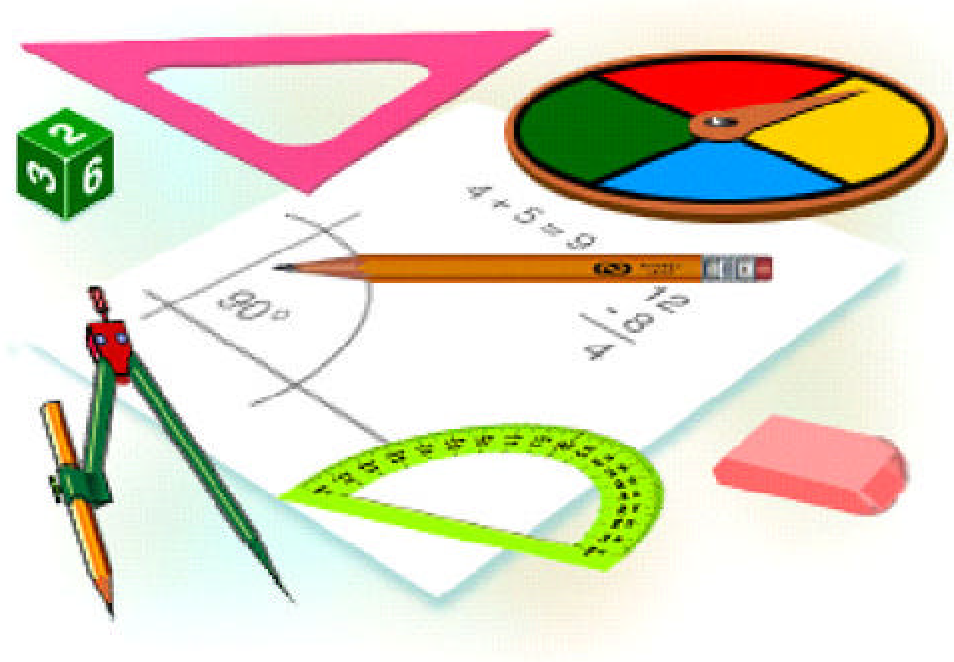
The Next Step

Mathematics Applications for Adults



Book 14018 - Geometry

OUTLINE

Mathematics - Book 14018

Geometry
Lines and Angles
identify parallel lines and perpendicular lines in a given selection of figures.
construct parallel lines.
determine angles when a transversal cuts parallel lines.
construct angles using a protractor, given a list of angle measurements.
identify types of angles: acute, right, obtuse, straight, complete, reflex.
illustrate with diagrams the following angle relations: complementary, adjacent, supplementary, exterior, and vertical or opposite.
Introduction to Geometric Figures
identify parts of a circle.
construct a circle and label its parts.
identify a variety of polygons.
identify a variety of polyhedrons.
use the Pythagorean Theorem to find length of one side of a triangle.

THE NEXT STEP

Book 14018

Geometry

Geometry is the branch of mathematics that explains how ***points***, ***lines***, ***planes***, and ***shapes*** are related.

Lines and Angles

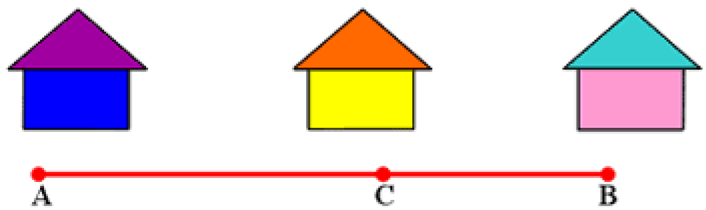
Points

Points have no size or dimensions, that is, no width, length, or height. They are an ***idea*** and cannot be seen. But, points are used to tell the position of lines and objects. Points are usually named with capital letters:

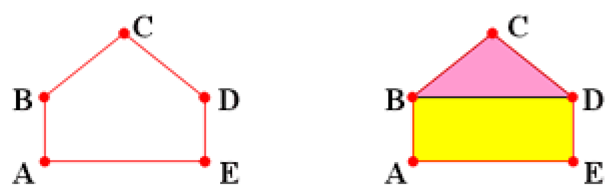
A, B, C, D and so on.

Points can describe where things begin or end.

Points can be used to measure distance.

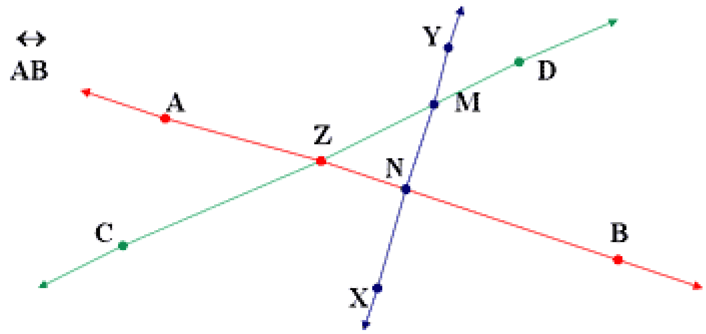


Points define the perimeter of shapes and objects.



Lines

Lines extend in opposite directions and go on without ending. Like points, lines have no volume, but they have infinite length. Lines are named by points with a line symbol written above.



Lines intersect at a point. Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point ***Z***.

⇒ Line $\overleftrightarrow{AB}$ can also be named $\overleftrightarrow{BA}$.

Line Segments

Line segments are parts of lines defined by two endpoints along the line. They have *length*.

Line segments are named by their two points with the line segment symbol written above:

$\overline{AB}$ (or $\overline{BA}$)

An infinite number of line segments can be located along a line:

$\overline{AB}$ $\overline{AE}$ $\overline{AC}$ $\overline{AD}$ $\overline{AF}$ $\overline{EC}$ $\overline{ED}$ $\overline{EF}$ …

$\Rightarrow$ ***Line segments of equal length are called congruent line segments.***

Rays

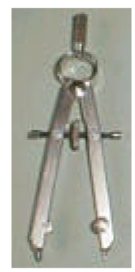
Rays are parts of lines that extend in one direction from one endpoint into infinity.

Rays are named by the endpoints and one other point with a ray symbol written above. The endpoint must always be named first.

$\overrightarrow{AB}$

The Compass

The compass at left is a typical golf-pencil compass that seems to be preferred by many students. It is not recommended. The whole point of a compass is to draw an arc with constant radius. This model tends to slip easily. Friction is the only thing holding the radius. As it wears out, it becomes even looser. Also, the point is not very sharp, so it will not hold its position well when drawing. Two advantages are that it is easy to find and it is inexpensive.



The compass on the right is a much better design. The wheel in the center allows for fine adjustment of the radius, and it keeps the radius from slipping. It has a much heavier construction, and will not easily bend or break.

Keep the compass lead sharpened for a nice, fine curve. There are special sharpeners made just for the leads that fit the compass, but it is a simpler matter to carry a small piece of sandpaper. Stroke the lead across it a few of times to give the tip a bevel.

Hold the compass properly. Use one hand, and hold it by the handle at the top. Do not hold it by the limbs. If you do that, there will be a tendency to change the radius as you draw.

This is especially a problem with the cheaper compasses that have no way of locking the radius. Tilt the compass back slightly, so that the lead is dragged across the page. If the compass is pushed toward the lead, it will cause the anchor point to lift up and slip out of position.

Do not be impatient with your work. When using a compass, there must be some well-defined point for the center point, such as the intersection of two lines. Center the compass precisely on that intersection. Depending on the complexity of the construction, small errors may be greatly magnified.

Parallel Lines

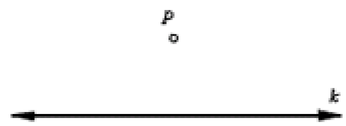
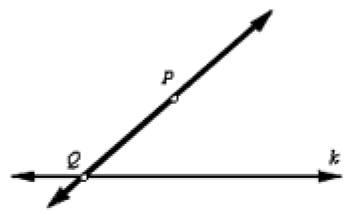
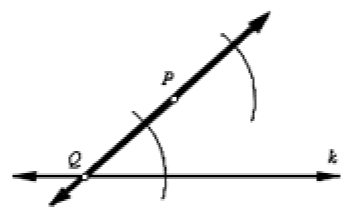
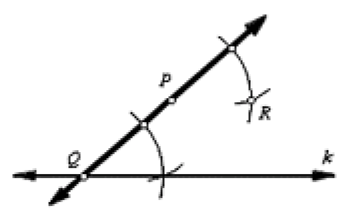
Parallel lines lie within the same plane and are always the same distance apart. Parallel lines continue to infinity without intersecting or touching at any point.

