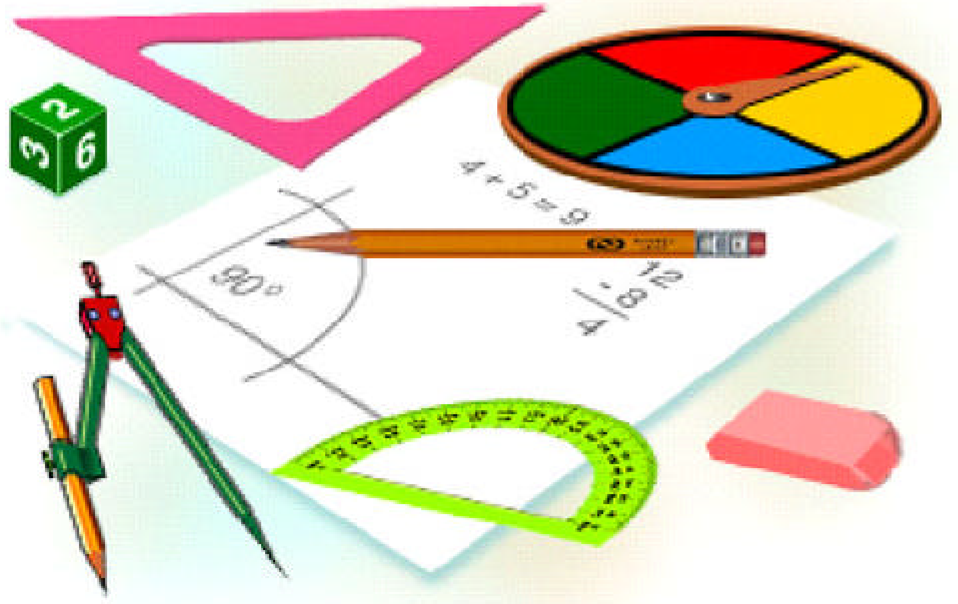
The Next Step

Mathematics Applications for Adults



Book 14018 – Number Operations

INTRODUCTION

Why Math?

The most important reason for learning math is that it teaches us how to think. Math is more than adding and subtracting, which can easily be done on a calculator; it teaches us how to organize thoughts, analyze information, and better understand the world around us.

Employers often have to re-educate their employees to meet the demands of our more complex technological society. For example, more and more, we must be able to enter data into computers, read computer displays, and interpret results. These demands require math skills beyond simple arithmetic.

Everyone Is Capable of Learning Math

There is no **type** of person for whom math comes easily. Even mathematicians and scientists spend a lot of time working on a single problem. Success in math is related to practice, patience, confidence in ability, and hard work.

It is true that some people can solve problems or compute more quickly, but speed is not always a measure of understanding. Being “faster” is related to **more practice or experience.**

For example, the reason why math teachers can work problems quickly is because they've done them so many times before, not because they have "mathematical minds".

Working with something that is familiar is natural and easy. For example, when cooking from a recipe we have used many times before or playing a familiar game, we feel confident. We automatically know what we need to do and what to expect. Sometimes, we don't even need to think. However, when using a recipe for the **first** time or playing a game for the **first** time, we must concentrate on each step. We double-check that we have done everything right, and even then we fret about the outcome. The same is true with math. When encountering problems for the very first time, **everyone must have patience** to understand the problem and work through it correctly.

It's Never Too Late to Learn

One of the main reasons people don't succeed in math is that they don't start at the right place. **IMPORTANT! You must begin where *you* need to begin.** Could you hit a homerun if you hadn't figured out which end of the bat had to make contact with the ball? Why should learning math be any different?

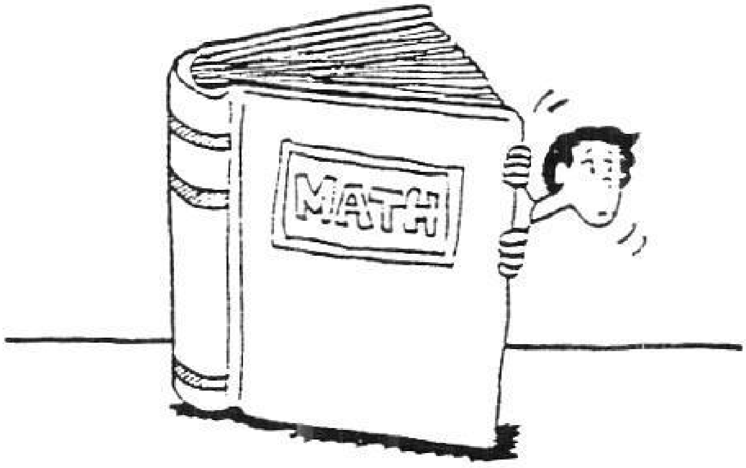
If it has been a while since your last math class, **you must determine what level math you should take.** A teacher or trained tutor can help determine this with a few placement tests and questions.

Sometimes a few tutoring sessions can help you fill gaps in your knowledge or help you remember some of the things you have simply forgotten. It could also be the case where your foundations may be weak and it would be better for you to relearn the basics. **Get some help** to determine what is best for you.

Feeling good about ourselves is what all of us are ultimately striving for, and nothing feels better than conquering something that gives us difficulty. This takes a great deal of courage and the ability to rebound from many setbacks. This is a natural part of the learning process, and when the work is done and we can look back at our success, nothing feels better.

Where's the best place to hide if you're scared?

Inside a math book because there is safety in numbers.



