{array}{r} 289 \\ \underline{+\ 30} \end{array}$	**8.**	$233 \overline{)20038}$
9.	$\begin{array}{r} 273 \\ \underline{-\ \ 78} \end{array}$	**10.**	$\begin{array}{r} 408 \\ \underline{+\ 50} \end{array}$	**11.**	$\begin{array}{r} 569 \\ \underline{+\ 76} \end{array}$	**12.**	$\begin{array}{r} 881 \\ \underline{\times\ 25} \end{array}$
13.	$\begin{array}{r} 525 \\ \underline{\times\ 75} \end{array}$	**14.**	$564 \overline{)5640}$	**15.**	$176 \overline{)4224}$	**16.**	$\begin{array}{r} 808 \\ \underline{-\ 19} \end{array}$

17. $\begin{array}{r} 385 \\ -\ 17 \\ \hline \end{array}$	**18.** $613\overline{)32489}$	**19.** $756\overline{)71064}$	**20.** $\begin{array}{r} 518 \\ +\ 29 \\ \hline \end{array}$
21. $\begin{array}{r} 574 \\ \times\ 46 \\ \hline \end{array}$	**22.** $\begin{array}{r} 334 \\ +\ 12 \\ \hline \end{array}$	**23.** $\begin{array}{r} 614 \\ -\ 53 \\ \hline \end{array}$	**24.** $\begin{array}{r} 276 \\ \times\ 84 \\ \hline \end{array}$

Order of Operations

Sometimes the order in which you add, subtract, multiply, and divide is very important. For example, how would you solve the following problem?

2 x 3 + 6

Would you group

(2 x 3) + 6 or **2 x (3 + 6) ?**

Which comes first, addition or multiplication? Does it matter?
Yes. Mathematicians have written two simple steps:

1. ***All multiplication and division operations are carried out first, from left to right, in the order they occur.***

2. ***Then all addition and subtraction operations are carried out, from left to right, in the order they occur.***

For example:

$$8 \div 2 + 2 \text{ x } 3 - 1 = 4 + 6 - 1 = 9$$

4 6 10

step 1 **step 2**

Þ ***Perform all operations with parentheses (brackets) and exponents before carrying out the remaining operations in an equation.***

$$8 \div (2 + 2) \text{ x } 3 - 1 =$$

$$8 \div 4 \text{ x } 3 - 1 =$$

$$2 \text{ x } 3 - 1 =$$

$$6 - 1 = 5$$

To remember the order of operations, simply remember BEDMAS: Brackets, Exponents, Division, Multiplication, Addition, Subtraction.

Example:

$10 \div (2 + 8) \times 2^3 - 4$ *Add inside parentheses.*

$10 \div 10 \times 2^3 - 4$ *Clear exponent.*

$10 \div 10 \times (2 \times 2 \times 2) - 4$

$10 \div 10 \times 8 - 4$ *Multiply and divide.*

$8 - 4$ *Subtract.*

4

Practice Exercise

1. 12 - 10 + 25	**2. 5 + 12 - 13**
3. 19 + 15 + 20	**4. 17 × 20 - 2**
5. 12 ÷ 2	**6. 11 + 21 × (11 + 23) × 18**
7. 14 ÷ 2 + 8 × 22	**8. 24 + 5 + (4 ÷ 2) + 16**
9. 10 ÷ 5 × 18 × 14 + 13	**10. 12 ÷ 6**

11. 14 × 10	**12. 14 ÷ 7 × 24 + 4**
13. 10 + 4^3 + 16 + 14	**14. 22 + 16 × 13 - 24**
15. 5 + (20 × 13 × 19)	**16. 18 ÷ 9 × 25**
17. 12 ÷ 3 + 18 - 9	**18. 18 + 5^3 + 6 + 17 - 20**
19. 4 ÷ 2 × 23 + 11	**20. 12^3 - 24 + 13**

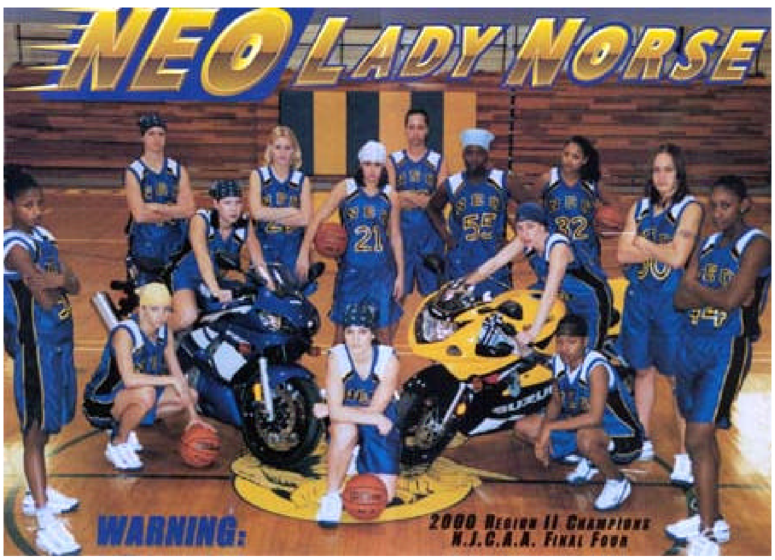
Averages

The most common way to find an ***average*** is to add up a list of numbers and divide the sum by the number of items on the list. Another word for average is ***mean***.

$3 + 4 + 6 + 8 + 9 = 30$ **number of addends**

sum — $30 \div 5 = 6$ So, the average of the numbers 3, 4, 6, 8, and 9 is 6.

When do you need to calculate an average? Your grades may be based on the average of all your test scores. In sports, you might want to find out the average height of players on your favorite basketball team.



The height of the starters for this team is:

Anita 60”

Jane 58”

Cathy 57”

Joy 52”

Tanya	**48”**	**The average height of these players is 55 inches.**

Medians

Average or mean is different from ***median***. The median is the middle number in a series of numbers stated in order from least to greatest. An average and a median can be the same number. The average of ***3***, ***5***, and *7* is ***5***:

$$3 + 5 + 7 = 15 \text{ and } 15 \div 3 = 5$$

and the median of ***3***, ***5***, and *7* is ***5***. But average and median are often different numbers.

Anita	**60”**	
Jane	**58”**	
Cathy	**57”**	
Joy	**52”**	
Tanya	**48”**	**The median height of these girls is 57 inches—Cathy’s height--because it is the middle number.**

If there is an even number of data items, the median is the average (mean) of the two middle numbers.

Example Amy's point totals for six games of basketball were 24, 16, 19, 22, 6, and 12 points. Find the median of her point totals.

Step 1 Arrange the data in order.

24, 22, **19, 16,** 12, 6

Step 2 The two middle numbers are 19 and 16. Average these to find the median.

$$19 + 16 = 35$$
$$35 \div 2 = 17.5$$

Amy's median point total is **17.5 points.**

Percentiles

Individual scores may be compared with all the other scores in a group by giving the score a positional standing or rank.

The percentile rank of a score indicates the percent of all the scores that are below this given score. If the rank of a particular score is the 60th percentile, it means that 60 % of all the scores are lower than this score.

Examples If Todd ranked fourth in a class of 16 students, there are 12 students of 16, or 75%, with a lower rank. He would have a percentile rank of 75 or a rank of the 75th percentile.

If Todd ranked fourth in a class of 40 students, there are 36 students of 40, or 90%, with a lower rank. He would have a percentile rank of 90 or a rank of the 90th percentile.

Mode

The number or numbers that occur most often in a collection of data; there can be more than one mode or none at all.

Examples:

2, 3, 4, 5, 5, 6, 7, **8, 8, 8**, 9, 11

The mode is 8.

2, 3, 4, **5, 5, 5**, 7, **8, 8, 8**, 9, 11

The modes are 5 and 8.

2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 17

There is no mode.

Practice Exercise

Calculating the Mean, Median, and Mode

Calculate the values to the nearest tenth.

1. 11, 12, 5, 5, and 5

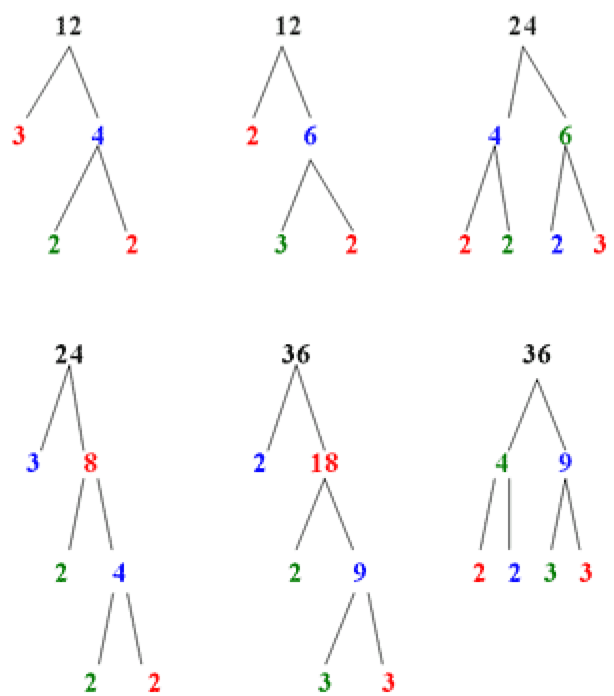
Write the median: ___________ Write the mean: ___________ Write the mode: ___________
2. 15, 24, 15, 6, and 6 Write the median: ___________ Write the mean: ___________ Write the mode: ___________
3. 8, 15, 14, 15, and 22 Write the median: ___________ Write the mean: ___________ Write the mode: ___________
4. 1, 11, 24, 24, and 16 Write the median: ___________ Write the mean: ___________ Write the mode: ___________
5. Students with the following GPAs applied for a job: 2, 2.2, 2.9, 2.3, 2.2, 2.8, 2.6, 2.9, 3.7, and 3.5 Write the median: ___________ Write the mean: ___________ Write the mode: ___________
6. The following temperatures were recorded: 5, 5, 62, 5, 39, -15, -13, 3, -9, and 70 Write the median: ___________ Write the mean: ___________ Write the mode: ___________
7. The following grades were posted on the latest exam: 90, 63, 68, 90, 84, 54, 63, 85, 85, and 63 Write the median: ___________ Write the mean: ___________ Write the mode: ___________

Factors and Prime Numbers

Factors are numbers that, when multiplied together, form a new number called a ***product***. For example, ***1*** and ***2*** are factors of ***2***, and ***3*** and ***4*** are factors of ***12***.

Every number except ***1*** has at least two factors: ***1*** and itself.

Composite numbers have more than two factors. In fact, every composite number can be written as the product of ***prime numbers***. You can see this on a ***factor tree***.



Prime numbers are counting numbers that can be divided by only two numbers---***1*** and themselves. A prime number can also be described as a counting number with only two factors, ***1*** and itself. The number ***1***, because it can be divided only by itself, is ***not*** a prime number.

Prime Numbers to 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,

53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Practice Exercise

Classify each number as prime or composite.

1. 9 ☐Prime ☐Composite	2. 52 ☐Prime ☐Composite	3. 20 ☐Prime ☐Composite	4. 97 ☐Prime ☐Composite
5. 8 ☐Prime ☐Composite	6. 45 ☐Prime ☐Composite	7. 21 ☐Prime ☐Composite	8. 60 ☐Prime ☐Composite
9. 5 ☐Prime ☐Composite	10. 42 ☐Prime ☐Composite	11. 19 ☐Prime ☐Composite	12. 96 ☐Prime ☐Composite

13. 67 ☐Prime ☐Composite	14. 17 ☐Prime ☐Composite	15. 47 ☐Prime ☐Composite	16. 69 ☐Prime ☐Composite
17. 10 ☐Prime ☐Composite	18. 43 ☐Prime ☐Composite	19. 59 ☐Prime ☐Composite	20. 44 ☐Prime ☐Composite
21. 81 ☐Prime ☐Composite	22. 36 ☐Prime ☐Composite	23. 51 ☐Prime ☐Composite	24. 76 ☐Prime ☐Composite

Find the prime factorization of each number.

1. 12 **2, 2, 3**
2. 10
3. 28
4. 30
5. 8
6. 4
7. 32
8. 96
9. 46
10. 36
11. 14

12. 26
13. 82
14. 48
15. 54
16. 40
17. 72
18. 84

The Greatest Common Factor

Common factors are numbers that are factors of two or more numbers. For example, ***2*** is a factor of ***12*** and ***36***, which makes ***2*** a common factor of ***12*** and ***36***. The common factor of two numbers with the greatest value is called the ***greatest common factor***. For example, ***2***, ***3***, ***4***, ***6***, and ***12*** are common factors of ***12*** and ***36***, but ***12*** is the greatest common factor.

Multiples

Find the ***multiples*** of a number by multiplying it by other whole numbers. The multiples of ***2***, for example, are:

0 x 2 = <u>0</u>	**2 x 3 = <u>6</u>**
1 x 2 = <u>2</u>	**2 x 4 = <u>8</u>**
2 x 2 = <u>4</u>	**2 x 5 = <u>10</u>**

…and so on.

As you can see, the multiples of ***2*** include ***0***, ***2***, ***4***, ***6***, ***8***, and ***10***. The list continues into infinity!

Some numbers share the same multiples. Those multiples are known as ***common multiples***.

Number Multiples

0	**0**	**0**	**0**	**0**	**0**	**0**
1	**0**	**1**	**2**	**3**	**4**	**5**
2	**0**	**2**	**4**	**6**	**8**	**10**
3	**0**	**3**	**6**	**9**	**12**	**15**
4	**0**	**4**	**8**	**12**	**16**	**20**
5	**0**	**5**	**10**	**15**	**20**	**25**
	0	**1**	**2**	**3**	**4**	**5**

The least multiple of two or more numbers is the least common multiple. For example, the least common multiple of ***2*** and ***3*** is ***6***.

2 x 1 = 2 **2 x 2 = 4** **2 x 3 = 6**
3 x 1 = 3 **3 x 2 = 6**

Practice Exercise

Find the greatest common factor (GCF) for the given numbers.

1. 5, 6 **1**
2. 12, 2

3. 12, 6
4. 4, 12
5. 10, 12
6. 9, 12
7. 6, 10
8. 8, 9
9. 3, 12
10. 14, 22
11. 12, 30
12. 8, 16
13. 12, 24
14. 30, 25
15. 13, 17
16. 6, 4
17. 16, 12
18. 9, 6
19. 45, 36
20. 36, 24
21. 48, 32
22. 20, 36
23. 40, 16
24. 24, 18
25. 17, 19
26. 45, 120
27. 32, 6
28. 110, 135
29. 6, 40

30. 25, 31

Find the least common multiple for the given numbers.

1. 2, 5 **10**
2. 12, 9
3. 10, 7
4. 3, 5
5. 12, 6
6. 4, 10
7. 7, 5
8. 6, 4
9. 8, 10
10. 18, 12
11. 5, 27
12. 24, 15
13. 16, 24
14. 18, 2
15. 6, 9
16. 18, 5
17. 19, 24
18. 18, 24
19. 24, 36
20. 27, 14

21. 25, 16
22. 9, 44
23. 12, 5
24. 36, 18
25. 6, 120
26. 6, 45
27. 14, 10
28. 42, 28
29. 36, 30
30. 16, 8

Exponents

Powers and Exponents

To find the ***powers*** of a number, multiply the number over and over by itself. The ***first power*** is the number. The ***second power*** is the product of the number multiplied once by itself or ***squared***. The ***third power*** is the number multiplied twice by itself or ***cubed***, and so on. For example:

2^1 = 2 x 1 **2^2 = 2 x 2** **2^3 = 2 x 2 x 2**

10^1 = 10 x 1 **10^2 = 10 x 10** **10^3 = 10 x 10 x 10**

The numbers above are written in expanded form.

5^2 can be read as "five to the second power" or "five squared".

5^3 can be read as "five to the third power" or "five cubed".

5^4 can be read as "five to the fourth power".

- ***There is a special way of writing the power of a number called an exponent. It's the tiny number written above and to the right of the number.***

- ***Sometimes you may see an exponent expressed like this, 2^3. This would be the same as 2^3. Both are examples of writing the power of a number in exponential form.***

Base

A number used as a repeated factor
Example:
$8^3 = 8 \times 8 \times 8$
The base is 8. It is used as a factor three times.

The exponent is 3.

Converting Numbers to Scientific (Exponential) Notation

Scientific notation is used to express very large or very small numbers. A number in scientific notation is written as the product of a number (integer or decimal) and a power of 10. This number is always 1 or more and less than 10.

For example, there are approximately 6,000,000,000 humans on earth. This number could be written in scientific notation as $6x10^9$. The number 6,000,000,000 is equivalent to 6x1,000,000,000. The number 1,000,000,000 is equivalent to 10^9 or 10x10x10x10x10x10x10x10x10.

Number	Scientific Notation?	Product of	Places after 1st Digit
1	$1.0x10^0$	1	0 places
10	$1.0x10^1$	1x10	1 places
100	$1.0x10^2$	1x10x10	2 places
1,000	$1.0x10^3$	1x10x10x10	3 places
10,000	$1.0x10^4$	1x10x10x10x10	4 places
100,000	$1.0x10^5$	1x10x10x10x10x10	5 places

1,000,000	$1.0x10^6$	1x10x10x10x10x10x10	6 places

A number can be converted to scientific notation by increasing the power of ten by one for each place the decimal point is moved to the left. In the example above, the decimal point was moved 9 places to the left to form a number more than 1 and less than 10.

Scientific notation numbers may be written in different forms. The number $6x10^9$ could also be written as 6e+9. The +9 indicates that the decimal point would be moved 9 places to the right to write the number in standard form.

Negative powers of 10 are useful for writing very small numbers. Any number to a negative power represents a fraction or decimal.

$$10^{-1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10} \times \frac{1}{10} = 0.01$$

$$10^{-3} = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.001$$

Example In a scientific experiment, the mass of a sample is 2^{-5} x 10 kilogram. Write the mass in standard notation.

Step 1 Write the given number with a string of zeros in front of it. You haven't changed the value.

0000002.

Step 2 Move the decimal point to the left by the number of places shown in the exponent. Discard extra zeros.

00.00002.

$2 \times 10^{-5} = 0.00002$

As you know a negative is the opposite of positive. You also know that multiplication is the opposite of division. Positive exponents show how many times the number 1 is multiplied by a number. Negative exponents show how many times 1 is divided by a number.

Take a look at the example below:

5^2
$1 \cdot 5 \cdot 5$
25

To find what 5 to the 2nd power is you simply multiply 1 by the number 5 two times. To find what 5 to the negative 2nd power is (as in the example below) you would do the same thing but only dividing instead of multiplying. Divide 1 by 5 two times:

5^{-2}
1 / 5 / 5 (The / symbol is used to represent division)
0.04

This method works well, but division is not always the easiest operation to do over and over again.

Take a look at the problem on the next page. This example is not important to commit to long-term memory, but it will help you understand how to avoid division.

1 / 2 / 2
.25

1 / (2 · 2)
.25

As you can see, the result of dividing 1 by 2, then by 2 again is the same as 1 divided by the product of 2 and 2. This rule can now be applied to negative exponents:

5^{-2}
1 / (5 · 5)
1 / 25
.04

If you are able to do the multiplication mentally, then there is no need to write out the (5 · 5) multiplication, you can simply work the problem like below.

5^{-2}
$1 / 5^2$
1 / 25
.04

As you can see above, the five is simplified to 1 divided by 5 to the same exponent [2]. But in this case the exponent's sign is changed to positive. Often, the work for simplifying negative exponents is shown with fractions in place of the / or division symbol.

Since a fraction is really just a division problem shown in a different way, the work to our problem [5^{-2}] can be shown in a fraction as below:

$$5^{-2}$$

$$\frac{1}{5^2}$$

$$\frac{1}{5 \cdot 5}$$

$$\frac{1}{25}$$

$$.04$$

The answer can be left as a simplified fraction (in the above problem this would be 1/25) or as a decimal (in the above problem .04).

Laws of Exponents

To multiply powers of the same base, *add their exponents.*

Example $2^2 \times 2^3 = 2^5$
This is true because $2^2 = 2 \times 2 = 4$; $2^3 = 2 \times 2 \times 2 = 8$; $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
$4 \times 8 = 32$

To divide powers of the same base, *subtract the exponent of the divisor from the exponent of the dividend.*

Example $3^5 \cdot 3^3 = 3^2$
This is true because $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$; $3^3 = 3 \times 3 \times 3 = 27$; $3^2 = 3 \times 3 = 9$
$243 \div 27 = 9$

You may need to find the value of a product or a quotient of powers that do not share the same base. In either case, first find the value of each power and then multiply or divide as indicated.

Example Find the value of the product $3^2 \bullet 2^4$.

Step 1 Find the value of each power.
$3^2 = 9$ and $2^4 = 16$

Step 2 Multiply the values found in Step 1.
$3^2 \bullet 2^4 = 9 \bullet 16 = 144$

Answer: 144

Example 2 Find the value of the quotient $4^3/2^2$.

Step 1 Find the value of each power.
$4^3 = 64$ and $2^2 = 4$

Step 2 Divide 64 by 4.
$64/4 = 16$

Answer: 16

You may need to find the value of a sum or a difference of powers. In either case, first find the value of each power and then add or subtract as indicated.

Example Find the value of the sum $7^2 + 3^3$.

Step 1 Find the value of each power.
$7^2 = 49$ and $3^3 = 27$

Step 2 Add the values found in Step 1.
$7^2 + 3^3 = 49 + 27 = 76$

Answer: 76

Example 2 Find the value of the difference $8^2 - 3^2$.

Step 1 Find the value of each power.
$8^2 = 64$ and $3^2 = 9$

Step 2 Subtract 9 from 64.
$64 - 9 = 55$

Answer: 55

Practice Exercise

Express each of the following numbers in scientific notation:

1. 60,000	**2. 308,000,000,000**	**3. 520,000**
4. .0018	**3. .00000079**	**4. .06356**

Use exponential form to write:

1. 2 x 2 x 2 x 2
2. 5 x 5 x 5 x 5 x 5 x 5 x 5 x 5 x 5 x 5
3. Express 36 as a power of 6.
4. Express 32 as a power of 2.

Solve each problem. The first one has been done for you.

1. 3^2 $3 \times 3 = 9$	**2.** 11^2

3. 5^{-3}	**4.** 8^{4}
5. 10^{-2}	**6.** 9^{3}
7. $3^{4} + 3^{5}$	**8.** $10^{3} - 8^{5} + 2^{4}$
9. $7 + 7^{5} - 7^{3} + 7^{3}$	**10.** $6^{4} - 6^{3} + 6^{4} - 6^{2}$
11. $3^{4} + 7^{5} + 6^{5}$	**12.** $4^{5} \div 8^{2}$
13. $4^{2} \times 4 \times 4^{-5} \times 4$	**14.** $10^{2} + 7^{4} + 6$
15. $9^{5} \div 9^{4}$	**16.** $8 + 6^{2} + 9^{4} + 7^{6}$
17. $4^{2} \times 4^{5}$	**18.** $6^{3} + 6^{4}$
19. $5^{4} - 5$	**20.** $3 \div 3^{4}$
21. $4^{5} + 8^{2} + 10^{2} + 12^{4}$	**22.** $5 + 5^{4} - 5^{2}$
23. $9 + 8^{4}$	**24.** 10×10

25. $8^5 - 8^5$	**26. $6 + 6^4 + 6^6 + 6^2$**

Squares and Square Roots

Raising a number to the second power is also called ***squaring*** a number.

For example, ***3 squared*** (3^2) is equal to ***9***, and the ***square root*** of ***9*** is ***3***.

⟹ **There is a shortcut for squaring any number ending in 5.**

Example **Evaluate 15^2**

15 lies between 10 and 20

$$15^2 = 10 \times 20 + 5^2$$
$$= 200 + 25$$
$$= 225$$

To find the square root of a number ask yourself, "What number times itself equals this number?"

There is a special way to write the symbol for a square root called a ***radical sign*** $\sqrt{}$.

2 squared: $2^2 = 2 \times 2 = 4$

3 squared: $3^2 = 3 \times 3 = 9$

4 squared: $4^2 = 4 \times 4 = 16$

Square root of 16: $\sqrt{16} = 4$

Square root of 9: $\sqrt{9} = 3$

Square root of 4: $\sqrt{4} = 2$

Table of Square Roots to 25

$\sqrt{1}$	1	$\sqrt{100}$	10	$\sqrt{361}$	19
$\sqrt{4}$	2	$\sqrt{121}$	11	$\sqrt{400}$	20
$\sqrt{9}$	3	$\sqrt{144}$	12	$\sqrt{441}$	21
$\sqrt{16}$	4	$\sqrt{169}$	13	$\sqrt{484}$	22
$\sqrt{25}$	5	$\sqrt{196}$	14	$\sqrt{529}$	23
$\sqrt{36}$	6	$\sqrt{225}$	15	$\sqrt{576}$	24
$\sqrt{49}$	7	$\sqrt{256}$	16	$\sqrt{625}$	25
$\sqrt{64}$	8	$\sqrt{289}$	17		
$\sqrt{81}$	9	$\sqrt{324}$	18		

Perfect Square

A number that has positive or negative whole numbers (integers) as its square roots

Example:
16 is a perfect square.

$\sqrt{16} = 4$ $\quad$ ${}^{-}\sqrt{16} = {}^{-}4$

Practice Exercise

Solve the problems below.

1. 6^2 = **36**	2. 2^2 =	3. 3^2 =	4. 11^2 =
5. 4^2 =	6. 7^2 =	7. 12^2 =	8. 13^2 =
9. 8^2 =	10. 9^2 =	11. 1^2 =	12. 10^2 =
13. 18^2 =	14. 23^2 =	15. 21^2 =	16. 25^2 =
17. 20^2 =	18. 24^2 =	19. 17^2 =	20. 22^2 =
21. $\sqrt{64}$ =	22. $\sqrt{81}$ =	23. $\sqrt{196}$ =	24. $\sqrt{169}$ =
25. $\sqrt{1}$ =	26. $\sqrt{25}$ =	27. $\sqrt{121}$ =	28. $\sqrt{36}$ =

29. $\sqrt{16}$ =	30. $\sqrt{4}$ =	31. $\sqrt{100}$ =	32. $\sqrt{49}$ =
33. $\sqrt{484}$ =	34. $\sqrt{144}$ =	35. $\sqrt{529}$ =	36. $\sqrt{256}$ =

Finding an Approximate Square Root

A number that is not a perfect square does not have a whole number square root. For example, the square root of 30 is between the whole numbers 5 and 6:

$\sqrt{30}$ is larger than 5, since $5^2 = 25$
$\sqrt{30}$ is smaller than 6, since $6^2 = 36$

To find the approximate square root of a number that is not a perfect square, follow the three steps of the Method of Averaging:

Example Find the approximate square root of 30.

Step 1 Choose a number that is close to the correct square root.

$$5 \approx \sqrt{30}$$
$$\text{since } 5 = \sqrt{25}$$

Step 2 Divide this chosen number into the number you're trying to find the square root of.

$$30 \div 5 = 6$$

Step 3 Average your choice from Step 1 with the answer from Step 2. The average of these two numbers is the approximate square root.

$$5 + 6 = 11$$
$$11 \div 2 = 5\ 1/2$$

Answer: 5 ½ = **5.5**
Check: $(5.5)^2 = 30.25$
Answer: 5.5 » √30
Note: The approximation sign "≈" means "is approximately equal to."

Square Root Algorithm

An algorithm is a special method that can be used to add subtract, or complete some mathematical operation. There is an algorithm for finding the square root.

Follow the steps below to find the square root of a number.

Example Find the square root of 676.

Step 1 Separate the numeral into groups of two figures each, starting at the decimal point. Attach zeros if needed.

$$\sqrt{6\ \ 76}$$

Step 2 Place the largest possible square under the first group at the left.

$$\begin{array}{l} \sqrt{6\ \ 76} \\ \ \ \mathbf{4} \end{array}$$

Step 3 Write the square root of the number in Step 2 above the first group.

$$\begin{array}{l} \ \ \ \mathbf{2} \\ \sqrt{6\ \ 76} \\ \ \ 4 \end{array}$$

Step 4 Subtract the square from the first group. Attach the next group to the remainder.

$$\begin{array}{l} \ \ \ 2 \\ \sqrt{6\ \ 76} \\ \ \ 4 \\ \sqrt{\mathbf{2\ \ 76}} \end{array}$$

Step 5 Form the trial divisor by doubling the root already found and leaving a space.

$$\begin{array}{rl} & \;\;2 \\ & \sqrt{6\ \ 76} \\ & \;\;4 \\ 4 & \sqrt{2\ \ 76} \end{array}$$

Step 6 Divide the dividend from Step 4 by the divisor in Step 5. Join the quotient to the root already found and put the same number in the space that you left next to the trial divisor to form the complete divisor.

$$\begin{array}{rl} & \;\;2\quad 6 \\ & \sqrt{6\ \ 76} \\ & \;\;4 \\ 46 & \sqrt{2\ \ 76} \end{array}$$

Step 7 Multiply the complete divisor by the new figure of the root.

$$\begin{array}{rl} & \;\;2\quad 6 \\ & \sqrt{6\ \ 76} \\ & \;\;4 \\ 46 & \sqrt{2\ \ 76} \\ & \;\;\underline{2\ \ 76} \end{array}$$

Step 8 Subtract this product from the dividend.

```
       2   6
      √6  76
       4
  46  √2  76
       2  76
       ‾‾‾‾‾
           0
```

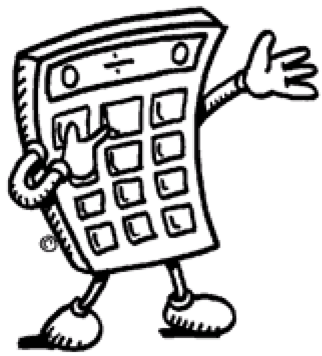
Step 9 Continue until all groups are used or the answer is of the desired accuracy.

Step 10 If necessary, place a decimal point in the root directly above the one in the given number.

Step 11 Check by squaring the square root to obtain the given number.

```
 26
 26
 ‾‾
156
52_
‾‾‾
676
```

Answer: The square root of 676 is **26**.



To find the square of a number, you multiply the number by itself. For example, $6^2 = 6 \times 6 = 36$. You can square numbers quickly using the {x^2} key on your calculator. You can also perform operations using squares. You will find this feature useful when solving problems involving the Pythagorean Theorem.

Examples

Solve: $8^2 = ?$ Enter: 8 {x^2} The answer is 64.

Solve: $12^2 - 7^2 = ?$ Enter: 12{x^2}{–}{7}{x^2}{=}95.

The square root function is the second operation assigned to the square key {x^2}. To find the square root of a number, enter the number, then press SHIFT and the square key.

Examples

Solve: What is the square root of 225?

Enter: 225{SHIFT}{x^2} The answer is 15.
Solve: $\sqrt{256} + \sqrt{81} = ?$
Enter: 256{SHIFT}{x^2}{+}81{SHIFT}{x^2}{=}25

Word Problems with Whole Numbers

Within every story (word) problem are several ***clue words***. These words tell you the kind of math sentence (equation) to write to solve the problem.

Addition Clue Words

add
sum
total
plus
in all
both
together
increased by
all together
combined

Subtraction Clue Words

subtract
difference
take away
less than
are not
remain
decreased by
have or are left
change (money problems)
more
fewer

Multiplication Clue Words

times

Division Clue Words

quotient of

product of
multiplied by
by (dimension)

divided by
half [or a fraction]
split
separated
cut up
parts
shared equally

⇒ ***Division clue words are often the same as subtraction clue words. Divide when you know the total and are asked to find the size or number of "one part" or "each part."***

Following a system of steps can increase your ability to accurately solve problems. Use these steps to solve word problems.

1. Read the problem carefully. Look up the meanings of unfamiliar words.

2. Organize or restate the given information.

3. State what is to be found.

4. Select a strategy (such as making a chart of working backward) and plan the steps to solve the problem.

5. Decide on an approximate answer before solving the problem.

6. Work the steps to solve the problem.

7. Check the final result. Does your answer seem reasonable?

The Problem Solving System was used to solve the following problem:

Mary has ten marbles. Lennie has thirteen. How many marbles do they have in all?

1. Mary has ten marbles. Lennie has thirteen. How many marbles do they have in all?

2. Mary – 10 marbles
Lennie – 13 marbles

3. How many marbles in all?

4. Add

5. A little over 20 marbles (10 + 10 = 20)

6.
$$\begin{array}{r} 10 \\ +13 \\ \hline 23 \end{array} \text{ marbles}$$

7. The final sum of 23 marbles is close to the estimated answer of 20 marbles. The final result is reasonable.

Þ ***Be sure to label answers whenever possible. For example: marbles, grams, pounds, feet, dogs, etc.***

Þ ***Some problems may require several steps to solve. Some may have more than one correct answer. And some problems may not have a solution.***

Have you ever tried to help someone else work out a word problem? Think about what you do. Often, you read the problem with the person, then discuss it or put it in your own words to help the person see what is happening. You can use this method---restating the problem---on your own as a form of "talking to yourself."

Restating a problem can be especially helpful when the word problem contains no key words. Look at the following example:

Example: Susan has already driven her car 2,700 miles since its last oil change. She still plans to drive 600 miles before changing the oil. How many miles does she plan to drive between oil changes?

Step 1: *question:* How many miles does she plan to drive between oil changes?

Step 2: *necessary information:* 2,700 miles, 600 miles

Step 3: *decide what arithmetic to use:* Restate the problem in your own words: "You are given the number of miles Susan has already driven and the number of miles more that she plans to drive. You need to add these together to find the total number of miles between oil changes."

Step 4: 2,700 miles + 600 miles = **3,300 miles** between oil changes.

Step 5: It makes sense that she will drive 3,300 miles between oil changes, since you are looking for a number larger than the 2,700 miles that she has already driven.

For some problems, you have to write two or three equations to solve the problem. For others, you may need to make charts or lists of information, draw pictures, find a pattern, or even guess and check. Sometimes you have to work backwards from a sum, product, difference, or quotient, or simply use your best logical thinking.

List/Chart

Marty's library book was six days overdue. The fine is $.05 the first day, $.10, the second, $.20 the third day, and so on. How much does Marty owe?

Marty's library book was six days overdue. The fine is $.05 the first day, $.10, the second, $.20 the third day, and so on. How much does Marty owe?

Days	**1**	**2**	**3**	**4**	**5**	**6**
Fine	**$.05**	**$.10**	**$.20**	**$.40**	**$.80**	**$1.60**

Answer: $1.60

Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?

Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?

Veronica	**Veronica**	**Archie**	**Archie**
Archie	**Betty**	**Veronica**	**Betty**
Betty	**Archie**	**Betty**	**Veronica**

Betty	**Betty**
Veronica	**Archie**
Archie	**Veronica**

Answer: 6 ways

Find a Pattern

Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X_______. How did Jenny fill in the blanks?

Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X_______. How did Jenny fill in the blanks?

Answer: 10, 11, 12 W

Draw a Picture

Mary is older than Jamie. Susan is older than Jamie, but younger than Mary. David is younger than Jamie. Who is oldest?

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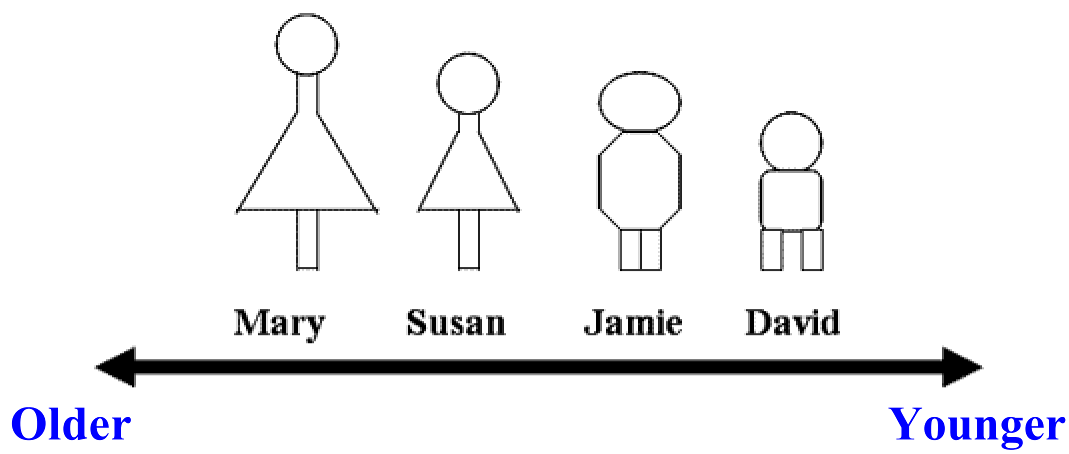


Answer: Mary is oldest.

Guess and Check

Farmer Joe keeps cows and chickens in the farmyard. All together, Joe can count 14 heads and 42 legs. How many cows and how many chickens does Joe have in the farmyard?

Farmer Joe keeps cows and chickens in the farmyard. All together, Joe can count 14 heads and 42 legs. How many cows and how many chickens does Joe have in the farmyard?

6 cows +8 chickens 14 heads	**Guess a number of cows. Then add the number of chickens to arrive at the sum of 14 heads. Then check the total legs.**	**6 cows = 24 legs +8 chickens = 16 legs 40 legs**
7 cows +7 chickens 14 heads	**Adjust your guesses. Then check again until you solve the problem.**	**7 cows = 28 legs +7 chickens = 14 legs 42 legs**

Answer: 7 cows and 7 chickens

Work Backwards

Marsha was banker for the school play. She took in $175 in ticket sales. She gave Wendy $75 for sets and costumes and Paul $17.75 for advertising and publicity. After paying for the props, Marsha had $32.25 left. How much did the props cost?

Marsha was banker for the school play. She took in $175 in ticket sales. She gave Wendy $75 for sets and costumes and Paul $17.75 for advertising and publicity. After paying for the props, Marsha had $32.25 left. How much did the props cost?

$ 175.00 tickets		$ 82.25
- 75.00 costumes		- 32.25
$ 100.00		$ 50.00 cost of props
- 17.75 advertising		
$ 82.25		

Logical Reasoning

Jim challenged Sheila to guess his grandmother's age in ten questions or less. It took her six. Here's what Sheila asked:

Jim challenged Sheila to guess his grandmother's age in ten questions or less. It took her six. Here's what Sheila asked:

"Is she less than fifty?" "No."	**50+ years old**
"Less than seventy-five?" "Yes."	**50 to 74 years old**
"Is her age an odd or even number?" "Odd."	**ends in 1, 3, 5, 7 or 9**

"Is the last number less than or equal to

five?" "No."	**ends in 7 or 9**
"Is it nine?" "No."	**ends in 7 – 57 or 67**
"Is she in her sixties?" "No."	**57 years old**

Not Enough Information

Now that you know how to decide whether to add, subtract, multiply, or divide to solve a word problem, you should be able to recognize a word problem that cannot be solved because not enough information is given.

Look at the following example:

Problem: At her waitress job, Sheila earns $4.50 an hour plus tips. Last week she got $65.40 in tips. How much did she earn last week?