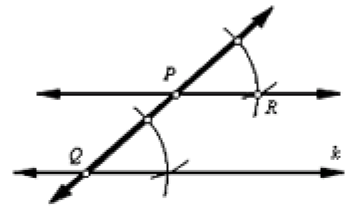
The symbol for parallel lines is $\parallel$ and is read “is parallel to.”

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Constructing Parallel Lines

Given a line and a point, construct a line through the point, parallel to the given line.

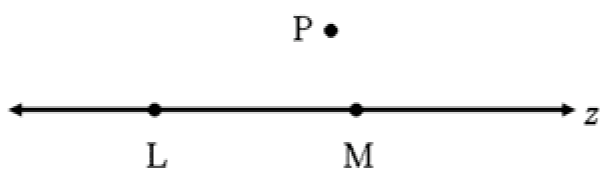
1. Begin with point *P* and line *k*.	
2. Draw an arbitrary line through point *P*, intersecting line *k*. Call the intersection point *Q*. Now the task is to construct an angle with vertex *P*, congruent to the angle of intersection.	
3. Center the compass at point *Q* and draw an arc intersecting both lines. Without changing the radius of the compass, center it at point *P* and draw another arc.	
4. Set the compass radius to the distance between the two intersection points of the first arc. Now center the compass at the point where the second arc intersects line *PQ*. Mark the arc intersection point *R*.	

5. Line *PR* is parallel to line *k*.	

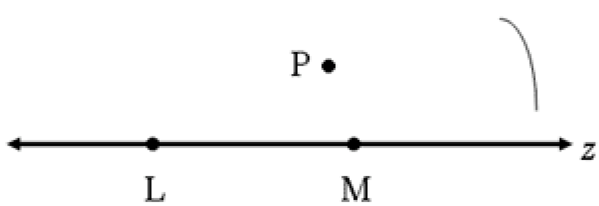
A second method follows these steps:

Step 1 On a given line *z*, create two points and label them.

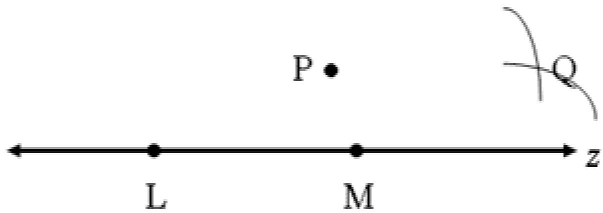
Step 2 Point P will be the point through which you will construct a line parallel to the given line *z*.



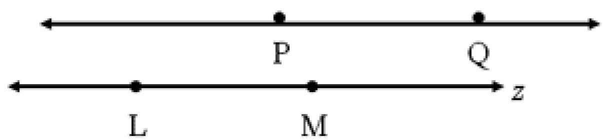
Step 3 Open compass to the length of LM. Put compass point at P and draw an arc.



Step 4 Open compass to the length of LP. Put compass point at M. Draw an arc to cut the previous arc. Label Q.



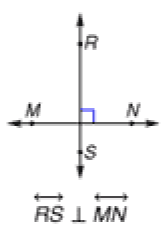
Step 5 Draw PQ. PQ || LM.



Perpendicular Lines

Lines that intersect to form 90° angles, or right angles

Example:



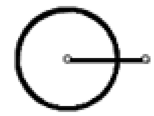
Read: Line *RS* is perpendicular to line *MN*

Construct the Perpendicular Bisector of a Line Segment

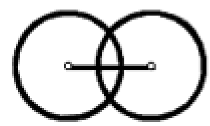
Definition: The *perpendicular bisector* of a segment is the line that is perpendicular (at a right angle) to the segment and goes through the midpoint of the segment.

Construction Steps

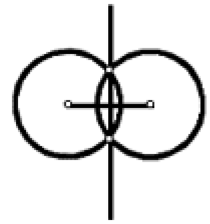
Use a compass to draw a circle whose center is one of the endpoints of the segment, and whose radius is more than half the length of the segment.



Draw another circle with the same radius, and center the other endpoint of the segment.



Draw the line through the two points where the circles intersect.



Note: **You don't have to draw the entire circle**, but just the arcs where the two circles intersect.

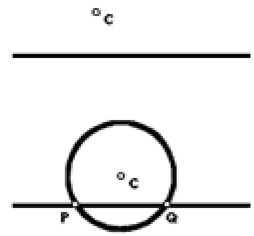
Construct the Perpendicular to a Line Through a Given Point

Given a line and a point, there is one and only one perpendicular to the line through the point.

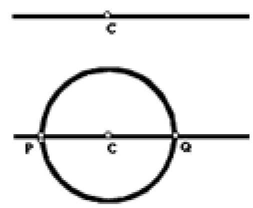
The main idea is to construct a line segment on the line, then construct the perpendicular bisector of this segment.

Construction Steps

A. If the point is not on the line, use a compass to draw a circle whose center is the given point, and whose radius is large enough so that the circle and line intersect in two points, P and Q.



B. If the point is on the line, draw a circle whose center is the given point; the circle and line intersect in two points, P and Q.



Construct the perpendicular bisector of segment PQ.

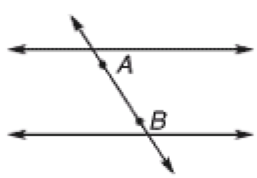
Lines that intersect but do not form 90° angles, or [right angles](), are simply called intersecting lines.

Transversal

A [line]() that intersects two or more lines

Example:

Line *AB* is a transversal.

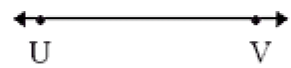


Practice Exercise

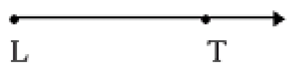
Name each figure.

1. 

- [x] line UY
- [ ] line segment UY
- [ ] Ray UY
- [ ] Ray YU

2. 

- [ ] line UV
- [ ] line segment UV
- [ ] Ray UV
- [ ] Ray VU

3. 

- [ ] line LT
- [ ] line segment LT
- [ ] Ray LT
- [ ] Ray TL

4. 

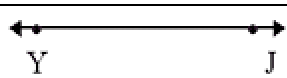
- [ ] line QS
- [ ] line segment QS
- [ ] Ray QS
- [ ] Ray SQ

5. 

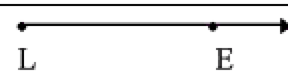
- [ ] line SD
- [ ] line segment SD
- [ ] Ray SD
- [ ] Ray DS

6. 

- [ ] line PQ
- [ ] line segment PQ
- [ ] Ray PQ
- [ ] Ray QP

7. 

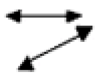
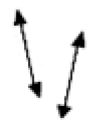
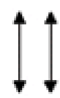
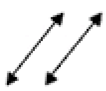
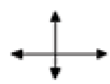
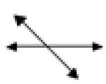
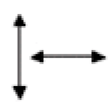
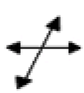
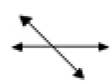
- [ ] line YJ
- [ ] line segment YJ

8. 

- [ ] line LE
- [ ] line segment LE

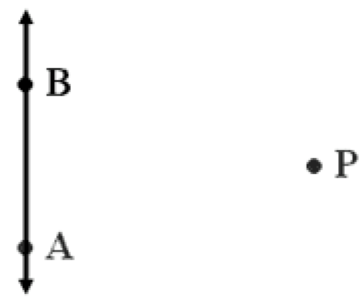
☐ Ray YJ ☐ Ray JY	☐ Ray LE ☐ Ray EL

Classify each group of lines.