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OUTLINE

Mathematics - Book 14018

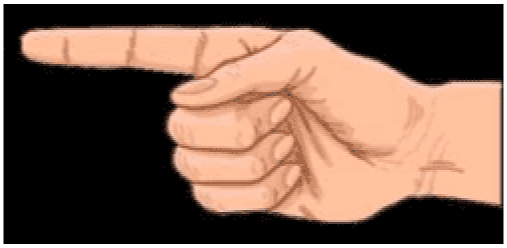
Number Operations
Mathematical Operations, Average, Median, and Mode
perform with accuracy and speed the four mathematical operations.
find average, median, and mode.
Factors and Prime Numbers
factor a given group of whole numbers.
determine which numbers are prime numbers.
find the Greatest Common Factor (GCM).
find the Least Common Multiple (LCM).
Exponents
express like factors using exponents.
express exponents using like factors.
perform certain mathematical operations involving exponents.
Squares and Square Roots
find the square and square root of whole numbers which have perfect squares.
Problem Solving With Whole Numbers
solve multi-step problems, with and without a calculator.

THE NEXT STEP

Book 14018

Number Operations

Mathematical Operations, Average, Median, Mode



Digit is a counting word. A digit is any of the numerals from **1** to **9.** The word “digit” is also the name for a finger. So number digits can be counted on finger digits.

Our modern system of counting or ***tallying*** probably came from counting on fingers. Fingers and hands were among the earliest known calculators!

The set of counting numbers has no end. It can go on forever. The idea that counting numbers can go on and on is called ***infinity***. Infinity has a special symbol:

∞

There is no such thing as the “largest number.” You can always add to or multiply a large number to make an even bigger number.

$$\infty + 3 = \infty$$

$$\infty \text{ x } 10 = \infty$$

If you began writing all the counting numbers today, you could continue writing every moment of every day for every day of the rest of your life and never be finished!

What's a googol?

A googol is a 1 with a hundred zeroes behind it. We can write a googol using exponents by saying a googol is 10^100 or 10 to the 100th power.

The biggest named number that we know is googolplex, ten to the googol power, or (10)^(10^100). That's written as a one followed by googol zeroes.

It’s funny that no one ever seems to ask, “What is the smallest number?” Again, there is really no such thing. You could always subtract from or divide a small number to make an even smaller number. As the number gets smaller and smaller, you would be approaching, but never reaching, negative infinity.

$$-\infty$$

The set of ***counting numbers***, or ***natural numbers***, begins with the number **1** and continues into infinity.

{1,2,3,4,5,6,7,8,9,10...}

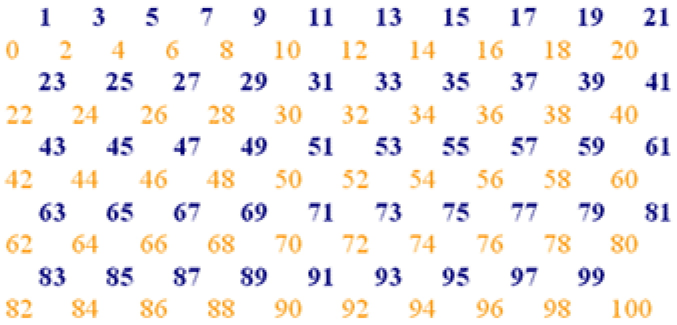
The set of ***whole numbers*** is the same as the set of counting numbers, except that it begins with **0**.

{0,1,2,3,4,5,6,7,8,9,10...}

☞ ***All counting numbers are whole numbers. Zero is the only whole number that is not a counting number.***

Even numbers include the numbers ***0*** and ***2*** and all numbers that can be divided evenly by ***2***. ***Odd numbers*** are all numbers that cannot be divided evenly by ***2***.

Odd and **Even** Numbers to 100

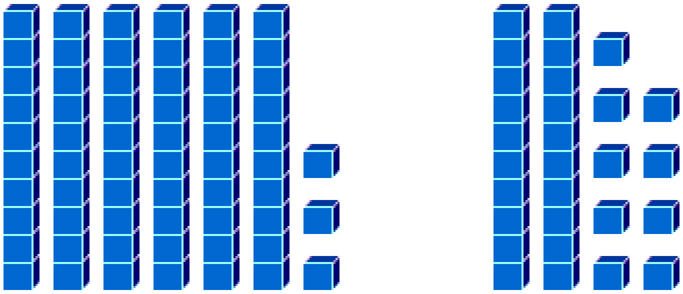


1	3	5	7	9	11	13	15	17	19	21
0	2	4	6	8	10	12	14	16	18	20
23	25	27	29	31	33	35	37	39	41	
22	24	26	28	30	32	34	36	38	40	
43	45	47	49	51	53	55	57	59	61	
42	44	46	48	50	52	54	56	58	60	
63	65	67	69	71	73	75	77	79	81	
62	64	66	68	70	72	74	76	78	80	
83	85	87	89	91	93	95	97	99		
82	84	86	88	90	92	94	96	98	100	

Ordering numbers means listing numbers from least to greatest, or from greatest to least. Two symbols are used in ordering.

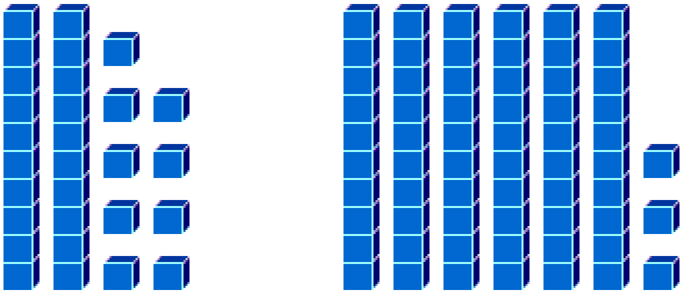
<	>
is less than	**is greater**
2 < 10	**10 > 2**

Greater Than >



63 is **greater than** 29.
63 > 29

Less Than <



29 is **less than** 63.
29 < 63

A ***mathematical operation*** is a process or action, such as addition, subtraction, multiplication, or division performed in a specified sequence and in accordance with specific rules.

Combining two or more numbers is called ***addition***. The term for addition is ***plus***, and the symbol for plus is +. The numbers that are combined in addition are called ***addends*** and together they form a new number called a ***sum***.

2 ---- addends ---- 3
+ 2 + 1
4 ---- sum --------- 4

Adding whole numbers is as simple as 2 + 2! To add two whole numbers, you can simply follow the number line and complete the addition fact.