Step 1: *question:* How much did she earn last week?
Step 2: *necessary information:* $4.50/hour, $65.40
Step 3: *decide what arithmetic to use:*

tips + (pay per hour **x** hours worked) = total earned

missing information: hours worked

At first glance, you might think that you have enough information since there are 2 numbers. But when the solution is set up, you can see that you need to know the number of hours Sheila worked to find out what she earned. **(Be Careful!!!)**

1. Sharon's rent has been increased $65 a month to $390 a month. What had she been paying?

2. Lily's allergy pills come in a 250 tablet bottle. She takes 4 tablets a day. How many tablets did she have left after taking the tablets for 30 days?

3. An oil truck carried 9,008 litres of oil. After making 7 deliveries averaging 364 litres each, how much oil was left in the truck?

4. Mary and Lucy disagree on the meaning of the expression $3^2 – 2^3$. Mary says it means 9 – 6, or 3. Lucy says it means 6 – 6, or 0. Who is right?

5. What is the average temperature for the full day if the daytime temperature is 23 degrees Celsius and the nighttime temperature is 15 degrees Celsius?

6. Joe scored 78 on his math test. The rest of Joe's math group scored 81, 85, 83, 92, 86, and 90. What was the median of the scores in Joe's math group?

7. After two strings of bowling, Sally's team had scores of 80, 75, 93, 81, 98, 93, 57, and 62. What is the mode of the scores for Sally's bowling team?

8. Which of the following numbers is a prime number: 14, 23, 39, 51, or 85?

9. Many of the things we take for granted were invented not so long ago. For example, pencils were first used in 1565. They were square until Joseph Dixon made them round in 1876. Round ones are easier to hold. For how many years did people use square pencils?

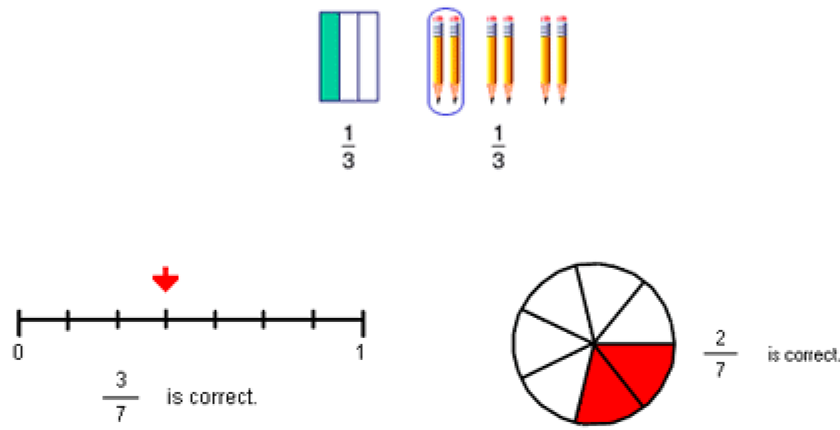
10. Ben’s trip to work is longer now that he has moved. He used to drive 5 kilometers to get to work. Now he drives 9 kilometers twice a day, 5 days a week, 4 weeks a month. How many extra kilometers does Ben now travel in a month?

Fractions, Decimals and Percent

Fractions

The word ***fraction*** means “part of a whole.” The word comes from the Latin word ***fractio***, meaning “to break into pieces.” In math, a fraction means one or more parts of a whole.

Example:



A fraction has two parts, a ***denominator*** and a ***numerator***. The denominator is the numeral written under the bar and tells the number of parts a whole is divided into. The numerator is the numeral written above the bar. The numerator tells the number of parts of the whole that are being counted. A ***proper fraction*** has a numerator that is smaller than its denominator.

numerator	**number of parts counted**	**1**
denominator	**total parts of the whole**	**17**

Improper Fractions

When the numerator of a fraction is greater than or equal to the denominator, the fraction is called an ***improper fraction***.

$$\frac{3}{2} \quad \frac{4}{3} \quad \frac{5}{4} \quad \frac{6}{5} \quad \frac{7}{6} \quad \frac{8}{8}$$

Þ ***The value of an improper fraction is always greater than or equal to one.***

Mixed Numerals

Mixed numerals combine whole numbers and fractions. The values of mixed numerals can also be written as ***improper fractions***. To write a mixed numeral as an improper fraction, multiply the whole number by the denominator of the fraction, then add the numerator. Use your answer as the new numerator and keep the original denominator.

$$1\tfrac{1}{2} = \frac{(2 \text{ x } 1) + 1}{2} = \frac{3}{2} \qquad 2\tfrac{3}{4} = \frac{(2 \text{ x } 4) + 3}{4} = \frac{11}{4}$$

To change an improper fraction to a mixed numeral, divide the numerator by the denominator. Then place the remainder over the old denominator.

$$\frac{3}{2} = \begin{array}{r} 1 \\ 2\,\overline{)\,3} \\ \underline{-2} \\ 1 \end{array} = 1\tfrac{1}{2} \qquad\qquad \frac{11}{4} = \begin{array}{r} 2 \\ 4\,\overline{)\,11} \\ \underline{-8} \\ 3 \end{array} = 2\tfrac{3}{4}$$

Practice Exercise

Express each fraction as a whole number or as a mixed number.

1. $\frac{84}{9} =$
2. $\frac{17}{2} =$
3. $\frac{26}{4} =$
4. $\frac{76}{8} =$
5. $\frac{28}{10} =$
6. $\frac{24}{3} =$
7. $\frac{101}{11} =$
8. $\frac{31}{6} =$
9. $\frac{58}{12} =$
10. $\frac{74}{11} =$
11. $\frac{38}{4} =$
12. $\frac{73}{10} =$
13. $\frac{11}{3} =$
14. $\frac{107}{9} =$
15. $\frac{36}{7} =$
16. $\frac{44}{5} =$

17. $\frac{21}{8} =$

18. $\frac{69}{6} =$

19. $\frac{36}{5} =$

20. $\frac{49}{6} =$

21. $\frac{10}{3} =$

22. $\frac{153}{12} =$

23. $\frac{46}{11} =$

24. $\frac{118}{10} =$

25. $\frac{4}{2} =$

26. $\frac{94}{9} =$

27. $\frac{64}{7} =$

28. $\frac{41}{7} =$

29. $\frac{66}{8} =$

30. $\frac{21}{10} =$

31. $\frac{19}{2} =$

32. $\frac{63}{5} =$

Common Denominators

Many fractions have ***common denominators***. That means that the numbers in their denominators are the same.

$$\frac{1}{2} \quad \frac{3}{2} \quad \frac{5}{2}$$

To find common denominators, (1) find the ***least common multiple*** for the denominators of the fractions you are comparing.

Compare:

$\frac{1}{2}$ and $\frac{2}{3}$ **Answer: least common multiple is 6**

(2) Divide the common multiple by the denominators.

$$2\,\overline{)\,6}\;=3 \qquad 3\,\overline{)\,6}\;=2$$

(3) Multiply the quotients by the old numerators to calculate the new numerators.

$$\begin{array}{r} 3 \\ \underline{\times\ 1} \end{array} \qquad \begin{array}{r} 2 \\ \underline{\times\ 2} \end{array}$$

3 **4**

(4) Place the new numerators over the common denominator.

$$\frac{3}{6} \qquad \frac{4}{6}$$

Þ ***To reduce a fraction to its lowest terms, divide both the numerator and the denominator by their greatest common denominator.***

$$\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$$

Equivalent Fractions

You know from experience that different fractions can have the same value.

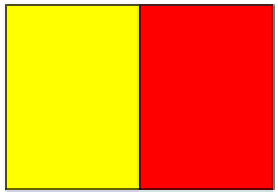
Since there are 100 pennies in a dollar, 25 pennies is equal to 25/100 of a dollar. The same amount also equals a quarter, or ¼of a dollar.

On a measuring cup, ½ cup is the same amount as 2/4 cup.

On an odometer, 5/10 of a mile is the same as ½ mile.

Out of a dozen doughnuts, six doughnuts equal 6/12, or ½ dozen.

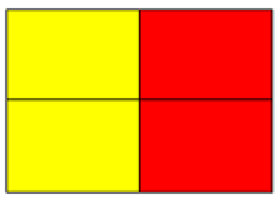
A napkin is folded into two parts. One part is yellow, the other red.



½ yellow

½ red

Then the napkin is folded again. Now there are two yellow parts and two red parts.



$\frac{2}{4}$ yellow

$\frac{2}{4}$ red

In this example, the red part of the napkin can be described as ½ red or 2/4 red. That makes ½ and 2/4 ***equivalent fractions***.

When solving math problems, reduce fractions to their lowest equivalent. Rather than describing the napkin as 2/4 yellow, call it ½ yellow.

Some Equivalent Fractions

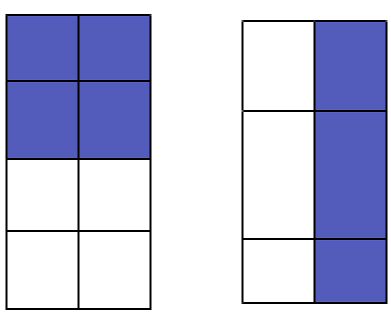
$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20}$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$$

You can tell if two fractions are equal by finding cross products.

Example Are 4/8 and 3/6 equal fractions?



Multiply diagonally as shown by the arrows below. If the cross products are equal, the fractions are equal.

$$\frac{4}{8} \quad \frac{3}{6} \qquad \begin{array}{l} \mathbf{4 \times 6 = 24} \\ \mathbf{8 \times 3 = 24} \end{array}$$

Since the cross products are equal, **4/8 = 3/6.**

Sometimes you need to find an equal fraction with higher terms. You raise a fraction to higher terms by multiplying both the numerator and the denominator by the same number (except 0).

5/8 and 20/32 are equal fractions because $\frac{5 \times 4}{8 \times 4} = \frac{20}{32}$

Often you will need to find an equal fraction with a specific denominator. To do this, think, "What number multiplied by the original denominator will result in the new denominator?" Then multiply the original numerator by the same number.

Example ¾ = ?/24

Since 4 x 6 = 24, multiply the numerator 3 by 6. $\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$

The fractions ¾ **and 18/24 are equal fractions.**

Comparing Fractions

When two fractions have the same number as the denominator, they are said to have a common denominator, and the fractions are called like fractions. When you compare like fractions, the fraction with the greater numerator is the greater fraction.

Example 1 Which fraction is greater, 3/5 or 4/5?

The fractions 3/5 and 4/5 are like fractions because they have a common denominator, 5. Compare the numerators.

Since 4 is greater than 3, **4/5 is greater than 3/5.**

Fractions with different denominators are called unlike fractions. To compare unlike fractions, you must change them to fractions with a common denominator.

The common denominator will always be a multiple of both of the original denominators. The multiples of a number are found by going through the times tables for the number. For instance, the multiples of 3 are 3, 6, 9, 12, 15, 18, and so on.

You can often find a common denominator by using mental math. If not, try these methods:

1. See whether the larger denominator could be a common denominator. In other words, if the smaller denominator can divide into the larger denominator evenly, use the larger denominator as the common denominator.

2. Go through the multiples of the larger denominator. The first one that can be divided evenly by the smaller denominator is the lowest common denominator.

Example 2 Which is greater, 5/6 or ¾?

Go through the multiples of the larger denominator: 6, 12, 18, 24, 30…. Since 12 can be divided evenly by both 4 and 6, 12 is the lowest common denominator.

Build equal fractions, each with the denominator 12: $\frac{5 \times 2}{6 \times 2} = \frac{10}{12}$ $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

Compare the like fractions. Since 10/12 > 9/12, the fraction **5/6 > ¾.**

Practice Exercise

Fraction Comparison

1. $\frac{1}{12}$ $<$ $\frac{5}{6}$	2. $\frac{2}{7}$ ____ $\frac{1}{10}$
3. $\frac{1}{5}$ ____ $\frac{3}{4}$	4. $\frac{3}{8}$ ____ $\frac{4}{7}$
5. $\frac{8}{13}$ ____ $\frac{3}{4}$	6. $\frac{4}{16}$ ____ $\frac{5}{14}$
7. $\frac{2}{4}$ ____ $\frac{34}{15}$	8. $\frac{4}{9}$ ____ $\frac{2}{3}$
9. $\frac{44}{45}$ ____ $\frac{2}{24}$	10. $\frac{20}{61}$ ____ $\frac{1}{7}$
11. $\frac{30}{45}$ ____ $\frac{10}{80}$	12. $\frac{11}{17}$ ____ $\frac{19}{22}$

13. $\frac{3}{7}$ ____ $\frac{1}{10}$	14. $\frac{43}{61}$ ____ $\frac{1}{6}$
15. $\frac{12}{84}$ ____ $\frac{9}{27}$	16. $\frac{24}{44}$ ____ $\frac{48}{88}$

Order each list of fractions from least to greatest.

1. $\frac{28}{40}$, $\frac{8}{32}$, $\frac{24}{42}$	2. $\frac{31}{39}$, $\frac{6}{39}$, $\frac{9}{39}$, $\frac{5}{39}$, $\frac{21}{39}$
3. $\frac{8}{11}$, $\frac{5}{6}$, $\frac{8}{11}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{3}{5}$	4. $\frac{1}{11}$, $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{5}$
5. $\frac{8}{11}$, $\frac{4}{6}$, $\frac{3}{8}$, $\frac{6}{7}$	6. $\frac{26}{20}$, $\frac{43}{49}$, $\frac{49}{45}$, $\frac{18}{27}$
7. $\frac{1}{11}$, $\frac{1}{9}$, $\frac{1}{5}$, $\frac{1}{2}$	8. $\frac{5}{55}$, $\frac{6}{36}$, $\frac{32}{24}$, $\frac{31}{56}$
9. $\frac{4}{5}$, $\frac{3}{7}$, $\frac{5}{9}$, $\frac{3}{4}$	10. $\frac{25}{36}$, $\frac{36}{76}$, $\frac{12}{24}$, $\frac{12}{72}$

Reduce each fraction to lowest terms.
(Hint: Divide its numerator and denominator by their GCF)

1. $\frac{9}{45} =$ 2. $\frac{24}{32} =$ 3. $\frac{12}{42} =$ 4. $\frac{8}{40} =$

5. $\frac{6}{24} =$ 6. $\frac{4}{20} =$ 7. $\frac{3}{33} =$ 8. $\frac{18}{24} =$

9. $\frac{12}{96} =$ 10. $\frac{25}{36} =$ 11. $\frac{8}{20} =$ 12. $\frac{43}{52} =$

13. $\frac{9}{63} =$ 14. $\frac{49}{74} =$ 15. $\frac{20}{45} =$ 16. $\frac{39}{36} =$

17. $\frac{34}{27} =$ 18. $\frac{5}{35} =$ 19. $\frac{30}{55} =$ 20. $\frac{32}{44} =$

21. $\frac{3}{18} =$ 22. $\frac{39}{15} =$ 23. $\frac{33}{53} =$ 24. $\frac{24}{60} =$

Adding Fractions

To add fractions, the fractions must have ***common denominators***. To add fractions with common denominators, simply add the numerators. The sum will become the numerator of your answer. The denominator will remain the same.

$$\frac{1}{3} + \frac{4}{3} = \frac{1+4}{3} = \frac{5}{3}$$

Unlike fractions have different denominators. Use these steps to add unlike fractions.

Step 1 Find a common denominator and change one or both of the fractions to make like fractions.

$$\tfrac{1}{2} + \tfrac{3}{4} = ?$$

$$\tfrac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Step 2 Add the like fractions

2/4 + ¾ = 5/4

Step 3 Reduce the answer if necessary. If the answer is an improper fraction, rewrite it as a whole or mixed number.

5/4 = 1 ¼

A mixed number is a whole number and a proper fraction. To add mixed numbers, work with each part separately and then combine the results.

Þ ***Adding fractions is impossible without first writing the fractions with common denominators.***

Step 1 Write the fractions with common denominators.

$$6\,1/3 = 6\,\frac{1 \times 4}{3 \times 4} = 6\,\frac{4}{12}$$

$$+\,4\,¾ = 4\,\frac{3 \times 3}{4 \times 3} = 4\,\frac{9}{12}$$

Step 2 Add the fractions first. Add the numerators and put the sum over the common denominator. Then add the whole numbers.

$$\begin{array}{r} 6\,\frac{4}{12} \\ +\;4\,\frac{9}{12} \\ \hline \end{array}$$

Step 3 Change the improper fraction to a mixed number. Add this to the whole number answer.

$$\frac{13}{12} = 1\,\frac{1}{12}$$

$$10 + 1\,\frac{1}{12} = 11\,\frac{1}{12}$$

Sometimes when you add the fraction parts, you get a whole number as an answer. If this happens, just add that whole number to the other one.

Example: $2\frac{3}{5} + 2\frac{2}{5}$

2 + 2 = 4

$\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$ **Remember that any number divided by itself is 1.**

4 + 1 = 5 **The answer is 5.**

Mixed numbers can be added to whole numbers by adding the whole numbers together and keeping the fraction. This makes sense because you are adding whole amounts plus another part of a whole.

Example: 3 + 2 ½ = 5 ½ 3 + 2 = 5, 5 + ½ = 5 ½

Subtracting Fractions

To subtract fractions, the fractions must have ***common denominators***. To subtract fractions with common denominators, simply subtract the numerators. The difference will become the numerator of your answer. The denominator will remain the same.

$$\frac{7}{8} - \frac{5}{8} = \frac{7-5}{8} = \frac{2}{8} = \frac{1}{4}$$

Unlike fractions have different denominators. Use these steps to subtract unlike fractions.

Step 1 Find a common denominator and change one or both of the fractions to make like fractions.

$$\frac{3}{4} - \frac{1}{2} = ?$$
$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Step 2 Subtract the like fractions.

$$\frac{3}{4} - 2/4 = \frac{1}{4}$$

Step 3 Reduce the answer if necessary. If the answer is an improper fraction, rewrite it as a whole or mixed number.

A mixed number is a whole number and a proper fraction. To subtract mixed numbers, work with each part separately and then combine the results.

Þ ***Subtracting fractions is impossible without first writing the fractions with common denominators.***

Step 1 Write the fractions with common denominators.

$$6\ 3/4 = 6\ \frac{3 \times 3}{3 \times 4} = 6\ \frac{9}{12}$$

$$-\ 4\ 1/3 = 4\ \frac{1 \times 4}{3 \times 4} = 4\ \frac{4}{12}$$

Step 2 Subtract the fractions first. Subtract the numerators and put the difference over the common denominator. Then subtract the whole numbers.

$$\begin{array}{r} 6\ \frac{9}{12} \\ -\ 4\ \frac{4}{12} \\ \hline 2\ \frac{5}{12} \end{array}$$

Step 3 If necessary, reduce to lowest terms.

When subtracting mixed numbers, sometimes the fraction you are subtracting from will be smaller than the fraction you are taking away. In this situation, you will need to regroup, or borrow, 1 from the whole number and rewrite it as a fraction. Remember, a fraction with the same numerator and denominator equals 1.

Example $$\begin{array}{r} 5\ \frac{1}{8} \\ -3\ \frac{3}{4} \\ \hline \end{array}$$

Step 1 Write the fractions with common denominators. The lowest common denominator is 8.

$$5\frac{1}{8} = 5\frac{1 \times 1}{8 \times 1} = 5\frac{1}{8}$$
$$-3\frac{3}{4} = 3\frac{3 \times 2}{4 \times 2} = 3\frac{6}{8}$$

Step 2 Because 1/8 is less than 6/8, you need to regroup, or borrow. Borrow 1 from the whole number 5, rewriting 5 as 4 8/8. Then add the fractional parts 1/8 and 8/8.

$$5\frac{1}{8} = 4\frac{8}{8} + \frac{1}{8} = 4\frac{9}{8}$$
$$-3\frac{6}{8} \qquad\qquad -3\frac{6}{8}$$
$$1\frac{3}{8}$$

Step 3 Subtract. If necessary, reduce the fraction to lowest terms

Sometimes when you subtract the fraction parts, you get a whole number as an answer. If this happens, just subtract that whole number from the other one.

Example: $4\frac{7}{5} - 2\frac{2}{5}$

4 - 2 = 2

$\frac{7}{5} - \frac{2}{5} = \frac{5}{5} = 1$ **Remember that any number divided by itself is 1.**

2 - 1 = 1 **The answer is 1.**

Mixed numbers can be subtracted from whole numbers by subtracting the whole numbers and keeping the fraction.

Example: 3 - 2 ½ = 1 ½ 3 - 2 = 1, 5 + ½ = 1 ½

Practice Exercise

Solve for each of the given problems.
Write the answer in lowest terms.

1. $\frac{3}{6} + \frac{1}{5}$	2. $\frac{6}{8} + \frac{3}{5}$	3. $\frac{1}{2} - \frac{1}{6}$	4. $\frac{3}{4} - \frac{1}{7}$
5. $\frac{2}{3} + 2$	6. $3\frac{2}{5} - \frac{10}{11}$	7. $\frac{2}{7} + 9$	8. $11\frac{4}{8} - 3$
9. $11\frac{3}{5} + 10\frac{3}{11}$	10. $\frac{1}{2} + 11\frac{5}{16}$	11. $5\frac{1}{3} + 5\frac{1}{15}$	12. $7\frac{1}{4} + 10\frac{11}{12}$

13. $\frac{6}{7}$ $+\ 10\frac{6}{9}$	**14.** $9\frac{4}{6}$ $+\ 9\frac{3}{13}$	**15.** $8\frac{1}{13}$ $-\ \frac{2}{3}$	**16.** $10\frac{8}{13}$ $-\ \frac{1}{3}$

Multiplying Fractions

To multiply a fraction by a whole number, change the whole number to a fraction by placing it over a denominator of one. (This does not change the value of the whole number.) Multiply the numerators then multiply the denominators to get the product.

$$\frac{1}{2} \times 1 = \frac{1}{2} \times \frac{1}{1} = \frac{1 \times 1}{2 \times 1} = \frac{1}{2}$$

$$\frac{2}{7} \times 3 = \frac{2}{7} \times \frac{3}{1} = \frac{2 \times 3}{7 \times 1} = \frac{6}{7}$$

$$\frac{8}{9} \times 6 = \frac{8}{9} \times \frac{6}{1} = \frac{8 \times 6}{9 \times 1} = \frac{48}{9} = 5\frac{3}{9} = 5\frac{1}{3}$$

Þ ***Change improper fractions to mixed numerals. Be sure the fraction part of the mixed numeral is written in the lowest possible terms.***

To multiply one fraction by another fraction, multiply the numerators. Their product will become the new numerator. Next, multiply the denominators. Their product will become the new denominator.

multiply the numerators

$$\frac{1}{2} \text{ x } \frac{1}{3} = \frac{1 \text{ x } 1}{2 \text{ x } 3} = \frac{1}{6}$$

multiply the denominators

$$\frac{7}{8} \text{ x } \frac{1}{3} = \frac{7}{24}$$

$$\frac{4}{3} \text{ x } \frac{1}{10} = \frac{4 \text{ x } 1}{3 \text{ x } 10} = \frac{4}{30}$$

To multiply mixed numerals by fractions, change the mixed numerals to improper fractions. Then multiply the fractions.

change the mixed numeral to an improper fraction

$$1\frac{6}{7} \text{ x } \frac{2}{3} = \frac{13}{7} \text{ x } \frac{2}{3} = \frac{13 \text{ x } 2}{7 \text{ x } 3} = \frac{26}{21} = 1\frac{5}{21}$$

$$2\,\frac{1}{8} \times 3\,\frac{1}{2} = \frac{17}{8} \times \frac{7}{2} = \frac{17 \times 7}{8 \times 2} = \frac{119}{16} = 7\,\frac{7}{16}$$

As you know, reducing a fraction means to divide the numerator and the denominator by the same number. You can use this principle to simplify before you work the problem. This process is called canceling.

Example Find 1/6 of 2/3.

Both the numerator of one fraction and the denominator of the other fraction can be divided by 2. Since 2 ¸ 2 = 1, draw a slash through the numerator 2 and write 1. Since 6 ¸ 2 = 3, draw a slash through the denominator 6 and write 3. Then multiply the simplified fractions.

$$\frac{1}{6} \times \frac{2}{3} = \frac{1}{\cancel{6}_{3}} \times \frac{\cancel{2}^{1}}{3} = \frac{1}{9}$$

Since you used canceling before multiplying, there is no need to reduce the answer: 1/6 of 2/3 is **1/9.**

When you cancel, make sure you divide a numerator and a denominator by the same number. The canceling shown in the following example is **incorrect**.

$$\frac{1}{6} \text{ x } \frac{2}{3} = \frac{1 \text{ x } 2}{\underset{2}{\cancel{6}} \quad \underset{1}{\cancel{3}}}$$

Although 6 and 3 can both be divided by 3, both numbers are in the denominator.

To multiply with mixed numbers, change the mixed numbers to improper fractions before you multiply.

Example Multiply 1 2/3 by 7 ½.

Step 1 Change to improper fractions.

$$1\frac{2}{3} \text{ x } 7\frac{1}{2} = \frac{5}{3} \text{ x } \frac{15}{2}$$

Step 2 Cancel and multiply.

$$\frac{5}{\underset{1}{\cancel{3}}} \text{ x } \frac{\overset{5}{\cancel{15}}}{2} =$$

Step 3 Write as a mixed number.

$$\frac{25}{2} = 12\frac{1}{2}$$

The product of 1 2/3 and 7 ½ is **12 ½.**

Dividing Fractions

To divide a fraction by a whole number, change the whole number to an improper fraction with a denominator of one. Invert the divisor fraction. Then multiply the fractions.

$$\frac{1}{2} \div 2 = \frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\frac{2}{7} \div 3 = \frac{2}{7} \div \frac{3}{1} = \frac{2}{7} \times \frac{1}{3} = \frac{2}{21}$$

To divide a whole number by a fraction or to divide a fraction by another fraction, ***invert*** the divisor fraction. Then multiply the fractions.

$$\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times \frac{3}{1} = \frac{1 \times 3}{2 \times 1} = \frac{3}{2} = 1\frac{1}{2}$$

Invert the divisor fraction and multiply

$$7 \div \frac{6}{8} = \frac{7}{1} \times \frac{8}{6} = \frac{7 \times 8}{1 \times 6} = \frac{56}{6} = 9\frac{2}{6} = 9\frac{1}{3}$$

To divide a mixed numeral by another mixed numeral, first change the mixed numerals to improper fractions. Then invert the divisor fraction and multiply.

$$4\frac{1}{2} \div 2\frac{1}{3} = \frac{9}{2} \div \frac{7}{3} = \frac{9}{2} \times \frac{3}{7} = \frac{27}{14} = 1\frac{13}{14}$$

$$7\frac{6}{8} \div 6\frac{1}{3} = \frac{62}{8} \div \frac{19}{3} = \frac{62}{8} \times \frac{3}{19} = \frac{186}{152} = 1\frac{34}{152} = 1\frac{17}{76}$$

Turn it Upside Down: Inverting

Inverting a fraction means turning it upside down, or reversing the numerator and the denominator.

$$\frac{1}{3} \text{ inverted is } \frac{3}{1} \qquad \frac{6}{8} \text{ inverted is } \frac{8}{6}$$

Inverting a whole number means to make it the denominator of a fraction with 1 as the numerator. 3 inverted is 1/3, 7 inverted is 1/7.

So, to solve the problem 1/3 ÷ 3,

$$\text{invert} \quad 3 \text{ or } \frac{3}{1} \text{ to } \frac{1}{3}$$

$$\text{then} \quad \frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9}$$

Practice Exercise

Solve for each of the given problems.
Write the answer in lowest terms.

1. $1\frac{5}{6} \times 3$	2. $3\frac{2}{4} \times 2\frac{2}{6}$
3. $\frac{1}{8} \times \frac{6}{7}$	4. $3\frac{2}{8} \times 1\frac{2}{4}$
5. $2\frac{5}{7} \times 3\frac{2}{4}$	6. $\frac{4}{6} \times 2$
7. $7\frac{2}{7} \times 3\frac{5}{8}$	8. $7\frac{4}{9} \times 1\frac{5}{9}$
9. $4\frac{6}{7} \times \frac{10}{11}$	10. $3\frac{3}{7} \times 4\frac{1}{6}$
11. $1\frac{6}{9} \times 10\frac{2}{7}$	12. $5\frac{1}{2} \times 14\frac{1}{4}$
13. $2\frac{4}{5} \times \frac{7}{11}$	14. $13\frac{5}{11} \times 6\frac{1}{4}$

15. $14\frac{2}{9} \times 12\frac{3}{11}$	16. $6\frac{12}{13} \times 3\frac{3}{13}$
17. $13\frac{8}{10} \times 15\frac{9}{13}$	18. $4\frac{1}{4} \times 10\frac{3}{9}$

Solve for each of the given problems.
Write the answer in lowest terms.

1. $2 \div 3\frac{2}{4}$	2. $\frac{1}{2} \div \frac{1}{2}$
3. $1\frac{2}{5} \div 2\frac{5}{8}$	4. $\frac{4}{5} \div 2\frac{4}{5}$
5. $3\frac{1}{5} \div 2\frac{6}{7}$	6. $\frac{6}{8} \div \frac{1}{9}$
7. $6\frac{6}{7} \div 11\frac{4}{6}$	8. $7\frac{3}{8} \div 2\frac{1}{2}$
9. $9\frac{4}{11} \div 5\frac{3}{9}$	10. $14\frac{2}{3} \div 13\frac{7}{9}$
11. $15\frac{6}{10} \div 11\frac{1}{6}$	12. $10\frac{4}{8} \div 9\frac{3}{4}$

13. $6\frac{9}{13} \div 2\frac{4}{9}$	**14.** $12\frac{12}{13} \div 5\frac{1}{11}$
15. $\frac{5}{8} \div 17$	**16.** $15\frac{4}{12} \div 3\frac{9}{13}$
17. $18\frac{3}{9} \div 14\frac{1}{2}$	**18.** $13\frac{1}{15} \div 18\frac{12}{15}$

Decimals

The numerals we use today are called ***decimal*** numerals. These numerals stand for the numbers in the decimal system. The decimal system is also known as the Arabic system. The decimal system was first created by Hindu astronomers in India over a thousand years ago. It spread into Europe around 700 years ago.

The ***decimal system*** uses ten symbols: ***0, 1, 2, 3, 4, 5, 6, 7, 8,*** and ***9***. The word "decimal" comes from the Latin root ***decem***, meaning "ten."

Comparing Decimals

Comparing decimals uses an important mathematical concept. You can add zeros to the right of the last decimal digit without changing the value of the number. Study these examples.

RULE When comparing decimals with the same number of decimal places, compare them as though they were whole numbers.

Example Which is greater, 0.364 or 0.329?
Both numbers have three decimal places.
Since 364 is greater than 329, the decimal
0.364 > 0.329.

The rule for comparing whole numbers in which the number with more digits is greater does not hold true for decimals. The decimal number with more decimal places is not necessarily the greater number.

RULE When decimals have a different number of digits, write zeros to the right of the decimal with fewer digits so the numbers have the same number of decimal places. Then compare.

Example Which is greater, 0.518 or 0.52?
Add a zero to 0.52.
Since 520 > 518, the decimal **0.52 > 0.518.**

RULE When numbers have both whole number and decimal parts, compare the whole numbers first.

Example 1 Compare 32.001 and 31.999.
Since 32 is greater than 31, the number **32.001 is greater than 31.999.** It does not matter that 0.999 is greater than 0.001.

Using the same rules, you can put several numbers in order according to value. When you have several numbers to compare, write the numbers in a column and line up the decimal points. Then add zeros to the right until all the decimals have the same number of decimal digits.

Example 2 A digital scale displays weight to thousandths of a pound.
Three packages weigh 0.094 pound, 0.91 pound, and 0.1 pound. Arrange the weights in order from greatest to least.

Step 1	Write the weights in a column, aligning the decimal point.	0.094 0.91**0**
Step 2	Add zeros to fill out the columns.	0.1**00**
Step 3	Compare as you would whole numbers.	

In order from greatest to least, the weights are **0.91, 0.1,** and **0.094 pound.**

Equivalent Decimals

Decimals that name the same number or amount

Example:

0.5 = 0.50 = 0.500

Practice Exercise

Compare the given decimals.

1.	0.2	>	0.02
2.	0.969	________	0.061
3.	4295.9	________	4292.2
4.	0.926	________	9.26
5.	0.32	________	0.041
6.	0.62	________	2.6
7.	0.11	________	0.74
8.	9.24	________	0.05
9.	6364.9	________	6364.8
10.	691142.763	________	691142.763
11.	0.996	________	0.088
12.	0.647	________	0.647
13.	0.71	________	0.077

14.	587	________	0.587
15.	0.439	________	43.9
16.	341165.051	________	341165.053
17.	1392296.740	________	1329296.74
18.	1874820.8	________	1874820.93
19.	975527.087	________	975527.04

Decimals and Place Value

Decimal

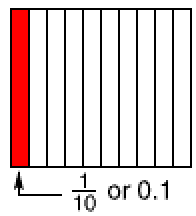
A number that uses place value and a decimal point to show values less than one, such as tenths and hundredths

Example:
3.47

	hundreds	tens	ones	Decimal point	tenths	hundredths	thousandths
$10\frac{1}{10}$		1	0	.	1		
$205\frac{3}{100}$	2	0	5	.	0	3	
$4\frac{9}{1000}$			4	.	0	0	9

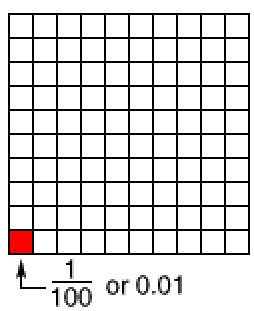
Tenth

One of ten equal parts
Example:



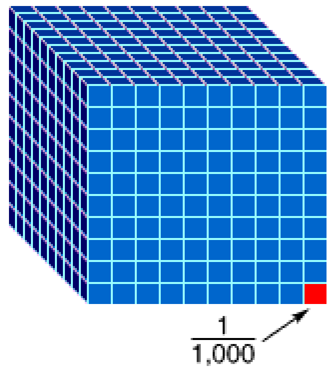
Hundredth

One of one hundred equal parts
Example:



Thousandth

One part of 1,000 equal parts
Example:



How do you write 16.034 in words?

Read the whole number part of the number. Say *and* to represent the decimal point. Read the digits to the right of the decimal point, and say the place name of the last digit on the right. Note that there are no commas setting off groups of three digits in the decimal part of the number to the right of the decimal point.

The number 16.034 is read ***sixteen and thirty-four thousandths.***

Þ **Be careful!!! Although most Canadians and Americans recognize the "." as a decimal point, the decimal point is expressed as a comma in many countries. Most French Canadians use the comma to represent the decimal point.**

Decimal Fractions and Decimal Numbers

Decimal fractions or ***decimals*** are fractions with denominators of ***10***, ***100***, ***1,000***, ***10,000***, and so on.

Decimal fractions are written using a decimal point:

$$\frac{1}{10} = .1 \qquad \frac{1}{100} = .01 \qquad \frac{1}{1000} = .001$$

Changing a Fraction to a Decimal

Any fraction can be written as a decimal by dividing the numerator by the denominator, and adding a decimal point in the correct place.

$$\frac{1}{10} = 10\,\overline{)1.0}^{\,.1} \qquad \frac{3}{5} = 5\,\overline{)3.0}^{\,.6} \qquad \frac{1}{4} = 4\,\overline{)1.00}^{\,.25}$$

Þ ***In decimal notation, a decimal point distinguishes whole numbers from decimal fractions:***

$$1 = 1.0$$
$$\frac{1}{10} = 0.1$$
$$1\frac{1}{10} = 1.1$$

Changing Decimals to Fractions

Both decimals and fractions can be used to show part of a whole. Sometimes it is easier to calculate using fractions. At other times, decimals are more useful. If you know how to change from one form to the other, you can solve any problem using the form that is best for the situation.

Example Change 0.375 to a fraction.

Step 1 Write the number without the decimal point as the numerator of the fraction.

$$0.375 = \frac{375}{?}$$

Step 2 Write the place value for the last decimal digit as the denominator.

$$0.375 = \frac{375}{1000}$$

Step 3 Reduce the fraction to lowest terms.

$$\frac{375 \div 125}{1000 \div 125} = \frac{3}{8}$$

The decimal 0.375 is equal to the fraction **3/8.**

Practice Exercise

Write each fraction in decimal format.

1. $\frac{94}{100}$ = **0.94**	2. $\frac{6}{10}$ = ____	3. $\frac{8}{100}$ = ____
4. $\frac{43}{0}$ = ____	5. $\frac{3}{4}$ = ____	6. $\frac{292}{400}$ = ____
7. $\frac{73}{20}$ = ____	8. $\frac{34}{5}$ = ____	9. $\frac{36}{40}$ = ____
10. $\frac{3}{30}$ = ____	11. $\frac{1}{5}$ = ____	12. $\frac{40}{25}$ = ____
13. $\frac{36}{45}$ = ____	14. $\frac{2}{4}$ = ____	15. $\frac{33}{15}$ = ____
16. $\frac{590}{100}$ = ____	17. $\frac{15}{20}$ = ____	18. $\frac{2}{5}$ = ____
19. $\frac{11}{55}$ = ____	20. $\frac{84}{50}$ = ____	21. $\frac{136}{20}$ = ____

Write each decimal as a fraction or mixed number in lowest terms.

1. 0.78 = __________
2. 0.31 = __________
3. 0.32 = __________
4. 0.8 = __________
5. 0.25 = __________
6. 0.97 = __________
7. 0.44 = __________
8. 0.99 = __________
9. 0.75 = __________
10. 0.84 = __________
11. 50.26 = __________
12. 28.9 = __________
13. 33.4 = __________
14. 84.75 = __________
15. 45.75 = __________
16. 85.75 = __________

Repeating Decimals

When ¼ is written as a decimal, the process is exact. The quotient is exactly .25 and the remainder is 0. This type of decimal is called a ***terminating decimal***.

Some fractions, when written as a division sentence, never reach a final digit. For example:

```
1 =          .333
3    3  / 1.000
          - 9
            10
          -  9
             10
```

Since the pattern in the quotient repeats, we write 1/3 as $.\overline{3}$ or .3…to show that the pattern continues forever.

Adding decimals is easy.

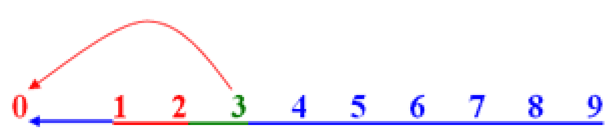
First, align the decimal points of the decimals. Then treat decimal fractions like whole numbers, aligning the decimal point in the sum. Adding decimals may look familiar---it's just like adding money.

```
     align decimal points
  1
  6.80
 +8.25
 15.05
     align decimal in sum
```

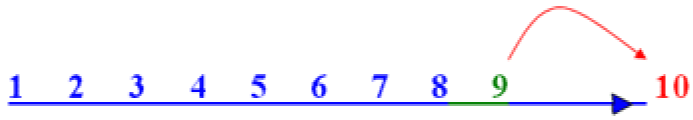
Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. You can estimate when you want to know if you have enough cash to pick up the three things you want at the grocery store or about how much each person should contribute to split the cost of lunch. In such cases, you can use amounts rounded to the nearest dollar (the ones place).