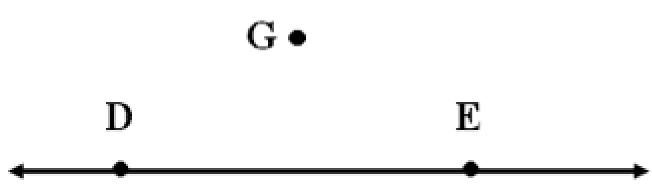
1.  ☐ Parallel ■ Intersecting ☐ Perpendicular	2.  ☐ Parallel ☐ Intersecting ☐ Perpendicular
3.  ☐ Parallel ☐ Intersecting ☐ Perpendicular	4.  ☐ Parallel ☐ Intersecting ☐ Perpendicular
5.  ☐ Parallel ☐ Intersecting ☐ Perpendicular	6.  ☐ Parallel ☐ Intersecting ☐ Perpendicular
7.  ☐ Parallel ☐ Intersecting ☐ Perpendicular	8.  ☐ Parallel ☐ Intersecting ☐ Perpendicular
9.  ☐ Parallel ☐ Intersecting ☐ Perpendicular	10.  ☐ Parallel ☐ Intersecting ☐ Perpendicular

Line Construction

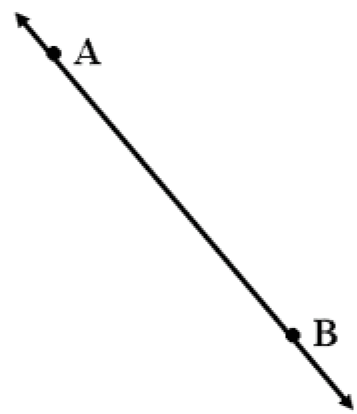
1. Using the following diagram, use a compass and a ruler to construct a line through P parallel to line AB.



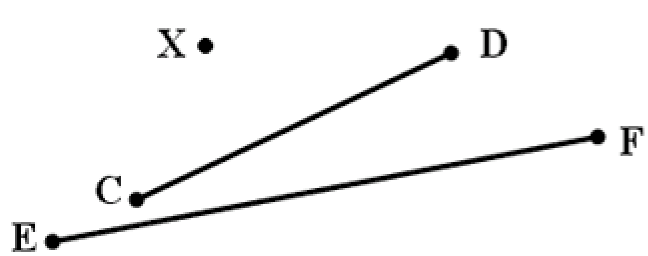
2. Using the following diagram, use a compass and a ruler to construct a line through G parallel to line DE.



3. Using the following diagram, use a compass and a ruler to construct the perpendicular from P to line AB.

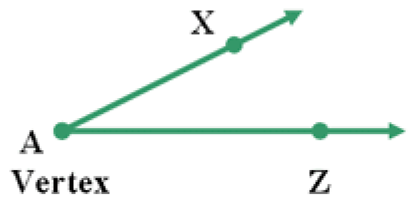


4. Using the following diagram, use a compass and a ruler to construct the perpendicular from X to line segment CD and from X to line segment EF.



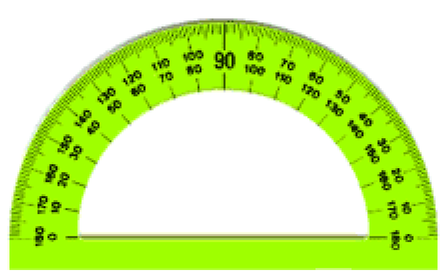
Angles

Angles are formed by two rays with a common endpoint called a ***vertex***.



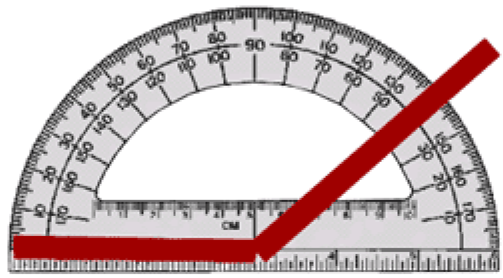
Angles are named by writing the names of three points on the set of lines after the symbol for angle, or by naming only the middle point after the angle symbol. The middle point always names the vertex.

Ð XAZ or **Ð** ZAX or **Ð** A
Angles come in different shapes and sizes. Some are narrow, some are wide. But all angles can be measured as part of a circle. To make calculations easy, scientists have developed the protractor, a kind of ruler for angles.



Angles are measured in degrees from 0 degrees to 180 degrees.

To measure with the protractor, line up the angle of the item to the center of the hole at the middle bottom. Make one edge of the angle line up with where there would be a 0 and then read on that scale where the other edge crosses.

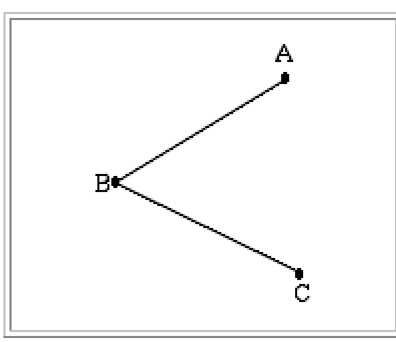


In this example, the angle is 140 degrees.

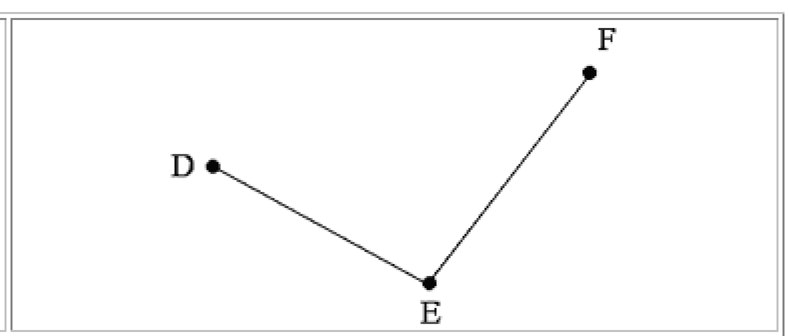
Practice Exercise

1.) Measure the following angles to the *nearest degree*:

(a)



(b)



2.) On a separate piece of paper, use a protractor and a ruler to construct an angle, *CAT,* with a 35° angle.

3.) On a separate piece of paper, use a protractor and a ruler to construct an angle, *DOG*, with a 125° angle.

4.) On a separate piece of paper, use a protractor and a ruler to construct an angle, WIN, with a 90° angle.

5.) On a separate piece of paper, use a protractor and a ruler to construct the angle, *SUN*, with *SU* = 7.5 cm, *UN* = 8 cm, and $\angle SUN$ measuring 70°.

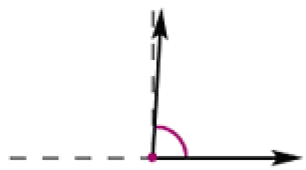
6.) On a separate piece of paper, use a protractor and a ruler to construct the angle, *BIG*, with *IG* = 10.4 cm, *BI* = 7.6 cm, and $\angle BIG$ measuring 110°.

7.) On a separate piece of paper, use a protractor and a ruler to construct the angle, *HAM*, with *HA* = 12.2 cm, *AM* = 9.4 cm, and $\angle HAM$ measuring 155°.

Acute Angle

An angle whose measure is greater than 0° and less than 90°

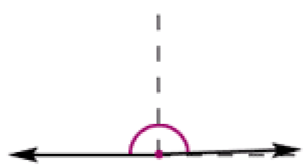
Example:



Obtuse Angle

An angle whose measure is greater than 90° and less than 180°

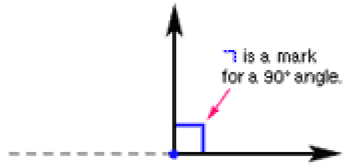
Example:



Right Angle

An angle whose measure is 90°

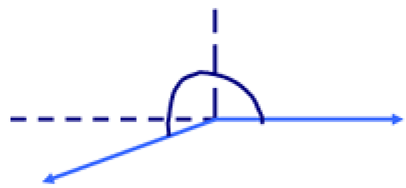
Example:



Reflex Angle

An angle whose measure is more than 180 degrees, but less than 360 degrees.

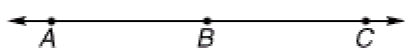
Example:



Straight Angle

An angle whose measure is 180°

Example:

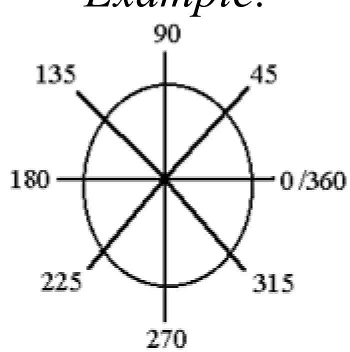


∠ ABC is a straight angle.

Complete Angle

An angle whose measure is 360 degrees (a circle)

Example:



Practice Exercise

Acute, Obtuse, Right, and Straight Angles

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1 A 35 degree angle would be classified as:

- [ ] An acute angle.
- [ ] An obtuse angle.
- [ ] A right angle.
- [ ] A straight angle.