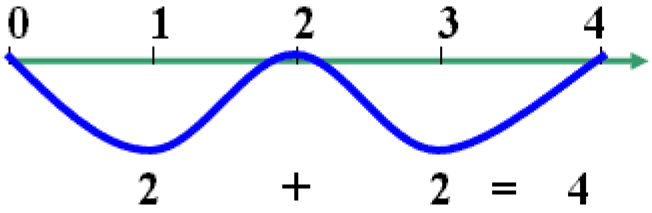
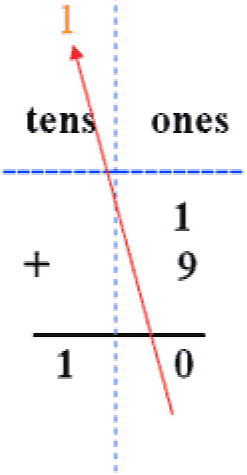


Table of Addition Facts

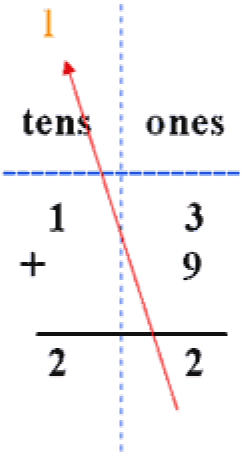
+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

Regrouping Numbers in Addition

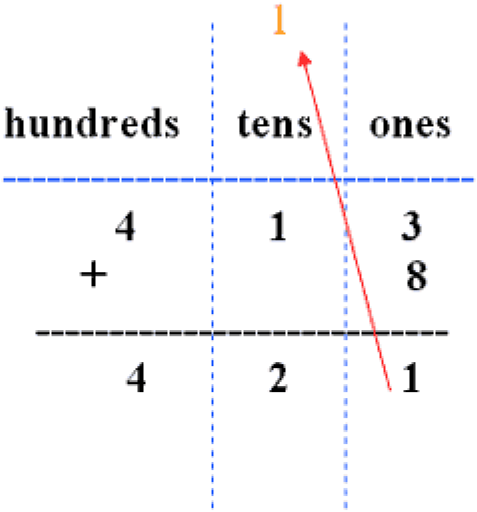
Addition often produces sums with a value greater than ***9*** in a given place. The value of ten is then ***regrouped*** (or ***carried***) to the next place.



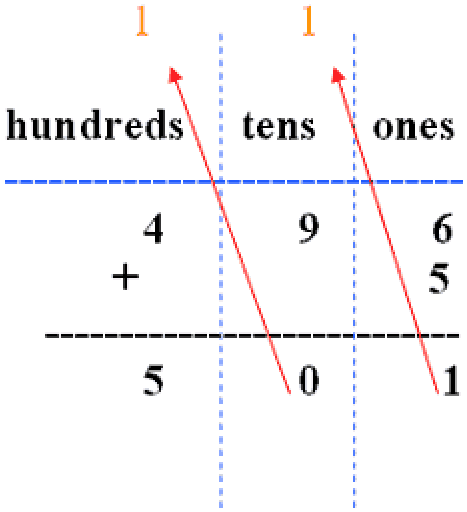
1	
tens	ones
	1
+	9
1	0



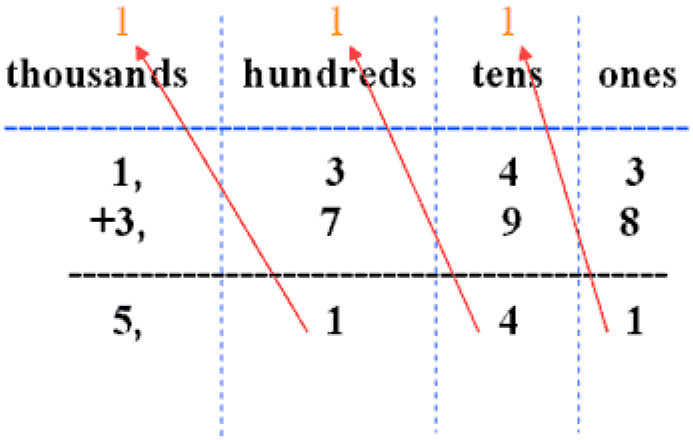
1	
tens	ones
1	3
+	9
2	2



	1	
hundreds	tens	ones
4	1	3
+		8
4	2	1



1	1	
hundreds	tens	ones
4	9	6
+		5
5	0	1



1	1	1	
thousands	hundreds	tens	ones
1,	3	4	3
+3,	7	9	8
5,	1	4	1

To explain addition another way, it can be done by adding the place value amounts separately.

e.g. 69
+ 8
17
60 (the 6 in the tens place means 6 tens or “60”)
77

$\Longrightarrow$ If there are not enough digits in each number to make even columns under each place value, then zeros may be used **before** a given number to make adding easier. Do **not** add zeros **after** a number because it changes the value of the whole number.

e.g. 69 + 8 + 125 could be added as:

069
008
+125

Commutative Property of Addition

The property which states that two or more addends can be added in any order without changing the sum

$$a + b = b + a$$

Examples:

$$c + 4 = 4 + c$$

$$(2 + 5) + 4r = 4r + (2 + 5)$$

Associative Property of Addition

The property which states that for all real numbers *a*, *b*, and *c*, their sum is always the same, regardless of their grouping:

$$(a + b) + c = a + (b + c)$$

Example:

$$(2 + 3) + 4 = 2 + (3 + 4)$$

"Taking away" one or more numbers from another number is called ***subtraction***. The term for subtraction is ***minus***, and the symbol for minus is **-**. The number being subtracted is called a ***subtrahend***. The number being subtracted from is called a ***minuend***. The new number left after subtracting is called a ***remainder*** or ***difference***.