Rounding means to express a number to the nearest given place. The number in the given place is increased by one if the digit to its right is 5 or greater. The number in the given place remains the same if the digit to its right is less than 5. When rounding whole numbers, the digits to the right of the given place become zeros (digits to the left remain the same). When rounding decimal numbers, the digits to the right of the given place are dropped (digits to the left remain the same).

If you are rounding 3 to the nearest tens place, you would round down to 0, because 3 is closer to 0 than it is to 10.



If you were rounding 9, you would round up to 10.



General Rule for Rounding to the Nearest 10, 100, 1,000, and Higher!

Round down from numbers under 5 and round up from numbers 5 and greater.

The same holds true for multiples of 10. Round to the nearest 100 by rounding down from 49 or less and up from 50 or greater. Round to the nearest 1,000 by rounding down from 499 or less and up from 500 or greater.

Example Using the following price list, about how much would Pat pay for a steering wheel cover, a wide-angle mirror, and an oil drip pan?

Auto Parts Price List	
Outside Wide-Angle Mirror	$13.45
Steering Wheel Cover	$15.95
Oil Drip Pan	$ 8.73
Windshield Washer Fluid	$ 2.85
Brake Fluid	$ 6.35

Round the cost of each item to the nearest dollar and find the total of the estimates.

Item	**Cost**	**Estimate**
Steering wheel cover	$15.95	$16
Wide-angle mirror	13.45	13
Oil drip pan	+ 8.73	+ 9
Total:	$38.13	$38

The best estimate is **$38** which is close to the actual cost of **$38.13.**

The steps for rounding decimals are similar to those you use for rounding whole numbers. The most important difference is that once you have rounded off your number, you must ***drop the remaining digits.***

Example Round 5.362 to the nearest tenth.

Step 1 Find the digit you want to round to.
It may help to circle, underline, or highlight it.

5.362

Step 2 Look at the digit immediately to the right of the highlighted digit.

5.3<u>6</u>2

Step 3 If the digit to the right is 5 or more, add 1 to the highlighted digit. If the digit to the right is less than 5, do not change the highlighted digit. ***Drop the remaining digits.***

5.4

Examples Round 1.832 to the nearest hundredth.

1.832 rounds to 1.83

Round 16.95 to the nearest tenth.

16.95 rounds to 17.0

Round 3.972 to the ones place.

3.972 rounds to 4

Practice Exercise

Round to the nearest dollar:

(1) $15.42 (2) $22.08 (3) $26.89

(4) $22.31 (5) $19.32 (6) $28.69

Round to tenths:

(7) 5.86 (8) 17.04 (9) 86.47

(10) 1.28 (11) 58.95 (12) 786.22

Round to hundredths:

(13) 0.055 (14) 1.202 (15) 12.566

(16) 0.879 (17) 1.79 (18) 79.316

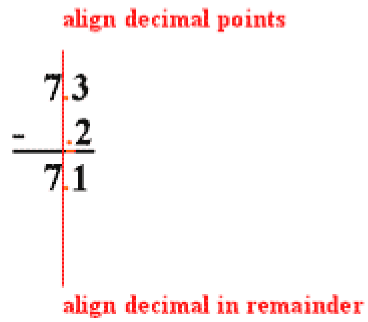
Round to thousandths:

(19) 0.9148 (20) 0.5045 (21) 0.4783

(22) 0.8762 (23) 0.0385 (24) 3.0629

Subtracting decimals is easy.

First, align the decimal points of the decimals. Then treat decimal fractions like whole numbers, aligning the decimal point in the remainder. Subtracting decimals may look familiar---it’s just like subtracting money.



To subtract decimals, if necessary, use place-holding zeros. Note: Whole numbers are understood to have a decimal point to the right of the ones place.

$$12 - 4.08 = \begin{array}{r} 12.00 \\ \underline{-4.08} \\ 7.92 \end{array}$$

Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. In such cases, you can use amounts rounded to the nearest dollar (the ones place).

Example Susan has $213 in a checking account. If she writes a check for $32.60, <u>about</u> how much will be left in the account?

Round the amount of the check off to the nearest dollar and find the difference.

$$\begin{array}{r} \$213.00 \\ \underline{-\quad \$\ 32.60} \\ \$180.40 \end{array} \qquad \begin{array}{r} \$213 \\ \underline{-\quad \$\ 33} \\ \$180 \end{array}$$

The best estimate is **$180** which is close to the actual amount of **$180.40.**

Practice Exercise

1. **27.811 + 21.023 = ______**
2. **69.9 - 62.03 = ______**
3. **32.11 + 12 = ______**
4. **17 - 15.02 = ______**
5. **92.49 - 47.3 = ______**
6. **30.038 + 11.669 = ______**
7. **42 - 7.28 = ______**
8. **12.566 + 15.4 = ______**
9. **62.01 - 13.083 = ______**
10. **30.05 + 9.531 = ______**
11. **(15 + 20.2) + 21.97 = ____**
12. **80.02 - 43.083 = ______**
13. **(16.794 + 1) + 21.8 = ____**
14. **21.007 + 26.28 = ______**
15. **63.029 - 61.52 = ______**
16. **32.528 - 21.02 = ______**
17. **27.6 + 31 = ______**
18. **59.009 - 49.65 = ______**
19. **76.08 - 21.092 = ______**
20. **25 + 13.3 + 30 = ______**
21. **5.078 + 13.075 = ______**
22. **(31.6 + 25.33) + 50 = _____**
23. **97 - 58.8 = ______**
24. **49.09 - 40.56 = ______**
25. **30.028 + 24.485 = ______**
26. **(7.6 + 2.054) + 19.5 = _____**
27. **63 - 47.798 = ______**
28. **60.048 - 37.06 = ______**
29. **40.07 - 37.9 = ______**
30. **16 + 14.642 = ______**
31. **18.55 + 29.66 + 8.04 = _____**
32. **6 - 3.895 = ______**
33. **45.2 - 41.049 = ______**
34. **26.7 + 5 = ______**
35. **99.63 - 6.75 = ______**
36. **11.058 + 29.09 + 5 = _____**
37. **69.03 - 40 = ______**
38. **34.92 + 12.45 = ______**

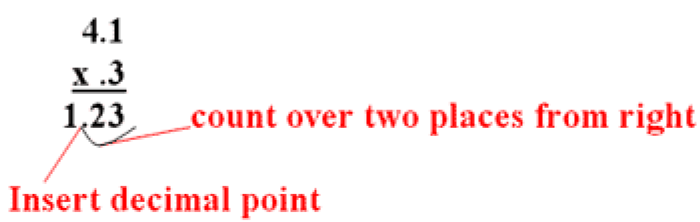
To multiply decimals, treat them as if they were whole numbers, at first ignoring the decimal point.

4.1
x .3
123

Next, count the number of places to the right of the decimal point in the multiplicand. Add this to the number of places to the right of the decimal point in the multiplier.

4.1	**multiplicand ------------**	**one place**
x .3	**multiplier ---------------**	**+one place**
		two places

Last, insert the decimal point in the product by counting over from the right the appropriate number of places.



Here are two other examples:

8.9	**65.003**
x 1.0	**x .025**
0 0	**325015**
8 9 0	**1300060**
8. 9 0	**1.625075**

Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. In such cases, round each factor to its greatest place. Then multiply.

Example Richard earns $7.90 per hour and works 38.5 hours each week. How much are his total earnings per week?

Round each factor to its greatest place and multiply.

38.5	40 hours
$7.90	$8 per hour
3950	$320 weekly wages, estimate
6320	
2370	
$304.150	

The best estimate is **$320** which is close to the actual solution of **$304.15.**

Multiplying Decimals by 10, 100, and 1,000

There are shortcuts you can use when multiplying decimals by 10, 100, and 1,000.

To multiply a decimal by 10, move the decimal point **one place to the right.**

Example .26 x 10

.26 x 10 = 2.6 = 2.6

To multiply a decimal by 100, move the decimal point **two places to the right.**

Example 3.7 x 100

3.7 x 100 = 3 70 = 370

To multiply a decimal by 1,000, move the decimal point **three places to the right.**

Example 1.4 x 1,000

1.4 x 1,000 = 1 400 = 1,400

Begin dividing decimals the same way you would divide whole numbers.

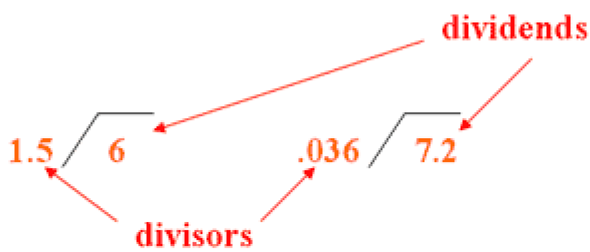
If the number in a division box (the dividend) has a decimal, but the number outside of the division box (the divisor) does not have a decimal, place the decimal point in the quotient (the answer) directly above the decimal point in the division box.

$$5\overset{0.002}{\overline{)0.010}}$$

If both the numbers inside and outside of the division box have decimals, count how many places are needed to move the decimal point outside of the division box (the divisor) to make it a whole number. Move the decimal point in the number inside of the division box (the dividend) the same number of places. Place the decimal point in the quotient (the answer) directly above the new decimal point.

$$0.05\overline{)0.01} = 5\overline{)1} = 5\overset{0.2}{\overline{)1.0}}$$

If the number outside of the division box has a decimal, but the number inside of the division box does not, move the decimal place on the outside number however many places needed to make it a whole number. Then to the right of the number in the division box (a whole number with an "understood decimal" at the end) add as many zeros to match the number of places the decimal was moved on the outside number. Place the decimal point in the quotient directly above the new decimal place in the division box.



(Note that 6 = 6.0.)

Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. In such cases, round the divisor to its greatest place, and round the dividend so that it can be divided exactly by the rounded divisor. Then divide.

Example If a plane flew 2,419.2 miles in 6.3 hours, what was its average speed in miles per hour?

Round the divisor to its greatest place, round the dividend so that it can be divided exactly by the rounded divisor, and divide.

6.3	6 hours
2,419.2	2,400 miles

2,400 ÷ 6 = 400 miles per hour, estimate
2,419.2 ÷ 6.3 = 384 miles per hour

The best estimate is **400 miles per hour** which is close to the actual answer of **384 miles per hour.**

Dividing Decimals by 10, 100, and 1,000

There are shortcuts you can use when dividing decimals by 10, 100, and 1,000.

To divide a decimal by 10, move the decimal point **one place to the left.**

Example 7.2 ÷ 10

7.2 ÷ 10 = 7.2 = .72

To divide a decimal by 100, move the decimal point **two places to the left.**

Example 364 ÷ 100

364 ÷ 100 = 3.64 = 3.64

To divide a decimal by 1,000, move the decimal point **three places to the left.**

Example 25.3 ÷ 1,000

25.3 ÷ 1,000 = .025 3 = .0253

Practice Exercise

Solve each problem.

1.	**3.3** **× 0.7**	**2.**	**0.3** **× 5**	**3.**	**0.6** **× 4**	**4.**	**9.2** **× 0.6**

5.	**4.6 × 0.92**	**6.**	**9.5 × 0.02**	**7.**	**3.6 × 0.24**	**8.**	**5.2 × 0.79**
9.	**7.8 × 0.14**	**10.**	**3.9 × 0.03**	**11.**	**4.3 × 0.53**	**12.**	**8.3 × 0.39**
13.	**24.69 × 19.324**	**14.**	**36.27 × 36.866**	**15.**	**22.03 × 4.9**	**16.**	**35 × 31.041**
17.	**38.98 × 5.396**	**18.**	**14.13 × 4.937**	**19.**	**34.36 × 22.782**	**20.**	**45.73 × 38**

Solve each problem.

(1) $16 \overline{)10462.4}$	**(2)** $191 \overline{)4847.58}$	**(3)** $61 \overline{)388.57}$
(4) $3.5 \overline{)46.585}$	**(5)** $6.2 \overline{)13.95}$	**(6)** $0.97 \overline{)640.879}$
(7) $4.1 \overline{)197.497}$	**(8)** $6.32 \overline{)437.344}$	**(9)** $6.93 \overline{)11434.5}$
(10) $6 \overline{)573.72}$	**(11)** $6.6 \overline{)1008.48}$	**(12)** $0.86 \overline{)478.504}$
(13) $6.6 \overline{)563.178}$	**(14)** $0.41 \overline{)36.695}$	**(15)** $2.99 \overline{)75.946}$
(16) $3.9 \overline{)12.051}$	**(17)** $0.62 \overline{)39667.6}$	**(18)** $1.75 \overline{)11782.75}$
(19) $0.8 \overline{)31.968}$	**(20)** $7.2 \overline{)406.728}$	**(21)** $9.3 \overline{)64.728}$
(22) $6.1 \overline{)39814.7}$	**(23)** $0.74 \overline{)23.014}$	**(24)** $4.31 \overline{)15248.78}$

Percent

The term ***percent*** means ***parts per hundred***. Any fraction with a denominator of ***100*** can be written as a percentage, using a percent sign, ***%***. So, if you ate ½ of a pie, you ate **50/100** or ***.50*** or ***50%*** of the pie.

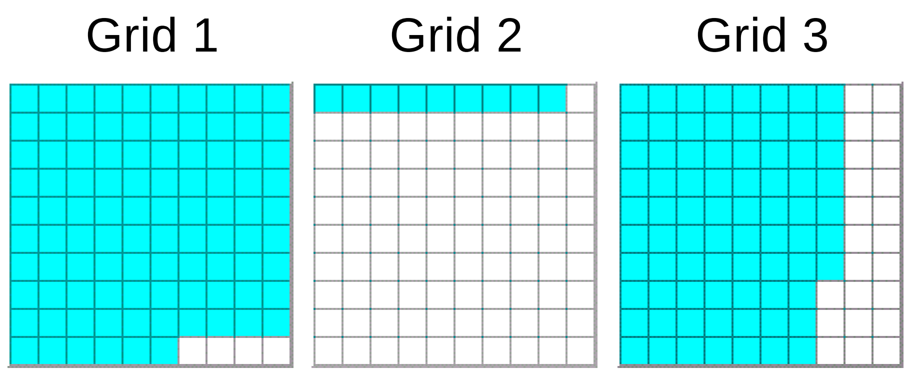
If you ate $\frac{1}{8}$ of the pie, you ate 12.5%

If you ate $\frac{3}{4}$ of the pie, you ate 75%

If you ate $\frac{1}{5}$ of the pie, you ate 20%

Þ **If you ate 1 whole pie, you ate 1.00 or 100% of the pie. You ate the whole thing!!!**

What fraction of each grid is shaded?



Each grid above has 100 boxes. For each grid, the ratio of the number of shaded boxes to the total number of boxes can be represented as a fraction.

Comparing Shaded Boxes to Total Boxes		
Grid	**Ratio**	**Fraction**
1	96 to 100	$\frac{96}{100}$
2	9 to 100	$\frac{9}{100}$
3	77 to 100	$\frac{77}{100}$

We can represent each of these fractions as a percent using the symbol %.

$\frac{96}{100}$ = 96% $\frac{9}{100}$ = 9% $\frac{77}{100}$ = 77%

Let's look at our comparison table again. This time the table includes percents.

Comparing Shaded Boxes to Total Boxes			
Grid	**Ratio**	**Fraction**	**Percent**
1	96 to 100	$\frac{96}{100}$	96%
2	9 to 100	$\frac{9}{100}$	9%
3	77 to 100	$\frac{77}{100}$	77%

It is easy to convert a fraction to a percent when its denominator is 100. If a fraction does not have a denominator of 100, you can convert it to an equivalent fraction with a denominator of 100, and then write the equivalent fraction as a percent.

Example 1: Write each fraction as a percent: $\frac{1}{2}, \frac{18}{20}, \frac{4}{5}$

Solution		
Fraction	**Equivalent Fraction**	**Percent**
$\frac{1}{2}$	$\frac{1 \times 50}{2 \times 50} = \frac{50}{100}$	50%
$\frac{18}{20}$	$\frac{18 \times 5}{20 \times 5} = \frac{90}{100}$	90%
$\frac{4}{5}$	$\frac{4 \times 20}{5 \times 20} = \frac{80}{100}$	80%

You may also change a fraction to a percentage by dividing the fraction.

$$\frac{2}{5} = 5\overline{)2.00}\quad .40$$

Then change the decimal to a fraction with ***100*** in the denominator.

$$.40 = \frac{40}{100} = 40\%$$

Unknown by *Busino*

To change a mixed number to a percent, change the mixed number to an improper fraction and multiply by 100.

Example Change 3 ¼ to a percent.

Step 1 Change the mixed number to an improper fraction.

$$3\,\frac{1}{4} = \frac{13}{4}$$

Step 2 Multiply by 100 and add the percent sign.

$$\frac{13}{4} \times 100$$

$$\frac{13}{\cancel{4}_{1}} \times \frac{\cancel{100}^{25}}{1} = 325\%$$

To change a percentage to a fraction or mixed number, write the percent as a fraction with a denominator of 100. Be sure to write the fraction in its lowest possible terms.

$$4\% = \frac{4}{100} = \frac{1}{25} \qquad 13\% = \frac{13}{100}$$

$$150\% = \frac{150}{100} = \frac{3}{2} = 1\,\frac{1}{2}$$

Converting percents with fraction parts requires extra steps.

Example Change 41 % to a fraction.

Write 41 2/3 over 100 and divide.

$$\frac{41\ 2/3}{100} = 41\ 2/3 \div 100 = \frac{\overset{5}{\cancel{125}}}{3} \times \frac{1}{\underset{4}{\cancel{100}}} = \frac{5}{12}$$

The percent 41 2/3% is equal to the fraction 5/12.

Writing Decimals as Percents

Problem:	What percent of a dollar is 76 cents?
	76 cents = .76
	.76 = 76%
Solution:	76 cents is 76% of a dollar.

The solution to the above problem should not be surprising, since both dollars and percents are based on the number 100. As a result, there is nothing complicated about converting a decimal to a percent. **To convert a decimal to a percent, move the decimal point two places to the right.** Look at the example below:

Example 1 Write each decimal as a percent:
.93, .08, .67, .41

Solution	
Decimal	**Percent**
.93	93%
.08	8%
.67	67%
.41	41%

Each of the decimals in Example 1 has two places to the right of the decimal point. However, a decimal can have any number of places to the right of the decimal point. Look at Example 2 and Example 3:

Example 2 Write each decimal as a percent:
.786, .002, .059, .8719

Solution	
Decimal	**Percent**
.786	78.6%
.002	.2%
0.59	5.9%
.8719	87.19%

Example 3 Write each decimal as a percent:
.1958, .007, .05623, .071362

Solution	
Decimal	**Percent**
.1958	19.58%
.007	.7%
.05623	5.623%
.071362	7.1362%

Writing Percents as Decimals

Problem: What is 35 percent of one dollar?

We know from the previous lesson that .35 = 35%. The word "of" means multiply. So we get the following:

35% x $1.00 = .35 x $1.00

.35 x $1.00 = .35 x 1 = .35

Solution: 35% of one dollar is $.35, or 35 cents.

The solution to the problem on page 86 should not be surprising, since percents, dollars and cents are all based on the number 100. **To convert a percent to a decimal, move the decimal point two places to the left.** Look at the example below:

Example 1 Write each percent as a decimal:
18%, 7%, 82%, 55%

Solution	
Percent	**Decimal**
18%	.18
7%	.07
82%	.82
55%	.55

In Example 1, note that for 7%, we needed to add in a zero. **To write a percent as a decimal, follow these steps:**

- Drop the percent symbol.
- Move the decimal point two places to the left, adding in zeros as needed.

Why do we move the decimal point 2 places to the left? Remember that percent means parts per hundred, so 18% equals $\frac{18}{100}$. From your knowledge of decimal place value, you know that $\frac{18}{100}$ equals eighteen hundredths (.18). So 18% must also equal eighteen hundredths (.18). In Example 2 below, we take another look at Example 1, this time including the fractional equivalents.

Example 2 Write each percent as a decimal:
18%, 7%, 82%, 55%

Solution		
Percent	**Fraction**	**Decimal**
18%	$\frac{18}{100}$	.18
7%	$\frac{7}{100}$	.07
82%	$\frac{82}{100}$	.82
55%	$\frac{55}{100}$	.55

Let's look at some more examples of writing percents as decimals.

Example 3 Write each percent as a decimal:
12.5%, 89.19%, 39.2%, 71.935%

Solution	
Percent	**Decimal**
12.5%	.125
89.19%	.8919
39.2%	.392
71.935%	.71935

Þ **To remember which way to move the decimal point when changing from a decimal to a percent or vice versa, think of your alphabet. Think of the decimal as “d” and the percent as “p”. To change from a decimal to a percent, move two places up your alphabet. Move two places down your alphabet to go from a percent to a decimal.**

Converting percents with fraction parts to decimals requires extra steps.

Example Change 15 ¼% to a decimal.
Change ¼ to a decimal.

$$\frac{1 \times 25}{4 \times 25} = \frac{25}{100}$$

$$\frac{25}{100} = .25$$

Combine the decimal with the original whole number part of the percent and then convert the percent to a decimal by moving the decimal point two places to the left.

15 ¼% = 15.25% = .1525

The percent 15 ¼% is equal to the decimal .1525.

Practice Exercise

Write each fraction as a percent.

1. $\frac{9}{10}$ = **9%**	2. $\frac{10}{100}$ = ____	3. $\frac{2}{10}$ = ____
4. $\frac{11}{20}$ = ____	5. $\frac{6}{50}$ = ____	6. $\frac{63}{90}$ = ____
7. $\frac{32}{100}$ = ____	8. $\frac{3}{20}$ = ____	9. $\frac{4}{5}$ = ____
10. $\frac{3}{6}$ = ____	11. $\frac{9}{15}$ = ____	12. $\frac{27}{90}$ = ____
13. $\frac{35}{50}$ = ____	14. $\frac{2}{5}$ = ____	15. $\frac{8}{20}$ = ____
16. $\frac{1}{5}$ = ____	17. $\frac{9}{20}$ = ____	18. $\frac{18}{25}$ = ____

19. $\frac{2}{4}$ = _____ 20. $\frac{36}{80}$ = _____ 21. $\frac{39}{50}$ = _____

Write each decimal as a percent.

1. 0.1 = **10%** 2. 0.6 = _____ 3. 0.96 = _____

4. 0.51 = _____ 5. 0.03 = _____ 6. 0.16 = _____

7. 0.58 = _____ 8. 0.93 = _____ 9. 0.26 = _____

10. 0.7 = _____ 11. 0.33 = _____ 12. 0.849 = _____

13. 0.35 = _____ 14. 0.37 = _____ 15. 0.07 = _____

16. 0.31 = _____ 17. 0.48 = _____ 18. 0.84 = _____

19. 0.73 = _____ 20. 0.06 = _____ 21. 0.94 = _____

22. 0.892 = _____ 23. 0.582 = _____ 24. 0.5 = _____

Write as a decimal

(1) 21% (2) 31% (3) $30\frac{1}{5}\%$

(4) 146.1% (5) 0.37% (6) 32%

(7) 16.58% (8) 272.2% (9) 17%

(10) $55\frac{1}{2}\%$ (11) 0.84% (12) 28%

Write as a fraction

(13) 144.7% (14) $2\frac{1}{4}\%$ (15) 19.96%

(16) 248.2% (17) 13.88% (18) 121%

(19) 40% (20) $97\frac{1}{2}\%$ (21) 85%

(22) 68% (23) 40.7% (24) 0.54%

Adding, subtracting, multiplying, and dividing percents follows the same procedure as whole numbers.

Examples **10% + 20% = 30%**
40% - 30% = 10%
50% x 60% = 3000%
80% ¸ 20% = 4%

Þ **You should know that, although it is possible to add and subtract percents, an initial discount of 15%, for example, followed by an additional discount of 25% does NOT create a 40% reduction in price but rather a discount of 36.25%.**

Example June was about to purchase a $250 coat at a 15% discount when the cashier told her that the store was having a special promotion. All June had to do was stick her hand into a box and pull out a circular disc. Whatever number was on the disc would be discounted from the price of the coat that she was going to buy. June put her hand in and pulled out a circle with "25%" marked on it. How much did June end up paying for the coat?

Step 1 Calculate the original amount to be discounted off the original price.

$250 x 15% =
$250 x .15 = $37.50

Step 2 Subtract the original amount to be discounted from the original price.

$$\begin{array}{r} \$250.00 \\ - \ \$\ \ 37.50 \\ \hline \$212.50 \end{array}$$

Step 3 Take the new discounted price and calculate how much more is to be taken off the discounted price of the coat.

$212.50 x 25% =
$212.50 x .25 = $53.125 = $53.13 (rounded to the nearest cent)

Step 4 Subtract the new amount from the discounted price.

$$\begin{array}{r} \$212.50 \\ - \ \$\ \ 53.13 \\ \hline \$159.37 \end{array}$$

June may now purchase the $250 coat for **$159.37.**

If the original discount had been 40% (15% + 25%), we could have multiplied $250 by .40 and the price would have been brought down by $100 to $150. At least, June can feel fortunate that she received her coat at a discount.

Problem Solving with Fractions, Decimals, and Percents

Using the Substitution Method

So far, you have solved addition, subtraction, multiplication, and division word problems using whole numbers. Many students can do these word problems with ease, but they worry when they see word problems using large whole numbers, fractions, or decimals.

The difficulty has to do with "math intuition," or the feel that a person has for numbers. You have a very clear idea of the correct answer to 4 – 3. It is more difficult to picture 7,483,251 + 29,983 or 6.45 – 5.5. And for most of us, our intuition totally breaks down for 3/8 – 1/3.

Changing only the numbers in a word problem does not change what must be done to solve the problem. By substituting small whole numbers in a problem, you can understand the problem and how to solve it.

Look at the following example:

Example: A floor is to be covered with a layer of ¾-in. fiberboard and 7/16-in. plywood. By how much will the floor level be raised?

Fractions, especially those with different denominators, are especially hard to picture. You can make the problem easier to understand by substituting small whole numbers for the fractions. You can substitute any numbers, but try to use numbers under 10. These numbers do not have to look like the numbers they are replacing.

In the example, try substituting 3 for ¾ and 2 for 7/16. The problem now looks like this:

> A floor is to be covered by a layer of 3-in. fiberboard and 2-in. plywood. By how much will the floor level be raised?

You can now read this problem and know that you must add.

Once you make your decision about *how* to solve the problem, you can return the original numbers to the word problem and work out the solution. With the substituted numbers, you decided to *add* 3 and 2. Therefore, in the original, you must *add* ¾ and 7/16.

$$\frac{3}{4} = \frac{12}{16}$$

$$+\frac{7}{16} = \frac{7}{16}$$

$$\frac{19}{16} = \mathbf{1\frac{3}{16}}$$

Remember: Choosing 3 and 2 was completely up to you. You could have used any small whole numbers.

Practice Exercise

1. Larry spent 2/3 of his allowance on movies and ¼ on snack foods. What fraction of his allowance did he spend on these items altogether?
2. Ms. Thomas fertilizes her lawn 3 times in a year. She uses 11 bags of fertilizer in all. How many bags does she use each time?
3. Sandra swam 5 ½ lengths of the pool in the same time as Alice swam 3 ¾ lengths. How many more lengths did Sandra swim?
4. At the L.P. Fisher Library, 128 people signed out books in one day. Of the people, 1/8 were children and ½ were women. How many men signed out books?
5. Peggy cut 10 sheets of Bristol board into eighths. How many pieces does she have?
6. The product of two numbers is .52206. One of the numbers is 33. What is the other number?
7. If 21% of the instruments need repair and 11% need to be replaced, what percent of the instruments are in satisfactory condition?

8. Shelly's basketball team won 12 games and lost 8 games. What percent of its games did Shelly's team win?
9. In a school election, Jake received 28% of the votes, Joan received 46%, and Louise received the remaining votes. What fraction of the vote did Louise capture?
10. In a recent year, 66 2/3% of the Canadian population spoke only English, 20% spoke only French, 13% spoke English and French, and the rest spoke neither English nor French. What fraction of the Canadian population spoke neither English nor French?

Ratio, Proportion, and Percent

Introduction to Ratio, Proportion and Percent

To find a percentage of a number, multiply the number by the percentage written in its decimal fraction form. Find 25% of 12.

.25 x 12 = 3

To find what percentage one number is of another, write the numbers as a fraction. Divide the fraction into its decimal form. Then change the decimal into its percentage form. ***12*** is what percent of ***48***?

$$\frac{12}{48} \text{ or } 48\overline{)12.00}\ \ .25 = 25\%$$

To find a number when a percentage of it is known, try this:

Nine is ***25%*** of what number?

$$\frac{25}{100} = \frac{9}{?}$$

25 x ? = 100 x 9
25 x ? = 900
? = 900 ÷ 25
? = 36

Nine is ***25%*** of 36.

Some people like to use a formula to find the percent of a number, what percent one number is of another, or a number when a percent is given. The formula looks like this:

$$\frac{r}{100} = \frac{P}{W}$$

r **= percent rate**
P **= part of the number**
W **= the whole (entire) number**

So, to solve the problem, nine is ***25%*** of what number, we would follow these steps.

Step 1 Write down the formula.

$$\frac{r}{100} = \frac{P}{W}$$

Step 2 Insert the necessary information in the correct places.

$$\frac{25}{100} = \frac{9}{?}$$

Step 3 Cross multiply.

$$25 \times ? = 9 \times 100$$
$$25 \times ? = 900$$

Step 4 Divide and solve.

$$? = 900 \div 25$$
$$? = 36$$

Therefore, nine is ***25%*** of 36.

Practice Exercise

1. 195 is 39% of what number?
2. What % of 37000 is 44.4?
3. What is 0.64% of 38000?
4. What is 12 ½% of 95?
5. What % of 2200 is 1100?
6. What is 4% of 58?
7. 313.5 is 0.95% of what number?
8. What is 8% of 3000?
9. What % of 27000 is 102.6?
10. What is 18 ½% of 43?
11. What is 7% of 50?
12. What % of 2400 is 1608?
13. 3139 is 73% of what number?
14. What is 72% of 2900?
15. 52.9 is 0.23% of what number?
16. What is 0.43% of 8000?
17. What is 3 ½% of 35?
18. What % of 41000 is 28.7?
19. What is 21 ½% of 61?
20. What is 0.71% of 16000?
21. What % of 820 is 123?
22. 2162 is 46% of what number?
23. 158.4 is 0.66% of what number?
24. What is 4% of 4600?
25. 273 is 0.65% of what number?
26. What % of 27000 is 59.4?
27. What is 47% of 4400?
28. What is 0.84% of 23000?

Ratios describe the size of things in comparison to each other. Ratios are sometimes written in the form of fractions. More often, the symbol : is used to separate the numerator and the denominator.

For example, if you ate ***2*** parts of a pie that had been cut into ***5*** parts, the ratio of pieces of pie you ate to the uneaten pieces of pie is ***2 to 3***. The ratio may be written as ***2:3*** or $\frac{2}{3}$.

Writing the ratio in words will help you keep the numbers in the correct order. The words will also help you remember the meaning of the numbers. Including labels in your final ratio is also helpful.

Example If Barbara earns $180 in 15 hours, how much does she earn per hour?

Write the ratio of earnings to hours. Then divide to simplify.

$$\frac{\text{dollars earned}}{\text{hours}} = \frac{\$180}{15} = \frac{180 \div 15}{15 \div 15} = \frac{\$12}{1\text{ hr}}$$

Barbara earns $12 for every 1 hour she works. In other words, she earns **$12 per hour.**

Reducing Ratios

Reducing a ratio means finding an equal, simplified version of the original. The ratio is reduced to lowest terms when there is no number other than 1 that will divide evenly into both of the numbers that make up the ratio.

To simplify a ratio, divide both of the numbers that make up the ratio by the same number, and write the new ratio.

Example In one hour, 10 customers visited Stuart's newsstand. Of those, 4 bought a magazine. What is the ratio of those who bought a magazine to those that did not?

Write a ratio: bought magazines:didn't buy magazines

4:6

Simplify the ratio by dividing both of the numbers in the ratio by 2.

4:6 = 2:3

You may also represent the ratios as fractions when simplifying.

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

$$\frac{2}{3} = 2:3$$

Calculating Equal Ratios

If one cherry pie is baked for every 4 apple pies, the ratio is 1:4, or ¼.
If the number of apple pies is increased to 12, how many cherry pies are needed to keep the same ratio?

To find the solution, write the ratios as an equation.

$$\frac{1}{4} = \frac{?}{12}$$

To solve, multiply (or divide) each term of the first ratio by the same number to make a true statement.

$$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$

1:4 and 3:12 are equal ratios.

You can also find the missing term by cross-multiplying and then dividing.

$$\frac{1}{4} \times \frac{?}{12} \qquad 1 \times 12 = 4 \times ?$$

$$\frac{12}{4} = \frac{4}{4} \times ?$$

$$3 = ?$$

When ratios are equal or equivalent, they are said to be **proportional** and can be referred to as **true proportions**. When ratios are not equal, they are said to be non-equivalent or disproportionate.

The **terms** of a proportion are:

$$\textbf{first} \rightarrow \frac{a}{b} = \frac{c}{d} \leftarrow \textbf{third}$$
$$\textbf{second} \rightarrow \quad \quad \leftarrow \textbf{fourth}$$

The **extremes** are the first and fourth terms of a proportion. The **means** are the second and third terms of a proportion.

Proportions are often written using a fraction bar that stands for "is compared to". For example, 2/3 = 6/9 means 2 compared to 3 is the same as 6 compared to 9.

A **direct proportion** is indicated when two quantities are so related that an increase in one causes a corresponding increase in the other or when a decrease in one causes a corresponding decrease in the other.

The following is a list of directly proportional relationships.

a. The faster the speed, the greater the distance covered.
b. The more people working, the greater amount of work done.
c. The slower the speed, the lower the number of revolutions.
d. The shorter the object, the shorter the shadow.

Practice Exercise

Mark True or False for Each of the Following

1. 4:6 = 20:30 T F
2. 7:3 = 14:6 T F
3. 18:40 = 6:20 T F
4. 14:36 = 7:18 T F
5. 8.4:6 = 28:20 T F

In each of the following proportions, solve for the unknown value.

1. $\frac{18}{15} = \frac{6}{?}$
2. $\frac{4}{12} = \frac{?}{24}$
3. $\frac{16}{10} = \frac{?}{5}$
4. $\frac{35}{?} = \frac{7}{9}$
5. $\frac{4}{?} = \frac{20}{45}$
6. $\frac{18}{36} = \frac{6}{?}$
7. $\frac{12}{21} = \frac{?}{49}$
8. $\frac{11}{?} = \frac{44}{8}$
9. $\frac{16}{?} = \frac{6}{9}$

10. $\frac{35}{?} = \frac{7}{12}$

11. $\frac{164}{?} = \frac{41}{18}$

12. $\frac{?}{72} = \frac{14}{16}$

13. $\frac{?}{84} = \frac{21}{98}$

14. $\frac{41}{19} = \frac{?}{38}$

15. $\frac{?}{28} = \frac{6}{14}$

16. $\frac{108.9}{47.7} = \frac{12.1}{?}$

17. $\frac{15}{20.9} = \frac{?}{125.4}$

18. $\frac{?}{21.7} = \frac{32.4}{130.2}$

19. $\frac{34}{33} = \frac{204}{?}$

20. $\frac{17}{10} = \frac{?}{20}$

21. $\frac{47}{46} = \frac{?}{230}$

Problem Solving with Percent, Ratio and Proportion

Identifying the Parts of and Solving a Percent Word Problem

Read the statement below:

The 8-ounce glass is 50% full. It contains 4 ounces.

This statement contains three facts:

the whole: the 8-ounce glass
the part: 4 ounces
the percent: 50%

A percent word problem would be missing one of these facts. When you are solving a percent word problem, the first step is to identify what you are looking for. As shown on page 159, you have three possible choices: *the part*, *the whole*, or *the percent*.

It is usually easiest to figure out that you are being asked to find the percent. Word problems asking for the percent usually ask for it directly, with a question such as "What is the percent?" or "Find the percent" or "3 is what percent?" Occasionally, other percent-type words are used, such as "What is the *interest rate*?"

Example 114 city employees were absent yesterday. This was 4% of the city work force. How many people work for the city?

Step 1: *question:* How many people work for the city?
Step 2: *necessary information:* 114 city employees, 4%
Step 3: You are given the number of city employees who were absent (114) and the percent of the work force that this represents (4%). You are looking for the total number of people who work for the city, the whole.

Once you identify what you are looking for in a percent word problem, set up the problem and solve it.

Percents are ratios: 15% means 15 out of 100. Percent word problems can be set up as a proportion by writing two ratios that are equal in the following form:

$$\frac{P}{W} = \frac{r}{100}$$

$$\frac{114}{?} = \frac{4}{100}$$

$$114 \times 100 = 4 \times ?$$

$$11400 = 4 \times ?$$

$$11400 \div 4 = ?$$

$$2850 = ?$$

2850 people work for the city.

Practice Exercise

(1) Jill worked 8.24 hours and received $70.04. What will Jill receive for working 9 hours?

(2) A six-pack of pop costs $11.40. What should it cost for 25 cans?

(3) If you want to enlarge a 4 inch wide by 9 inch long

photo to be 16 inches wide, how long will the length be?

(4) At the local market 5 bananas are $6. How much will 14 cost?

(5) If your purchase comes to $765.38, including tax for a $710 purchase, how much will the tax be on a $370 purchase?

(6) If your purchase comes to $140 + tax, how much do you owe?
Tax is 7% and the total should be rounded to the nearest penny.

(7) Paul, a salesman, sold a house for $112,000. Paul earns a 9% commission when selling a house. How much did Paul earn by selling this house?

(8) 11 ½% is the interest rate a bank pays. If you invest $8,800 for one year, how much interest will you earn?

(9) The final exam your teacher is giving has 150 questions. If you need 70% to pass, how many questions must you get correct?

(10) How much must be sold to earn $950.95, if the commission on sales is 13%?

Geometry

Geometry is the branch of mathematics that explains how ***points***, ***lines***, ***planes***, and ***shapes*** are related.

Lines and Angles

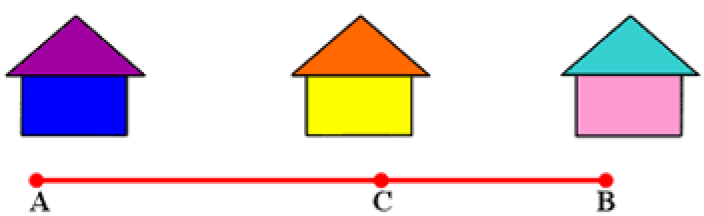
Points

Points have no size or dimensions, that is, no width, length, or height. They are an ***idea*** and cannot be seen. But, points are used to tell the position of lines and objects. Points are usually named with capital letters:

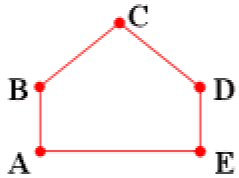
A, B, C, D and so on.