---

2 A 175 degree angle would be classified as:

- [ ] An acute angle.
- [ ] An obtuse angle.
- [ ] A right angle.
- [ ] A straight angle.

---

3 A 180 degree angle would be considered:

- [ ] An acute angle.
- [ ] An obtuse angle
- [ ] A right angle.
- [ ] A straight angle.

4 An 80 degree angle would be considered:

- [ ] An acute angle.
- [ ] An obtuse angle.
- [ ] A right angle.
- [ ] A straight angle.

---

5 A 90 degree angle would be considered:

- [ ] An acute angle.
- [ ] An obtuse angle.
- [ ] A right angle.
- [ ] A straight angle.

---

6 When you cut a pizza into four equal pieces, the tip of each piece creates one of these angles.

- [ ] Acute Angle
- [ ] Obtuse Angle
- [ ] Right Angle
- [ ] Straight Angle

---

7 Which angle is impossible to have in a triangle?

- [ ] Acute Angle

- [ ] Obtuse Angle
- [ ] Right Angle
- [ ] Straight Angle

---

8 Which angle is found in squares?

- [ ] Acute Angle
- [ ] Obtuse Angle
- [ ] Right Angle
- [ ] Straight Angle

---

9 Which angle is the smallest?

- [ ] Acute Angle
- [ ] Obtuse Angle
- [ ] Right Angle
- [ ] Straight Angle

---

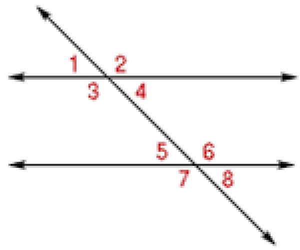
10 How many right angles, when put together, would make a straight angle?

- [ ] 0
- [ ] 1
- [ ] 2
- [ ] 3

Interior Angles

Angles on the inner sides of two lines cut by a transversal

Example:

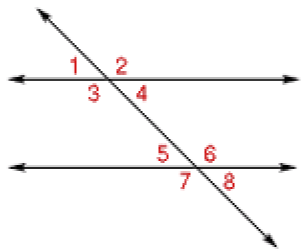


Angles 3, 4, 5, and 6 are interior angles.

Exterior Angles

The angles on the outer sides of two lines cut by a transversal

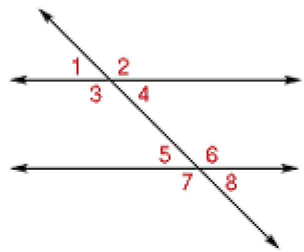
Example:



Angles 1, 2, 7, and 8 are exterior angles.

Alternate Exterior Angles

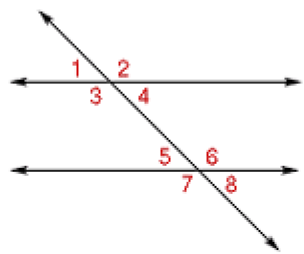
A pair of angles on the outer sides of two lines cut by a transversal, but on opposite sides of the transversal
Example:



∠ 1 and ∠ 8 and ∠ 2 and ∠ 7 are alternate exterior angles.

Alternate Interior Angles

A pair of angles on the inner sides of two lines cut by a transversal, but on opposite sides of the transversal
Example:

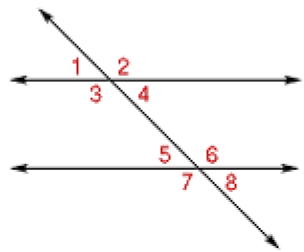


∠ 3 and ∠ 6 and ∠ 4 and ∠ 5 are alternate interior angles.

Corresponding Angles

Angles that are in the same position and are formed by a transversal cutting two or more lines

Example:

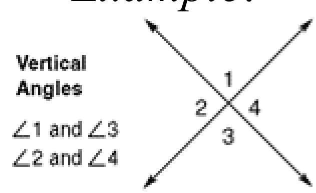


∠ 2 and ∠ 6 are corresponding angles.

Vertical or Opposite Angles

A pair of opposite congruent angles formed by intersecting lines

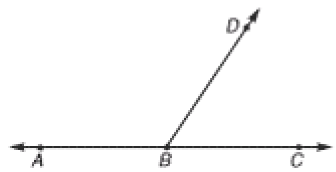
Example:



Adjacent Angles

Angles that share a common side, have the same vertex , and do not overlap

Example:

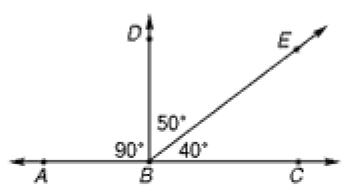


$\angle ABD$ is adjacent to $\angle DBC$.

Complementary Angles

Two angles whose measures have a sum of 90°

Example:

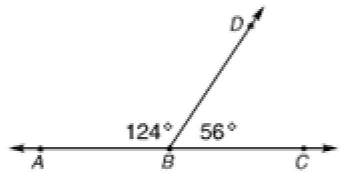


$\angle DBE$ and $\angle EBC$ are complementary.

Supplementary Angles

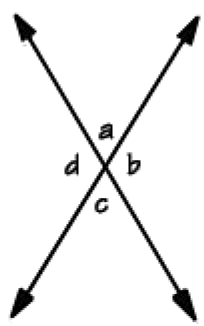
Two angles whose measures have a sum of 180°

Example:



$m\angle ABD + m\angle DBC = 124° + 56° = 180°$

Adjacent Angles



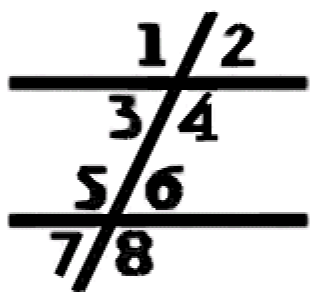
Use the picture above to answer the following questions?

Name the angles adjacent to <b.
Name the angles adjacent to <c.
Name the angles adjacent to <d.
Name the angles not adjacent to <a.

Name the angles not adjacent to <b.
Name the angles not adjacent to <c.

Complementary and Supplementary Angles

1. **What angle would be supplementary to a 105 degree angle?**
2. **What angle would be complementary to a 56 degree angle?**
3. **What angle would be complementary to 17 degree angle?**
4. **What angle would be supplementary to a 121 degree angle?**

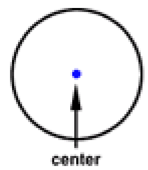


Using the diagram above:
a) **identify two interior angles.**
b) **identify two exterior angles.**
c) **identify two vertical or opposite angles.**

Introduction to Geometric Figures

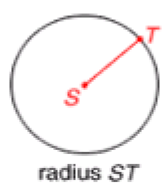
Circle

A closed curve with all points on the curve an equal distance from a given point called the center of the circle
Example:



Radius

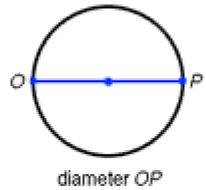
A line segment with one endpoint at the center of a circle and the other endpoint on the circle
Example:



Diameter

A chord or line segment with endpoints on a circle that passes through the center of a circle

Example:

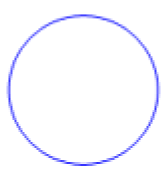


The ***diameter*** of a circle is a line that crosses the circle through its center from one side to the other. It also measures the distance across the circle. The ***radius*** of a circle is a line from the center of the circle to any point on the curve of the circle. A radius is half the distance across a circle. In other words, a radius is half of the diameter of a circle.

Circumference

The distance around a circle.

The perimeter of a circle is called circumference.



The formula for the circumference of a circle is $C = \pi d$, where C = circumference, $\pi \approx 3.14$ or $\frac{22}{7}$, and d = diameter

Pi **(π)**

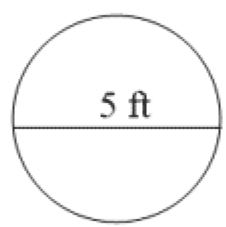
The ratio of the circumference of a circle to the length of its diameter;

$\pi \approx 3.14$ or $\frac{22}{7}$

$\approx$ is the symbol that means "approximately equal to".

$\Rightarrow$ It is useful to be familiar with *both* values of π, because in some problems a fraction is easier to use, while in others a decimal will make the computation easier.

Example Find the circumference of the circle shown below.