4 ---- minuend ---- 4
- 2 --subtrahend - - 1
2 – difference ---- 3

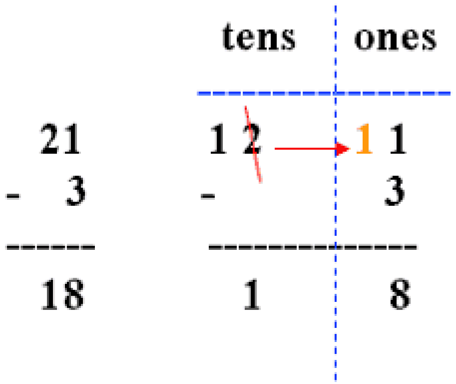
The complete addition or subtraction "sentence" is called an ***equation***. An equation has two parts. The two parts are separated by the ***equal sign***, **=**. For example, ***the minuend minus the subtrahend equals the difference***. An ***addition fact*** or a ***subtraction fact*** is the name given to specific addition or subtraction equations.

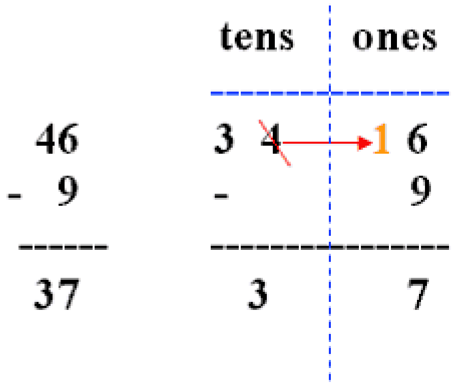
0 + 1 = 1	**1 – 1 = 0**
1 + 1 = 2	**2 – 1 = 1**
2 + 1 = 3	**3 – 1 = 2**
3 + 1 = 4	**4 – 1 = 3**
4 + 1 = 5	**5 – 1 = 4**

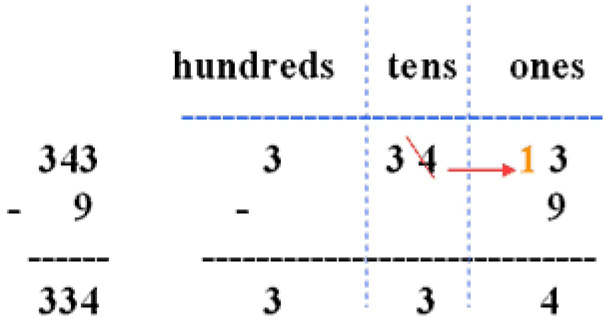
5 + 1 = 6	**6 – 1 = 5**
6 + 1 = 7	**7 – 1 = 6**
7 + 1 = 8	**8 – 1 = 7**
8 + 1 = 9	**9 – 1 = 8**

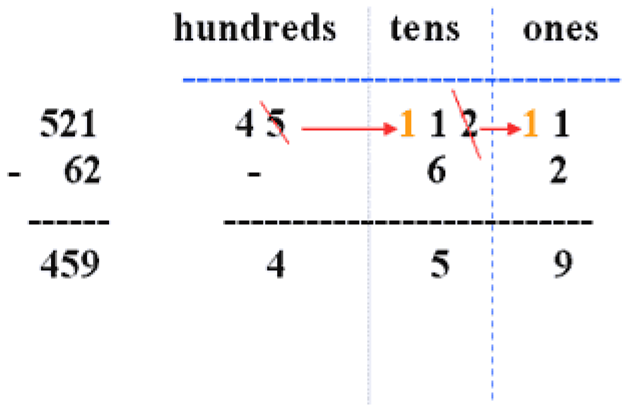
Regrouping in Subtraction

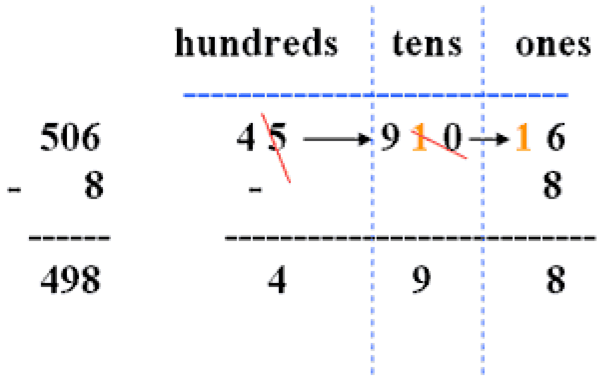
Regrouping, sometimes called ***borrowing***, is used when the subtrahend is greater than the minuend in a given place. Regrouping means to take a group of tens from the next greatest place to make a minuend great enough to complete the subtraction process.











Multiplication is a quick form of addition. By multiplying numbers together, you are really adding a series of one number to itself. For example, you can add ***2*** plus ***2***. Both ***2 plus 2*** and ***2 times 2*** equal ***4***.

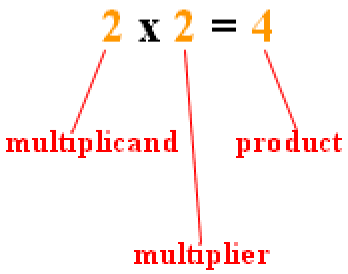
2 + 2 = 4
2 x 2 = 4

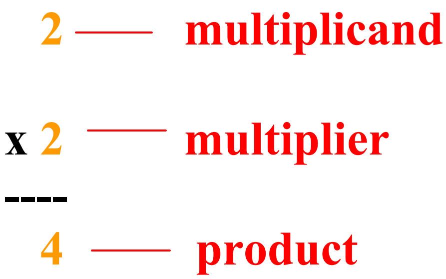
```
  2      2
+ 2    x 2
----   ----
  4      4
```

But what if you wanted to calculate the number of days in five weeks? You could add *7* days + *7* days + *7* days + *7* days + *7* days or you could multiply *7* days times ***5***. Either way you arrive at ***35***, the number of days in five weeks.