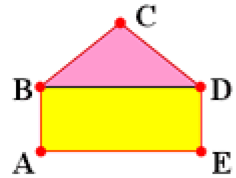
Points can describe where things begin or end.

Points can be used to measure distance.



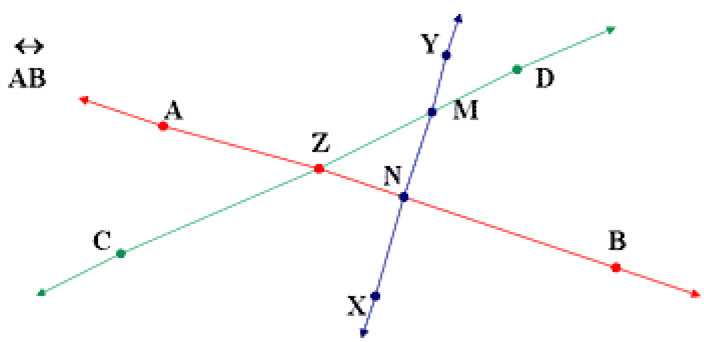
Points define the perimeter of shapes and objects.





Lines

Lines extend in opposite directions and go on without ending. Like points, lines have no volume, but they have infinite length. Lines are named by points with a line symbol written above.



Lines intersect at a point. Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ***Z***.

⇒ **Line $\overleftrightarrow{AB}$ can also be named $\overleftrightarrow{BA}$.**

Line Segments

Line segments are parts of lines defined by two endpoints along the line. They have *length*.

Line segments are named by their two points with the line segment symbol written above:

$\overline{AB}$ (or $\overline{BA}$)

An infinite number of line segments can be located along a line:

$\overline{AB}$ $\overline{AE}$ $\overline{AC}$ $\overline{AD}$ $\overline{AF}$ $\overline{EC}$ $\overline{ED}$ $\overline{EF}$ …

$\Longrightarrow$ ***Line segments of equal length are called congruent line segments.***

Rays

Rays are parts of lines that extend in one direction from one endpoint into infinity.

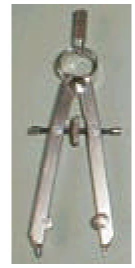
Rays are named by the endpoints and one other point with a ray symbol written above. The endpoint must always be named first.

$\overrightarrow{AB}$

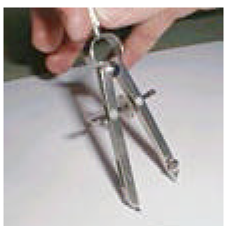
The Compass

The compass at left is a typical golf-pencil compass that seems to be preferred by many students. It is not recommended. The whole point of a compass is to draw an arc with constant radius. This model tends to slip easily. Friction is the only thing holding the radius. As it wears out, it becomes even looser. Also, the point is not very sharp, so it will not hold its position well when drawing. Two advantages are that it is easy to find and it is inexpensive.

The compass on the right is a much better design. The wheel in the center allows for fine adjustment of the radius, and it keeps the radius from slipping. It has a much heavier construction, and will not easily bend or break.



Keep the compass lead sharpened for a nice, fine curve. There are special sharpeners made just for the leads that fit the compass, but it is a simpler matter to carry a small piece of sandpaper. Stroke the lead across it a few of times to give the tip a bevel.



Hold the compass properly. Use one hand, and hold it by the handle at the top. Do not hold it by the limbs. If you do that, there will be a tendency to change the radius as you draw. This is especially a problem with the cheaper compasses that have no way of locking the radius. Tilt the compass back slightly, so that the lead is dragged across the page. If the compass is pushed toward the lead, it will cause the anchor point to lift up and slip out of position.

Do not be impatient with your work. When using a compass, there must be some well-defined point for the center point, such as the intersection of two lines. Center the compass precisely on that intersection. Depending on the complexity of the construction, small errors may be greatly magnified.

Parallel Lines

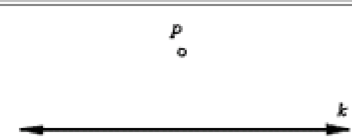
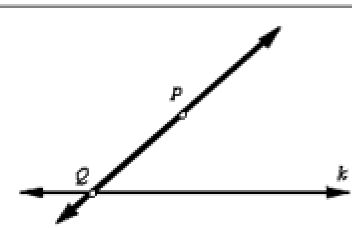
Parallel lines lie within the same plane and are always the same distance apart. Parallel lines continue to infinity without intersecting or touching at any point.

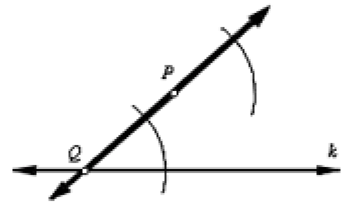
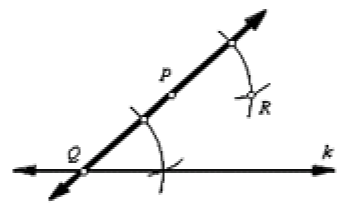
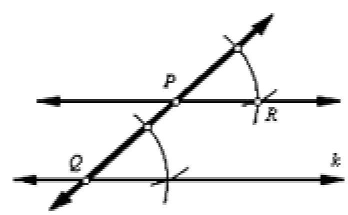
The symbol for parallel lines is $\parallel$ and is read "is parallel to."

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Constructing Parallel Lines

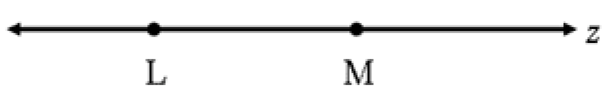
Given a line and a point, construct a line through the point, parallel to the given line.

1. Begin with point *P* and line *k*.	
2. Draw an arbitrary line through point *P*, intersecting line *k*. Call the intersection point *Q*. Now the	

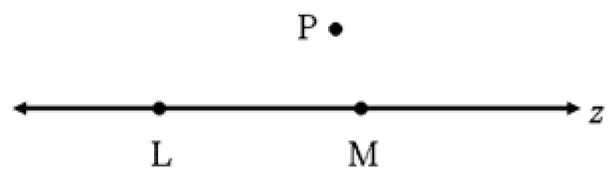
task is to construct an angle with vertex *P*, congruent to the angle of intersection.	
3. Center the compass at point *Q* and draw an arc intersecting both lines. Without changing the radius of the compass, center it at point *P* and draw another arc.	
4. Set the compass radius to the distance between the two intersection points of the first arc. Now center the compass at the point where the second arc intersects line *PQ*. Mark the arc intersection point *R*.	
5. Line *PR* is parallel to line *k*.	

A second method follows these steps:

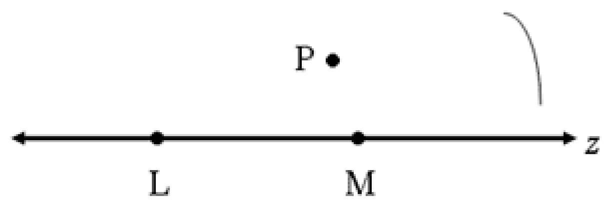
Step 1 On a given line *z*, create two points and label them.



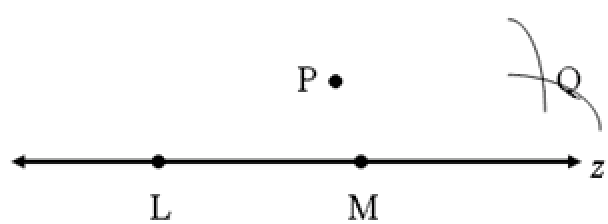
Step 2 Point P will be the point through which you will construct a line parallel to the given line *z*.



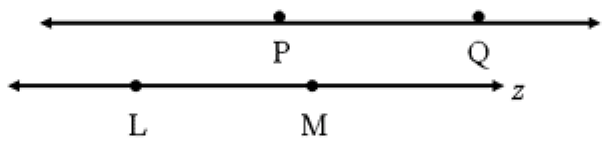
Step 3 Open compass to the length of LM. Put compass point at P and draw an arc.



Step 4 Open compass to the length of LP. Put compass point at M. Draw an arc to cut the previous arc. Label Q.



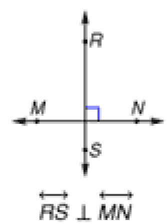
Step 5 Draw PQ. PQ || LM.



Perpendicular Lines

Lines that intersect to form 90° angles, or right angles

Example:



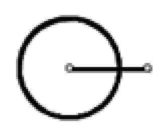
Read: Line *RS* is perpendicular to line *MN*

Construct the Perpendicular Bisector of a Line Segment

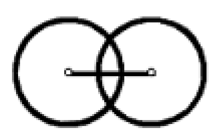
Definition: The *perpendicular bisector* of a segment is the line that is perpendicular (at a right angle) to the segment and goes through the midpoint of the segment.

Construction Steps

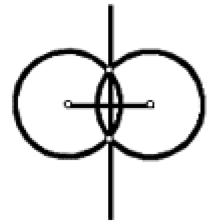
Use a compass to draw a circle whose center is one of the endpoints of the segment, and whose radius is more than half the length of the segment.



Draw another circle with the same radius, and center the other endpoint of the segment.



Draw the line through the two points where the circles intersect.



Note: **You don't have to draw the entire circle**, but just the arcs where the two circles intersect.

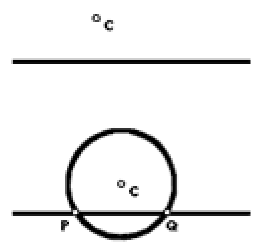
Construct the Perpendicular to a Line Through a Given Point

Given a line and a point, there is one and only one perpendicular to the line through the point.

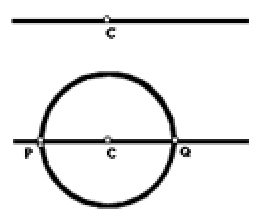
The main idea is to construct a line segment on the line, then construct the perpendicular bisector of this segment.

Construction Steps

A. If the point is not on the line, use a compass to draw a circle whose center is the given point, and whose radius is large enough so that the circle and line intersect in two points, P and Q.



B. If the point is on the line, draw a circle whose center is the given point; the circle and line intersect in two points, P and Q.

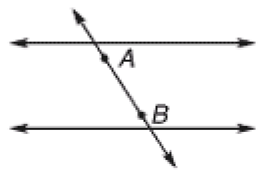


Construct the perpendicular bisector of segment PQ.

Lines that intersect but do not form 90° angles, or right angles, are simply called intersecting lines.

Transversal

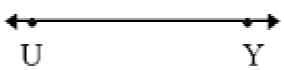
A line that intersects two or more lines
Example:
Line *AB* is a transversal.



Practice Exercise

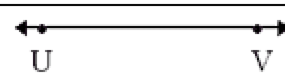
Name each figure.

1.



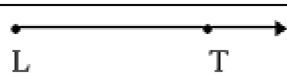
■ line UY
□ line segment UY
□ Ray UY
□ Ray YU

2.



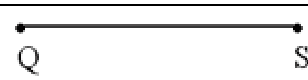
□ line UV
□ line segment UV
□ Ray UV
□ Ray VU

3.



□ line LT
□ line segment LT
□ Ray LT

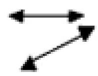
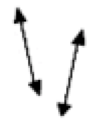
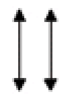
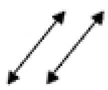
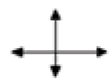
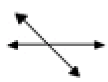
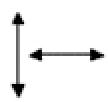
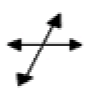
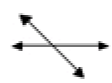
4.



□ line QS
□ line segment QS
□ Ray QS

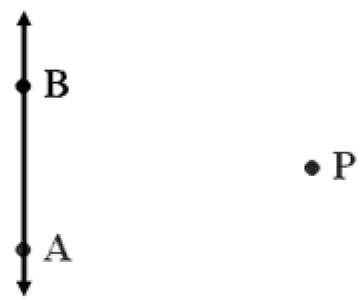
☐ Ray TL	☐ Ray SQ
5.  ☐ line SD ☐ line segment SD ☐ Ray SD ☐ Ray DS	6.  ☐ line PQ ☐ line segment PQ ☐ Ray PQ ☐ Ray QP
7.  ☐ line YJ ☐ line segment YJ ☐ Ray YJ ☐ Ray JY	8.  ☐ line LE ☐ line segment LE ☐ Ray LE ☐ Ray EL

Classify each group of lines.

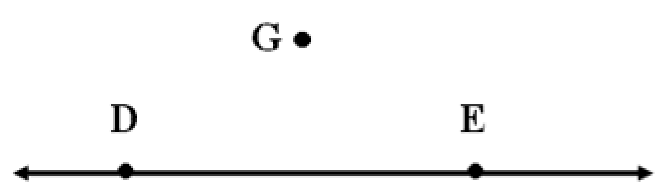
1.  ☐ Parallel ■ Intersecting ☐ Perpendicular	2.  ☐ Parallel ☐ Intersecting ☐ Perpendicular
3.  ☐ Parallel ☐ Intersecting ☐ Perpendicular	4.  ☐ Parallel ☐ Intersecting ☐ Perpendicular
5.  ☐ Parallel ☐ Intersecting ☐ Perpendicular	6.  ☐ Parallel ☐ Intersecting ☐ Perpendicular
7.  ☐ Parallel ☐ Intersecting ☐ Perpendicular	8.  ☐ Parallel ☐ Intersecting ☐ Perpendicular
9.  ☐ Parallel ☐ Intersecting ☐ Perpendicular	10.  ☐ Parallel ☐ Intersecting ☐ Perpendicular

Line Construction

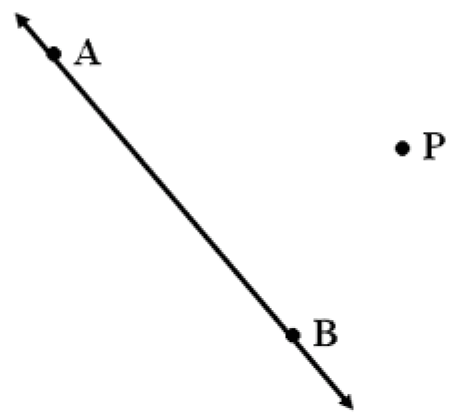
1. Using the following diagram, use a compass and a ruler to construct a line through P parallel to line AB.



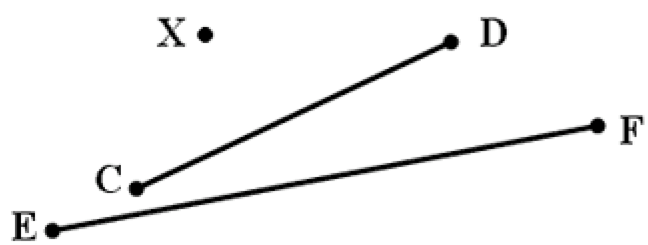
2. Using the following diagram, use a compass and a ruler to construct a line through G parallel to line DE.



3. Using the following diagram, use a compass and a ruler to construct the perpendicular from P to line AB.

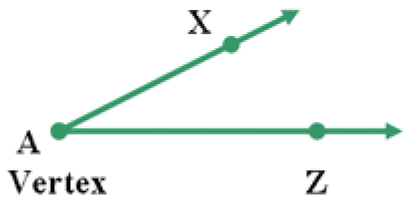


4. Using the following diagram, use a compass and a ruler to construct the perpendicular from X to line segment CD and from X to line segment EF.



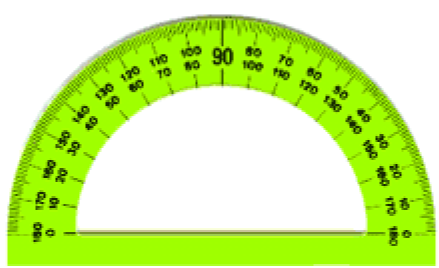
Angles

Angles are formed by two rays with a common endpoint called a ***vertex***.



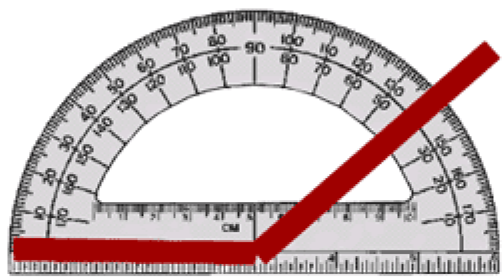
Angles are named by writing the names of three points on the set of lines after the symbol for angle, or by naming only the middle point after the angle symbol. The middle point always names the vertex.

Ð XAZ or **Ð** ZAX or **Ð** A
Angles come in different shapes and sizes. Some are narrow, some are wide. But all angles can be measured as part of a circle. To make calculations easy, scientists have developed the protractor, a kind of ruler for angles.



Angles are measured in degrees from 0 degrees to 180 degrees.

To measure with the protractor, line up the angle of the item to the center of the hole at the middle bottom. Make one edge of the angle line up with where there would be a 0 and then read on that scale where the other edge crosses.

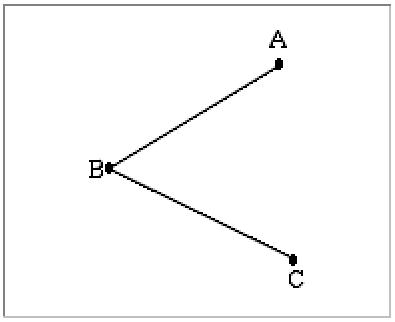


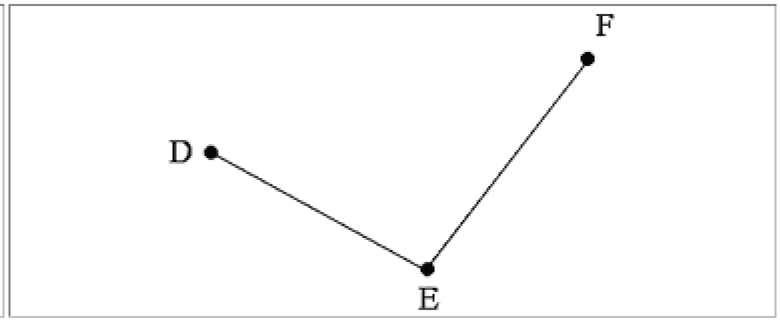
In this example, the angle is 140 degrees.

Practice Exercise

1.) Measure the following angles to the *nearest degree*:

(a) (b)





2.) On a separate piece of paper, use a protractor and a ruler to construct an angle, *CAT,* with a 35° angle.

3.) On a separate piece of paper, use a protractor and a ruler to construct an angle, *DOG*, with a 125° angle.

4.) On a separate piece of paper, use a protractor and a ruler to construct an angle, WIN, with a 90° angle.

5.) On a separate piece of paper, use a protractor and a ruler to construct the angle, *SUN*, with *SU* = 7.5 cm, *UN* = 8 cm, and $\angle$ *SUN* measuring 70°.

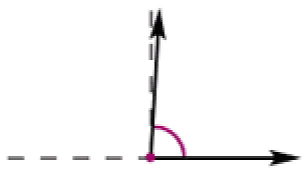
6.) On a separate piece of paper, use a protractor and a ruler to construct the angle, *BIG*, with *IG* = 10.4 cm, *BI* = 7.6 cm, and $\angle$ *BIG* measuring 110°.

7.) On a separate piece of paper, use a protractor and a ruler to construct the angle, *HAM*, with *HA* = 12.2 cm, *AM* = 9.4 cm, and $\angle$ *HAM* measuring 155°.

Acute Angle

An angle whose measure is greater than 0° and less than 90°

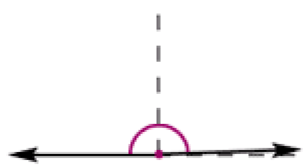
Example:



Obtuse Angle

An angle whose measure is greater than 90° and less than 180°

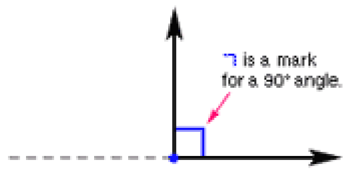
Example:



Right Angle

An angle whose measure is 90°

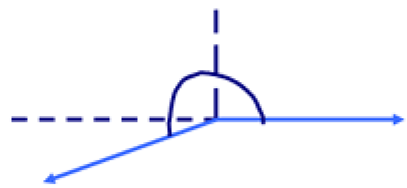
Example:



Reflex Angle

An **angle** whose measure is more than 180 degrees, but less than 360 degrees.

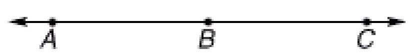
Example:



Straight Angle

An angle whose measure is 180°

Example:

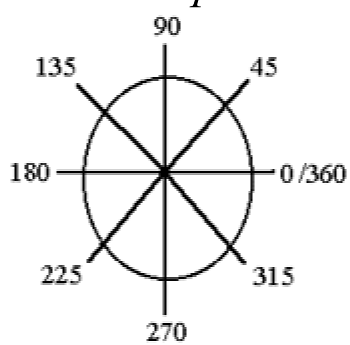


∠ *ABC is a straight angle.*

Complete Angle

An angle whose measure is 360 degrees (a circle)

Example:



Practice Exercise

Acute, Obtuse, Right, and Straight Angles

---

1 A 35 degree angle would be classified as:

- ☐ An acute angle.
- ☐ An obtuse angle.
- ☐ A right angle.
- ☐ A straight angle.

---

2 A 175 degree angle would be classified as:

- ☐ An acute angle.
- ☐ An obtuse angle.
- ☐ A right angle.
- ☐ A straight angle.

---

3 A 180 degree angle would be considered:

- ☐ An acute angle.
- ☐ An obtuse angle
- ☐ A right angle.
- ☐ A straight angle.

---

4 An 80 degree angle would be considered:

- [ ] An acute angle.
- [ ] An obtuse angle.
- [ ] A right angle.
- [ ] A straight angle.

---

5 A 90 degree angle would be considered:

- [ ] An acute angle.
- [ ] An obtuse angle.
- [ ] A right angle.
- [ ] A straight angle.

---

6 When you cut a pizza into four equal pieces, the tip of each piece creates one of these angles.

- [ ] Acute Angle
- [ ] Obtuse Angle
- [ ] Right Angle
- [ ] Straight Angle

---

7 Which angle is impossible to have in a triangle?

- [ ] Acute Angle

- [ ] Obtuse Angle
- [ ] Right Angle
- [ ] Straight Angle

---

8 Which angle is found in squares?

- [ ] Acute Angle
- [ ] Obtuse Angle
- [ ] Right Angle
- [ ] Straight Angle

---

9 Which angle is the smallest?

- [ ] Acute Angle
- [ ] Obtuse Angle
- [ ] Right Angle
- [ ] Straight Angle

---

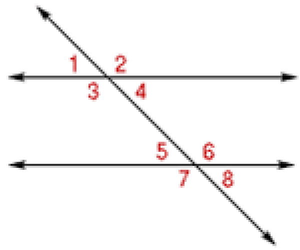
10 How many right angles, when put together, would make a straight angle?

- [ ] 0
- [ ] 1
- [ ] 2
- [ ] 3

Interior Angles

Angles on the inner sides of two lines cut by a transversal

Example:

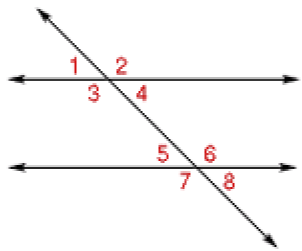


Angles 3, 4, 5, and 6 are interior angles.

Exterior Angles

The angles on the outer sides of two lines cut by a transversal

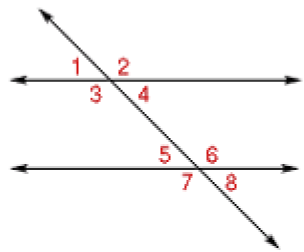
Example:



Angles 1, 2, 7, and 8 are exterior angles.

Alternate Exterior Angles

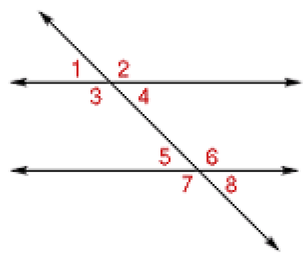
A pair of angles on the outer sides of two lines cut by a transversal, but on opposite sides of the transversal
Example:



∠ 1 and ∠8 and ∠2 and ∠7 are alternate exterior angles.

Alternate Interior Angles

A pair of angles on the inner sides of two lines cut by a transversal, but on opposite sides of the transversal
Example:

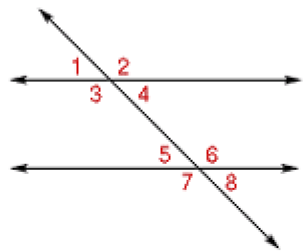


∠ 3 and ∠6 and ∠4 and ∠5 are alternate interior angles.

Corresponding Angles

Angles that are in the same position and are formed by a transversal cutting two or more lines

Example:

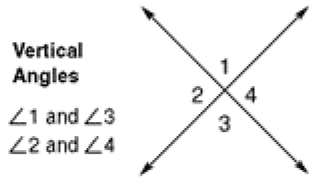


∠ 2 and ∠6 are corresponding angles.

Vertical or Opposite Angles

A pair of opposite congruent angles formed by intersecting lines

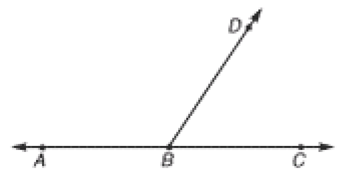
Example:



Adjacent Angles

Angles that share a common side, have the same vertex , and do not overlap

Example:

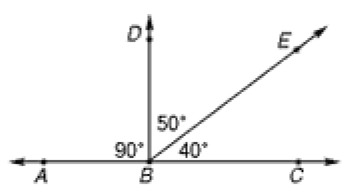


$\angle ABD$ is adjacent to $\angle DBC$.

Complementary Angles

Two angles whose measures have a sum of 90°

Example:

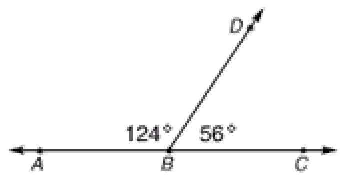


$\angle$ DBE and $\angle EBC$ are complementary.

Supplementary Angles

Two angles whose measures have a sum of 180°

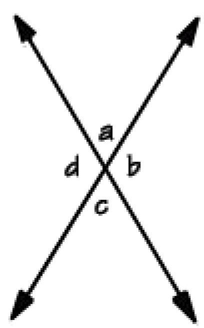
Example:



$m\angle ABD + m\angle DBC = 124° + 56° = 180°$

Practice Exercise

Adjacent Angles



Use the picture above to answer the following questions?

Name the angles adjacent to <b.

Name the angles adjacent to <c.

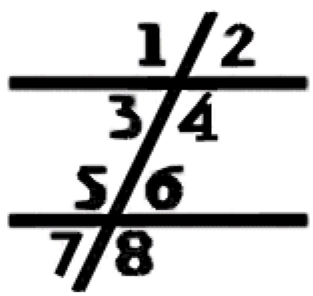
Name the angles adjacent to <d.

Name the angles not adjacent to <a.

Name the angles not adjacent to <b.
Name the angles not adjacent to <c.

Complementary and Supplementary Angles

1. **What angle would be supplementary to a 105 degree angle?**
2. **What angle would be complementary to a 56 degree angle?**
3. **What angle would be complementary to 17 degree angle?**
4. **What angle would be supplementary to a 121 degree angle?**



Using the diagram above:

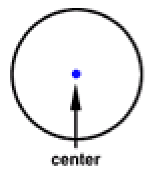
a) **identify two interior angles.**
b) **identify two exterior angles.**
c) **identify two vertical or opposite angles.**

Introduction to Geometric Figures

Circle

A closed curve with all points on the curve an equal distance from a given point called the center of the circle

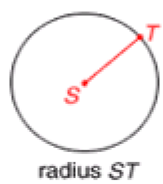
Example:



Radius

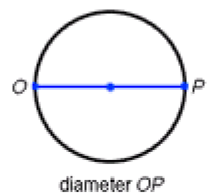
A line segment with one endpoint at the center of a circle and the other endpoint on the circle

Example:



Diameter

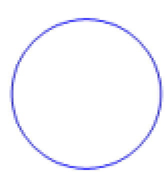
A chord or line segment with endpoints on a circle that passes through the center of a circle
Example:



The ***diameter*** of a circle is a line that crosses the circle through its center from one side to the other. It also measures the distance across the circle. The ***radius*** of a circle is a line from the center of the circle to any point on the curve of the circle. A radius is half the distance across a circle. In other words, a radius is half of the diameter of a circle.

Circumference

The distance around a circle.
The perimeter of a circle is called circumference.



The formula for the circumference of a circle is $C = \pi d$, where C = circumference, $\pi \approx 3.14$ or $\frac{22}{7}$, and d = diameter

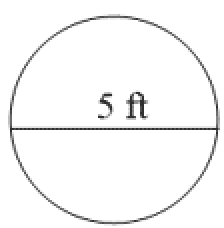
Pi (π)

The ratio of the circumference of a circle to the length of its diameter;
$\pi \approx 3.14$ or $\frac{22}{7}$

$\approx$ is the symbol that means "approximately equal to".

$\Rightarrow$ It is useful to be familiar with *both* values of π, because in some problems a fraction is easier to use, while in others a decimal will make the computation easier.

Example Find the circumference of the circle shown below.

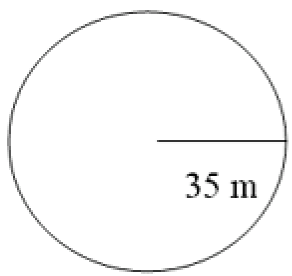


$C = \pi d$
$C = 3.14 \text{ x } 5 \text{ ft}$
$C = 15.70 = 15.7 \text{ ft}$

Solution: Replace π with 3.14 and *d* with 5 ft in the formula $C = \pi d$. The circumference of the circle is **15.7 feet.**

When finding the circumference of a circle, if only the radius is given, you must multiply the radius by 2 to find the diameter, and then use the formula.

Example Find the circumference of the circle shown below. Use 22/7 for π.



Step 1 Notice that the picture shows the radius of the circle. To find the diameter, multiply the radius by 2.

$$d = 2 \times 35 = 70 \text{ m}$$

Step 2 Replace π with 22/7 and *d* with 70 m in the formula $C = \pi d$.

$$C = \pi d$$
$$C = 22/7 \times 70$$
$$C = 220\ m$$

Answer: The circumference of the circle is **220 m.**

To find the diameter of a circle, if only the circumference is given, divide the circumference by π.

To find the radius of a circle, if only the circumference is given, divide the circumference by π to find the diameter. Then divide the diameter by 2 to find the radius.

If you already know the diameter of the circle, you can find the radius by dividing the diameter by 2.

If you already know the radius of the circle, you can find the diameter by multiplying the radius by 2.

Practice Exercise

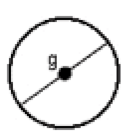
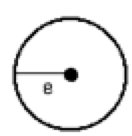
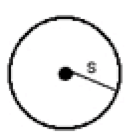
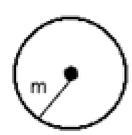
Complete the table for each circle.
Round to the nearest hundredth. Use 3.14 for π

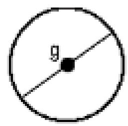
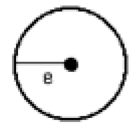
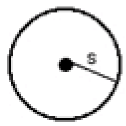
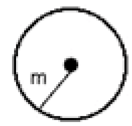
For problems 11-14, use $3\frac{1}{7}$ for π.

	radius	*diameter*	*circumference*
1.	8 ft	16 ft	__________ ft
2.	9 m	__________ m	__________ m
3.	__________ mm	4 mm	__________ mm
4.	7 mi	14 mi	__________ mi
5.	10 km	20 km	__________ km

6.	6.7 cm	13.4 cm	__________ cm
7.	__________ in	__________ in	61.54 in
8.	__________ yd	__________ yd	57.78 yd
9.	11.4 mm	__________ mm	__________ mm
10.	__________ m	12.4 m	__________ m
11.	$5\frac{1}{5}$ km	$10\frac{2}{5}$ km	__________ km
12.	__________ in	$14\frac{2}{5}$ in	__________ in
13.	$7\frac{7}{10}$ yd	$15\frac{2}{5}$ yd	__________ yd
14.	$6\frac{4}{5}$ cm	$13\frac{3}{5}$ cm	__________ cm
15.	19.41 mi	__________ mi	__________ mi

Find the Circumference for each.
Round to the nearest hundredth. Assume π = 3.14

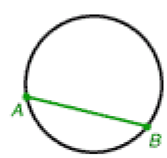
1.		g = 21.24 yd **138.16 ft**	2.		e = 38 cm __________
3.		s = 5 mi __________	4.		m = 23 yd __________

5.		g = 21.24 yd ____________	6.		e = 9.6 cm ____________
7.		s = 4 c m ____________	8.		m = 12.69 in ____________

Parts of a Circle

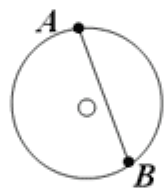
Chord

A line segment with endpoints on a circle
Example:



Segment

A straight set of points that has two endpoints.
Example:

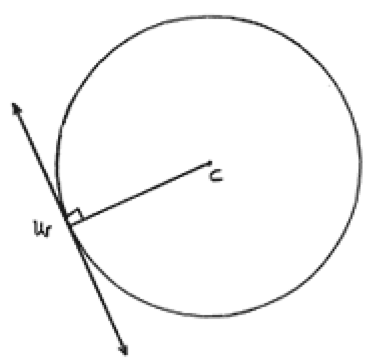


AB is a segment. In this picture, it is a straight set of points with two endpoints. Since both of the endpoints are on the circle this segment is also a **chord**.

Tangent

Tangent lines are perpendicular to the radius that has an endpoint on the point of tangency.

Example:



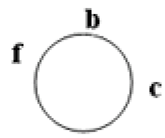
Line J is a tangent line that meets the radius line CW at an endpoint W on the circle that forms a 90 degree angle. Therefore, Line J is perpendicular to line CW.

Arc

A section of a circle.

Think about a circular pizza that has been cut like a pie is cut. The crust acts like the circumference of the pizza. That would make the crust on one piece of pizza an arc because it is just a section of the whole circle.

Example:



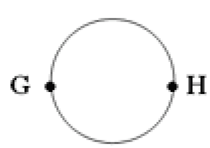
There are many arcs shown here. Can you see them all?

1) Small arc fb and big arc fb.
2) Small arc fc and big arc fc.
3) Small arc bc and big arc bc.
4) Arc cfb

Semicircle

The **arc** that goes halfway around a circle is called a semicircle.

Example:



Sector

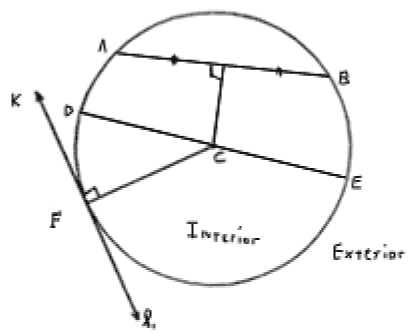
A region in a circle that is created by a central angle and its intercepted arc.

A piece of pie.

Example:

The piece of pie that the number 1 is in is called a sector.

Practice Exercise



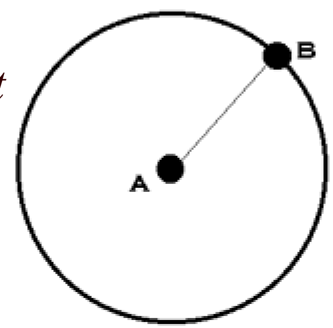
Using the diagram above, identify the following circle parts.

1. Name two radiuses.
2. Name a diameter.
3. Name a chord.
4. Name two arcs.
5. Name a segment.
6. Name a sector.
7. Name a tangent.
8. Name a semicircle.

CIRCLE CONSTRUCTIONS

Center/point construction:

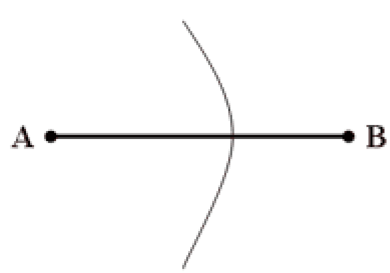
Procedure: *Center point A and linear point B are the endpoints of a given radius. Set the point of the compass on A and the lead on B and draw the circle.*



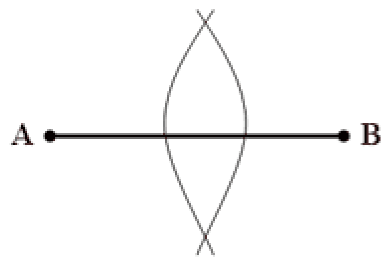
Diameter construction:

Procedure: *Bisect given diameter AB by placing the compass point first on Point A and opening your compass so that the lead touches a point on the line that is more than midway towards Point B.*

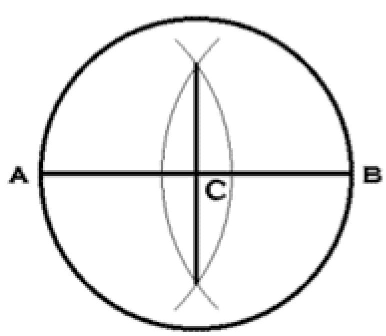
Using this setting, make an arc above and below the line



Now, put your compass point on Point B, and using the same compass setting, make an arc above and below the line that intercepts the arc made from point A.



Where the arcs intercept, join the two points to form a line that will bisect line AB. Since C denotes the midpoint of AB, then AC and BC are radii of the circle and either can be used to set the compass.



Practice Exercise

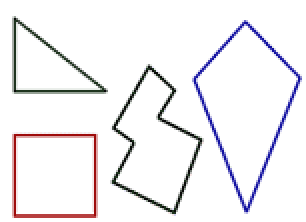
1. On a separate piece of paper, use a compass to construct a circle with center O and radius 8 cm. Draw a sector in your circle labeled AON.

2. On a separate piece of paper, use a compass to construct a circle with a diameter 11 cm. Label the diameter as RS.

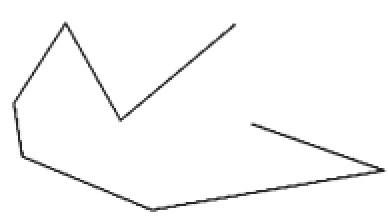
Polygon

A closed plane figure formed by three or more line segments

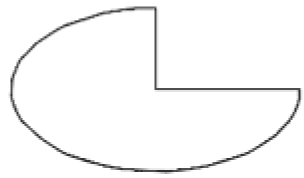
Examples:



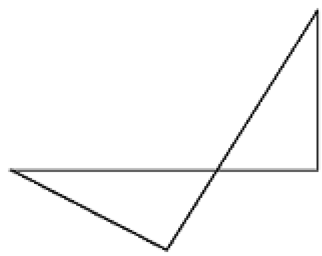
The figure below is not a polygon, since it is not a closed figure:



The figure below is not a polygon, since it is not made of line segments:



The figure below is not a polygon, since its sides do not intersect in exactly two places each:

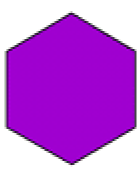


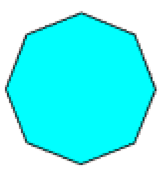
Regular Polygon

A regular polygon is a polygon whose sides are all the same length, and whose angles are all the same.