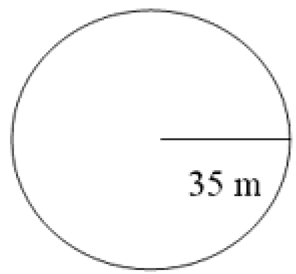


$C = \pi d$
$C = 3.14 \times 5$ ft
$C = 15.70 = 15.7$ ft

Solution: Replace π with 3.14 and d with 5 ft in the formula $C = \pi d$. The circumference of the circle is **15.7 feet.**

When finding the circumference of a circle, if only the radius is given, you must multiply the radius by 2 to find the diameter, and then use the formula.

Example Find the circumference of the circle shown below. Use 22/7 for π.



Step 1 Notice that the picture shows the radius of the circle. To find the diameter, multiply the radius by 2.

$$d = 2 \text{ x } 35 = 70 \text{ m}$$

Step 2 Replace π with 22/7 and d with 70 m in the formula $C = \pi d$.

$$C = \pi d$$
$$C = 22/7 \text{ x } 70$$
$$C = 220\ m$$

Answer: The circumference of the circle is **220 m.**

To find the diameter of a circle, if only the circumference is given, divide the circumference by π.

To find the radius of a circle, if only the circumference is given, divide the circumference by π to find the diameter. Then divide the diameter by 2 to find the radius.

If you already know the diameter of the circle, you can find the radius by dividing the diameter by 2.

If you already know the radius of the circle, you can find the diameter by multiplying the radius by 2.

Practice Exercise

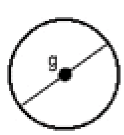
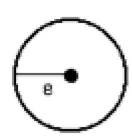
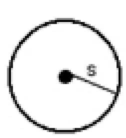
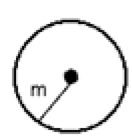
Complete the table for each circle.
Round to the nearest hundredth. Use 3.14 for π

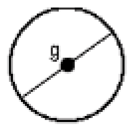
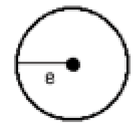
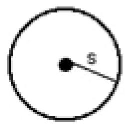
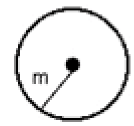
For problems 11-14, use $3\frac{1}{7}$ for π.

	radius	*diameter*	*circumference*
1.	8 ft	16 ft	__________ ft
2.	9 m	__________ m	__________ m
3.	__________ mm	4 mm	__________ mm
4.	7 mi	14 mi	__________ mi
5.	10 km	20 km	__________ km
6.	6.7 cm	13.4 cm	__________ cm

7.	__________ in	__________ in	61.54 in
8.	__________ yd	__________ yd	57.78 yd
9.	11.4 mm	__________ mm	__________ mm
10.	__________ m	12.4 m	__________ m
11.	$5\frac{1}{5}$ km	$10\frac{2}{5}$ km	__________ km
12.	__________ in	$14\frac{2}{5}$ in	__________ in
13.	$7\frac{7}{10}$ yd	$15\frac{2}{5}$ yd	__________ yd
14.	$6\frac{4}{5}$ cm	$13\frac{3}{5}$ cm	__________ cm
15.	19.41 mi	__________ mi	__________ mi

Find the Circumference for each.
Round to the nearest hundredth. Assume π = 3.14

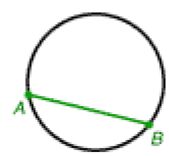
1.  g = 21.24 yd **138.16 ft**	2.  e = 38 cm __________
3.  s = 5 mi __________	4.  m = 23 yd __________

5.		g = 21.24 yd ____________	6.		e = 9.6 cm ____________
7.		s = 4 c m ____________	8.		m = 12.69 in ____________

Parts of a Circle

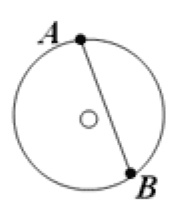
Chord

A line segment with endpoints on a circle
Example:



Segment

A straight set of points that has two endpoints.
Example:

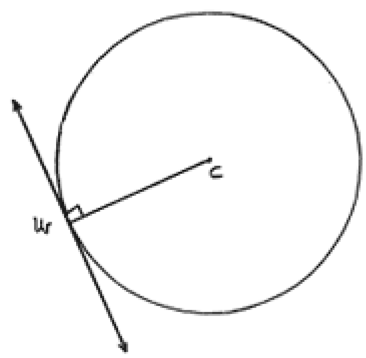


AB is a segment. In this picture, it is a straight set of points with two endpoints. Since both of the endpoints are on the circle this segment is also a chord.

Tangent

Tangent lines are perpendicular to the radius that has an endpoint on the point of tangency.

Example:

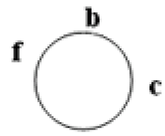


Line J is a tangent line that meets the radius line CW at an endpoint W on the circle that forms a 90 degree angle. Therefore, Line J is perpendicular to line CW.

Arc

A section of a circle.
Think about a circular pizza that has been cut like a pie is cut. The crust acts like the circumference of the pizza. That would make the crust on one piece of pizza an arc because it is just a section of the whole circle.

Example:

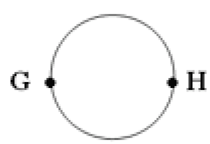


There are many arcs shown here. Can you see them all?
1) Small arc fb and big arc fb.
2) Small arc fc and big arc fc.
3) Small arc bc and big arc bc.
4) Arc cfb

Semicircle

The arc that goes halfway around a circle is called a semicircle.

Example:



Sector

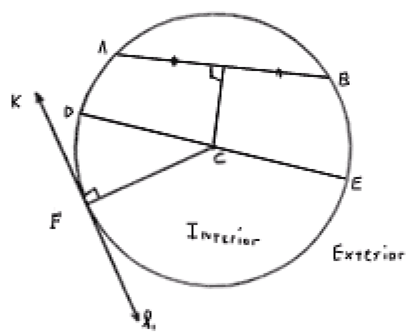
A region in a circle that is created by a central angle and its intercepted arc.

A piece of pie.

Example:

The piece of pie that the number 1 is in is called a sector.

Practice Exercise



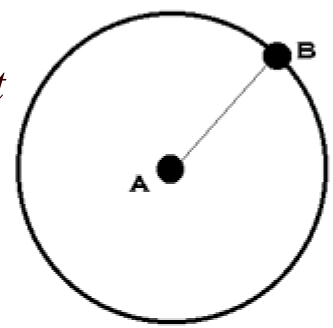
Using the diagram above, identify the following circle parts.

1. Name two radiuses.
2. Name a diameter.
3. Name a chord.
4. Name two arcs.
5. Name a segment.
6. Name a sector.
7. Name a tangent.
8. Name a semicircle.

CIRCLE CONSTRUCTIONS

Center/point construction:

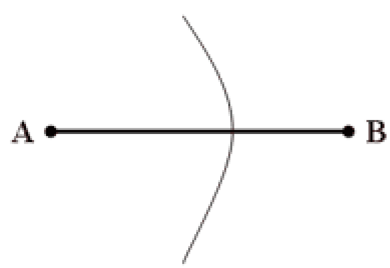
Procedure: *Center point A and linear point B are the endpoints of a given radius. Set the point of the compass on A and the lead on B and draw the circle.*



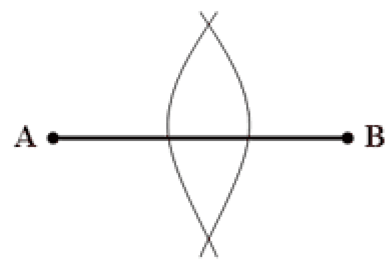
Diameter construction:

Procedure: *Bisect given diameter AB by placing the compass point first on Point A and opening your compass so that the lead touches a point on the line that is more than midway towards Point B.*

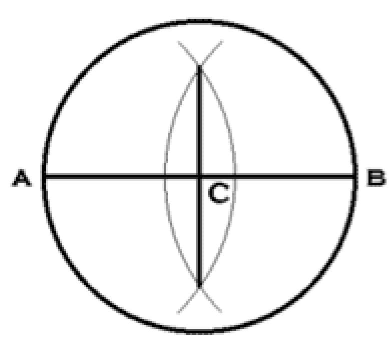
Using this setting, make an arc above and below the line



Now, put your compass point on Point B, and using the same compass setting, make an arc above and below the line that intercepts the arc made from point A.



Where the arcs intercept, join the two points to form a line that will bisect line AB. Since C denotes the midpoint of AB, then AC and BC are radii of the circle and either can be used to set the compass.