7 + 7 + 7 + 7 + 7 = 35
5 x 7 = 35

Although multiplication is related to addition, the parts of multiplication are not known as addends. Instead, the parts are known as ***multiplicands*** and ***multipliers***. A multiplication sentence, like an addition sentence, is called an ***equation***. But a multiplication sentence results in a ***product***, not a sum.





X	0	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Multiplication, Step-by-Step

When the multiplicand and the multiplier are numbers with two or more digits, multiplication becomes a step-by-step process.

Look at 15 x 13:

$$\begin{array}{rr} 1 & 5 \\ \text{x} & 3 \\ \hline 1 & 5 \end{array}$$

First, multiply the ones – 3 x 5. Write down the product so the ones columns line up.

$$\begin{array}{rr} 1 & 5 \\ \text{x} & 3 \\ \hline 1 & 5 \\ & \\ 3 & 0 \\ \hline \end{array}$$

Next, multiply the tens – 3 x 1 ten. Line up the product with the tens column.

— Zero is the place holder.

$$\begin{array}{rrr} & 1 & 5 \\ \times & & 3 \\ \hline & 1 & 5 \\ + & 3 & 0 \\ \hline & 4 & 5 \end{array}$$

Last, add the ones and tens to find the product of the equation.

Here is a shorter way:

$$\begin{array}{rrr} & 1 & \\ & 1 & 5 \\ \times & & 3 \\ \hline & 4 & 5 \end{array}$$

1. Multiply the ones: 3 x 5 = 15. Put the 5 in the ones column and regroup the 1 to the tens column.

2. Multiply the tens: 3 x 1 = 3.

3. Add the 1 that you regrouped to the 3, put the sum in the tens column.

Look at 265 x 23:

```
     265

x     23
---------
      15
     180
     600
```

First, multiply the multiplicand by the ones in the multiplier – 3 x 5, 3 x 6, and 3 x 2. Zero is the place holder.

```
   265

x   23
--------
    15
   180
   600
--------
       100
     1,200
     4,000
```

Next, multiply by the tens – 2 x 5, 2 x 6, and 2 x 2. Zero is the place holder.

```
       265

  x     23
  ---------
        15
+      180
+      600
  ---------
+      100
+    1,200
+    4,000
  ---------
     6,095
```

Last, add.

Here is a shorter way:

```
     11
     11
    265
x    23
---------
    795

   5300
---------
  6,095
```

1. Multiply the ones: 3 x 265
   3 x 5 = 15 regroup the 1
   3 x 6 = 18 plus the regrouped 1 = 19; regroup the 1
   3 x 2 = 6 plus the regrouped 1 = 7

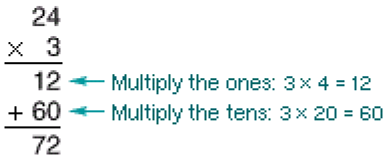
2. Multiply the tens: 2 x 265
   0 is the place holder
   2 x 5 = 10 regroup the 1
   2 x 6 = 12 plus the regrouped 1 = 13; regroup the 1
   2 x 2 – 4 plus the regrouped 1 = 5

3. Add 795 + 5300 = 6,095

Partial Product

A method of multiplying where the ones, tens, hundreds, and so on are multiplied separately and then the products added together

Example:



```
   24
 ×  3
 ----
   12  ← Multiply the ones: 3 × 4 = 12
 + 60  ← Multiply the tens: 3 × 20 = 60
 ----
   72
```

36 x **17 = 42 + 210 + 60 + 300 = 612**

When you multiply whole numbers, the ***product*** usually has a greater value than either the ***multiplicand*** or the ***multiplier***.

But there are exceptions:

A number multiplied by ***1*** is always equal to itself.

$$\begin{array}{r} \mathbf{1} \\ \underline{\mathbf{x\ 1}} \\ \mathbf{1} \end{array} \qquad \mathbf{21\ x\ 1 = 21} \qquad \begin{array}{r} \mathbf{36} \\ \underline{\mathbf{x\ 1}} \\ \mathbf{36} \end{array}$$

A number multiplied by ***0*** is always equal to ***0***.

$$\begin{array}{r} \mathbf{1} \\ \underline{\mathbf{x\ 0}} \\ \mathbf{0} \end{array} \qquad \mathbf{21\ x\ 0 = 0} \qquad \begin{array}{r} \mathbf{36} \\ \underline{\mathbf{x\ 0}} \\ \mathbf{0} \end{array}$$

To multiply a number by 10, add a 0 to the right of the number.

EXAMPLE

$$25\ X\ 10 = 250 \qquad \text{or} \qquad \begin{array}{r} 25 \\ \underline{x10} \\ 250 \end{array}$$

To multiply a number by 100, add two 0’s to the right of the number.

EXAMPLE

$$36\ X\ 100 = 3{,}600 \qquad \text{or} \qquad \begin{array}{r} 36 \\ \underline{x100} \\ 3{,}600 \end{array}$$

Commutative Property of Multiplication

The property which states that two or more factors can be multiplied in any order without changing the product

$$a \cdot b = b \cdot a$$

Examples:

$$3 \cdot c = c \cdot 3$$

$$4 \cdot 5 \cdot y7 = 5 \cdot 4 \cdot y7$$

Associative Property of Multiplication

The property which states that for all real numbers a, b, and c, their product is always the same, regardless of their grouping:

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

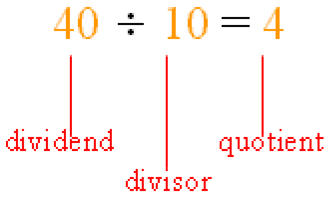
Example:

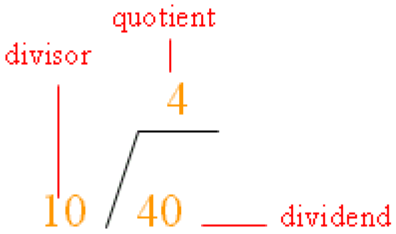
$$(5 \cdot 6) \cdot 7 = 5 \cdot (6 \cdot 7)$$