Examples:

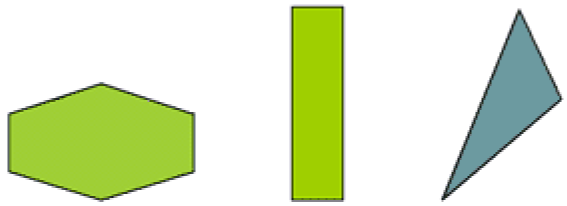
The following are examples of regular polygons:





Examples:

The following are not examples of regular polygons:

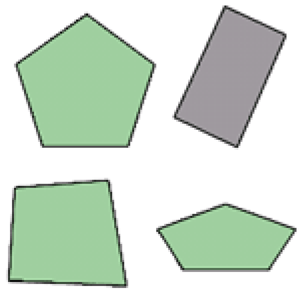


Convex Polygon

A figure is convex if every line segment drawn between any two points inside the figure lies entirely inside the figure.

Example:

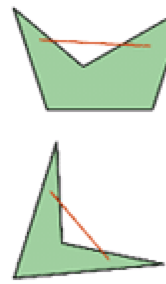
The following figures are convex.



Concave Polygon

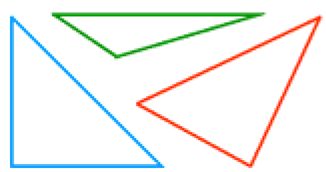
A figure that is not convex is called a concave figure. A concave polygon has at least one side that is curved inward.

The following figures are concave. Note the red line segment drawn between two points inside the figure that also passes outside of the figure.



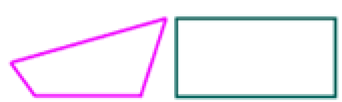
Triangle

A three-sided polygon
Examples:



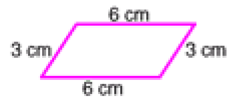
Quadrilateral

A four-sided polygon
Examples:



Parallelogram

A quadrilateral whose opposite sides are parallel and congruent
Example:

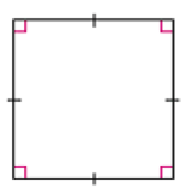


Trapezoid

A quadrilateral with only one pair of parallel sides
Example:

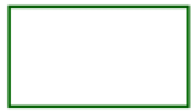
Square

A rectangle with 4 congruent sides
Example:



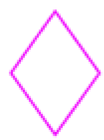
Rectangle

A parallelogram with 4 right angles
Example:



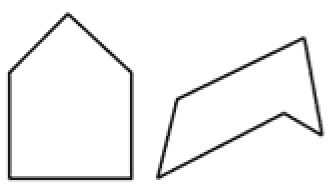
Rhombus

A parallelogram whose four sides are congruent and whose opposite angles are congruent
Example:



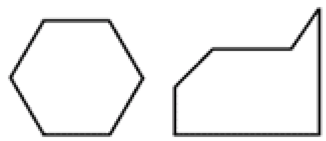
Pentagon

A five-sided polygon
Examples:



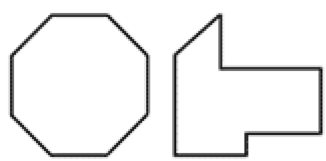
Hexagon

A six-sided polygon
Examples:



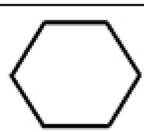
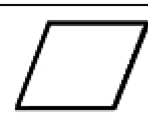
Octagon

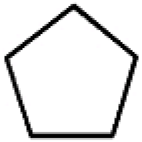
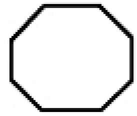
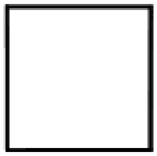
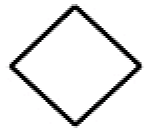
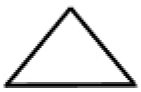
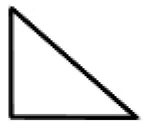
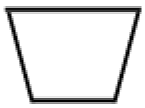
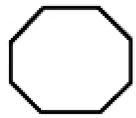
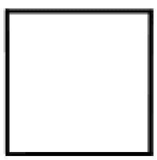
An eight-sided polygon
Examples:



Practice Exercise

Write down the name for each polygon.

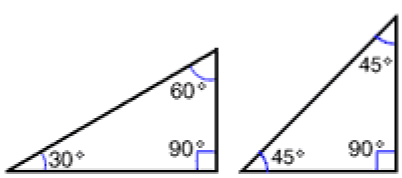
1.  ________	2.  ________

3.  ______	4.  ______
5.  ______	6.  ______
7.  ______	8.  ______
9.  ______	10.  ______
11.  ______	

Right Triangle

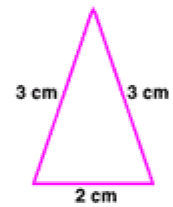
A triangle with exactly one right angle

Examples:



Isosceles Triangle

A triangle with two congruent sides
Example:

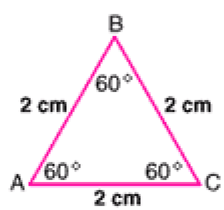


Scalene Triangle

A triangle with no congruent sides
Example:

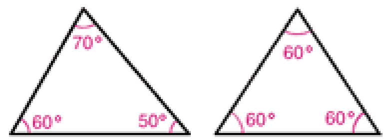
Equilateral Triangle

A triangle with three congruent sides and three congruent angles
Example:



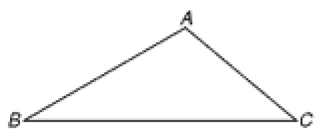
Acute Triangle

A triangle in which all three angles are acute
Example:



Obtuse Triangle

A triangle containing exactly one obtuse angle
Example:

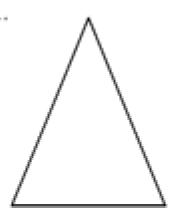


$\angle A$ is obtuse so $\triangle ABC$ is an obtuse triangle

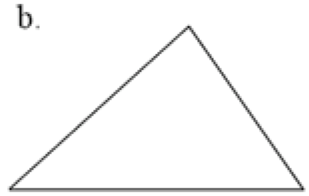
Practice Exercise

1. Measure the sides and classify the triangle as equilateral, isosceles, or scalene.

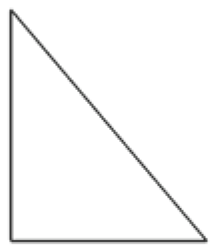
a.



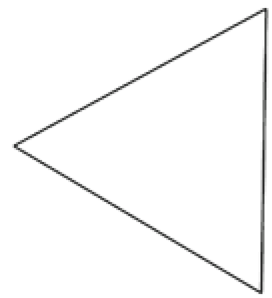
b.



c.

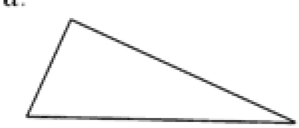


d.

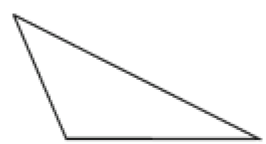


2. Measure the largest angle, then classify the triangle as acute, obtuse, or right.

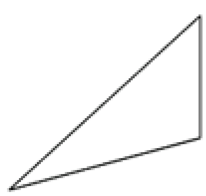
a.



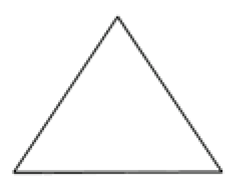
b.



c.



d.

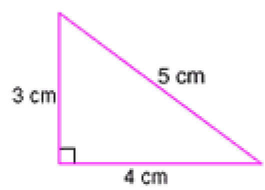


Pythagorean Theorem (Pythagorean Property)

Pythagoras was a Greek philosopher and mathematician. His ideas influenced great thinkers throughout the ages, and he is well known to math students. His Pythagorean Theorem is a simple rule about the proportion of the sides of right triangles: ***The square of the hypotenuse (the longest side)of a right triangle is equal to the sum of the square of the other two sides (legs).***

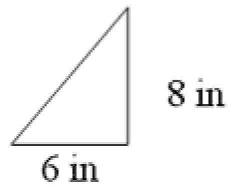
In any right triangle, if a and b are the lengths of the legs and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$

Example:



$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$

Example Find the length of the hypotenuse in the triangle below.



Step 1 Replace a with 6 and b with 8 in the formula $a^2 + b^2 = c^2$.

$$a^2 + b^2 = c^2$$

$$6^2 + 8^2 = c^2$$

$$36 + 64 = c^2$$

$$100 = c^2$$

Step 2 The formula gives the value of the hypotenuse squared. To find the length of the hypotenuse, find the square root of 100.

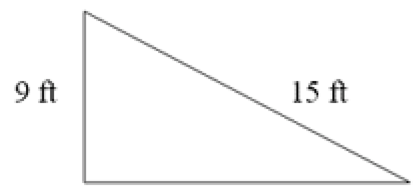
$$c = \sqrt{100}$$

$c =$ **10 in**

Answer: The length of the hypotenuse in the given triangle is **10 in.**

In some problems you may be given the length of the hypotenuse and the length of one of the legs. To find the length of the other leg, you can still use the Pythagorean theorem.

Example Find the length of the missing leg in the triangle below.



Step 1 Write down the Pythagorean theorem and substitute in the values you know.

$$a^2 + b^2 = c^2$$

$$a^2 + 9^2 = 15^2$$

Step 2 Find the values of the squares.

$$a^2 + 81 = 225$$

Step 3 To get the unknown, *a*, alone on one side, subtract 81 from both sides.

$$a^2 + 81 - 81 = 225 - 81$$

$$a^2 = 144$$

Step 4 To find *a*, find the square root of both sides of the equation.

$$a = \sqrt{144}$$

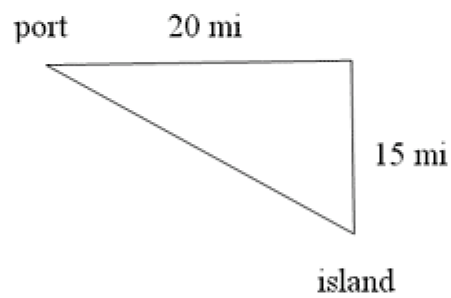
$$\boldsymbol{a = 12}$$

Answer: The length of the missing leg in the given triangle is **12 ft.**

In some problems you will have to recognize that a figure is a right triangle. The picture or problem may say nothing about a right triangle, the hypotenuse, or legs. Drawing a picture may help you see that the problem involves a right-triangle relationship.

Example A boat sails 20 miles east of port and then 15 miles south to an island. How far is the boat from the port if you measure in a straight line?

Step 1 Make a drawing to see how to solve the problem. East is normally to the right on a map, and south is toward the bottom. Notice that the actual distance from the port is the hypotenuse of a right triangle.



Step 2 Replace a with 20 and b with 15 in the formula $a^2 + b^2 = c^2$.

$$a^2 + b^2 = c^2$$

$$20^2 + 15^2 = c^2$$

$$400 + 225 = c^2$$

$$625 = c^2$$

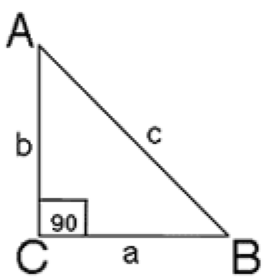
Step 3 Find the square root of 625.

$$\sqrt{625} = c$$

$$\mathbf{25 = c}$$

Answer: The boat is **25 miles** from the port.

Practice Exercise



Use ABC as shown on left to help you complete each question.
Round to the nearest hundredth.

1. If a = 9 and b = 40, then c = __________	2. If a = 5 and b = 12, then c = __________
3. If a = 3 and b = 4, then c = __________	4. If a = 24 and b = 45, then c = __________
5. If a = 11 and b = 7, then c = __________	6. If a = 11 and b = 10, then c = __________

7. If a = 4 and b = 2, then c = __________	8. If a = 8 and b = 12, then c = __________
9. If a = 6 and b = 5, then c = __________	10. If a = 3 and b = 6, then c = __________
11. If a = 9 and b = 2, then c = __________	12. If a = 7 and b = 2, then c = __________
13. If a = 4 and b = 6, then c = __________	14. If a = 6 and b = 6, then c = __________
15. If a = 9 and b = 3, then c = __________	16. If a = 2 and b = 8, then c = __________
17. If a = 12 and b = 12, then c = __________	18. If a = 4 and b = 11, then c = __________
19. If a = 11 and b = 20, then c = __________	20. If a = 18 and b = 18, then c = __________
21. If a = 16 and b = 19, then c = __________	22. If a = 21 and b = 16, then c = __________
23. If a = 19 and b = 13, then c = __________	24. If a = 10 and b = 16, then c = __________
25. If a = 10.6 and b = 7.2, then c = __________	26. If a = 9.4 and b = 12.8, then c = __________
27. If a = 8.4 and b = 10.8, then c = __________	28. If a = 4.9 and b = 5.5, then c = __________

29. If c = 37 and b = 35 then a = __________	30. If c = 17 and b = 15 then a = __________
31. If c = 13 and a = 12 then b = __________	32. If c = 29 and a = 21 then b = __________
33. If c = 51 and b = 45 then a = __________	34. If c = 25 and a = 24 then b = __________
35. If c = 65 and a = 56 then b = __________	36. If c = 20 and b = 10 then a = __________
37. If c = 30 and b = 12 then a = __________	38. If c = 38 and a = 14 then b = __________
39. If c = 39 and a = 21 then b = __________	40. If c = 34 and b = 24 then a = __________
41. If c = 21 and a = 9 then b = __________	42. If c = 25 and a = 14 then b = __________
43. If c = 39 and b = 13 then a = __________	44. If c = 32 and b = 22 then a = __________
45. If c = 40 and b = 30 then a = __________	46. If c = 49 and a = 20 then b = __________
47. If c = 50 and b = 31 then a = __________	48. If c = 22 and a = 12 then b = __________
49. If c = 49 and b = 28 then a = __________	50. If c = 61 and b = 51 then a = __________

51. If c = 90 and a = 63 then b = __________	52. If c = 59 and a = 39 then b = __________
53. If c = 8.22 and b = 6.2 then a = __________	54. If c = 11.46 and a = 4.6 then b = __________
55. If c = 11.95 and a = 10.9 then b = __________	56. If c = 6.83 and b = 5.8 then a = __________

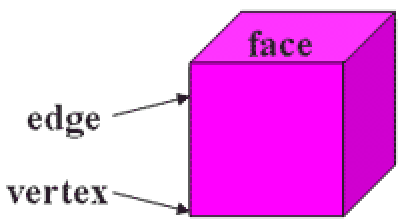
Using the Pythagorean theorem, solve the problems presented below.

1. A ladder rests against the side of Kate's house. The bottom of the ladder is 8 meters from the house, and the top just reaches a window that's 15 meters above ground. How long is the ladder?

2. A forest ranger at Oak Ridge Lookout spotted a fire 24 kilometers west of his location. If the town of Dairy is 38 miles due south of the lookout tower, how far is Dairy from the fire?

Polygons and circles are flat, or two-dimensional. They have only length and width. But ***cubes***, ***prisms***, ***pyramids***, and ***spheres*** are solid. They have a third dimension known as height or, sometimes, depth. These solids are also called ***space figures*** or ***polyhedrons***.

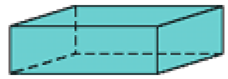
Cubes, prisms, pyramids, and other solids have sides called ***faces***. These faces are flat surfaces that are in the shapes of polygons. Faces meet at edges. The edges are line segments, which meet in vertexes. The vertexes are points.



Prism

A polyhedron whose two bases are congruent, parallel polygons in parallel planes and whose lateral faces are parallelograms

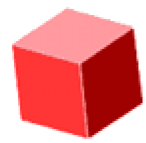
Example:



rectangular prism

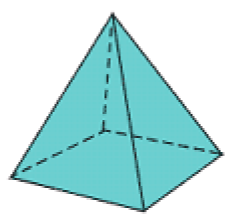
Cube

A square prism with six congruent square faces
Example:



Pyramid

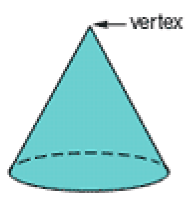
A polyhedron with a base that is a polygon and with lateral faces that are triangles which share a common vertex
Example:



square pyramid

Cone

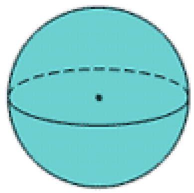
A solid figure with a circular base and one vertex
Example:



Sphere

A solid figure with all points the same distance from the center

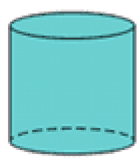
Example:



Cylinder

A solid figure with two parallel, congruent circular bases connected by a curved surface

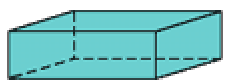
Example:



Practice Exercise

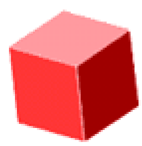
Match the solid shapes with the common three-dimensional objects listed below them.

A



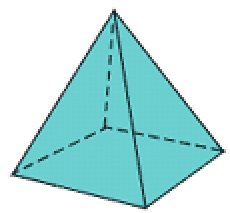
Rectangular Prism

B



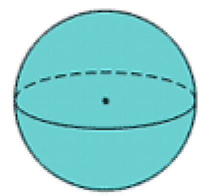
Cube

C



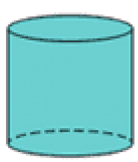
Cone

D



Sphere

E



Cylinder

1. _____	Dice	2. _____	Fish Bowl
3. _____	Balloon	4. _____	Marble
5. _____	Light Bulb	6. _____	Bowling Ball
7. _____	Mug	8. _____	AA Battery
9. _____	Tool Box	10. _____	Soccer Ball
11. _____	Funnel	12. _____	Pill Bottle
13. _____	Paper Towel Roll	14. _____	Pen

15. ______	Megaphone	16. ______	Ice Cube
17. ______	Spray Can	18. ______	Cement Block
19. ______	Child's Block	20. ______	Suitcase
21. ______	Fish Tank	22. ______	Cereal Box
23. ______	Tepee	24. ______	Planet Earth
25. ______	Can of Paint	26. ______	Hockey Puck
27. ______	Can of Peas	28. ______	Water Pipe

Measurement

The Metric System

In the 1790s, French scientists worked out a system of measurement based on the ***meter*** that they called the Systeme International (International System). The meter is one ten-millionth of the distance between the North Pole and the Equator. The French scientists made a metal rod equal to the length of the standard meter.

By the 1980s, the French metal bar was no longer a precise measure for the meter. Scientists figured out a new standard for the meter. They made it equal to 1/299,792,548 of the distance light travels in a vacuum in one second. Since the speed of light in a vacuum never changes, the distance of the meter will not change.

The French scientists developed the ***metric*** system to cover measurement of length, area, volume, and weight.

Metric Length Equivalents

Metric Unit	Abbreviation	Metric Equivalent
millimeter	mm	.1 centimeter
centimeter	cm	10 millimeters
decimeter	dm	10 centimeters
meter	m	100 centimeters
dekameter	dam	10 meters
hectometer	hm	100 meters
kilometer	km	1000 meters

Metric Weight Equivalents

Metric Unit	Abbreviation	Metric Equivalent
milligram	mg	.001 gram
centigram	cg	10 milligrams
decigram	dg	10 centigrams
gram	g	1,000 milligrams
decagram	dag	10 grams
hectogram	hg	100 grams
kilogram	kg	1,000 grams

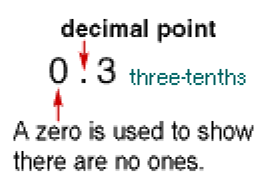
Metric Volume Measures

Metric Unit	Abbreviation	Metric Equivalent
milliliter	ml	.001 liter
centiliter	cl	10 milliliters
deciliter	dl	10 centiliters
liter	l	1,000 milliliters
dekaliter	dal	10 liters
hectoliter	hl	100 liters
kiloliter	kl	1,000 liters

Decimal Point

A period that separates the whole numbers from the fractional part of a number; or that separates dollars from cents

Example:



Kilometers Hectometers Decameters Meters Decimeters Centimeters Millimeters

Kilograms Hectograms Decagrams Grams Decigrams Centigrams Milligrams

Kiloliters Hectoliters Decaliters Liters Deciliters Centiliters Milliliters

To use the chart above, if a question asks you how many grams that you can get from 200 centigrams, for example, try this:

Start by putting down the number:

200

If we don't see a decimal point, the number is a whole number; and therefore, a decimal point may be inserted to the right of the last digit:

200.

Now, using your chart, start at centigrams and count back to grams (two spaces to the left).

Move the decimal point in your number the same amount of spaces in the same direction:

2.00

The answer to the question is that 200 centigrams is equal to 2 grams.

If a question asks you to tell how many millimeters are is 8.3 decimeters, try this:

Write down the number:

8.3

We already see a decimal point, so there is no need to guess where to place it:

8.3

Now, using your chart, start at decimeters and count forward to millimeters (two spaces to the right).

Move the decimal point in your number the same amount of spaces in the same direction:

830.

The answer to the question is that 830 millimeters is equal to 8.3 decimeters.

⇒ Change larger to smaller units by multiplying.

3 meters = ? cm
3 x 100 (100 centimeters to a meter) = 300 centimeters

Þ Change smaller to larger units by dividing

5000 grams = ? kg
5000 ¸ 1000 grams = 5 kg

Practice Exercise

Fill in the answer.

1. 2200 m = ________ km
2. 11.43 cg = ________ mg
3. 6.419 g = ________ cg
4. 10258 g = ________ kg
5. 6.869 cm = ________ mm
6. 170 cl = ________ L
7. 2 cl = ________ ml
8. 12866 L = ________ kl
9. 1000 cm = ________ m
10. 30 mm = ________ cm
11. 11220 g = ________ kg
12. 6 cm = ________ mm
13. 5500 m = ________ km
14. 1 cl = ________ ml
15. 8800 L = ________ kl
16. 3 m = ________ cm
17. 5 g = ________ mg
18. 12.772 kg = ________ g
19. 91 mg = ________ cg
20. 301.2 cl = ________ L
21. 9 m = ________ mm

22. 11.67 cl = ________ ml

23. 8 L = ________ ml

24. 12 L = ________ cl

25. 8.6 L = ________ ml

26. 1 L = ________ cl

27. 6.9 cl = ________ ml

28. 11 L = ________ cl

29. 4780 mg = ________ g

30. 8.094 cl = ________ ml

31. 9.564 L = ________ ml

32. 4000 m = ________ km

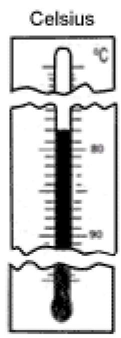
33. 7275 ml = ________ L

The Centigrade Scale

In 1742, Swedish astronomer Anders Celsius (1701 – 1744) invented a scale for measuring heat. His scale is called the ***centigrade*** or ***Celsius*** scale. Celsius's scale is based on the freezing and boiling points of water. The freezing point of water is equal to 0 degrees Celsius. The boiling point is 100 degrees Celsius. While the Fahrenheit scale is used in the United States, the centigrade scale is used in most countries throughout the world. It is the scale preferred by scientists.

The markings on a thermometer are in degrees.
We read the degrees as:
above zero +1, +2, +3, ……
below zero -1, -2, -3, ……..

The temperature on the Celsius thermometer below is -78 degrees. This can be written as -78℃.



A degree Celsius memory device:

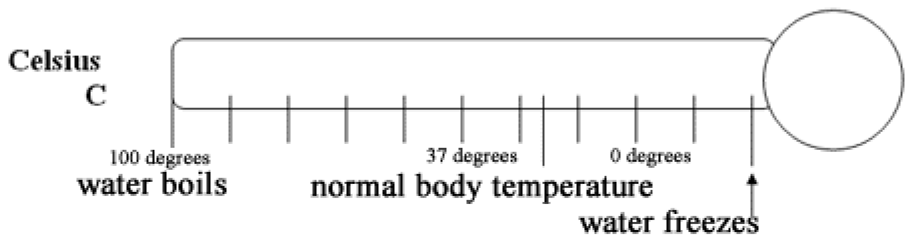
There are several **memory aids** that can be used to help the novice understand the degree Celsius temperature scale. **One such device** is:

*When it's **zero** it's **freezing**,*
*when it's **10** it's **not**,*
*when it's **20** it's **warm**,*
*when it's **30** it's **hot**!*

Or, another one to remember:

30's hot
20's nice

10's cold
zero's ice

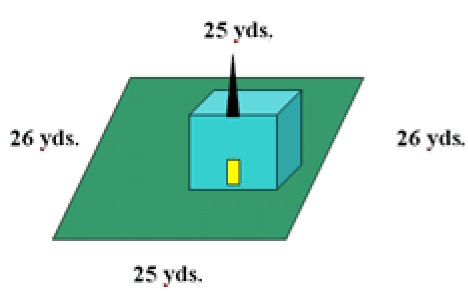


Area, Perimeter, and Volume

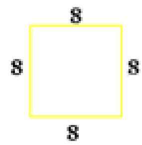
To measure flat spaces we calculate ***perimeter***. Perimeter is the distance around a two-dimensional or flat shape.

Calculating Perimeter

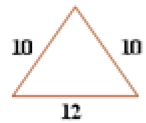
Perimeter is calculated in different ways, depending upon the shape of the surface. The perimeter of a surface outlined by straight lines is calculated by adding together the lengths of its sides.



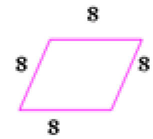
25 + 26 + 25 + 26 = 102 yds. perimeter of the rectangular lot



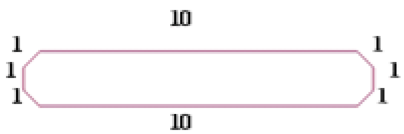
8 + 8 + 8 + 8 = 4 x 8 = 32
4s (4 sides) = perimeter of a square



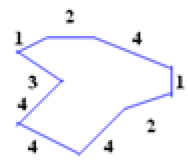
10 + 10 + 12 = 32
3s (3 sides) = perimeter of a triangle



8 + 8 + 8 + 8 = 4 x 8 = 32
4s = perimeter of a rhombus



1 + 1 + 10 + 1 + 1 + 1 + 10 + 1 = 26
8s = perimeter of an irregular octagon

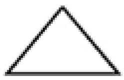
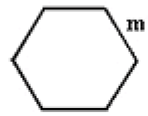
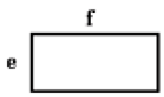
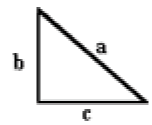
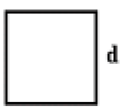
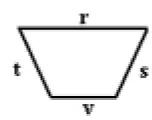
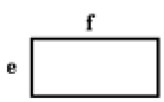
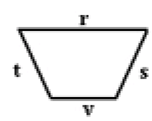
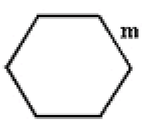
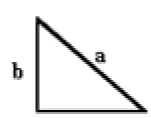


4 + 1 + 2 + 4 + 4 + 4 + 3 + 1 + 2 = 25
all sides = perimeter of an irregular polygon

A regular polygon is a polygon whose sides are all the same length, and whose angles are all the same. To calculate the perimeter of regular polygons like squares or rhombuses, multiply the number of sides by the length of a side. This is possible, because all the sides are the same length.

Practice Exercise

Find the perimeter.

1.		All sides equal 8 m **24 m**	2.		m = 20 in All sides are equal __________
3.		e = 8 m f = 11 m __________	4.		a = 3 mi c = 1 mi b = c __________
5.		The side d of this square is 28 mi __________	6.		v = 5 m t = 9 m r = 13 m s = t __________
7.		e = 5 mi f = 11 mi __________	8.		v = 8 mi t = 10 mi r = 15 mi s = t __________
9.		m = 16 mi All sides are equal __________	10.		a = 6 ft c = 5 ft b = c __________

Complete the table for each rectangle.
Round to the nearest hundredth.

	length	*width*	*perimeter*
1.	2 mi	8 mi	__________ mi
2.	2 km	2 km	__________ km
3.	2 yd	7 yd	__________ yd
4.	5 in	7 in	__________ in
5.	11 m	6 m	__________ m
6.	12 cm	8 cm	__________ cm
7.	$12\frac{1}{4}$ mm	$6\frac{1}{4}$ mm	__________ mm
8.	6 ft	11.2 ft	__________ ft
9.	17.5 km	__________ km	63 km
10.	$19\frac{4}{5}$ ft	$10\frac{1}{10}$ ft	__________ ft
11.	14.6 mm	__________ mm	55.4 mm
12.	$10\frac{1}{2}$ m	$17\frac{1}{2}$ m	__________ m
13.	17.9 in	13 in	__________ in
14.	$9\frac{1}{2}$ mi	$9\frac{3}{10}$ mi	__________ mi
15.	8.6 yd	__________ yd	43.8 yd

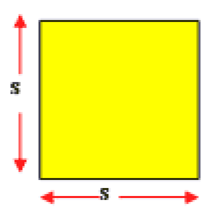
Calculating Area

The ***area*** of a figure is the size of the region it covers.

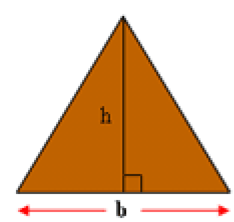
Area is a measurement of only ***two*** dimensions, usually length and width.

Area is calculated in different ways, depending on the shape of the surface. Area is expressed in squares: square inches, square meters, etc.

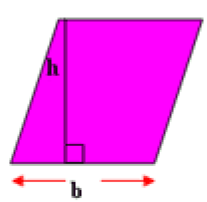
An area with a perimeter made up of straight lines is calculated in different ways for different shapes.



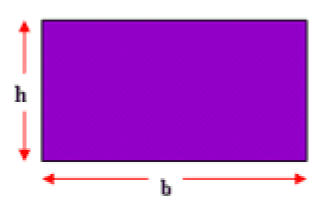
S^2 = area of a square



$\frac{\text{base x height}}{2}$ = area of a triangle



base x height = area of a rhombus



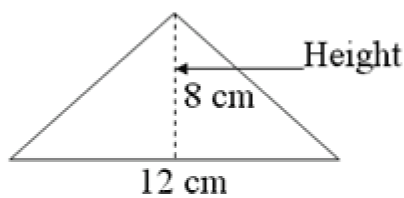
b x h = area of a rectangle

Þ ***The area of a rectangle or square can also be referred to as length x width (l x w).***

Þ ***The area of a triangle is also expressed as ½ the base x height (1/2 b x h).***

The area formula for triangles is used to find the area of all triangles, ***not just right triangles***, and the height that appears in the formula must be a line perpendicular to the base of the triangle or to its extension. Perpendicular lines meet at right angles.

Example What is the area of the triangle below?



Step 1 Identify the base (b) and the height (h).

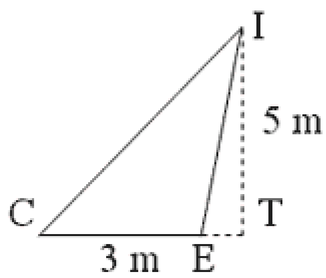
$$b = 12 \text{ cm and } h = 8 \text{ cm}$$

Step 2 Substitute 12 for b and 8 for h in the area formula.

$$A = \tfrac{1}{2}bh$$
$$A = \tfrac{1}{2} \times 12 \times 8$$
$$A = \mathbf{48}$$

Answer: The area is 48 square centimeters.

Example What is the area of triangle ICE?



Note that the line drawn to show height is an extension of the base.

Step 1 Identify the base (b) and the height (h). Side CE is the base and the dotted line IT is the height.

b = 3m and h = 5m

Step 2 Substitute 3 for b and 5 for h in the area formula.

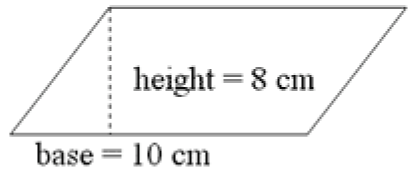
A = ½bh
A = ½ x 3 x 5
A = 15/2 = **7 ½**

Answer: The area is 7 ½ square meters.

A ***parallelogram*** has 4 sides and the opposite sides are parallel. The area of a parallelogram is found by multiplying the length of the base by the height. ***Height*** is the distance straight down from a point on one non-slanting side to its opposite side, or the ***base.***

The formula for the area of a parallelogram can be written:
A = bh, where b = base and h = height.

Example Find the area of the parallelogram below.



Use the formula for finding the area of a parallelogram:

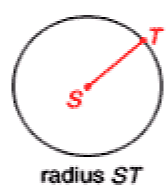
$$\begin{aligned} A &= bh \\ &= 10 \times 8 \\ &= 80 \text{ sq cm} \end{aligned}$$

Answer: The area of the parallelogram is **80 sq cm.**

The area of a circle has a special calculation:

$$\mathbf{a} = \pi r^2$$

The equation is read "***area equals pi times radius squared***."



Step 1 Substitute 22/7 for π, and 7 for r in the area formula.

$$A = \frac{22}{7} \times 7 \times 7$$

Step 2 Cancel the first two "7's."

$$A = \frac{22}{\not{7}_{1}} \times \not{7}^{1} \times 7$$

Step 3 Multiply 22 by 7.

$$A = 22 \times 7 = \mathbf{154}$$

Answer: 154 square centimeters.

Practice Exercise

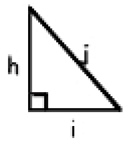
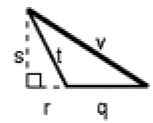
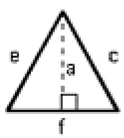
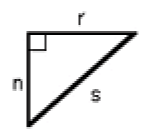
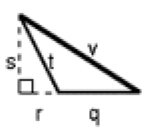
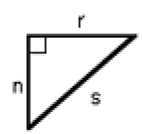
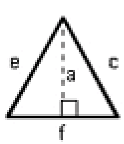
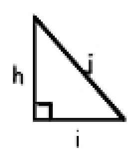
Find the area for each.

1.	20 m; 14 m	**36 squared cm**
2.	4 m; 12 m	____________________
3.	◇	All sides are 12 cm ____________________
4.	17 cm; 17 cm	____________________
5.	29 ft; 26 ft	____________________

Complete the table for each rectangle.

	length	*width*	*area*
1.	9 yd	4 yd	__________ square yd
2.	5 km	6 km	__________ square km
3.	7 in	5 in	__________ square in
4.	7 cm	8 cm	__________ square cm
5.	7 ft	11 ft	__________ square ft
6.	16 mi	8 mi	__________ square mi
7.	14 m	20.7 m	__________ square m
8.	$8\frac{1}{2}$ mm	$8\frac{1}{2}$ mm	__________ square mm
9.	$10\frac{2}{5}$ mi	$16\frac{1}{2}$ mi	__________ square mi
10.	__________ ft	14 ft	155.4 square ft
11.	13.3 in	__________ in	182.21 square in
12.	$6\frac{1}{2}$ m	$7\frac{1}{2}$ m	__________ square m
13.	$16\frac{4}{5}$ mm	$13\frac{1}{2}$ mm	__________ square mm
14.	__________ km	18.9 km	245.7 square km
15.	__________ yd	16 yd	200 square yd

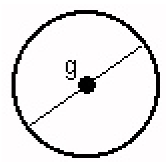
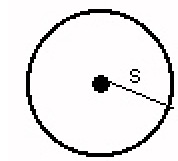
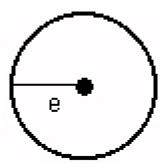
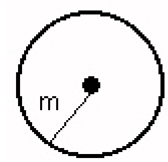
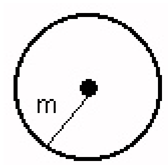
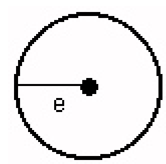
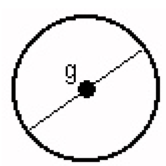
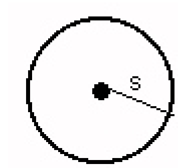
Find the area for each.

1.  i = 5 ft j = 13 ft h = 12 ft **30 ft^2**	2.  s = 15 in r = 8 in t = 17 in q = 16 in v = 29 in ___________
3.  a = 28 in c = 53 in f = 90 in e = c ___________	4.  s = 41 m r = 9 m n = 40 m ___________
5.  s = 4 m r = 3 m t = 5 m q = 6 m v = 10 m ___________	6.  s = 17 yd r = 8 yd n = 15 yd ___________
7.  a = 3 in c = 5 in f = 8 in e = c ___________	8.  i = 24 cm j = 51 cm h = 45 cm ___________

Complete the table for each triangle.
Round to the nearest hundredth.

	base	*height*	*area*	
1.	4 in	5 in	__________	square in
2.	2 mi	3 mi	__________	square mi
3.	3 cm	5 cm	__________	square cm
4.	3 yd	7 yd	__________	square yd
5.	7 mm	9 mm	__________	square mm
6.	12 km	12 km	__________	square km
7.	17.6 ft	8 ft	__________	square ft
8.	7 m	20.3 m	__________	square m
9.	16.4 m	__________ m	81.18	square m
10.	$14\frac{1}{2}$ yd	$6\frac{1}{2}$ yd	__________	square yd
11.	$16\frac{1}{4}$ mi	$9\frac{1}{4}$ mi	__________	square mi
12.	$11\frac{1}{4}$ cm	$20\frac{1}{4}$ cm	__________	square cm
13.	__________ ft	13.7 ft	78.78	square ft
14.	$12\frac{1}{5}$ km	$12\frac{3}{10}$ km	__________	square km
15.	19 in	__________ in	112.1	square in

Find the Area for each.
Round to the nearest hundredth. Assume π = 3.14

1.  g = 6 ft **28.26 ft²**	2.  s = 4 mi ____________
3.  e = 39 yd ____________	4.  m = 71 yd ____________
5.  m = 41.589 cm ____________	6.  e = 18.5 m ____________
7.  g = 8.28 yd ____________	8.  s = 2 cm ____________

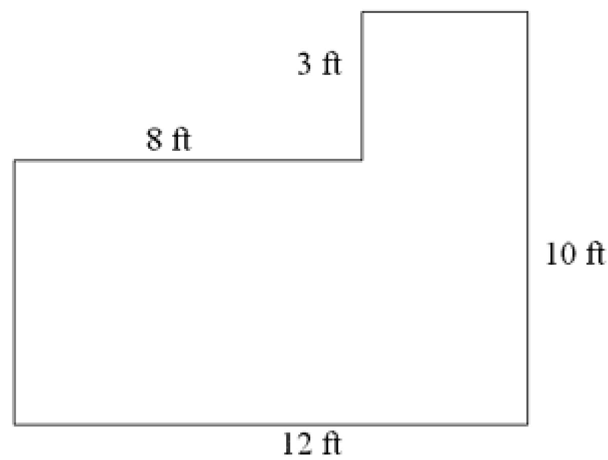
Complete the table for each circle. Assume π = 3.14
Round to the nearest hundredth. Assume π = 3 1/7 for questions 11-14.

	radius	*diameter*	*area*
1.	2 cm	4 cm	__________ cm^2
2.	__________ in	12 in	__________ in^2
3.	8 yd	__________ yd	__________ yd^2
4.	5 mm	10 mm	__________ mm^2
5.	10 mi	20 mi	__________ mi^2

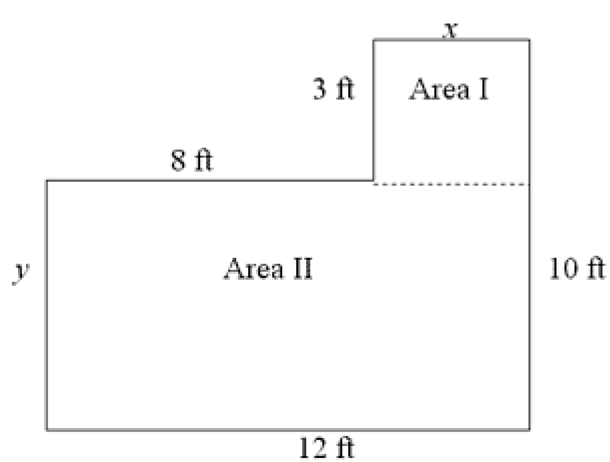
6.	__________ m	19.4 m	__________ m^2
7.	__________ km	__________ km	38.47 km^2
8.	8.5 ft	__________ ft	__________ ft^2
9.	__________ yd	__________ yd	408.07 yd^2
10.	9.2 m	18.4 m	__________ m^2
11.	$7\frac{1}{2}$ mm	__________ mm	__________ mm^2
12.	$7\frac{3}{5}$ ft	$15\frac{1}{5}$ ft	__________ ft^2
13.	__________ cm	$10\frac{4}{5}$ cm	__________ cm^2
14.	$5\frac{1}{10}$ in	$10\frac{1}{5}$ in	__________ in^2
15.	16.73 mi	33.46 mi	__________ mi^2

In some problems you may be asked to find the area of irregular shapes. These figures are often made up of two or more simple figures.