Practice Exercise

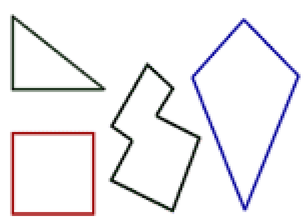
1. On a separate piece of paper, use a compass to construct a circle with center O and radius 8 cm. Draw a sector in your circle labeled AON.

2. On a separate piece of paper, use a compass to construct a circle with a diameter 11 cm. Label the diameter as RS.

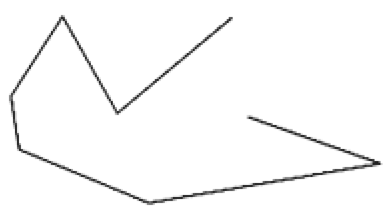
Polygon

A closed plane figure formed by three or more line segments

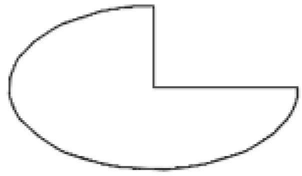
Examples:



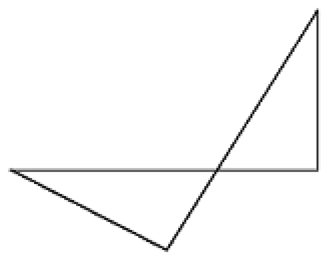
The figure below is not a polygon, since it is not a closed figure:



The figure below is not a polygon, since it is not made of line segments:



The figure below is not a polygon, since its sides do not intersect in exactly two places each:

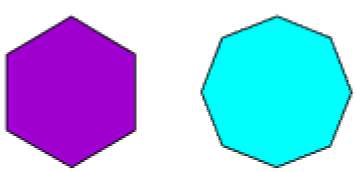


Regular Polygon

A regular polygon is a polygon whose sides are all the same length, and whose angles are all the same.

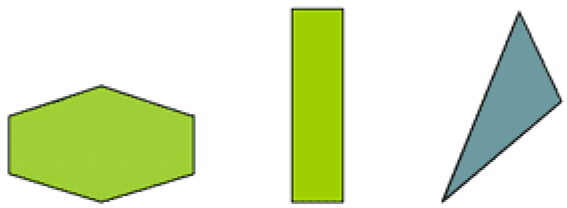
Examples:

The following are examples of regular polygons:



Examples:

The following are not examples of regular polygons:

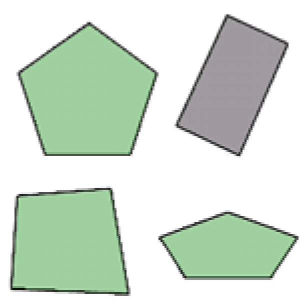


Convex Polygon

A figure is convex if every line segment drawn between any two points inside the figure lies entirely inside the figure.

Example:

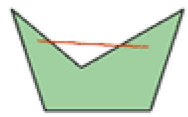
The following figures are convex.

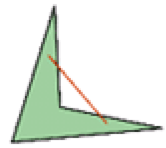


Concave Polygon

A figure that is not convex is called a concave figure. A concave polygon has at least one side that is curved inward.

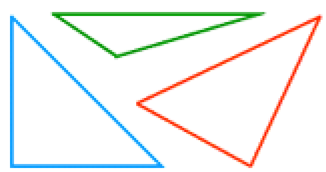
The following figures are concave. Note the red line segment drawn between two points inside the figure that also passes outside of the figure.





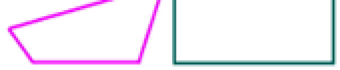
Triangle

A three-sided polygon
Examples:



Quadrilateral

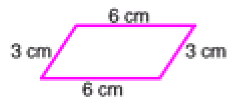
A four-sided polygon
Examples:



Parallelogram

A quadrilateral whose opposite sides are parallel and congruent

Example:



Trapezoid

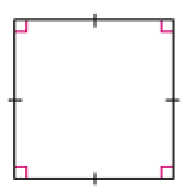
A quadrilateral with only one pair of parallel sides

Example:

Square

A rectangle with 4 congruent sides

Example:

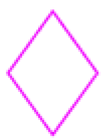


Rectangle

A parallelogram with 4 right angles
Example:

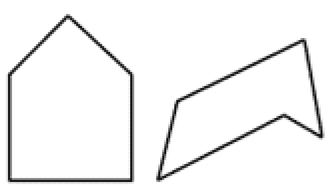
Rhombus

A parallelogram whose four sides are congruent and whose opposite angles are congruent
Example:



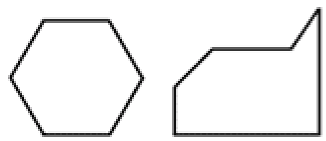
Pentagon

A five-sided polygon
Examples:



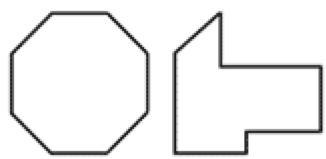
Hexagon

A six-sided polygon
Examples:



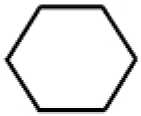
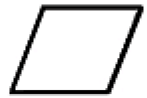
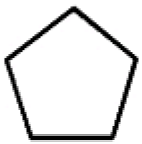
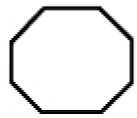
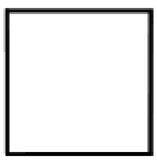
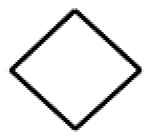
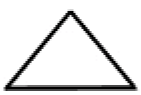
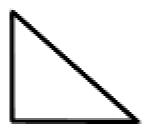
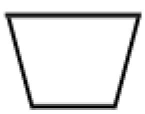
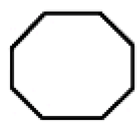
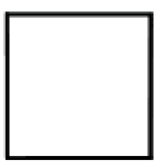
Octagon

An eight-sided polygon
Examples:



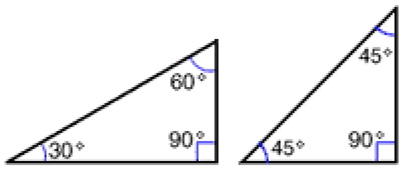
Practice Exercise

Write down the name for each polygon.

1.  ________	2.  ________
3.  ________	4.  ________
5.  ________	6.  ________
7.  ________	8.  ________
9.  ________	10.  ________
11.  ________	

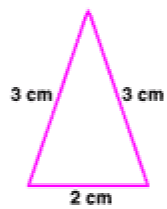
Right Triangle

A triangle with exactly one right angle
Examples:



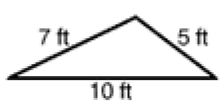
Isosceles Triangle

A triangle with two congruent sides
Example:



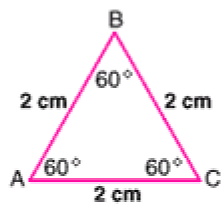
Scalene Triangle

A triangle with no congruent sides
Example:



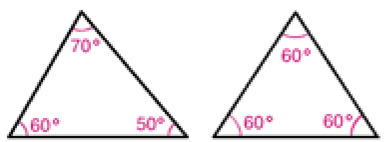
Equilateral Triangle

A triangle with three congruent sides and three congruent angles
Example:



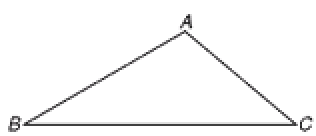
Acute Triangle

A triangle in which all three angles are acute
Example:



Obtuse Triangle

A triangle containing exactly one obtuse angle
Example:

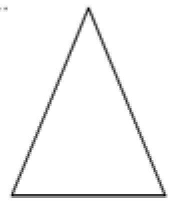


$\angle A$ is obtuse so $\angle ABC$ is an obtuse triangle

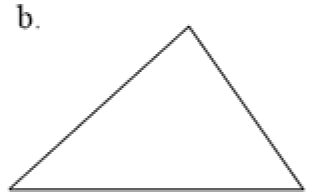
Practice Exercise

1. Measure the sides and classify the triangle as equilateral, isosceles, or scalene.

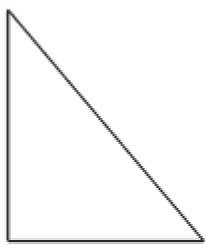
a.