Division is the process of finding out how many times one number, the ***divisor***, will fit into another number, the ***dividend***. The division sentence results in a ***quotient***. The signs of division are ÷ and ⟌, and mean ***divided by***. You can think of division as a series of repeated subtractions. For example, ***40*** ÷ ***10*** could also be solved by subtracting ***10*** from ***40*** four times:

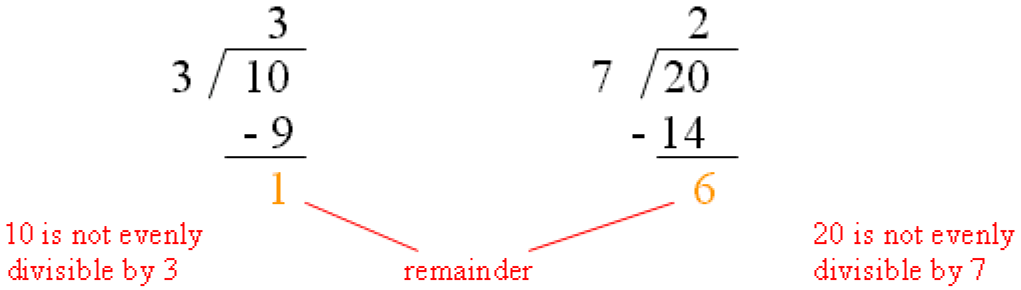
40 – 10 – 10 – 10 – 10 = 0

Because ***10*** can be subtracted four times, you can say that ***40*** can be divided by ***10*** four times, or ***40*** ÷ ***10 = 4***.

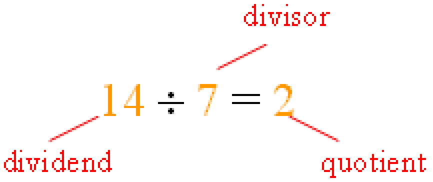


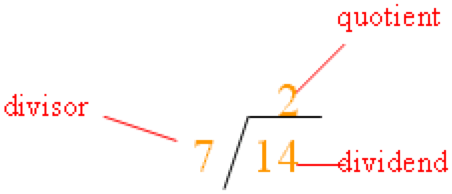


Many numbers do not fit evenly into other numbers. They are ***not evenly divisible by*** those numbers, and the number left over is called the ***remainder***.



To divide whole numbers, reverse the process of multiplication. For example, if ***2 x 7 = 14*** in a multiplication equation, then in a division sentence, ***14*** is the ***dividend*** and ***7*** is the ***divisor*** with a ***quotient*** of ***2***.





A whole number divided by ***1*** will always equal itself.

1 ¸ 1 = 1

$$1\overline{)21}\quad 21$$

36 ¸ 1 = 36

Zero divided by a whole number will always equal ***0***.

$$\begin{array}{r} 0 \\ 0 \div 12\overline{)=0} \end{array} \qquad \mathbf{3} \quad \mathbf{0} \qquad \mathbf{0/7 = 2}$$

Division, Step-by-Step

Where the dividend and divisor are numbers with two or more digits, division becomes a step-by-step process.

$$\begin{array}{rr} & 2 \\ 8 & \overline{)208} \\ - & 16 \\ \cline{2-2} & 4 \end{array}$$

First, round the divisor up - 8 rounds up to 10 - and estimate the number of 10s in 20. Answer: 2. Multiply the divisor – 8 x 2 – and subtract the product from the dividend.

$$\begin{array}{rr} & 26 \\ 8 & \overline{)208} \\ - & 16 \\ \cline{2-2} & 48 \\ - & 48 \\ \cline{2-2} & 0 \end{array}$$

Next, pull down the next digit from the dividend – 8 – and repeat the estimation and subtraction process.

$$\begin{array}{r} 26 \\ 8\overline{)208} \\ -\underline{16} \\ 48 \\ -\underline{48} \\ 0 \end{array}$$

Last, when you can subtract no more you’ve found the quotient.

No remainder

$$\begin{array}{r} 1 \\ 23\overline{)276} \\ -\underline{23} \\ 4 \end{array}$$

First, round 23 to 25 and estimate the number of 25s in 27. Answer: 1. Multiply the divisor by 1 – 23 x 1 – and subtract.

$$\begin{array}{r} 12 \\ 23\overline{)276} \\ -\underline{23} \\ 46 \\ -\underline{46} \end{array}$$

Next, pull down the next digit from the dividend – 6 – and repeat the estimation and subtraction process.

$$\begin{array}{r} 12 \\ 23\overline{)276} \\ -\underline{23} \\ 46 \\ -\underline{46} \\ 0 \end{array}$$

Then, pull down the next digit, estimate, and subtract, until you can subtract no more.

No remainder

Inverse (opposite) operations are used to simplify an equation for solving.
One operation is involved with the unknown and the inverse operation is used to solve the equation.

Addition and subtraction are inverse operations.
Given an equation such as $7 + x = 10$, the unknown x and 7 are *added*. Use the inverse operation subtraction. To solve for n, subtract 7 from 10. The unknown value is therefore 3.

Examples for addition and subtraction

Addition Problem	Solution
$x + 15 = 20$	$x = 20 - 15 = 5$

Subtraction Problem	Solution
$x - 15 = 20$	$x = 20 + 15 = 35$

Multiplication and division are inverse operations.
Given an equation $7x = 21$. x and 7 are multiplied to create a value of 21. To solve for x, divide 21 by 7 for an answer of 3.

Examples for division and multiplication.

Multiplication Problem	Solution
$3x = 21$	$x = 21 \div 3 = 7$

Division Problem	Solution
$x \div 12 = 3$	$y = 3 x 12 = 36$

Practice Exercise

Mixed Problems

Solve each problem.