Example Find the area of the figure shown below.



Step 1 Separate the figure into two familiar shapes---in this case rectangles. Decide what measurements you are missing and label them (x and y in this figure).



Step 2 Find the missing lengths of the sides by subtracting values you do know.

Side x is 12 ft – 8 ft = 4 ft

Side y is 10 ft – 3 ft = 7 ft

Step 3 Find the area of each rectangle by using the correct formula. First replace l with 4 and w with 3 in the formula $A = lw$. Then replace l with 12 and w with 7 in the formula $A = lw$.

Area I = lw

= 4 x 3

= 12 sq ft

Area II = lw

= 12 x 7

= 84 sq ft

Step 4 Add the areas of the two rectangles.

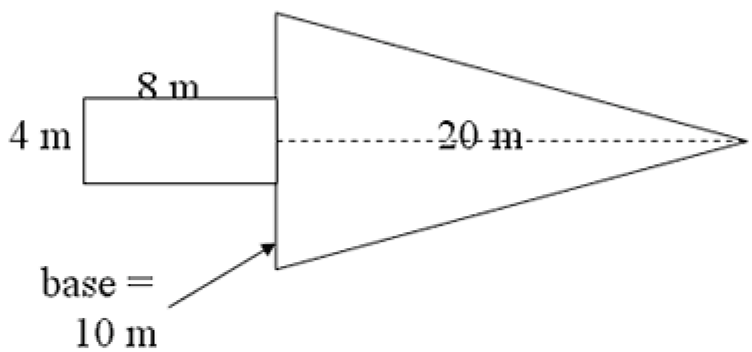
Area I + Area II = 12 + 84 = 96 sq ft

Answer: The total area of the figure is **96 sq ft.**

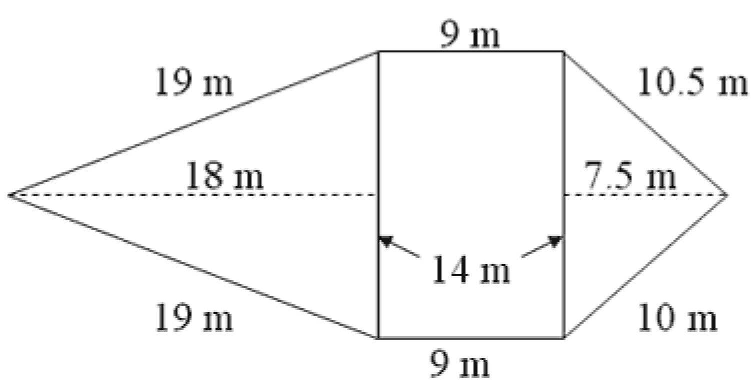
Practice Exercise

Find the area of each figure.

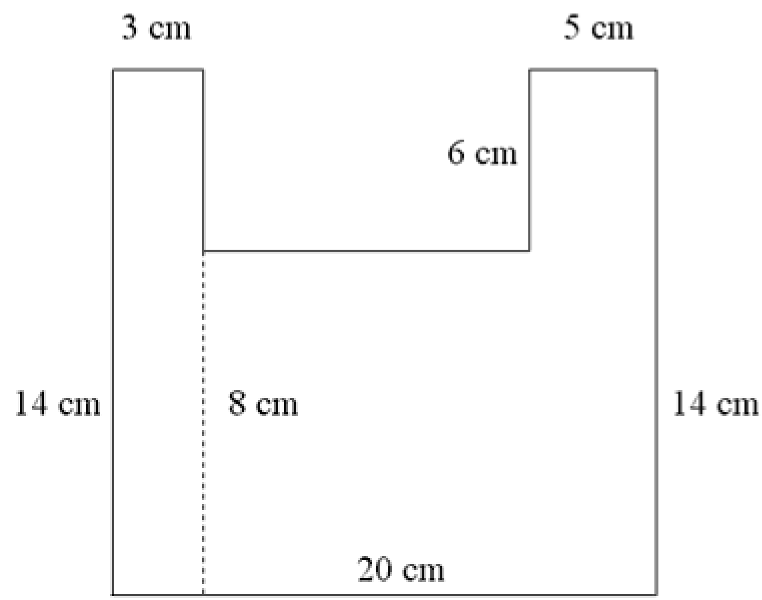
1.



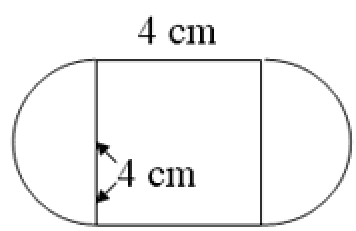
2.



3.



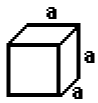
4.



Surface Area

The ***surface area*** is the sum of the areas of the outside surface of a three-dimensional shape.

Surface Area of a Cube = 6 a^2



(a is the length of the side of each edge of the cube)

In words, the surface area of a cube is the area of the six squares that cover it. The area of one of them is aa, or a^2. Since these are all the same, you can multiply one of them by six, so the surface area of a cube is 6 times one of the sides squared.

Example Find the total area of a cube whose edges measure 15 inches.

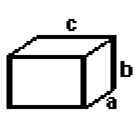
$A = 6a^2$

$A = 6 \times (15)^2$

$A = 6 \times 225$

A = 1,350 square inches.

Surface Area of a Rectangular Prism = 2ab + 2bc + 2ac



(a, b, and c are the lengths of the 3 sides)

In words, the surface area of a rectangular prism is the area of the six rectangles that cover it. But we don't have to figure out all six because we know that the top and bottom are the same, the front and back are the same, and the left and right sides are the same.

The area of the top and bottom (side lengths a and c) = ac. Since there are two of them, you get 2ac. The front and back have side lengths of b and c. The area of one of them is bc, and there are two of them, so the surface area of those two is 2bc. The left and right side have side lengths of a and b, so the surface area of one of them is ab. Again, there are two of them, so their combined surface area is 2ab.

Example Find the total area of a rectangular prism 9 meters long, 6 meters wide, and 7 meters high.

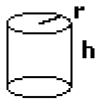
$A = 2ab + 2bc + 2ac$

$A = 2 \times 9 \times 6 + 2 \times 9 \times 7 + 2 \times 6 \times 7$

$A = 108 + 126 + 84$

$A = 318$ square meters.

Surface Area of a Cylinder = 2(*pi* r ^{2}) + (2 pi r)h



(h is the height of the cylinder, r is the radius of the top)

Surface Area = Areas of top and bottom + Area of the side

Surface Area = 2(Area of top) + (circumference of top) x height

Surface Area = $2(\pi r^{2}) + (2\pi r)h$

In words, the easiest way is to think of a can. The surface area is the areas of all the parts needed to cover the can. That's the top, the bottom, and the paper label that wraps around the middle.

You can find the area of the top (or the bottom). That's the formula for area of a circle (πr^2). Since there is both a top and a bottom, that gets multiplied by two.

The side is like the label of the can. If you peel it off and lay it flat it will be a rectangle. The area of a rectangle is the product of the two sides. One side is the height of the can, the other side is the circumference of the circle, since the label wraps once around the can. So the area of the rectangle is $(2\pi r)h$.

Add those two parts together and you have the formula for the surface area of a cylinder.

Surface Area = $2(\pi r^{2}) + (2 \pi r)h$

Example Find the total area of a cylinder whose radius is 21 feet and whose height is 30 feet.

$A = 2(\pi r^{2}) + (2 \pi r)h$

$A = 2 \times 22/7 \times (21)^{2} + 2 \times 22/7 \times 21 \times 30$

$A = 2{,}772 + 3{,}960$

$A = 6{,}732$ square feet

Practice Exercise

Find the total areas of each of the following rectangular prisms.

1. Find the total area of a rectangular prism measuring 4 feet by 3 feet by 2 feet.

2. Find the total area of a rectangular prism measuring 18 meters by 8 meters by 4 meters.

3. Find the total area of a rectangular prism measuring 20 centimeters by 14 centimeters by 11 centimeters.

4. Find the total area of a rectangular prism measuring 6.4 meters by 3.7 meters by 4.5 meters.

Find the total area of each cube.

1. Find the total area of a cube whose edges measure 4 centimeters.

2. Find the total area of a cube whose edges measure 1.5 feet.

3. Find the total area of a cube whose edges measure 5.6 meters.

4. Find the total area of a cube whose edges measure 2 inches.

Find the total area of each cylinder.

1. Find the total area of a cylinder whose diameter is 4 inches and whose height is 10 inches.

2. Find the total area of a cylinder whose radius is 4 inches and whose height is 7 inches.

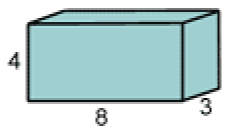
3. Find the total area of a cylinder whose radius is 20 centimeters and whose height is 25 centimeters.

4. Find the total area of a cylinder whose radius is 6 meters and whose height is 4 meters.

Calculating Volume

Volume is the amount of space contained in a three-dimensional shape. Area is a measurement of only ***two*** dimensions, usually length and width. Volume is a measurement of ***three*** dimensions, usually ***length***, ***width***, and ***height***, and is measured in cubic units.

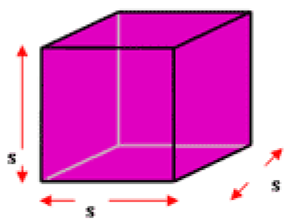
To find the volume of a ***cube*** or a ***rectangular prism***, multiply length by width by height.



l x w x h = volume of a rectangular prism

8 x 3 x 4 = 96

Since a cube has sides of equal length, multiply the length of one side by itself three times, S^3 :

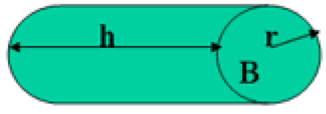


S^3 = volume of a cube

To find the volume of a ***cylinder***, multiply the area of the base (B) (or πr^2) by the height of the cylinder.

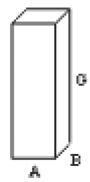
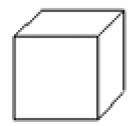
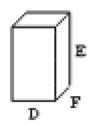
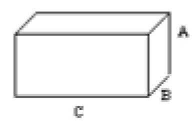
B x h = volume of a cylinder

πr^2 **x h =**



Practice Exercise

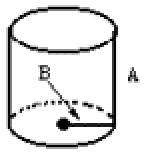
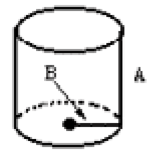
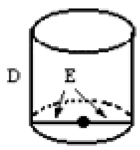
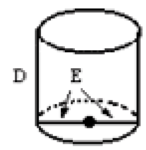
Find the volume.

1. 	A = 15 mm B = 8 mm G = 32 mm __________	2. 	All sides are 7 m __________
3. 	D = 26 yd E = 10 yd F = 4 yd __________	4. 	A = 8 mm B = 3 mm C = 18 mm __________

Fill in the missing spaces and complete the table.
Round to the nearest hundredth.

	length	*width*	*height*	*volume*	
5.	16 mm	5 mm	10 mm	_______	cubic millimeters
6.	3 cm	15 cm	7 cm	_______	cubic centimeters
7.	28 mm	26 mm	8 mm	_______	cubic millimeters
8.	48 yd	24 yd	32 yd	_______	cubic yards
9.	11 mm	_____ mm	10 mm	990	cubic millimeters
10.	_____ m	6 m	9 m	270	cubic meters
11.	6 in	13 in	_____ in	234	cubic inches
12.	3 ft	16.1 ft	14 ft	_______	cubic feet
13.	10 mm	15 mm	11.3 mm	_______	cubic millimeters
14.	12.41 yd	8.39 yd	14 yd	_______	cubic yards
15.	9.02 in	14 in	11.45 in	_______	cubic inches

Find the volume.
Use 3.14 for π. Round to the nearest hundredth.

1.  B = 4 in A = 8 in	2.  B = 4 cm A = 6 cm
3.  B = 5 mm A = 8 mm	4.  E = 10 in D = 17 in

Fill in the missing spaces and complete the table.
Use 3.14 for π. Round to the nearest hundredth.

	diameter	*radius*	*height*	*volume*	
5.	8 ft	4 ft	9 ft	**452.16**	cubic feet
6.	_____ ft	8 ft	4 ft	_______	cubic feet
7.	14 in	7 in	3 in	_______	cubic inches
8.	30 ft	15 ft	9 ft	_______	cubic feet
9.	10 cm	5 cm	_____ cm	785	cubic centimeters
10.	12 ft	6 ft	_____ ft	452.16	cubic feet
11.	32 ft	16 ft	8.6 ft	________	cubic feet
12.	_____ in	13 in	5.6 in	_______	cubic inches
13.	_____ ft	11.5 ft	10.9 ft	_______	cubic feet
14.	19 yd	9.5 yd	8 yd	_______	cubic yards
15.	19.6 cm	9.8 cm	11 cm	_______	cubic centimeters

FORMULAS	
Perimeter	
Polygon	P = sum of the lengths of the sides
Rectangle	$P = 2(l + w)$
Circumference	
Circle	$C = 2\pi r$, or $C = \pi d$
Area	
Circle	$A = \pi r^2$
Parallelogram	$A = bh$
Rectangle	$A = lw$
Square	$A = s^2$
Triangle	$A = \frac{1}{2}bh$
Rectangular Prism	$A = 2ab + 2bc + 2ac$
Cube	$A = 6a^2$
Cylinder	$A = 2(\pi r^2) + (2\pi r)\text{h}$
Volume	
Cube	$V = s^3$
Cylinder	$V = Bh$, or $V = \pi r^2 h$
Prism	$V = Bh$

Problem Solving Involving Measurement

1. Jeff and Mary are remodeling their home. On the roof, Jeff wants to put shingles on a rectangle 25 feet by 20 feet. What is the area of that part of the roof? What is the perimeter of that part of the roof?

2. Jeff bought enough shingles to cover 575 square feet. How many shingles will be left over after he covers the 25-foot by 20-foot rectangle on the roof?

3. Mary is installing new windows. Each window is 2 feet by 4 feet. She installed 120 square feet of windows. How many windows did she install?

4. If 6 people share a large pizza that has a 14 inch diameter, how much pizza will each person get? Give your answer to the nearest square inch.

5. Bill has a job with a company that delivers ice. He delivers 25-pound cubes of ice to various restaurants in town. Each side of each cube is 300 millimeters. What is the volume of one cube of ice?

6. A cylindrical water tower is half full of water. How many cubic feet of water are in the tower if it measures 20 feet high and has a radius of 7 feet?

7. Gloria has a new bathtub. The bathtub is 60 inches long, 31 inches wide, and 20 inches high. If she fills the tub full, what is the volume of water in the tub?

8. A swimming pool cover measures 5 meters long by 3 meters wide. At $5.25 per square meter, what is the cost of the cover?

9. Lou made a round oak table that he wants to cover with glass. How many square feet of glass are needed if the distance across the center of the table is 4 feet?

10. Jim cut a piece of plywood into the shape of a triangle. To the nearest square foot, find the area of the plywood if the base measures 8 feet and the height measures 6¼ feet.

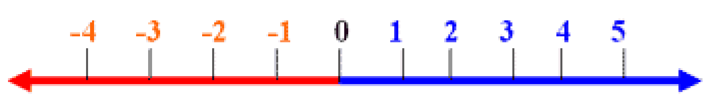
Integers

Introduction to Integers

The set of ***integers*** includes ***0***, all of the counting numbers (called ***positive*** whole numbers), and the whole numbers less than ***0*** (called ***negative*** numbers). Integers are shown below on a number line.

⇒ ***All counting numbers and whole numbers are integers.***

Negative Numbers   Positive Numbers



Numbers less than 0 are negative numbers. Numbers greater than 0 are positive.

Number lines show numbers in order. If you follow the number line to the right, the numbers get larger and larger.

If you follow the number line to the left, the numbers get smaller and smaller.

⇒ ***To remember the order of negative and positive numbers on a number line, think of the alphabet (n,o,p = negative, zero, positive).***

It's important to understand the number line because it shows you that every number has an opposite. The famous German mathematician Leopold Kronecker once said: "God made the positive integers; everything else is the work of man." Why, then, did we confuse things with negative numbers? As it turns out, there are many, many everyday problems where negative numbers are useful. For example, we can both gain and lose weight.

The temperature can rise or fall. Locations on the earth can be above sea level, or below sea level.

Integers can be understood both as signs of operation and signs of quantity.

Examples +5 stands for 0 + 5 (operation), but it can also stand for a positive amount or a gain as in measurements such as temperature or weight (quantity).

-5 stands for 0 – 5 (operation), but it can also stand for a negative amount or a drop as in measurements such as temperature or weight (quantity).

Practice Exercise

Write an integer for each description.
(Hint: include a negative sign for a description that is below zero)

1.	**137**	A deposit of $137
2.	______________________	17 units to the left of 1 on a number line
3.	______________________	19 units to the right of -5 on a number line
4.	______________________	Withdraw $268 from an ATM machine
5.	______________________	The opposite of -16
6.	______________________	10 units to the right of 4 on a number line
7.	______________________	A loss of 19 pounds
8.	______________________	42 degrees below zero

9. ________________________ A profit of $210 dollars

10. ________________________ An altitude of 9000 ft

11. ________________________ 8 yards short for first down

12. ________________________ The stock market went up 291 points today

13. ________________________ A gain of 18 yards

14. ________________________ 582 ft below sea level

15. ________________________ The temperature dropped 20 degrees overnight

16. ________________________ A gain of 20 pounds

17. ________________________ The opposite of 17

18. ________________________ 4 inches taller

1.	4	<	6	2.	2	________	-4
3.	-4	________	-4	4.	1	________	0
5.	-5	________	7	6.	20	________	-10
7.	-35	________	-35	8.	42	________	-42
9.	22	________	11	10.	22	________	-21
11.	22	________	-99	12.	12	________	44
13.	-15	________	-32	14.	33	________	17
15.	-66	________	32	16.	-14	________	-26
17.	-78	________	-78	18.	-14	________	-11
19.	-5	________	5	20.	24	________	-6
21.	73	________	-86	22.	15	________	-15
23.	-17	________	-39	24.	-50	________	11
25.	2	________	-11	26.	-81	________	-81
27.	-61	________	59	28.	47	________	54
29.	23	________	25	30.	-44	________	-17
31.	-13	________	11	32.	-88	________	18
33.	-3	________	10	34.	17	________	-78
35.	-68	________	-68	36.	-45	________	45

Adding and Subtracting Integers

Adding and subtracting positive integers works the same way as adding and subtracting whole numbers. Adding and subtracting negative numbers works differently.

When you add a negative integer to a positive integer, you are actually subtracting the value of the negative integer from the positive integer.

4 + -2 = 4 – 2 = 2
7 + 3 + -2 = 7 + 3 – 2 = 8
11 + -6 + 4 + -2 = 11 – 6 + 4 – 2 = 7

When you add a negative integer to another negative integer, you add the values of the integers and then add a negative sign in front of them.

⟹ ***Adding a negative number to another negative number results in a sum less than either negative addend.***

-4 + -2 = - (4 + 2) = -6
-7 + -3 = - (7 + 3) = -10
-11 + -6 + -4 + -2 = - (11 + 6 + 4 + 2) = -23

⇒ ***Adding a positive number to a positive number always results in a sum greater than either addend.***

2 + 3 = 5

Addition Property of Opposites

The property which states that the sum of a number and its opposite is zero

Examples:

$5 + {}^{-}5 = 0$ $\qquad$ ${}^{-}15 + 15 = 0$

When you subtract a negative number from a negative integer, you are actually adding a positive integer to the negative integer.

-4 - -2 = -4 + 2 = -2
-7 - -3 - -2 = -7 + 3 + 2 = -2
-11 - -6 - -4 - - 2 = -11 + 6 + 4 + 2 = 1

When you subtract a positive integer of greater value from another positive integer, the difference will be a negative integer.

2 – 4 = -2
3 – 7 – 2 = -6
11 – 6 – 4 – 2 = -1

Practice Exercise

1.	-13 - 4	=	-17	2.	3 + -7	=	______
3.	7 + -5	=	______	4.	3 - (-4)	=	______
5.	11 + 5	=	______	6.	18 - (-8)	=	______
7.	-17 + 8	=	______	8.	16 + -1	=	______
9.	-9 - (-8)	=	______	10.	12 - 11	=	______
11.	6 + -2	=	______	12.	-6 - (-5)	=	______
13.	15 + 7	=	______	14.	-4 - (-1)	=	______
15.	17 + -11	=	______	16.	4 + -10	=	______
17.	-15 + -12	=	______	18.	-11 - 4	=	______
19.	-15 + 12	=	______	20.	6 - (-13)	=	______
21.	13 - 13	=	______	22.	19 + -4	=	______
23.	-19 - 9	=	______	24.	-47 + -7	=	______
25.	26 - 49	=	______	26.	18 + -13	=	______
27.	-46 - (-9)	=	______	28.	-21 + 9	=	______
29.	36 + -13	=	______	30.	15 - (-11)	=	______
31.	1 - 22	=	______	32.	-27 + 15	=	______
33.	19 + -9	=	______	34.	500 - 130	=	______
35.	8 + -1	=	______	36.	-250 - (-313)	=	______

Multiplying and Dividing Integers

Multiplying and dividing integers works the same way as multiplying and dividing whole numbers, unless one or more of the integers is a negative number.

The product or quotient of a positive integer multiplied or divided by another positive integer will always be a positive integer. Positive integers may or may not be written with a positive sign: +8 = 8. Negative integers are ***always*** written with the minus sign.

4 x 2 = 8
24 ¸ 6 = 4

The product or quotient of a positive integer multiplied or divided by a negative integer will always be a negative integer.

4 x –2 = -8
24 ¸ -6 = -4

The product or quotient of a negative integer multiplied or divided by a positive integer will always be a negative integer.

-4 x 2 = -8
-24 ¸ 6 = -4

The product or quotient of a negative integer multiplied or divided by a negative integer will always be a positive integer.

-4 x –2 = 8
-24 ¸ -6 = 4

The rule for multiplying or dividing integers is if the signs are the same, the answer is positive. If the signs are different, the answer is negative.

⇒ ***Remember:***

positive x positive or positive ÷ positive = positive
positive x negative or positive ÷ negative = negative
negative x positive or negative ÷ positive = negative
negative x negative or negative ÷ negative = positive

Practice Exercise

1.	**-21 ÷ -3 =**	**7**	**2.**	**11 × -11 =**	
3.	**72 ÷ -8 =**		**4.**	**-72 ÷ -9 =**	
5.	**-9 × -10 =**		**6.**	**12 × -4 =**	
7.	**40 ÷ -4 =**		**8.**	**7 × -10 =**	
9.	**28 ÷ -4 =**		**10.**	**-3 × 10 =**	
11.	**-20 ÷ 4 =**		**12.**	**-4 × -9 =**	
13.	**-66 ÷ 11 =**		**14.**	**-12 × -9 =**	
15.	**-6 × -5 =**		**16.**	**5 × -11 =**	
17.	**162 ÷ 9 =**		**18.**	**-24 ÷ 2 =**	
19.	**14 × 6 =**		**20.**	**63 ÷ -7 =**	
21.	**-33 ÷ -11 =**		**22.**	**-6 × 3 =**	
23.	**-4 × -2 =**		**24.**	**-48 ÷ -8 =**	
25.	**-133 ÷ 19 =**		**26.**	**11 × -20 =**	
27.	**69 ÷ 23 =**		**28.**	**-5 × -13 =**	
29.	**1222 ÷ 26 =**		**30.**	**-3 × -13 =**	
31.	**-10 × -6 =**		**32.**	**6 × -18 =**	
33.	**-22 × -9 =**		**34.**	**45 ÷ -15 =**	
35.	**39 × 5 =**		**36.**	**100 ÷ -10 =**	

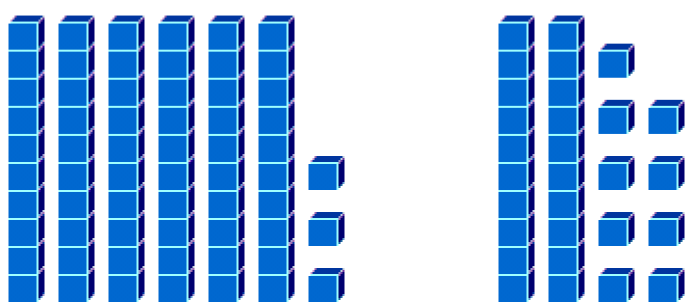
Equations and Inequalities

Introduction to Equations: Equalities and Inequalities

Ordering numbers means listing numbers from least to greatest, or from greatest to least. Two symbols are used in ordering.

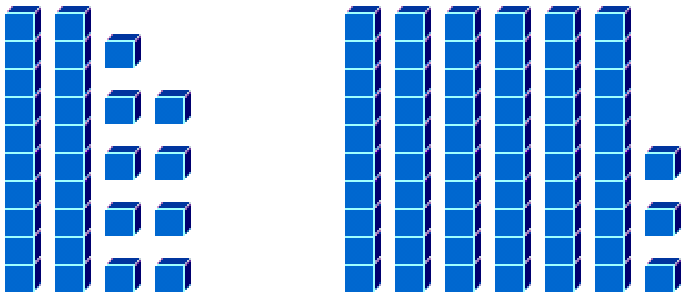
<	>
is less than	**is greater**
2 < 10	**10 > 2**

Greater Than >



63 is **greater than** 29.
63 > 29

Less Than <



29 is **less than** 63.

29 < 63

Sometimes numbers in a set can be "***greater than or equal to***" members of another set. Likewise, members of a set are sometimes "***less than or equal to***" members of another set. A bar is added to ***less than*** and ***greater than*** symbols to show that they are also equal.

≤	≥
is less than or equal to	**is greater than or equal to**

Algebraic or number sentences use the symbols =, ≠, <, >, ≤, or ≥ to show the relationship between two quantities.

Any sentence using the symbol = is called an ***equality*** or ***equation.***

$$\mathbf{4} + \mathbf{8} = \mathbf{2} \times \mathbf{6}$$
$$\mathbf{3x} \div \mathbf{2} = \mathbf{17}$$

Any sentence using the symbol ≠, <, >, ≤, or ≥, is called an ***inequality***.

$$\mathbf{15} > \mathbf{7}$$
$$\mathbf{6} \neq \mathbf{3} + \mathbf{1}$$
$$\mathbf{x} + \mathbf{2} \leq \mathbf{12}$$

SYMBOLS	
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$\sqrt{\ }$	positive square root
$\neq$	is not equal to
+	plus, add
-	minus, subtract
X	multiplied by, multiply
.	Multiplied by, multiply
$\div$	divided by, divide
=	equal to

Algebra is a division of mathematics designed to help solve certain types of problems quicker and easier. Algebra operates on the idea that an equation represents a scale such as the one shown above. Instead of keeping the scale balanced with weights, we use numbers, or constants. These numbers are called constants because they constantly have the same value. For example the number 47 always represents 47 units or 47 multiplied by an unknown number. It never represents another value.

In algebra, we often use letters to represent numbers. A letter that stands for a number is called a ***variable*** or ***unknown***.

A variable can be used to represent numbers in addition, subtraction, multiplication, or division problems. The symbols used in algebra are "+" for addition and "-" for subtraction. Multiplication is indicated by placing a number next to a variable; no multiplication sign is used. Division is indicated by placing a number or variable over the other.

An equation is made up of ***terms***. Each term is a number standing alone or an unknown multiplied by a ***coefficient*** (i.e. $7a$, $5x$, $3y$…).

For example, $3y + 5y = 32$ would be considered a three term equation. $12y – 11y – 9 = 17$ would be considered a four term equation.

Factors are numbers that are multiplied together. For instance, the factors of 12 are 3 and 4, 2 and 6, and 1 and 12. In the algebraic term, $7x$, 7 and x are factors.

An ***algebraic expression*** consists of two or more numbers or variables combined by one or more of the operations---addition, subtraction, multiplication, or division.

The following are examples of algebraic expressions:

Operation	Algebraic Expression	Word Expression
Addition	$x + 2$	x plus 2
Subtraction	$y - 3$	y minus 3
	$3 – y$	3 minus y
Multiplication	$4z$	4 times z
Division	$n/8$	n divided by 8
	$8/n$	8 divided by n

Many algebraic expressions contain more than one of the operations of addition, subtraction, multiplication, or division. Placing a number or variable outside of an expression in parentheses (brackets) means that the whole expression is to be multiplied by the term on the outside.

For example, look at the difference in meaning between $3y + 7$ and $3(y + 7)$. If the number 2 were substituted for y in each expression, the following solutions would result:

$$3y + 7 = 3 \cdot 2 + 7 = 6 + 7 = \mathbf{13}$$

but

$$3(y + 7) = 3(2 + 7) = 3(9) = \mathbf{27}$$

Algebraic Expression	Word Expression
$3y – 7$	3 times y minus 7
$3(y – 7)$	3 times the quantity y minus

	7
- $x + 5$	Negative x plus 5
- $(x + 5)$	Negative times the quantity x plus 5
$3x^2 + 2$	3 x-squared plus 2
$3(x^2 + 2)$	3 times the quantity x-squared plus 2

Practice Exercise

Express each of the following problems algebraically. (Hint: Use n as the unknown number and create an equation from the problem)

1. The product of 8 and a number is 24 **8n = 24**	2. A number minus 57 is -3
3. Twice the sum of a certain number and 97 is 214	4. A number increased by 20 is 111
5. eleven less than eleven times what number is 33	6. 5 more than 3 times a number is 23
7. eight times a number, less 3, is 77	8. 20 less than a number equals -15

9. 11 less than twice a number is 7	10. 5 less than what number equals 62
11. twelve times the sum of a number and 4, is 84	12. ten times what number added to 8 is 38
13. 10 less than the product of 9 and a number is 71	14. The sum of 12 and the product of 6 and a number is 42
15. The sum of 81 and a number is 112	16. four times what number equals 40
17. The sum of what number and seven times the same number is 96	18. One-fifth of a number is 45
19. 73 less than a number equals 19	20. The sum of 8 and the product of 2 and a number is 26

Solving an algebraic equation means finding the value of the unknown or variable that makes the equation a true statement. The ***solution*** is the value of the unknown that solves the equation.

To check if a possible value for the unknown is the solution of an equation, follow these two steps:

Step 1 Substitute the value for the unknown into the original equation.

Step 2 Simplify (do the arithmetic) and compare each side of the equation.

Example 1 Is $y = 5$ the solution for $3y - 9 = 6$?

Step 1 Substitute 5 for *y*. $3(5) - 9 = 6$?

Step 2 Simplify and compare. $15 - 9 = 6$?
$6 = 6$?

Since **6 = 6, *y* = 5 is a solution** of the equation.

Example 2 Is $x = 23$ the solution for $x - 7 = 14$?

Step 1 Substitute 23 for *x*. $(23) - 7 = 14$?
Step 2 Simplify and compare. $16 = 14$?

Since 16 is not equal to 14, ***x* = 23 is not a solution** of the equation.

Sometimes the order in which you add, subtract, multiply, and divide is very important. For example, how would you solve the following problem?

2 x 3 + 6

Would you group

(2 x 3) + 6 or **2 x (3 + 6) ?**

Which comes first, addition or multiplication? Does it matter?
Yes. Mathematicians have written two simple steps:

1. ***All multiplication and division operations are carried out first, from left to right, in the order they occur.***

2. ***Then all addition and subtraction operations are carried out, from left to right, in the order they occur.***

For example:

8 ÷ 2 + 2 x 3 – 1 = 4 + 6 – 1 = 9

4 6 10

step 1 step 2

Þ ***Perform all operations with parentheses (brackets) and exponents before carrying out the remaining operations in an equation. Parentheses or brackets may come in these forms: ( ), { }, or [ ].***

8 ÷ (2 + 2) x 3 – 1 =

8 ÷ 4 x 3 – 1 =

2 x 3 – 1 =

6 – 1 = 5

To remember the order of operations, simply remember BEDMAS: Brackets, Exponents, Division, Multiplication, Addition, Subtraction.

Example:

$10 \div (2 + 8) \times 2^3 - 4$ *Add inside parentheses.*
$10 \div 10 \times 2^3 - 4$ *Clear exponent.*
$10 \div 10 \times 8 - 4$ *Multiply and divide.*
$8 - 4$ *Subtract.*
4

Distributive Property

The distributive property says this: ***Multiplication and addition can be linked together by "distributing" the multiplier over the addends in an equation.***

3 x (1 + 4) = (3 x 1) + (3 x 4)
3 x 5 = 3 + 12
15 = 15

Associative Property of Addition

The property which states that for all real numbers *a*, *b*, and *c*, their sum is always the same, regardless of their grouping:

$$(a + b) + c = a + (b + c)$$

Example:

$$(2 + 3) + 4 = 2 + (3 + 4)$$

Associative Property of Multiplication

The property which states that for all real numbers *a*, *b*, and *c*, their product is always the same, regardless of their grouping:

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Example:

$$(5 \cdot 6) \cdot 7 = 5 \cdot (6 \cdot 7)$$

Simplify

To combine like terms

Example:

Simplify. $n^2 + 4n - 9n + 7 - 5$

$n^2 + 4n - 9n + 7 – 5$ *Collect like terms by using*

$n^2 + (4n - 9n) + (7 - 5)$ *the Associative Property.*

$n^2 + (^-5n) + 2$ *Combine like terms.*

$n^2 - 5n + 2$

Solving Equations using Inverse Operations

Inverse operations are used in algebra to simplify an equation for solving.

One operation is involved with the unknown and the inverse operation is used to solve the equation.

Addition and subtraction are inverse operations.

Given an equation such as 7 + x = 10, the unknown x and 7 are *added*. Use the inverse operation subtraction. To solve for n, subtract 7 from 10. The unknown value is therefore 3.

Multiplication and division are inverse operations.

Given an equation 7x = 21. x and 7 are multiplied to create a value of 21. To solve for x, divide 21 by 7 for an answer of 3.

Examples for addition, subtraction, division, and multiplication.

Addition Problem	Solution
$x + 15 = 20$	$x = 20 - 15 = 5$
Subtraction Problem	Solution
$x - 15 = 20$	$x = 20 + 15 = 35$
Multiplication Problem	Solution
$3x = 21$	$x = 21 \div 3 = 7$
Division Problem	Solution
$x \div 12 = 3$	$y = 3 \text{ x } 12 = 36$

Solving Equations using Rules of Equality

The Rule of Equality
The same fundamental operation can be made on both sides of the equation, using the same number and the equation will remain equal to the original. (This does not apply to division by 0)

This rules means that what you do on one side of an equation, has to be done on the other side. For example, it means you can add 5 to both the left and right side of the equations.

Given: $x - 5 = 40$
1. Add 5 to both sides of the equation which gives you: $x - 5 + 5 = 40 + 5$
2. Simplify for $x - 5 + 5$ and $40 + 5$, to give you the equation $x = 45$

Practice Exercise

Solve each equation.

1. $x + 9 = 86$	**77**	**2.** $38 = a - 57$	______
3. $65 + y = 118$	______	**4.** $x - 34 = 46$	______
5. $x + 42 = 48$	______	**6.** $33 + y = 34$	______
7. $x - 39 = 51$	______	**8.** $13 = a - 3$	______
9. $x + 100 = 114$	______	**10.** $x - 73 = 21$	______
11. $83.1 + y = 141.4$	______	**12.** $x + 47 = 51$	______
13. $25 + y = 77$	______	**14.** $x + 92 = 107$	______
15. $x - 5 = 95$	______	**16.** $31 = a - 36$	______

17. $44 = a - 14$ ______ $18.38 + y = 138$ ______

Solve each equation.

1. $7b = 56$ 8

2. $n \div 21 = 5$ ______

3. $3237 = 39a$ ______

4. $\frac{b}{8} = 11$ ______

5. $2b = 8$ ______

6. $\frac{b}{4} = 5$ ______

7. $6b = 6$ ______

8. $\frac{b}{8} = 7$ ______

9. $n \div 35 = 6$ ______

10. $6818.4 = 94.7a$ ______

11. $2115 = 47a$ ______

12. $4b = 28$ ______

13. $720 = 12a$ ______

14. $\frac{b}{11} = 6.5$ ______

15. **-6130 = 61.3a** ______ **16.** **204 = -34a** ______

17. **6851.7 = 99.3a** ______ **18.** **12b = 120** ______

Solving Equations using Two or More Operations

Using your knowledge of applying the rule of equality and the inverse operations, you can solve more complicated equations that require more than one operation.

For example, $2x + 7 = 11$, cannot be completed in one step. You need to use division and subtraction.

The easy approach for this equation is to subtract 7 on each side which gives you $(2x + 7) - 7 = 11 - 7$, or $2x = 4$. You can then divide each side by 2 which gives you $(2x) \div 2 = 4 \div 2$ or $x = 2$!

Of course you can apply the division to $2x + 7 = 11$ first, which would be $(2x + 7) \div 2 = 11 \div 2$. The problem with that is you have to divide the entire $(2x + 7)$, which cannot be easily simplified. Using subtraction first makes solving the equation much easier.