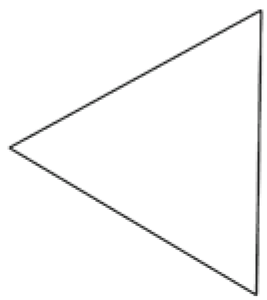
b.



c.

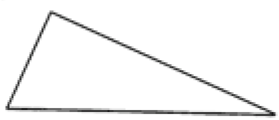


d.

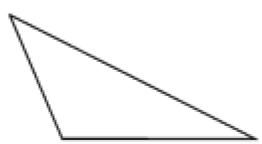


2. Measure the largest angle, then classify the triangle as acute, obtuse, or right.

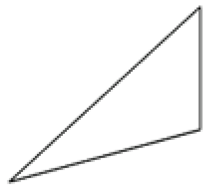
a.



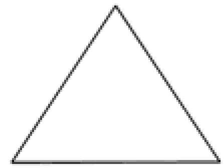
b.



c.



d.

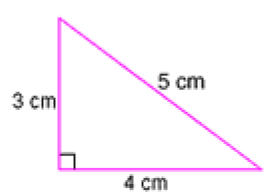


Pythagorean Theorem (Pythagorean Property)

Pythagoras was a Greek philosopher and mathematician. His ideas influenced great thinkers throughout the ages, and he is well known to math students. His Pythagorean Theorem is a simple rule about the proportion of the sides of right triangles: ***The square of the hypotenuse (the longest side)of a right triangle is equal to the sum of the square of the other two sides (legs).***

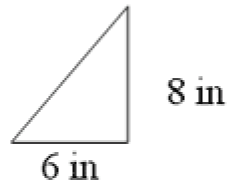
In any right triangle, if a and b are the lengths of the legs and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$

Example:



$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$

Example Find the length of the hypotenuse in the triangle below.



Step 1 Replace a with 6 and b with 8 in the formula $a^2 + b^2 = c^2$.

$$a^2 + b^2 = c^2$$

$$6^2 + 8^2 = c^2$$

$$36 + 64 = c^2$$

$$100 = c^2$$

Step 2 The formula gives the value of the hypotenuse squared. To find the length of the hypotenuse, find the square root of 100.

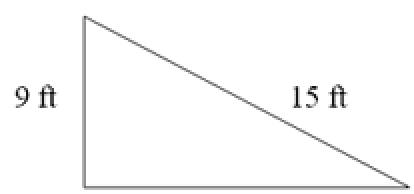
$$c = \sqrt{100}$$

c = **10 in**

Answer: The length of the hypotenuse in the given triangle is **10 in.**

In some problems you may be given the length of the hypotenuse and the length of one of the legs. To find the length of the other leg, you can still use the Pythagorean theorem.

Example Find the length of the missing leg in the triangle below.



Step 1 Write down the Pythagorean theorem and substitute in the values you know.

$$a^2 + b^2 = c^2$$

$$a^2 + 9^2 = 15^2$$

Step 2 Find the values of the squares.

$$a^2 + 81 = 225$$

Step 3 To get the unknown, *a*, alone on one side, subtract 81 from both sides.

$$a^2 + 81 - 81 = 225 - 81$$

$$a^2 = 144$$

Step 4 To find *a*, find the square root of both sides of the equation.

$$a = \sqrt{144}$$

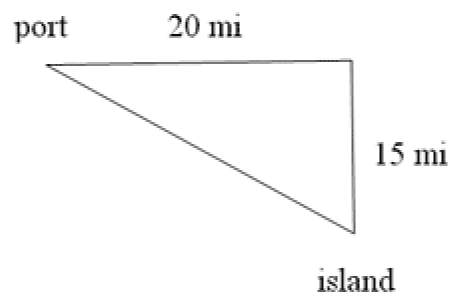
$$\boldsymbol{a = 12}$$

Answer: The length of the missing leg in the given triangle is **12 ft.**

In some problems you will have to recognize that a figure is a right triangle. The picture or problem may say nothing about a right triangle, the hypotenuse, or legs. Drawing a picture may help you see that the problem involves a right-triangle relationship.

Example A boat sails 20 miles east of port and then 15 miles south to an island. How far is the boat from the port if you measure in a straight line?

Step 1 Make a drawing to see how to solve the problem. East is normally to the right on a map, and south is toward the bottom. Notice that the actual distance from the port is the hypotenuse of a right triangle.



Step 2 Replace *a* with 20 and *b* with 15 in the formula $a^2 + b^2 = c^2$.

$$a^2 + b^2 = c^2$$

$$20^2 + 15^2 = c^2$$

$$400 + 225 = c^2$$

$$625 = c^2$$

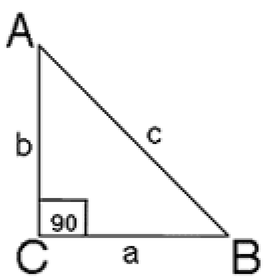
Step 3 Find the square root of 625.

$$\sqrt{625} = c$$

$$\boldsymbol{25 = c}$$

Answer: The boat is **25 miles** from the port.

Practice Exercise



Use ABC as shown on left to help you complete each question.
Round to the nearest hundredth.

1. If a = 9 and b = 40, then c = __________	2. If a = 5 and b = 12, then c = __________
3. If a = 3 and b = 4, then c = __________	4. If a = 24 and b = 45, then c = __________
5. If a = 11 and b = 7, then c = __________	6. If a = 11 and b = 10, then c = __________

7. If a = 4 and b = 2, then c = __________	8. If a = 8 and b = 12, then c = __________
9. If a = 6 and b = 5, then c = __________	10. If a = 3 and b = 6, then c = __________
11. If a = 9 and b = 2, then c = __________	12. If a = 7 and b = 2, then c = __________
13. If a = 4 and b = 6, then c = __________	14. If a = 6 and b = 6, then c = __________
15. If a = 9 and b = 3, then c = __________	16. If a = 2 and b = 8, then c = __________
17. If a = 12 and b = 12, then c = __________	18. If a = 4 and b = 11, then c = __________
19. If a = 11 and b = 20, then c = __________	20. If a = 18 and b = 18, then c = __________
21. If a = 16 and b = 19, then c = __________	22. If a = 21 and b = 16, then c = __________
23. If a = 19 and b = 13, then c = __________	24. If a = 10 and b = 16, then c = __________
25. If a = 10.6 and b = 7.2, then c = __________	26. If a = 9.4 and b = 12.8, then c = __________
27. If a = 8.4 and b = 10.8, then c = __________	28. If a = 4.9 and b = 5.5, then c = __________

29. If c = 37 and b = 35 then a = __________	30. If c = 17 and b = 15 then a = __________
31. If c = 13 and a = 12 then b = __________	32. If c = 29 and a = 21 then b = __________
33. If c = 51 and b = 45 then a = __________	34. If c = 25 and a = 24 then b = __________
35. If c = 65 and a = 56 then b = __________	36. If c = 20 and b = 10 then a = __________
37. If c = 30 and b = 12 then a = __________	38. If c = 38 and a = 14 then b = __________
39. If c = 39 and a = 21 then b = __________	40. If c = 34 and b = 24 then a = __________
41. If c = 21 and a = 9 then b = __________	42. If c = 25 and a = 14 then b = __________
43. If c = 39 and b = 13 then a = __________	44. If c = 32 and b = 22 then a = __________
45. If c = 40 and b = 30 then a = __________	46. If c = 49 and a = 20 then b = __________
47. If c = 50 and b = 31 then a = __________	48. If c = 22 and a = 12 then b = __________
49. If c = 49 and b = 28 then a = __________	50. If c = 61 and b = 51 then a = __________

51. If c = 90 and a = 63 then b = __________	52. If c = 59 and a = 39 then b = __________
53. If c = 8.22 and b = 6.2 then a = __________	54. If c = 11.46 and a = 4.6 then b = __________
55. If c = 11.95 and a = 10.9 then b = __________	56. If c = 6.83 and b = 5.8 then a = __________

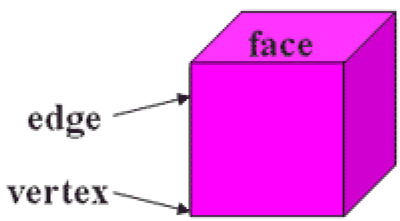
Using the Pythagorean theorem, solve the problems presented below.

1. A ladder rests against the side of Kate's house. The bottom of the ladder is 8 meters from the house, and the top just reaches a window that's 15 meters above ground. How long is the ladder?