1. $\begin{array}{r} 287 \\ +\ 34 \\ \hline \end{array}$

2. $506\overline{)48576}$

3. $\begin{array}{r} 532 \\ -\ \ 98 \\ \hline \end{array}$

4. $\begin{array}{r} 330 \\ \times 23 \\ \hline \end{array}$

5. $\begin{array}{r} 722 \\ \times\ 78 \\ \hline \end{array}$

6. $\begin{array}{r} 873 \\ -\ \ 33 \\ \hline \end{array}$

7. $\begin{array}{r} 289 \\ +\ 30 \\ \hline \end{array}$

8. $233\overline{)20038}$

9. $\begin{array}{r} 273 \\ -\ \ 78 \\ \hline \end{array}$

10. $\begin{array}{r} 408 \\ +\ 50 \\ \hline \end{array}$

11. $\begin{array}{r} 569 \\ +\ 76 \\ \hline \end{array}$

12. $\begin{array}{r} 881 \\ \times\ 25 \\ \hline \end{array}$

13. $\begin{array}{r} 525 \\ \times\ 75 \\ \hline \end{array}$

14. $564\overline{)5640}$

15. $176\overline{)4224}$

16. $\begin{array}{r} 808 \\ -\ 19 \\ \hline \end{array}$

17. $\begin{array}{r} 385 \\ -\ \ 17 \\ \hline \end{array}$

18. $613\overline{)32489}$

19. $756\overline{)71064}$

20. $\begin{array}{r} 518 \\ +\ 29 \\ \hline \end{array}$

21. $\begin{array}{r} 574 \\ \times\ 46 \\ \hline \end{array}$

22. $\begin{array}{r} 334 \\ +\ 12 \\ \hline \end{array}$

23. $\begin{array}{r} 614 \\ -\ \ 53 \\ \hline \end{array}$

24. $\begin{array}{r} 276 \\ \times\ 84 \\ \hline \end{array}$

Order of Operations

Sometimes the order in which you add, subtract, multiply, and divide is very important. For example, how would you solve the following problem?

2 x 3 + 6

Would you group

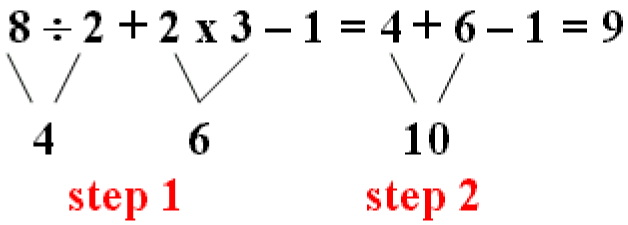
(2 x 3) + 6 or **2 x (3 + 6) ?**

Which comes first, addition or multiplication? Does it matter?
Yes. Mathematicians have written two simple steps:

1. ***All multiplication and division operations are carried out first, from left to right, in the order they occur.***

2. ***Then all addition and subtraction operations are carried out, from left to right, in the order they occur.***

For example:



Þ ***Perform all operations with parentheses (brackets) and exponents before carrying out the remaining operations in an equation.***

8 ÷ (2 + 2) x 3 – 1 =

8 ÷ 4 x 3 – 1 =

2 x 3 – 1 =

6 – 1 = 5

To remember the order of operations, simply remember BEDMAS: Brackets, Exponents, Division, Multiplication, Addition, Subtraction.

Example:

10 ÷ (2 + 8) X 2^3 - 4 *Add inside parentheses.*
10 ÷ 10 X 2^3 – 4 *Clear exponent.*

10 ÷ 10 X (2 X 2 X2) - 4
10 ÷ 10 X 8 - 4 *Multiply and divide.*
8 - 4 *Subtract.*
4

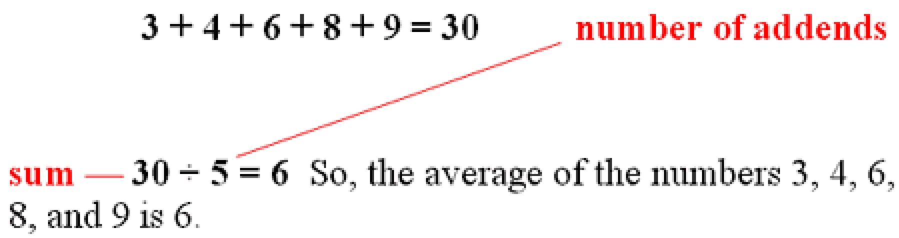
Practice Exercise

1. 12 - 10 + 25
2. 5 + 12 - 13
3. 19 + 15 + 20
4. 17 × 20 - 2
5. 12 ÷ 2
6. 11 + 21 × (11 + 23) × 18
7. 14 ÷ 2 + 8 × 22
8. 24 + 5 + (4 ÷ 2) + 16
9. 10 ÷ 5 × 18 × 14 + 13
10. 12 ÷ 6
11. 14 × 10
12. 14 ÷ 7 × 24 + 4
13. $10 + 4^3 + 16 + 14$
14. 22 + 16 × 13 - 24
15. 5 + (20 × 13 × 19)
16. 18 ÷ 9 × 25
17. 12 ÷ 3 + 18 - 9
18. $18 + 5^3 + 6 + 17 - 20$

19. $4 \div 2 \times 23 + 11$	**20.** $12^3 - 24 + 13$

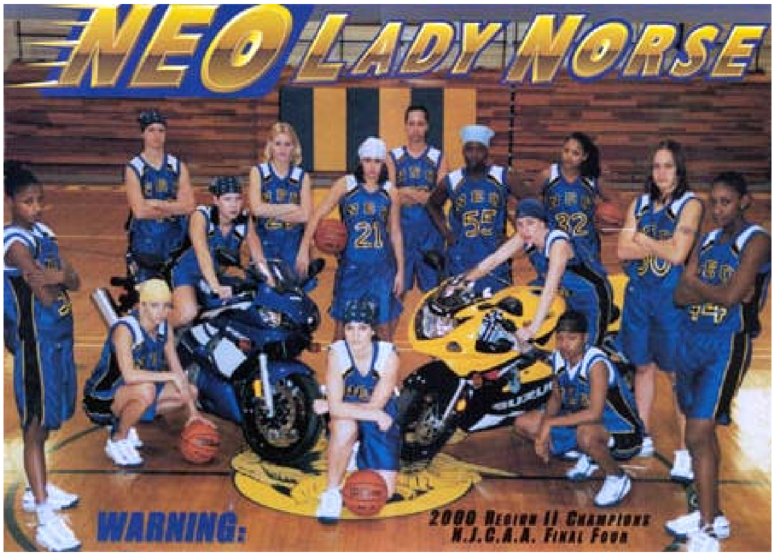
Averages

The most common way to find an ***average*** is to add up a list of numbers and divide the sum by the number of items on the list. Another word for average is ***mean***.