Terms on Both Sides of an Equation

Terms that contain an unknown can appear on both sides of an equation. Usually, we move all terms containing an unknown to the left side of the equation. To move a term containing an unknown from the right side of the equation to the left side, change its sign and place it next to the unknown on the left side.

Rule 1: If the term is preceded by a positive sign, remove the term from the right side and subtract it from the left side.

Rule 2: If the term is preceded by a negative sign, remove the term from the right side and add it to the left side.

EXAMPLE 1 Solve for x in $3x = 2x + 7$

Step 1 Write down the equation $3x = 2x + 7$. Subtract $2x$ from $3x$.

$$3x = 2x + 7$$
$$3x - 2x = 7$$

Step 2 Combine the x's.

$$\boldsymbol{x = 7}$$

Answer: $\boldsymbol{x = 7}$

EXAMPLE 2 Solve for y in $2y - 6 = -3y + 24$

Step 1 Write down the equation $2y - 6 = -3y + 24$. Add $3y$ to $2y - 6$.

$$2y - 6 = -3y + 24$$
$$2y + 3y - 6 = 24$$

Step 2 Combine the y's.

$$5y - 6 = 24$$

Step 3 Add 6 to 24.

$$5y = 24 + 6$$
$$5y = 30$$

Step 4 Divide 30 by 5.

$$y = 30 \div 5$$
$$\boldsymbol{y = 6}$$

Answer: $\boldsymbol{y = 6}$

Practice Exercise

Solve each problem.

1.	$21 + o = 47$	$o = 26$	2.	$69 - s = 14$	______
3.	$j - 5 = 51$	______	4.	$r \times 10 = 30$	______
5.	$r + 18 = 37$	______	6.	$r + 5 \times 25 \times 4 = 508$	______
7.	$6 \div 3 + g \times 9 = 137$	______	8.	$o \div 4 + 20 = -23$	______
9.	$6 + t = 19$	______	10.	$3 \times 17 \times (p - 13) = 0$	______
11.	$-12 \times g = 84$	______	12.	$p \times 18 = 270$	______
13.	$y + 6 = 14$	______	14.	$o + 14 = 24$	______
15.	$18 + y \times 9 = 117$	______	16.	$(8 + 4) \times f = 168$	______
17.	$-18 \div y \times 15 = 45$	______	18.	$a \times 17 \times 18 = 1224$	______
19.	$21 \times i = 210$	______	20.	$p \times 11 + 4 = 268$	______

Solve each equation.

1. $0.2x = 2$	2. $-6 = 5x - 2x$
3. $2a + 4a + 8 = 44$	4. $\frac{b}{6} = 12$
5. $x - \frac{1}{6}x = 20$	6. $1x = 5$
7. $0.08x = 0.48$	8. $8m + 19 - 3m = 54$
9. $12 = 6x - 4x$	10. $121 = 5x + 6x$
11. $x - \frac{1}{9}x = 16$	12. $0.4x = -3.6$
13. $0.6x = 7.2$	14. $2y + 3 = y - 7$
15. $12x = 108$	16. $3x - 2x + 4 = 7$
17. $0.6x = 6.6$	18. $0.3x = 0.3$

Solve each equation.

1. $\frac{108}{x} = 9$	2. $8x - 17 = 7$
3. $4y + 6 = y + 18$	4. $\frac{b}{4} + 7.8 = 17.8$
5. $\frac{b}{4} + 6 = 17$	6. $2x - 5.3 = 8.7$
7. $15x + 17 - 5x = 3x + 101$	8. $2x + 56 = 66$
9. $\frac{b}{5} - 8 = 2$	10. $22 - \frac{1}{3} b = 13$
11. $3z – z – 5 = z + 8$	12. $9x + 8.4 = 80.4$
13. $2x - 51 + 5x = 19$	14. $10n = 48 - 2n$
15. $39 + \frac{6}{9} b = 87$	16. $7x - 24 = 11$
17. $4x - 66 + 7x = 11$	18. $\frac{18}{x} = 2$

19. $37 + \frac{1}{6}b = 48$	**20.** -8x + 55.4 = -32.6
21. $\frac{b}{6} + 9.9 = 19.9$	**22.** 7x = -27 – 2x
23. $\frac{b}{12} - 8 = 3$	**24.** 5a = -3a + 24

Solving an Equation With Parentheses

Parentheses or *brackets* are commonly used in algebraic equations. Parentheses are used to identify terms that are to be multiplied by another term, usually a number.

The first step in solving an equation is to remove the parentheses by multiplication. Then, combine separated unknowns and solve for the unknown.

Follow these four steps to solve an algebraic equation:

Step 1 Remove parentheses by multiplication.
Step 2 Combine separated unknowns.
Step 3 Do addition or subtraction first.
Step 4 Do multiplication or division last.

To remove parentheses, multiply each term inside the parentheses by the number outside the parentheses. If the parentheses are preceded by a negative sign or number, remove the parentheses by changing the sign of each term within the parentheses.

For example, $+4(x + 3)$ becomes $4 \bullet x + 4 \bullet 3 = 4x + 12$, and $-(3z + 2)$ becomes $-3z - 2$.

EXAMPLE 1 Solve for x in $4(x + 3) = 20$

Step 1 Write down the equation. Remove parentheses by multiplication.

$$4(x + 3) = 20$$
$$4x + 12 = 20$$

Step 2 Subtract 12 from 20.

$$4x = 20 - 12$$
$$4x = 8$$

Step 3 Divide 8 by 4.

$$x = 8 \div 4$$
$$\mathbf{x = 2}$$

Answer: ***x* = *2***

EXAMPLE 2 Solve for z in $4z - (3z + 2) = 5$

Step 1 Write down the equation. Remove parentheses by multiplication.

$$4z - (3z + 2) = 5$$
$$4z - 3z - 2 = 5$$

Step 2 Combine the z's.

$$z - 2 = 5$$

Step 3 Add 2 to 5.

$$z = 5 + 2$$
$$\mathbf{z = 7}$$

Answer: $\mathbf{z = 7}$

Parentheses may appear on both sides of an equation. Remove both sets of parentheses as your first step in solving for the unknown.

EXAMPLE Solve for x in $3(x - 6) = 2(x + 3)$

Step 1 Write down the equation. Remove both sets of parentheses by multiplication.

$$3(x - 6) = 2(x + 3)$$
$$3x - 18 = 2x + 6$$

Step 2 Subtract $2x$ from $3x - 18$.

$$3x - 2x - 18 = 6$$
$$x - 18 = 6$$

Step 3 Add 18 to 6.

$$x = 6 + 18$$
$$\boldsymbol{x = 24}$$

Answer: $\boldsymbol{x = 24}$

Practice Exercise

Solve each of the following equations.

1. $2(a + 3) = 16$
2. $4(b - 2) = 8$
3. $2m - 3(m + 3) = -13$
4. $-3y + 2(2y - 1) = -6$
5. $5(z - 1) = 4(z + 4)$
6. $7(a - 2) = 6(a + 1)$
7. $3(y + 1) = 9 - (y + 2)$
8. $4(z - 2) = -2(z - 5)$

Problem Solving with Equations, Equalities and Inequalities

Equations may be used to solve word problems. To solve a word problem, read the whole problem carefully and then follow these three steps:

Step 1 Represent the unknown with a letter.
Step 2 Write an equation that represents the problem.
Step 3 Solve the equation for the unknown.

EXAMPLE 1 Seven times a number is equal to 147.
What is the number?

Step 1 Let x equal the unknown number.
Step 2 Write an equation for the problem.

$$7x = 147$$

Step 3 Solve the equation. Divide each side by 7.

$$\frac{7x}{7} = \frac{147}{7}$$
$$\mathbf{x = 21}$$

Answer: $\mathbf{x = 21}$
The unknown number is 21.

EXAMPLE 2 Six times a number plus 7 is equal to 55.
What is the number?
Step 1 Let x equal the unknown number.
Step 2 Write an equation for the problem.

$$6x + 7 = 55$$

Step 3 Solve the equation.
a) Subtract 7 from 55.
b) Divide 48 by 6.

$$6x = 55 - 7$$
$$6x = 48$$
$$x = 48 \div 6$$
$$\mathbf{x = 8}$$

Answer: The number is 8.

EXAMPLE 3 Three times the quantity a number minus 4 is equal to two times the sum of the number plus 3. What is the number?

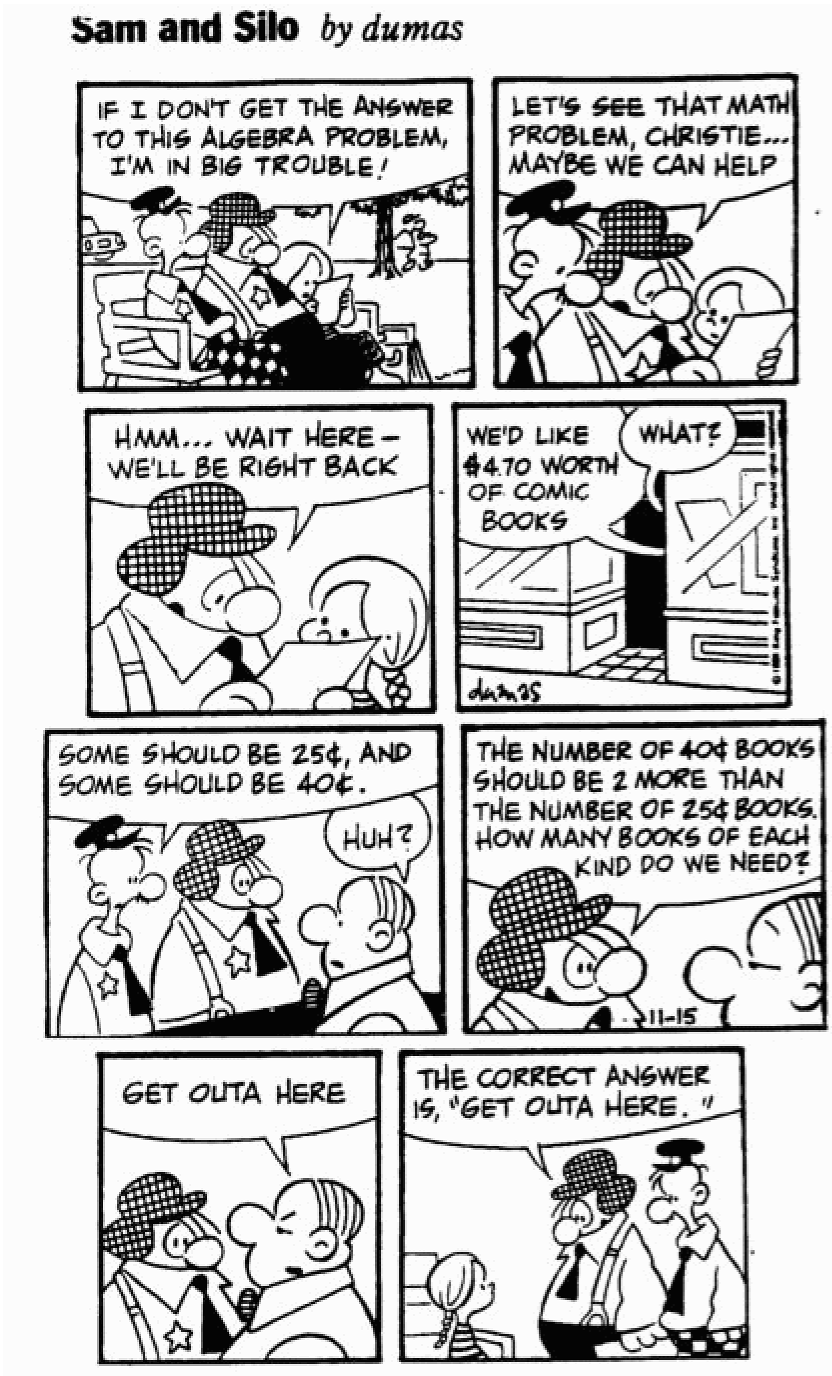
Step 1 Let x equal the unknown number.
$3(x – 4)$ is three times the quantity x minus 4
$2(x + 3)$ is two times the sum of x plus 3
Step 2 Write an equation for the problem.

$$3(x - 4) = 2(x + 3)$$

Step 3 Solve the equation.
a) Remove parentheses.
b) Subtract $2x$ from $3x – 12$.
c) Add 12 to 6.

$$3x - 12 = 2x + 6$$
$$3x - 2x - 12 = 6$$
$$x - 12 = 6$$
$$x = 6 + 12$$
$$\mathbf{x = 18}$$

Answer: The number is 18.



Sam & Silo by *Dumas*

Examples 4, 5 and 6 show how to set up a word or story problem in algebra. Study these carefully.

EXAMPLE 4 Bill saves of his monthly paycheck. If his monthly savings is \$92, how much does he earn each month?

Step 1 Let x = monthly income because this is the unknown quantity that you must find.
\$92 = monthly savings
= fraction saved

Step 2 Write an equation for the problem.
Fraction saved times income = savings

$$x = \$92$$

Step 3 Solve the equation. Multiply each side by 8.

$$(8) \quad x = \$92(8)$$
$$\boldsymbol{x = \$736}$$

Answer: $\boldsymbol{x}$ **= \$736**
Bill earns \$736 each month

EXAMPLE 5 Jack and Steve do yard work. Because Jack provides the truck, gas, and yard equipment, he receives twice the money that Steve does. If they collect \$540, how much does each receive?

Step 1 Let x = Steve's share
$2x$ = Jack's share (We know that Jack receives twice Steve's share.)

Step 2 Write an equation for the problem.
Jack's share plus Steve's share = $540

$$2x + x = 540$$

Step 3 Solve the equation.
a) Combine the *x*'s.
b) Divide 540 by 3.

$$3x = 540$$
$$x = 540 \div 3$$
$$\mathbf{x = \$180}$$
$$\mathbf{2x} = 2(180) = \mathbf{\$360}$$

Answer: $\mathbf{x = \$180}$ **Steve's share**
$\mathbf{2x = \$360}$ **Jack's share**

EXAMPLE 6 Mary, Anne, and Sally share living expenses. Anne pays $25 less rent than Mary. Sally pays twice as much rent as Anne. If the total rent is $365, how much rent does each pay?

Step 1 *Hint:* Since you know nothing about how much Mary pays for rent, let Mary's rent equal *x*.
Let x = Mary's rent
$x - 25$ = Anne's rent
$2(x - 25)$ = Sally's rent

Step 2 Write an equation for the problem.

Mary's + Anne's + Sally's = total rent.

$$x + (x - 25) + 2(x - 25) = 365$$

Step 3 Solve the equation.

a) Remove parentheses.
b) Combine the x's and the numbers.
c) Add 75 to 365.
d) Divide 440 by 4.

$$x + x - 25 + 2x - 50 = 365$$
$$4x - 75 = 365$$
$$4x = 365 + 75$$
$$4x = 440$$
$$x = 440 \div 4$$
$$\mathbf{x = 110}$$

Answer: $\boldsymbol{x}$ **= $110, Mary's rent**

$\boldsymbol{x}$ **- 25 = $85, Anne's rent**

2($\boldsymbol{x}$ **– 25) = $170, Sally's rent**

Practice Exercise

Solve for the number.

1. If 8 is added to a certain number, the sum is 19. What is the number?
2. If a large number is divided by 12, the answer is 121. What is the large number?

3. Joan pays 1/12 of her total monthly income in property taxes. If she paid $45 last month in taxes, what was her monthly income?
4. When a certain number is decreased by 12, the result is 9. Find the number.
5. Jim pays 2/7 of his monthly earnings for the rent of his truck. If his truck rental averages $220 a month, what is his average monthly income?
6. Eight times a number plus 9 is equal to 73. What is the number?
7. Two-thirds of a number plus one-sixth of the same number is equal to 25. What is the number?
8. Susan and Terry run a day-care center. Since they use Susan's house, it was agreed that her share is to be twice Terry's share. If they earn $225, how much is each person's share?
9. Five times a number minus 7 is equal to three times the same number plus 19. What is the number?
10. Lucy earns $400 a month in salary and she receives a commission of $18 for each appliance she sells. If last month Lucy earned a total of $886, how many appliances did she sell?
11. Three times the sum of a number plus 1 is equal to two times the sum of the number plus 4. What is the number?
12. Maria, Amy, and Sarah share food expenses. Amy pays $10 a month less than Maria. Sarah pays twice as much as Amy. If the monthly food bill is $310, how much does each pay?
13. Five times the quantity x minus 1 is equal to three times the quantity x plus 9. What is x?

14. Frank, Sam, and Lou went to lunch. Sam's meal cost $.45 less than Frank's. Lou's meal cost twice as much as Sam's. If the bill came to $9.25, how much does each one owe?
15. Two-thirds times the quantity *y* minus 3 is equal to one-third *y*. What is *y*?

Graphs

Introduction to Graphs

Plotting Information

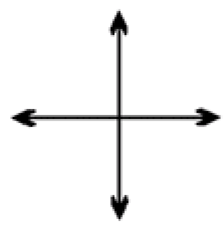
A ***graph*** is a kind of drawing or diagram that shows ***data***, or information, usually in numbers. In order to make a graph, you must first have data.

Making a Coordinate Graph

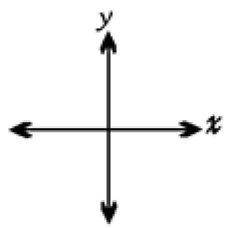
Many graphs show information on a ***grid***. The grid is made up of lines that intersect to create a screen pattern. The bottom line of the grid is called the ***horizontal axis*** and the vertical line on the left or right is called the ***vertical axis***.

The Plane

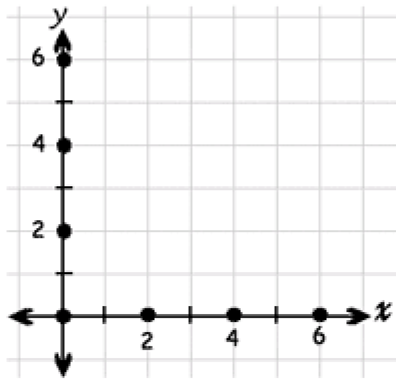
Here is a picture of a plane. Two lines are drawn inside the plane. Each of these lines is an axis. (Together they are called axes.) The axes are like landmarks that we can use to find different places in the plane.



We can label the axes to make them easier to tell apart. The axis that goes from side to side is the x-axis, and the axis that goes straight up and down is the y-axis.



Let's zoom in on one corner of the plane. (This corner is called the first quadrant.)



We have marked some of the points on each axis to make them easier to find. The point where the two axes cross has a special name: it is called the origin.

The gray lines will help us find points. When you make your own graphs, you can use the lines on your graph paper to help you.

Finding Points in the Plane

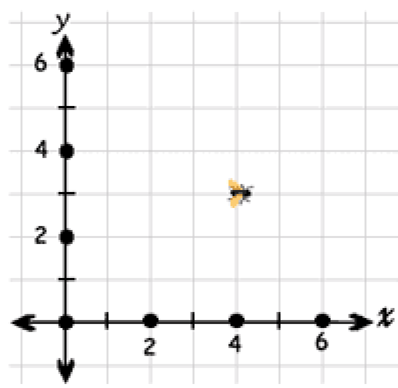
We can find every point in the plane using two numbers. These numbers are called coordinates. We write a point's coordinates inside parentheses, separated by a comma, like this: (5, 6). Sometimes coordinates written this way are called an ordered pair.

> **The first number in an ordered pair is called the** x-coordinate**. The x-coordinate tells us how far the point is along the x-axis.**

The second number is called the y-coordinate**. The y-coordinate tells us how far the point is along the y-axis.**

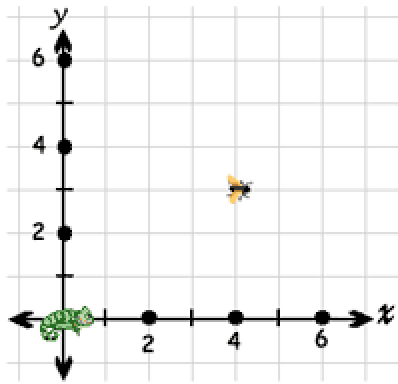
Let's try an example.

A fly is sitting in the plane.

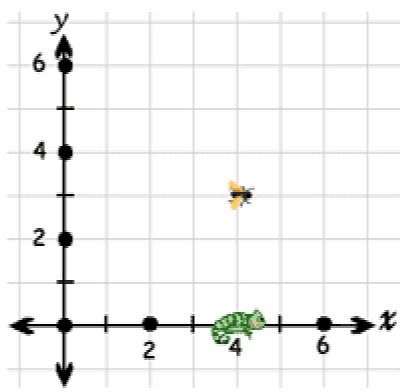


Sam knows that the fly is at point (4, 3). What should he do?

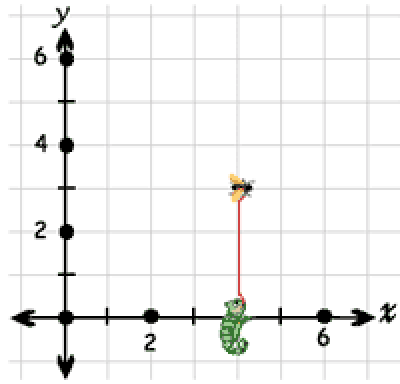
Sam starts at the origin. So far, he has not moved along the x-axis or the y-axis, so he is at point (0, 0).



Because he wants to find (4, 3), Sam moves four units along the x-axis.



Next, Sam turns around and shoots his tongue three units. Sam's tongue goes straight up, in the same direction that the y-axis travels.



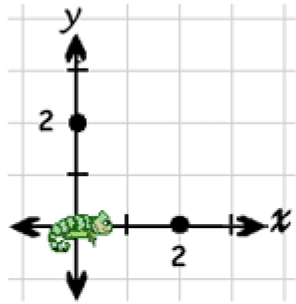
Sam has found point (4, 3). **He eats the fly happily**.

Graphing Points in the Plane

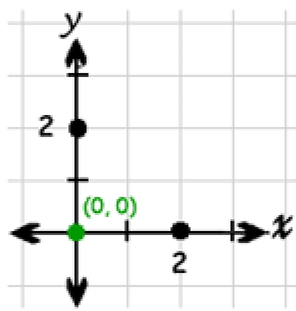
You can graph points the same way that Sam found the fly. Let's practice graphing different points in the plane.

We'll begin by graphing point (0, 0).

Sam starts at the origin and moves 0 units along the x-axis, then 0 units up. He has found (0,0) without going anywhere!



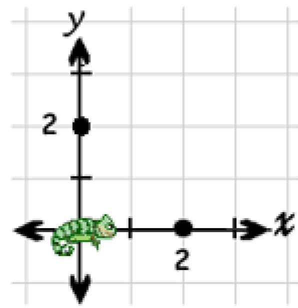
Sam marks the point with a green dot, and labels it with its coordinates.



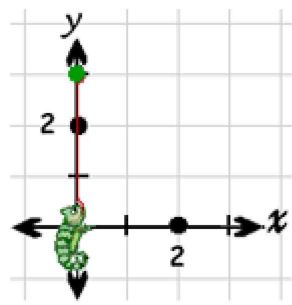
Sam has finished graphing point (0, 0).

Next, let's graph point (0, 3).

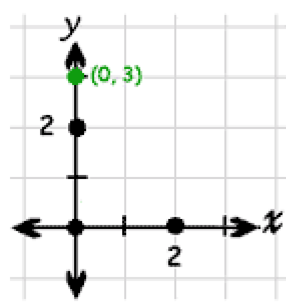
Sam starts at the origin, just like always. He moves 0 units along the x-axis, because the x-coordinate of the point he is trying to graph is 0.



Sam uses his tongue to move a green dot 3 units straight up.



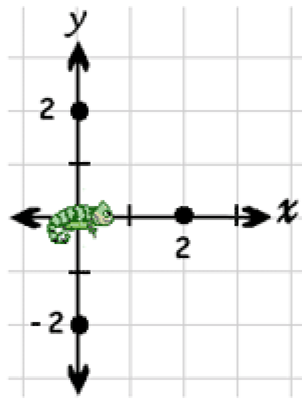
The final step is labeling the point.



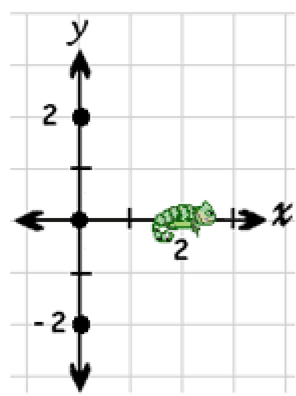
Notice that point (0, 3) is *on the y-axis* and its x-coordinate is 0. Every point on the y-axis has an x-coordinate of 0, because you don't need to move sideways to reach these points. Similarly, every point on the x-axis has a y-coordinate of 0.

Let's end with a more complicated example: graphing point (2, -2).

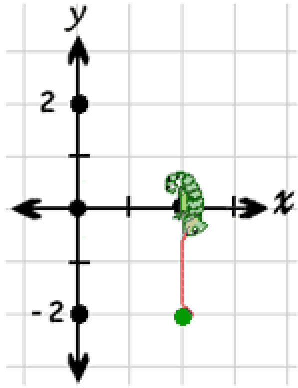
Sam begins at point (0, 0).



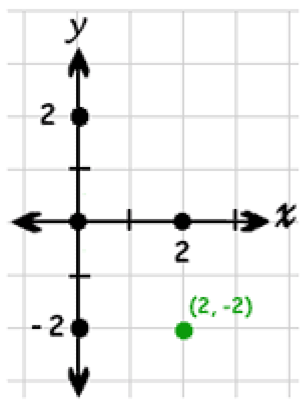
He moves 2 units along the x-axis.



The y-coordinate of the point Sam wants to graph is -2. Because the number is negative, Sam sticks his tongue down two units. This makes sense, because negative numbers are the opposite of positive numbers, and down is the opposite of up.



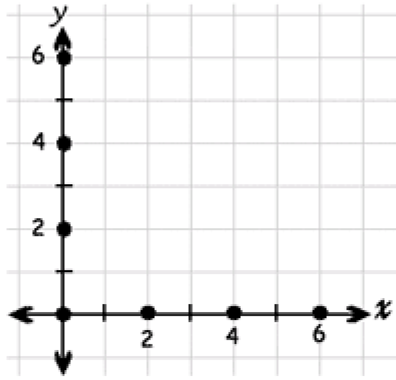
Before he leaves, Sam labels the point he graphed.



Scale

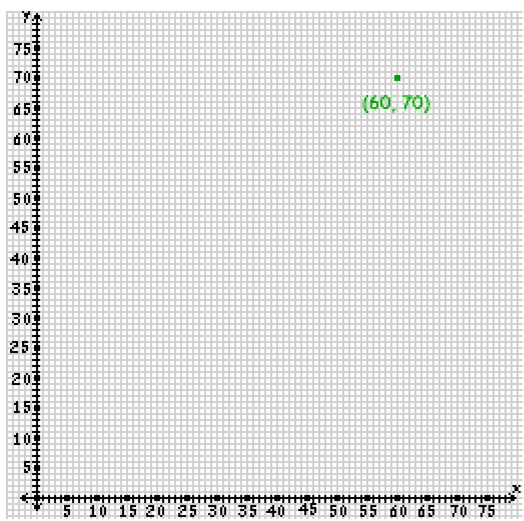
How would you graph the point (60, 70)?

We could start with this graph,



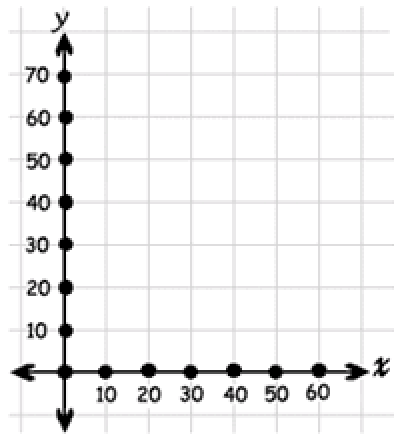
make the x and y axes much longer, and then graph our point. If we tried that, though, the graph would never fit on this screen.

We could try shrinking the axes, and then graphing the point:

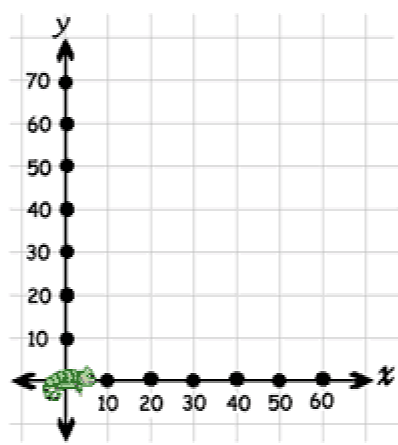


This graph is so small that it is hard to understand.

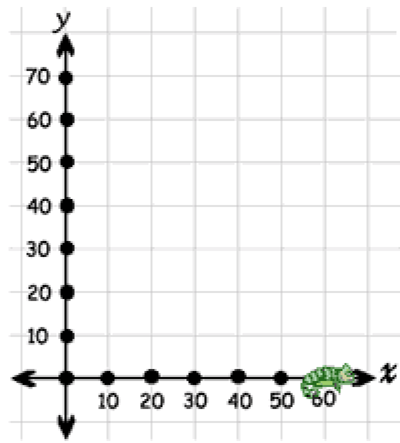
Instead of trying to mark every whole number on the axes, let's count by tens. When we change the distance between points on our graph like this, we say that we are changing the scale of the graph.



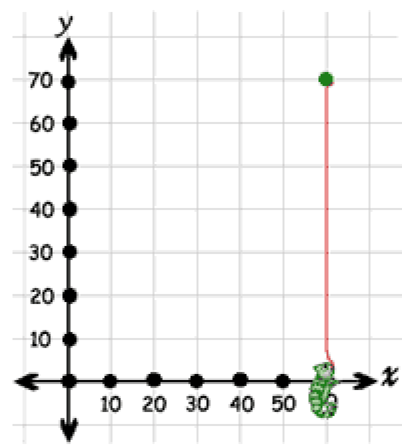
Now, let's watch Sam graph the point (60, 70) on this graph. Sam always starts at the origin.



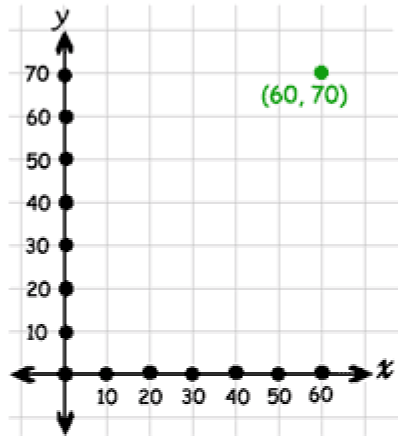
The x-coordinate of the point is 60, so Sam counts to 60 by tens.



Since the point's y-coordinate is 70, Sam must use his tongue to count to 70 by tens, moving straight up.



Before he leaves, Sam labels the point he graphed.



Practice Exercise

Graph the line segment using the following end points.

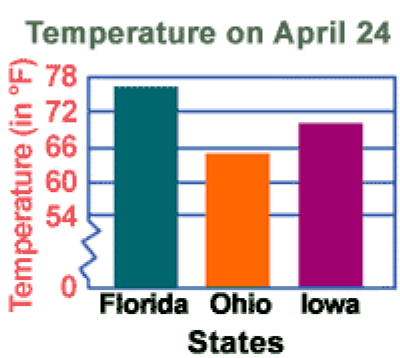
1. (4,-5) and (4,3)	2. (-8,-7) and (-2,-7)
3. (5,-2) and (-2,-2)	4. (-2,2) and (-2,4)
5. (7,2) and (7,-1)	6. (-4,9) and (-1,9)
7. (-1,8) and (2,8)	8. (-1,-2) and (7,-2)
9. (6,-7) and (6,-3)	10. (-2,6) and (-2,0)

11. (16,-17) and (-9,-17)	12. (-17,2) and (-17,18)
13. (-10,11) and (8,11)	14. (4,0) and (16,0)
15. (-8,-4) and (-8,-17)	16. (18,-8) and (18,19)
17. (-5,-19) and (-3,-19)	18. (-6,-10) and (-6,13)

Bar Graph

A graph that uses separate bars (rectangles) of different heights (lengths) to show and compare data. They are generally more complicated to read than other types of graphs.

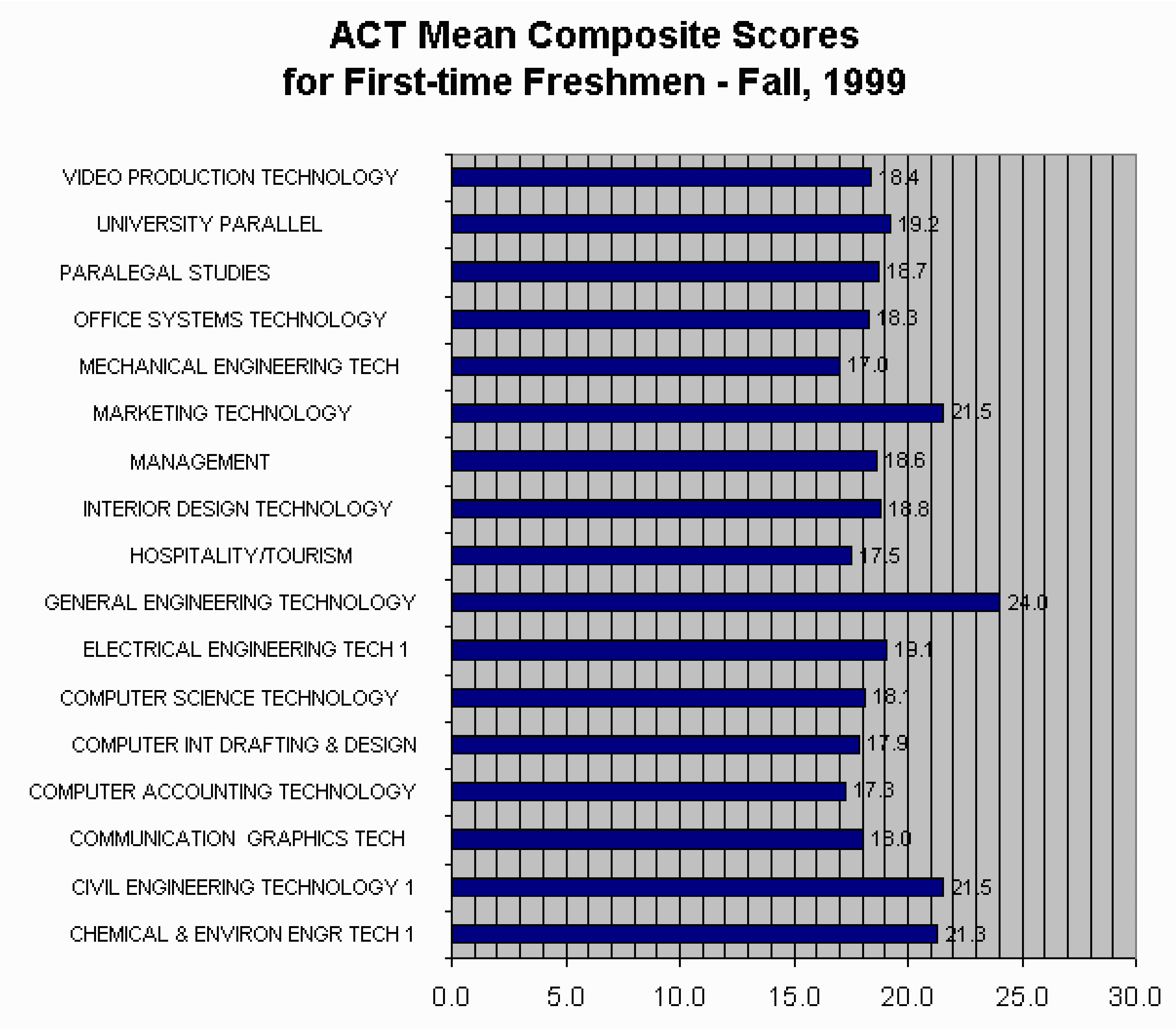
Example:



Bar graphs are usually drawn in one of two different directions:

1) With the bars ***running up and down*** like the graph above. The bars are placed at equal distances along the ***horizontal axis*** that runs across the bottom of the graph.

2) With the bars ***running from left to right*** like the graph on the next page. The bars are placed at equal distances along the ***vertical axis*** on the left side of the graph.



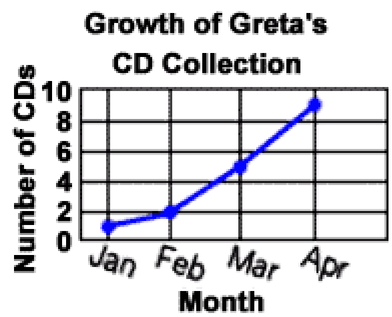
Bar graphs may also use a key to show additional information.

Sometimes, a graph may have a break in the vertical axis and an open space running across the graph. This means that some values have been left off to save space on the graph.

Line Graph

A graph in which line segments are used to show changes over time. Like the bar graph, a line graph is drawn using values along a horizontal and a vertical axis.

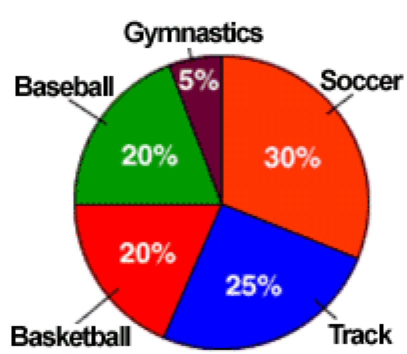
Example:



Circle Graph

A graph using a circle that is divided into pie-shaped sections showing percents or parts of the whole. A part of a circle graph is called a **segment** or a **section** and has its own name and value. The segments of a circle add up to a whole or 100% of the topic.

Example:

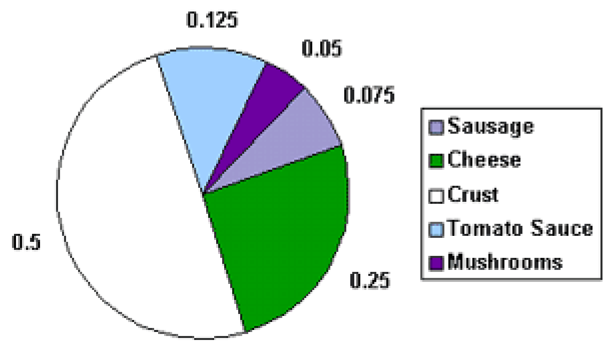


Circle graphs are often used to illustrate budgets and expenses.

Pie Charts

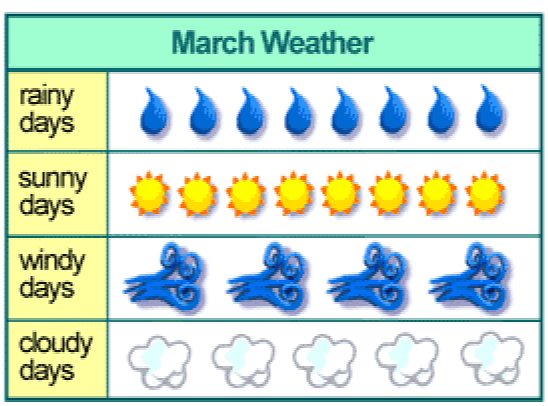
A pie chart is a circle graph divided into pieces, each displaying the size of some related piece of information. Pie charts are used to display the sizes of parts that make up some whole.