2. A forest ranger at Oak Ridge Lookout spotted a fire 24 kilometers west of his location. If the town of Dairy is 38 miles due south of the lookout tower, how far is Dairy from the fire?

Polygons and circles are flat, or two-dimensional. They have only length and width. But ***cubes***, ***prisms***, ***pyramids***, and ***spheres*** are solid. They have a third dimension known as height or, sometimes, depth. These solids are also called ***space figures*** or ***polyhedrons***.

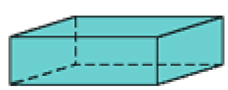
Cubes, prisms, pyramids, and other solids have sides called ***faces***. These faces are flat surfaces that are in the shapes of polygons. Faces meet at edges. The edges are line segments, which meet in vertexes. The vertexes are points.



Prism

A polyhedron whose two bases are congruent, parallel polygons in parallel planes and whose lateral faces are parallelograms

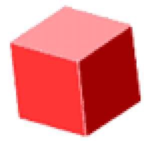
Example:



rectangular prism

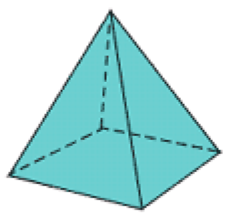
Cube

A square prism with six congruent square faces
Example:



Pyramid

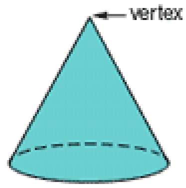
A polyhedron with a base that is a polygon and with lateral faces that are triangles which share a common vertex
Example:



square pyramid

Cone

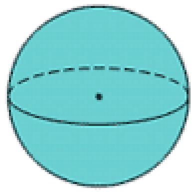
A solid figure with a circular base and one vertex
Example:



Sphere

A solid figure with all points the same distance from the center

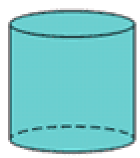
Example:



Cylinder

A solid figure with two parallel, congruent circular bases connected by a curved surface

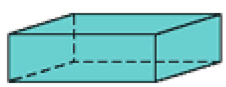
Example:



Practice Exercise

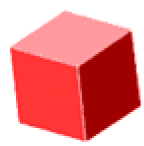
Match the solid shapes with the common three-dimensional objects listed below them.

A



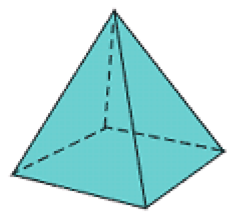
Rectangular Prism

B



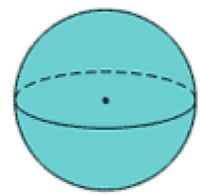
Cube

C



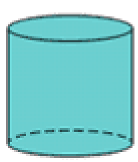
Cone

D



Sphere

E



Cylinder

1. ______ Dice
2. ______ Fish Bowl
3. ______ Balloon
4. ______ Marble
5. ______ Light Bulb
6. ______ Bowling Ball
7. ______ Mug
8. ______ AA Battery
9. ______ Tool Box
10. ______ Soccer Ball
11. ______ Funnel
12. ______ Pill Bottle
13. ______ Paper Towel Roll
14. ______ Pen

15. ______ Megaphone

16. ______ Ice Cube

17. ______ Spray Can

18. ______ Cement Block

19. ______ Child’s Block

20. ______ Suitcase

21. ______ Fish Tank

22. ______ Cereal Box

23. ______ Tepee

24. ______ Planet Earth

25. ______ Can of Paint

26. ______ Hockey Puck

27. ______ Can of Peas

28. ______ Water Pipe

Answer Key

Book 14018 - Geometry

Page 14 **2.** line UV **3.** Ray LT
4. line segment QS **5.** line segment SD
6. Ray QP **7.** line YJ **8.** Ray LE

Page 15 **2.** Intersecting **3.** Parallel **4.** Parallel
5. Parallel **6.** Perpendicular
7. Intersecting **8.** Perpendicular
9. Intersecting **10.** Intersecting

Page 16 **Make sure that all of the instructions are followed to complete the constructions in questions 1 – 4.**

Page 19 **1. a.** 59º **b.** 98º
Make sure that all of the instructions are followed to complete the constructions in questions 2 – 7.

Page 23 **1.** An acute angle **2.** An obtuse angle
3. A straight angle **4.** An acute angle
5. A right angle **6.** Acute Angle
7. Straight Angle **8.** Right Angle
9. Acute Angle **10.** 2

Page 30 **(adjacent angles)**

angle a and angle c
angle b and angle d
angle a and angle c
angle c
angle d
angle a

Page 31 (complementary and supplementary angles)

1. a 75 degree angle **2.** a 34 degree angle
3. a 73 degree angle **4.** a 59 degree angle

a. any of angles 3, 4, 5 and 6
b. any of angles 1, 2, 7, and 8
c. angle 1 and angle 4 **or** angle 2 and angle 3 **or** angle 6 and angle 7 **or** angle 5 and angle 8

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1. 50.24 **2.** 18; 56.52 **3.** 2; 12.56
4. 43.96 **5.** 62.8 **6.** 42.08
7. 9.80; 19.60 **8.** 9.20; 18.40
9. 22.8; 71.59 **10.** 6.2; 38.94
11. 32 24/35 **12.** 7 1/5; 45 9/35
13. 16 34/35 **14.** 42 26/35
12. 38.82; 121.89

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2. 238.64 cm **3.** 31.4 in **4.** 144.44 yd
5. 66.69 yd **6.** 60.29 cm **7.** 25.12 cm
3. 79.69 in

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1. EC **or** DC **or** CF **or** FC **or** CD **or** CE
2. DE **or** ED **3.** AB **or** BA

4. AB **or** BA **or** DE **or** ED **or** AE **or** EA **or** BE **or** EB **or** BD **or** DB **or** FE **or** EF **or** FB **or** BF **or** FA **or** AF **or** FD **or** DF **or** AD **or** DA **5.** AB **or** BA **or** DE **or** ED **6.** DCF **or** FCD **or** ECF **or** FCE **7.** JK **or** KJ **8.** DE **or** ED

Page 44 **Make sure that all of the instructions are followed to complete the constructions in questions 1 – 2.**

Page 51 **1.** Hexagon **2.** Rhombus **3.** Pentagon **4.** Octagon **5.** Square **or** Rectangle **6.** Square **or** Rectangle **7.** Triangle **8.** Triangle **9.** Trapezoid **10.** Hexagon **11.** Square **or** Rectangle

Page 54 **1. a.** isosceles **b.** scalene **c.** scalene **d.** isosceles **2. a.** right **b.** obtuse **c.** obtuse **d.** acute

Page 60 **1.** 41 **2.** 13 **3.** 5 **4.** 51 **5.** 13.04 **6.** 14.87 **7.** 4.47 **8.** 14.42 **9.** 7.81 **10.** 6.71 **11.** 9.22 **12.** 7.28 **13.** 7.21 **14.** 8.49 **15.** 9.49 **16.** 8.25 **17.** 16.97 **18.** 11.70 **19.** 22.83 **20.** 25.46 **21.** 24.84 **22.** 26.40 **23.** 23.02 **24.** 18.87 **25.** 12.81 **26.** 15.88 **27.** 13.68 **28.** 7.37 **29.** 12 **30.** 8 **31.** 5 **32.** 20 **33.** 24 **34.** 7 **35.** 33 **36.** 17.32 **37.** 27.50 **38.** 35.33

39. 32.86 **40.** 24.08 **41.** 18.97
42. 20.71 **43.** 36.77 **44.** 23.24
45. 26.46 **46.** 44.73 **47.** 39.23
48. 18.44 **49.** 40.21 **50.** 33.47
51. 64.27 **52.** 44.27 **53.** 5.40 **54.** 10.50
55. 4.90 **56.** 3.61

Page 67 **1.** Cube **2.** Sphere **3.** Sphere
4. Sphere **5.** Sphere **6.** Sphere
7. Cylinder **8.** Cylinder
9. Rectangular Prism **10.** Sphere
11. Cone **12.** Cylinder **13.** Cylinder
14. Cylinder **15.** Cone **16.** Cube
17. Cylinder **18.** Rectangular Prism
19. Cube **20.** Rectangular Prism
21. Rectangular Prism
22. Rectangular Prism **23.** Cone
24. Sphere **25.** Cylinder **26.** Cylinder
27. Cylinder **28.** Cylinder