The pie chart below shows the ingredients used to make a sausage and mushroom pizza. The fraction of each ingredient by weight is shown in the pie chart below. We see that half of the pizza's weight comes from the crust. The mushrooms make up the smallest amount of the pizza by weight, since the slice corresponding to the mushrooms is smallest. Note that the sum of the decimal sizes of each slice is equal to 1 (the "whole" pizza").



Pictographs (picture graphs) are graphs that use pictures called ***icons*** to display data. Pictographs are used to show data in a small space. Pictographs, like bar graphs, compare data. Because pictographs use icons, however, they also include keys, or definitions of the icons. Parts of symbols are often used to represent a fractional amount of a quantity shown in the key.

Pictographs are often not as exact as other types of graphs, but they are the easiest to read. All you need to do is count the symbols on a line and compute their value.



How to Create Eye-catching, Information-packed Graphs

A woman stands up in a crowded city council meeting and reads the research. "We recently asked a random sampling of 250 citizens how often they use the new toll road. Five percent say they use it four or more times a week, eight percent say they use it one to three times a week, 12 percent say..." and continues on.

The point she would eventually get to, could have been stated in a simple declarative sentence, in half the time, with twice the effect: "We completed some eye-opening research this week--over one third of the people in the city don't use the new toll road because they can't afford it!"

The same is true on paper. Instead of getting mired in statistical detail, you can make your point with a simple, informative graph.

STEP-BY-STEP GRAPHS

Creating graphs is easy if you divide it into a few manageable steps.

STEP 1: *Gather and present accurate information*

You will need to seek out facts and figures from reputable sources. It is imperative, too, that your proportions be reasonably accurate. If, for example, you intentionally exaggerate a bar that represents 50% to look like 60%, you risk the reader dismissing your whole argument because of it. Be sure to add a caption line that credits your source.

Where possible, gather and present your information in a ***table of values***. This table will allow you to organize your data in rows and columns.

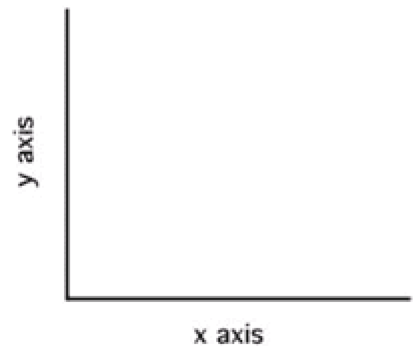
STEP 2: *Focus on a single point*
Don't make the mistake the speaker did. Sort through the details and decide on a single point to be made and organize everything around it. Simplify as much as you can: Consolidate several nonessential categories into one. Choose units of measure that are most easily understood. And round off values.

STEP 3: *Find the most relevant image*
Be obvious--you have just a moment to make the connection with your reader.

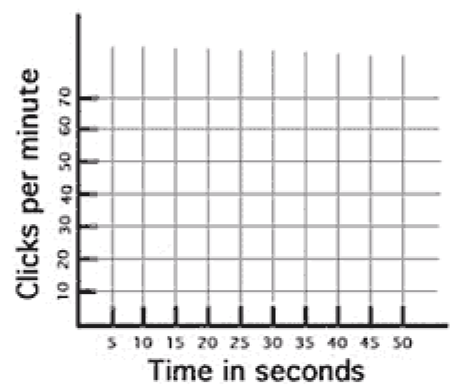
STEP 4: *Use words sparingly*
Your reader should get your point with as few words as possible. Use a title of five words or less to telegraph the theme, and a short subhead to fill in the details. Use labels economically and let the shapes, colors, and proportions do the work.

How to Construct a Line Graph.

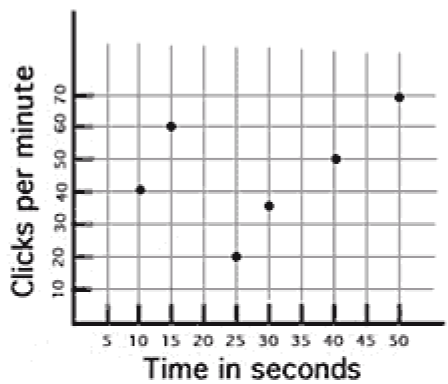
1. Draw a pair of axes (x-axis and y-axis).



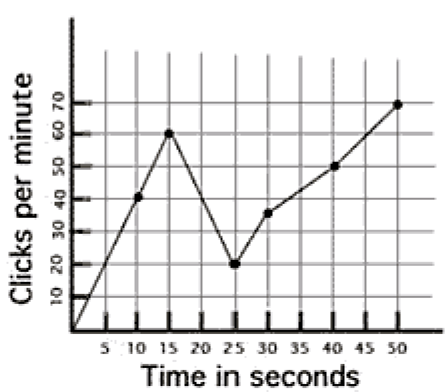
2. Label each axis with a scale.



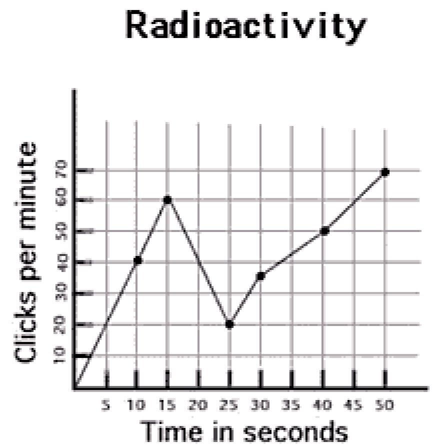
3. Plot the data points for each pair of data. First go over on the x-axis and then the y-axis.



4. After all the data points are plotted, connect them.

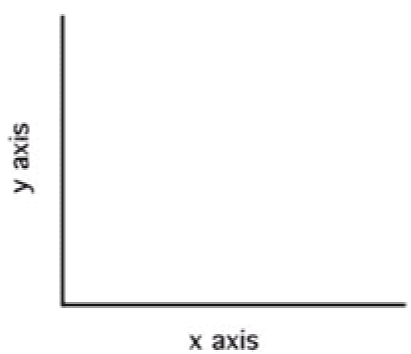


5. Give the graph a title.

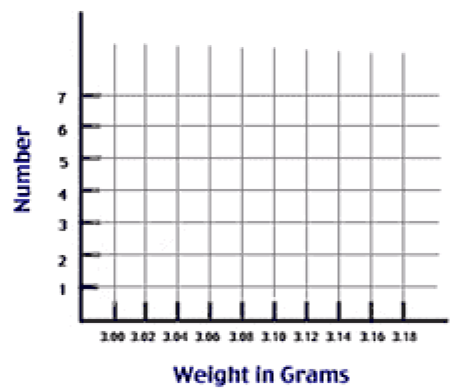


How to Construct a Bar Graph.

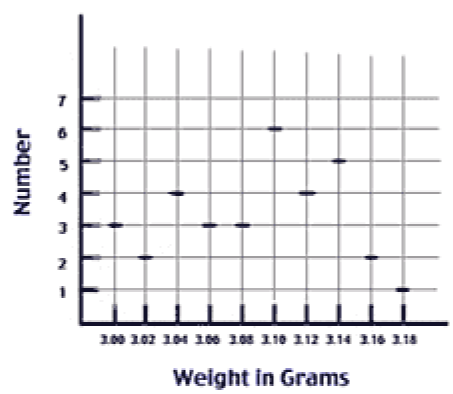
1. Draw a pair of axis (x-axis and y-axis).



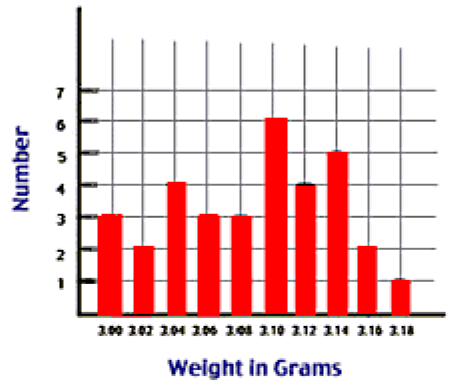
2. Label each axis with a scale.



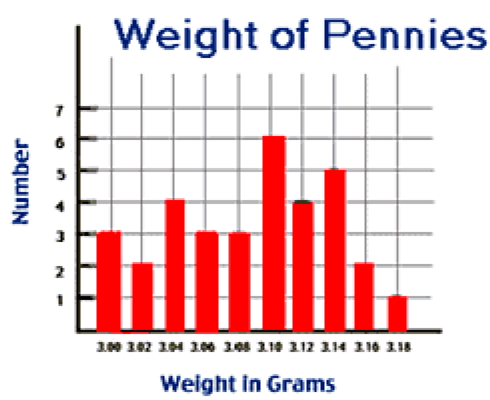
3. Plot the data points for each pair of data. First go over on the x-axis then up on the y-axis.



4. Draw a thick bar from the x-axis up to an imaginary point where the y-axis would intersect the bar.



5. Give the graph a title.



Constructing Circle Graphs

When constructing a circle graph, follow the steps below (NOTE: If the data is not already in a table, put it into tabular form. This will be your table of values.):

1. **Is the Data Suitable**--Determine if there is a "whole" for the data. Then determine what the different parts, or data groups, of the whole are.
2. **Calculate Percentages**--For data that is not already given as a percentage, convert the amounts for each part, or data group size, into a percentage of the whole.
3. **Draw the Graph**--Draw a circle and draw in a sector for each data group. Try to make the sector sizes look as close to the percentage of the circle as the percentage of the data group.
4. **Title and Label the Graph**--Label the sectors with the data group name and percentage. Then add a title to the graph. This is the same as the title of the table of values.

Example

Construct the circle graph for the data in the table of values on the next page.

Sneakers Sold for November 1997 at The Shoe Source

Brand Name	Number Sold
Adidas	150
Nike	192
Reebok	60
Asics	108
Other	90

1. **Is the Data Suitable:**

Determine whether there is a "whole" for the data. Then determine what the different parts, or data groups, of the whole are.

- Define the whole--In this table of values, the whole is the total number of sneakers sold for the month of November 1997.
- How many different parts, or groups, are there--There are five parts to the whole. Each data group is a category of sneaker brands **(1)** Adidas, **(2)** Nike, **(3)** Reebok, **(4)** Asics, **(5)** Other.

From the table we can calculate the whole, and we do have different parts. This means the data could be displayed in a circle graph.

2. **Calculate Percentages**

For data that is not already given as percentages, convert the amounts for each part, or data group, into a percentage of the whole.

- Calculate the whole--This total can be found by adding up the numbers sold for each type of sneaker.
  There are five different parts to our whole. The table of values lists the number sold in each part. To find the total sold we add up these parts. When we do this, we find the total number is 600.

150 + 192 + 60 + 108 + 90 = 600 **total shoes sold**

- Calculate the percentage for each part--This means we must calculate the percentage of the whole for each of the five data groups. Since we now have the total number of sneakers sold, and we have the amount of each category sold, we can calculate the percentage of sales for each data group.
  Once we find the total, we can convert each Number Sold into a percentage of the whole. Let's go through the calculation for Adidas shoes. If we look at Adidas shoes, 150 of the 600 shoes sold

are Adidas.

Brand Name	Number Sold
Adidas	150

- This means the fraction of shoes sold that are Adidas is 150/600. To convert this fraction to a decimal, we divide the numerator by the denominator and then multiply by 100.

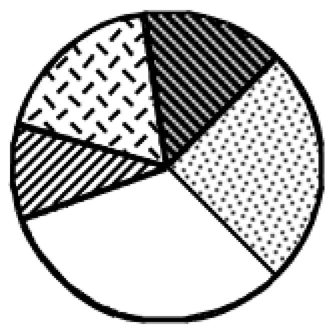
150/600 x 100 = 25

- So Adidas accounts for 25% of the sneakers sold in November 1997. The below has the percentages added for each category of sneakers

Brand Name	Number Sold	Percentage Sold
Adidas	150	25
Nike	192	32
Reebok	60	10
Asics	108	18
Other	90	15

3. **Draw the Graph**

First, using a compass, draw a circle. Then, use a protractor to draw in the sectors of the circle. We need to try to make each sector correspond to the percentage of the whole that it represents. For this circle our sectors need to be 32%, 25%, 18%, 15%, and 10%.

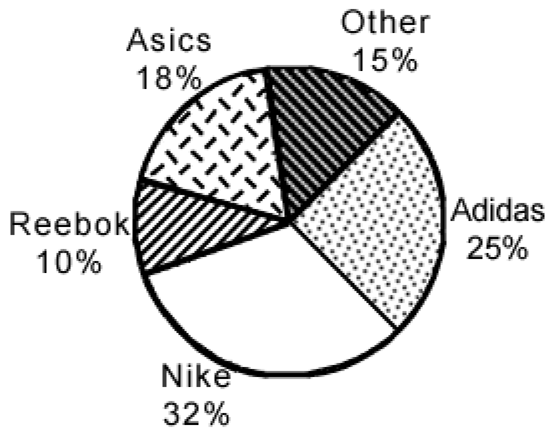


You can also add shading to the sectors. This helps to make them easier to distinguish.

4. **Title and Label the Graph**

Label the sectors with the data group name and percentage. Then add a title to the graph. This is the same as the title of the table of values.

Sneakers Sold for November 1997 at The Shoe Source



Now, we have the completed circle graph. This allows us to evaluate the relative sizes of each group quickly. We can see that Nike has the largest percentage of sales, and that, of the top four sellers, Reebok has the smallest, approximately 1/3 that of Nike.

Problem Solving Using Graphs

1. Given the table of values on the next page, create an accurate circle graph which represents the same information.

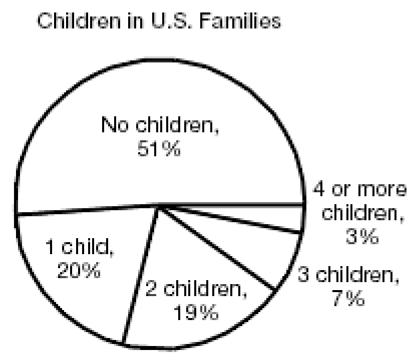
Shares of Stock Owned by an Investor

Type of Stock	Number of Shares
Coca-Cola	8
Pepsi	10
IBM	4
Exxon	8
General Motors	20

2. Given the table of values below, displaying the annual income of four employees, construct a bar graph that displays the same information.

Annual Income of Four Employees	
Employee	**Income (in dollars)**
Sue	25,000
Brian	38,000
Dan	30,000
Nancy	35,000

Use the Children in U.S. Families graph to answer Exercises 3–4.



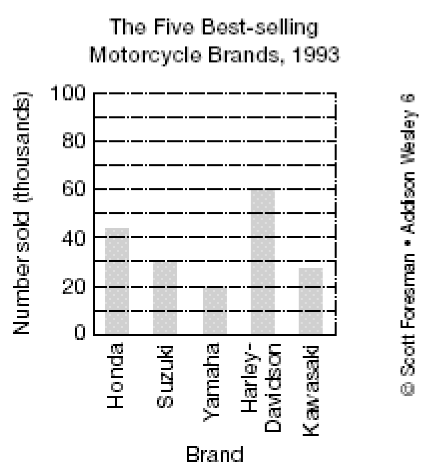
3. What percent of families include exactly 2 children?

____

4. Of the categories shown, which is the largest?

____

Use the graph of the Five Best-selling Motorcycle Brands to answer Exercises 5-8.



5. Which brand sold the most motorcycles?

____

6. Which brand (of the five shown) sold the fewest?

   ____

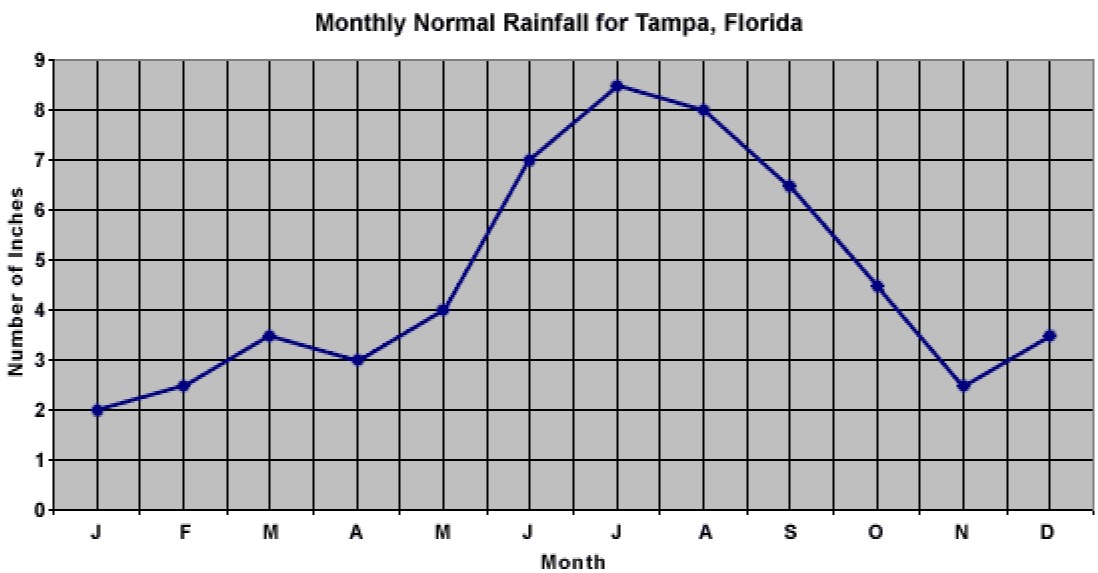
7. About how many Honda motorcycles were sold?

   ____

8. Which brand sold more motorcycles, Kawasaki or Suzuki?

   ____

Use the Monthly Normal Rainfall for Tampa graph to answer Exercises 9-14.



9. In which month is the amount of rainfall greatest?

   ____

10. In which month is the amount of rainfall least?

    ____

In which month is the amount of rainfall closest to each of the following months?

11. September __________

12. October __________

13. July __________

14. What is the normal amount of rainfall for the month of March?

    ____

15. John had read, "Chariot of the Gods" and decided to construct a large sign to indicate to travellers from space that we humans are friendly people. He intended to use a piece of twine as his unit length and with his house as the origin construct the sign by placing a large rock at each point and painting the rock a bright orange. He prepared the following diagram of his proposed sign on a sheet of grid paper. Plot the points and see what the sign said.

1. (-10, 2)	2. (-9, -4)	3. (4, 9)	4. (-8, -6)
5. (10, 2)	6. (-8, 6)	7. (9, -4)	8. (-6, -2)
9. (-2, -6)	10. (0, -10)	11. (-2, 10)	12. (-10, 0)
13. (-7, -7)	14. (6, -8)	15. (-2, 6)	16. (-3, 6)
17. (10, -2)	18. (-2, 5)	19. (-4, 9)	20. (-1, -10)
21. (-3, 3)	22. (2, -6)	23. (-4, 6)	24. (0, -6)
25. (-5, 5)	26. (7, 7)	27. (5, -4)	28. (-10, -2)
29. (8, 6)	30. (-4, 3)	31. (-4, -5)	32. (-5, 4)

33. (-2, 4)	34. (4, -5)	35. (2, 6)	36. (3, 6)
37. (-7, 7)	38. (2, 5)	39. (6, 8)	40. (10, 0)
41. (3, 3)	42. (7, -7)	43. (4, 6)	44. (4, -9)
45. (5, 5)	46. (-4, -9)	47. (-6, 8)	48. (-6, -8)
49. (9, 4)	50. (4, 3)	51. (2, 10)	52. (5, 4)
53. (2, 4)	54. (8, -6)	55. (-9, 4)	56. (0, 10)
57. (-5, -4)	58. (6, -2)	59. (1, -10)	

CURRICULUM OBJECTIVES

MATHEMATICAL OPERATIONS, AVERAGE, MEDIAN, AND MODE			
Review of Operations	1	define: whole numbers, even numbers, odd numbers, digit	
	2	define: addition, subtraction, multiplication, division	
	3	define and use: addend, sum, difference, multiple	
	4	define and use: operations, product, dividend, divisor	
	5	define and use: quotient, remainder	
	6	explain relation between: counting and addition	
	7	explain relation between: addition and subtraction	
	8	explain relation between: addition and multiplication	
	9	explain relation between: subtraction and division	
	10	addition, subtraction, division facts and times tables	
	11	perform addition and subtraction of whole numbers	
	12	perform multiplication and division of whole numbers	
	13	perform borrowing and carrying	
	14	use signs: +, -, x, ÷, =, <, >	
	15	perform mathematical operations in columns	
	16	explain importance of neat columns and legibility	
	17	explain importance of accuracy, checking for errors	
	18	checking for errors using inverse operations	
	19	explain zero and its effects on mathematical operations	
Average (Mean)	20	find the average of two numbers	
	21	find average of a group of numbers	
Median	22	find the median of three numbers	
	23	find the median of two numbers	
	24	find the median of an odd number of items	
	25	find the median of an even number of items	
	26	explain percentile	
	27	tallying	
Mode	28	finding the mode of a group of numbers	
	29	tallying	
Order of Operations	30	following the order of operations	
	31	mnemonic: BEDMAS	
FACTORS AND			

PRIME NUMBERS			
Terms	1	define factor	
	2	define product	
	3	define multiple	
	4	define GCF (Greatest Common Factor)	
	5	define LCM (Least Common Multiple)	
	6	define prime number and prime factor	
	7	define prime factorization	
Factors and Prime Numbers	8	use factor trees to find factors	
	9	use factor trees to explain prime factors	
	10	use divisibility test to find factors	
	11	find GCM	
	12	find LCM	
EXPONENTS			
Terms	1	define exponential form	
	2	define power, exponent, base	
	3	define exponential notation	
Exponents	4	express like factors using exponents	
	5	express exponents using factors, expanded form	
	6	identify the base and the exponent in power form	
	7	read power form as squared, cubed, to the fourth power, or fourth power	
	8	find the value of exponents	
	9	explain the Law of Exponents (briefly)	
	10	perform addition and subtraction involving exponents	
	11	perform multiplication and division involving exponents	
SQUARES AND SQUARE ROOTS			
Terms	1	define square	
	2	define square root	
	3	define perfect square	
	4	define algorithm	
Square and Square Roots	5	inverse relationship between squares and square roots	
	6	recall the perfect squares from 1-25	
	7	methods for determining square root: estimation and averaging	
	8	methods for determining square root: algorithms	
	9	methods for determining square root: calculator	
PROBLEM SOLVING WITH WHOLE NUMBERS			
Types of Problems	1	any combination of mathematical operations with whole	

		numbers	
	2	any combination of mathematical operations with averages, medians, modes, factors, prime numbers	
	3	any combination of mathematical operations with exponents, squares of numbers, square roots	
Strategies	4	develop good work habits	
	5	read all parts of question carefully	
	6	determine what is asked for or required	
	7	separate information given from question being asked	
	8	record information given and solution required separately	
	9	decide what arithmetic process will solve the problem	
	10	work neatly and arrange work in rows where possible	
	11	label the answer in terms of values given in question	
	12	estimate an answer	
	13	check every step and compare with estimated answer	
	14	compare estimated answer with answer found	
	15	translate English statements into mathematical expressions	
	16	draw pictures of problem	
	17	supply missing information if necessary	
	18	write full statements to answer questions	
	19	develop calculator skills	
	20	use clue words to solve word problems; e.g. total, sum, how much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at, divide, and each	
FRACTIONS			
Terms	1	define fraction, numerator, denominator	
	2	define mixed number, proper fraction	
	3	define improper fractions	
	4	define common denominator	
Fractions	5	the proper way to write fractions	
	6	compare and reduce fractions	
	7	write equivalent fractions	
	8	add fractions: like and unlike denominators	
	9	add fractions: find common denominators	
	10	subtract fractions: like and unlike denominators	
	11	subtract fractions: find common denominators	
	12	reduce fractions to lowest terms	
	13	cancelling fractions	
	14	multiply fractions	
	15	divide fractions	
	16	use division rule: cancel, invert 2nd fraction, then multiply	
	17	change mixed numbers to improper fractions, as appropriate	
	18	change improper fractions to mixed numbers, as appropriate	
	19	report answer in lowest terms or mixed numbers, as appropriate	

DECIMALS			
Terms	1	define: decimal, decimal system	
	2	define mixed decimal	
	3	define terminating decimals	
	4	define repeating decimals	
	5	define lowest common multiple (LCM)	
Decimals	6	use of the decimal point	
	7	convert mixed numbers to decimals	
	8	multiply and divide by powers of 10	
	9	zero as a place holder	
	10	add and subtract decimals	
	11	place decimal points under each other	
	12	borrowing and carrying decimals	
	13	multiply decimals and placement of decimal in final answer	
	14	divide decimals and placement of decimal in final answer	
	15	expressing remainders as decimals	
	16	round off decimals	
	17	estimate when working with decimals	
	18	work with money	
	19	compare decimals and fractions	
	20	convert decimals to fractions	
	21	convert fractions to decimals	
PERCENT			
Terms	1	define percent	
	2	use of the "%" sign	
Percent	3	add and subtract with percents	
	4	multiply and divide with percents	
	5	convert fraction to percent	
	6	convert percent to fraction	
	7	convert decimals to percents	
	8	convert percents to decimals	
	9	convert fractions to decimals to percents	
PROBLEM SOLVING WITH FRACTIONS, DECIMALS, PERCENTS			
Types of Problems	1	any combination of math operations involving fractions, decimals, and/or percents	
Strategies	2	develop good work habits	
	3	read all parts of question carefully	
	4	determine what is asked for or required	
	5	separate information given from question being asked	
	6	record information given and solution required separately	
	7	decide what arithmetic process will solve the problem	

	8	work neatly and arrange work in rows where possible	
	9	label the answer in terms of values given in question	
	10	estimate an answer	
	11	check every step and compare with estimated answer	
	12	compare estimated answer with answer found	
	13	translate English statements into mathematical expressions	
	14	draw pictures of problem	
	15	supply missing information if necessary	
	16	write full statements to answer questions	
	17	develop calculator skills	
	18	use clue words to solve word problems; e.g. total, sum, how much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at, and each	
INTRODUCTION TO RATIO, PROPORTION, AND PERCENT			
Percent	1	definition and calculation of percent	
	2	use formula “$r/100 = P/W$” to find percent of a number	
	3	use formula “$r/100 = P/W$” to find what percent one number is of another	
	4	use formula “$r/100 = P/W$” to find a number when a percent is given	
	5	discuss other terms: “r” represents Percent rate	
	6	discuss other terms: “P” represents part of the number	
	7	discuss other terms: “W“ represents the whole (entire) number	
Ratio	8	define ratio	
	9	how to write ratios	
	10	reduce ratios	
	11	distinguish between equivalent and non-equivalent ratios	
	12	compare and write equivalent ratios	
Proportion	13	define proportion	
	14	explain relation between ratio and proportion	
	15	how to write proportions	
	16	discuss proportional	
	17	discuss mean	
	18	discuss extreme	
	19	discuss product	
	20	discuss true proportion	
	21	discuss direct proportion	
PROBLEM SOLVING WITH PERCENT, RATIO, AND			

PROPORTION			
Types of Problems	1	requiring any combination of mathematical operations involving ratio, percent, and proportion	
Strategies	2	develop good work habits	
	3	read all parts of question carefully	
	4	determine what is asked for or required	
	5	separate information given from question being asked	
	6	record information given and solution required separately	
	7	decide what arithmetic process will solve the problem	
	8	work neatly and arrange work in rows where possible	
	9	label the answer in terms of values given in question	
	10	estimate an answer	
	11	check every step	
	12	compare estimated answer with answer found	
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	15	supply missing information if necessary	
	16	write full statements to answer questions	
	17	develop calculator skills	
	18	use clue words to solve word problems; e.g. total, sum, how much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at, divide, and each	
LINES AND ANGLES			
Terms	1	define lines: point, line, line segment, ray	
	2	define lines: vertex, angle, perpendicular, parallel lines	
	3	define angles: acute, right, obtuse, straight, complete, reflex	
	4	define transversal lines	
	5	define alternate angles	
	6	define corresponding angles	
	7	define interior angles	
	8	define angle relations: complementary, supplementary	
	9	define angle relations: adjacent, vertical, opposite, exterior	
	10	investigate angle relations when transversal intersects two parallel lines	
Construction	11	draw perpendicular lines and 90 degree angles	
	12	construct parallel lines	
Angles	13	label angles: 3 capital letters, middle one is vertex	
	14	find relation of angles when transversal cuts parallel lines	
	15	discuss angles, using circle as a base for measuring angles	
	16	discuss degree as unit of measure for angles	
	17	use protractor to measure angles	
	18	classify angles and angle relation	
Protractor	19	use three step approach to measuring an angle: center point of protractor on vertex of angle, one arm of angle on base	

		line of protractor, and decide on measurement units/scale	
INTRODUCTION TO GEOMETRIC FIGURES			
Definition	1	define geometry	
Circles	2	define and/or diagram: circle, radius , diameter	
	3	define and/or diagram: circumference, chord, arc, segment, sector	
	4	define and/or diagram: tangent, semi-circle	
	5	measure radius, diameter, and circumference	
	6	investigate relation between radius, diameter, circumference	
	7	explain and use π	
	8	use compass to construct circle, given radius and diameter	
Polygons	9	define polygon	
	10	types of polygons: triangle, quadrilateral, pentagon	
	11	types of polygons: hexagon, octagons	
	12	recognize that polygons are named by number of sides	
	13	distinguish between polygons and non-polygons	
	14	distinguish between regular and irregular polygons	
	15	identify concave, convex, and regular polygons	
	16	types of triangles: scalene, isosceles, equilateral	
	17	types of triangles: acute, obtuse, right triangles; hypotenuse	
	18	explain Pythagorean Theorem	
	19	types of quadrilaterals and characteristics: trapezoid	
	20	parallelograms (rectangle, square, rhombus)	
Three-dimensional	21	define polyhedron	
	22	explain relation between polyhedrons and polygons	
	23	types: cube, prism, pyramid, cones, spheres, cylinders	
Working with Geometric Figures	24	circle: find radius/diameter, given circumference	
	25	circle: find circumference, given radius/diameter	
	26	triangle: use Pythagorean Theorem to find length of one side of a triangle	
	27	triangle: use Pythagorean Theorem to confirm that triangle is right triangle	
THE METRIC SYSTEM			
Metric System	1	explain metric system and its base of ten	
	2	explain International System (SI Units)	
	3	fundamental units: length – metre (m)	
	4	fundamental units: mass – gram (g)	
	5	fundamental units: capacity – litre (L)	
	6	fundamental units: time – second (s)	
	7	fundamental units: temperature (degrees C)	
	8	metric prefixes and abbreviations	

	9	milli, (m) e.g. mm, mg mL	
	10	centi, (c) e.g. cm, cg, cL	
	11	deci, (d) e.g. dm, dg, dL	
	12	unit (metre, gram, litre) m, g, L	
	13	deka, (da) e.g. dam, dag, daL	
	14	hecto, (h) e.g. hm, hg, hL	
	15	kilo, (k) e.g. km, kg, kL	
	16	derived units such as area (square m.)	
	17	derived units such as volume (cubic cm.)	
	18	derived units such as capacity (cubic dm)	
	19	concept of place value	
	20	convert one metric unit of measure into another	
AREA, PERIMETER, AND VOLUME			
Perimeter	1	define perimeter	
	2	explain that circumference is the perimeter of a circle	
	3	formula for finding perimeter of polygons: Perimeter (P) = sum of length of all sides	
	4	formula for finding perimeter of regular polygons: Perimeter (P) = number of sides (n) x length of sides (s)	
	5	practice finding perimeter of a variety of figures: square, rectangle, pentagon, decagon, equilateral, irregular shapes	
Area	6	define area	
	7	measure area in square units	
	8	formula: area of square: A = side x side = s^2	
	9	formula: area of a rectangle: A = length x width = l x w	
	10	formula: area of triangle: A = ½ base x height = ½ x b x h	
	11	formula: area of parallelogram: A = base x height = b x h	
	12	formula: area of circle: A = pi x radius squared = πr^2	
	13	area of irregular shapes	
	14	surface area of 3-dimensional figures	
	15	application of area: e.g. flooring coverings, paint, etc.	
	16	practice finding area of rectangle, square, triangle	
	17	practice finding area of cube, parallelogram, circle, irregular shapes	
Volume	18	define volume	
	19	measure volume in cubic units	
	20	formula: volume of a cube: V = side x side x side = s^3	
	21	formula: volume of a rectangular prism: V = length x width x height = l x w x h	
	22	formula: volume of a cylinder: V = pi x radius squared x height = $\pi \times r^2 \times h$	

	23	Applications of volume: e.g. amount of gravel to buy, capacity of fuel tank, etc.	
PROBLEM SOLVING INVOLVING MEASUREMENT			
Types of Problems	1	requiring any combination of mathematical operations involving the metric system	
	2	requiring any combination of mathematical operations involving area, perimeter, and volume	
Strategies	3	develop good work habits	
	4	read all parts of question carefully	
	5	determine what is asked for or required	
	6	separate information given from question being asked	
	7	record information given and solution required separately	
	8	decide what arithmetic process will solve the problem	
	9	work neatly and arrange work in rows where possible	
	10	label the answer in terms of values given in question	
	11	estimate an answer	
	12	check every step and compare with estimated answer	
	13	compare estimated answer with answer found	
	14	translate English statements into mathematical expressions	
	15	draw pictures of problem	
	16	supply missing information if necessary	
	17	write full statements to answer questions	
	18	develop calculator skills	
	19	use clue words to solve word problems; e.g. total, sum, how much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at, divide, and each	
INTRODUCTION TO INTEGERS			
Integers	1	review of thermometer temperature reading	
	2	definition of integers	
	3	using a number line	
	4	standard form of integers: signs of operation	
	5	standard form of integers: signs of quantity	
	6	use of negative and positive integers: + shows gain	
	7	use of negative and positive integers: - shows loss	
	8	order integers from least to greatest and vice versa	
	9	add, subtract, multiply, divide with integers	
	10	practical applications of integers (golf, banking, etc.)	
PROBLEM SOLVING WITH INTEGERS			
Types of Problems	1	requiring any combination of mathematical operations	

		involving integers	
Strategies	2	develop good work habits	
	3	read all parts of question carefully	
	4	determine what is asked for or required	
	5	separate information given from question being asked	
	6	record information given and solution required separately	
	7	decide what arithmetic process will solve the problem	
	8	work neatly and arrange work in rows where possible	
	9	label the answer in terms of values given in question	
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	12	compare estimated answer with answer found	
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	14	draw pictures of problem	
	15	supply missing information if necessary	
	16	write full statements to answer questions	
	17	develop calculator skills	
	18	use clue words to solve word problems; e.g. total, sum, how much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at, divide, and each	
INTRODUCTION TO EQUATIONS: EQUALITIES AND INEQUALITIES			
Terms	1	define equation	
	2	use and understand: variable, constant	
	3	use and understand: algebraic expressions, term, factors, coefficient	
	4	use and understand: replacement and solution	
	5	order of operations (BEDMAS)	
	6	symbols: +, -, x, ÷, =, and √	
	7	symbols: ( ), [ ], { }	
	8	equality and inequality	
	9	use of "•" in place of "x" for multiplying	
Equations	10	use letters to represent numbers	
	11	order of operations	
	12	solve equations	
	13	use opposite operations to isolate the variable	
	14	Principle of Equations: doing same thing on both sides	
	15	use distributive property	
	16	build equations	
	17	solve equations with integers	
	18	combine like terms to solve equations	
	19	translate English statements into mathematical statements	
	20	use mathematical symbols appropriate to grade level	

PROBLEM SOLVING WITH EQUATIONS AND EQUALITIES			
Types of Problems	1	requiring any combination of mathematical operations involving equations (equalities)	
Strategies	2	develop good work habits	
	3	read all parts of question carefully	
	4	determine what is asked for or required	
	5	separate information given from question being asked	
	6	record information given and solution required separately	
	7	decide what arithmetic process will solve the problem	
	8	work neatly and arrange work in rows where possible	
	9	label the answer in terms of values given in question	
	10	estimate an answer	
	11	check every step and compare with estimated answer	
	12	compare estimated answer with answer found	
	13	translate English statements into mathematical expressions	
	14	draw pictures of problem	
	15	supply missing information if necessary	
	16	write full statements to answer questions	
	17	develop calculator skills	
	18	use clue words to solve word problems; e.g. total, sum, how much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at, divide, and each	
INTRODUCTION TO GRAPHS			
Types of Graphs	1	bar graph, line graph, pictograph	
	2	circle graphs, co-ordinate graphs	
Steps to Creating Good Graphs	3	determine what type of graph to use	
	4	collect and organize information	
	5	prepare graph outline, name horizontal and vertical scales	
	6	name the graph and interpret graph	
	7	good use of space, scales accurate, neatness	
	8	descriptive titles, data expressed accurately with tally	
Line Graphs	9	define line graph	
	10	shows changes and relationships between quantities	
	11	construct line graph: determine scale	
	12	construct line graph: draw and label lines, name the graph	
	13	interpret line graphs	
	14	practical applications: e.g. comparing retail sales by month	
Bar Graphs	15	define bar graph	
	16	length of solid vertical bars shows its value	
	17	construct bar graph: determine scale	

	18	construct bar graph: draw and label "x" and "y" axis, plot the data	
	19	construct bar graph: draw bars (equal width), label and name graph	
	20	interpret bar graphs	
	21	practical applications: e.g. comparing profits by year	
Circle Graphs	22	define circle graph	
	23	emphasizes relative size of parts to whole	
	24	construct circle graph: make table showing facts given, convert facts to %, calculate % of 360 degrees, then construct circle with compass; use protractor to construct number of degrees in each central angle, label parts and name graph	
Co-ordinate Graphs	25	define co-ordinate graph	
	26	define coordinate axes, origin, quadrants, grid	
	27	define ordered pairs, table of values	
	28	choose scale, plot points, find co-ordinates of a point	
	29	Practical applications: textile design, longitude/latitude	
PROBLEM SOLVING USING GRAPHS			
Types of Problems	1	requiring any combination of mathematical operations involving graphs	
	2	read and produce bar graphs and line graphs	
	3	read and produce circle graphs and co-ordinate graphs	
Strategies	4	develop good work habits	
	5	read all parts of question carefully	
	6	determine what is asked for or required	
	7	separate information given from question being asked	
	8	record information given and solution required separately	
	9	decide what arithmetic process will solve the problem	
	10	work neatly and arrange work in rows where possible	
	11	label the answer in terms of values given in question	
	12	estimate an answer	
	13	check every step and compare with estimated answer	
	14	compare estimated answer with answer found	
	15	translate English statements into mathematical expressions	
	16	draw pictures of problem	
	17	supply missing information if necessary	
	18	write full statements to answer questions	
	19	develop calculator skills	
	20	use clue words to solve word problems; e.g. total, sum, how much, how many, increased, altogether, less, fewer, more, difference, left, remains, times, at, divide, and each	