$3 + 4 + 6 + 8 + 9 = 30$

$30 \div 5 = 6$ So, the average of the numbers 3, 4, 6, 8, and 9 is 6.

When do you need to calculate an average? Your grades may be based on the average of all your test scores. In sports, you might want to find out the average height of players on your favorite basketball team.



The height of the starters for this team is:

Anita	**60"**
Jane	**58"**
Cathy	**57"**
Joy	**52"**
Tanya	**48"**

The average height of these players is 55 inches.

Medians

Average or mean is different from ***median***. The median is the middle number in a series of numbers stated in order from least to greatest. An average and a median can be the same number. The average of ***3***, ***5***, and ***7*** is ***5***:

3 + 5 + 7 = 15 and **15 ÷ 3 = 5**

and the median of ***3***, ***5***, and *7* is ***5***. But average and median are often different numbers.

Anita	**60”**
Jane	**58”**
Cathy	**57”**
Joy	**52”**
Tanya	**48”**

The median height of these girls is 57 inches—Cathy’s height-- because it is the middle number.

If there is an even number of data items, the median is the average (mean) of the two middle numbers.

Example Amy’s point totals for six games of basketball were 24, 16, 19, 22, 6, and 12 points. Find the median of her point totals.

Step 1 Arrange the data in order.

24, 22, **19, 16,** 12, 6

Step 2 The two middle numbers are 19 and 16. Average these to find the median.

19 + 16 = 35

$$35 \div 2 = 17.5$$

Amy's median point total is **17.5 points.**

Percentiles

Individual scores may be compared with all the other scores in a group by giving the score a positional standing or rank.

The percentile rank of a score indicates the percent of all the scores that are below this given score. If the rank of a particular score is the 60th percentile, it means that 60 % of all the scores are lower than this score.

Examples If Todd ranked fourth in a class of 16 students, there are 12 students of 16, or 75%, with a lower rank. He would have a percentile rank of 75 or a rank of the 75th percentile.

If Todd ranked fourth in a class of 40 students, there are 36 students of 40, or 90%, with a lower rank. He would have a percentile rank of 90 or a rank of the 90th percentile.

Mode

The number or numbers that occur most often in a collection of data; there can be more than one mode or none at all.

Examples:

2, 3, 4, 5, 5, 6, 7, **8, 8, 8**, 9, 11

The mode is 8.

2, 3, 4, **5, 5, 5**, 7, **8, 8, 8**, 9, 11

The modes are 5 and 8.

2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 17

There is no mode.

Practice Exercise

Calculating the Mean, Median, and Mode

Calculate the values to the nearest tenth.

1. 11, 12, 5, 5, and 5

Write the median: ___________ Write the mean: ___________ Write the mode: ___________

2. 15, 24, 15, 6, and 6

Write the median: ___________ Write the mean: ___________ Write the mode: ___________

3. 8, 15, 14, 15, and 22

Write the median: ___________ Write the mean: ___________ Write the mode: ___________

4. 1, 11, 24, 24, and 16

Write the median: Write the mean: Write the mode:

__________ __________ __________

5. Students with the following GPAs applied for a job: 2, 2.2, 2.9, 2.3, 2.2, 2.8, 2.6, 2.9, 3.7, and 3.5

Write the median: Write the mean: Write the mode:

__________ __________ __________

6. The following temperatures were recorded: 5, 5, 62, 5, 39, -15, -13, 3, -9, and 70

Write the median: Write the mean: Write the mode:

__________ __________ __________

7. The following grades were posted on the latest exam: 90, 63, 68, 90, 84, 54, 63, 85, 85, and 63

Write the median: Write the mean: Write the mode:

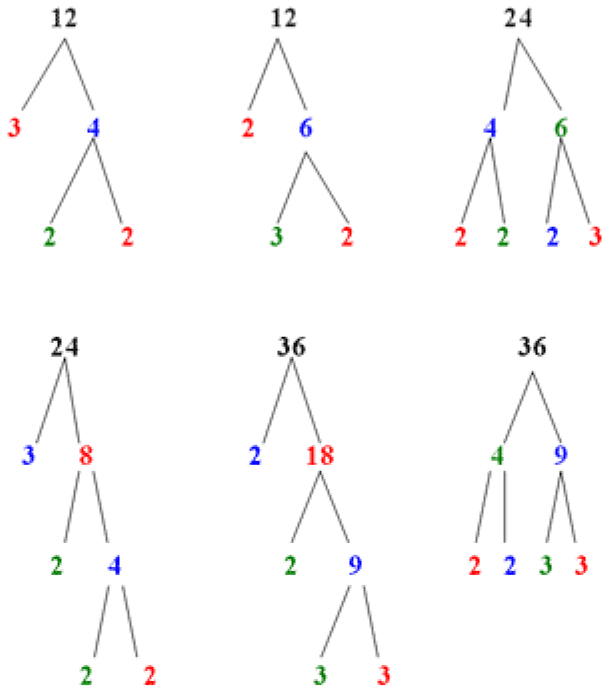
__________ __________ __________

Factors and Prime Numbers

Factors are numbers that, when multiplied together, form a new number called a ***product***. For example, ***1*** and ***2*** are factors of ***2***, and ***3*** and ***4*** are factors of ***12***.

Every number except ***1*** has at least two factors: ***1*** and itself.

Composite numbers have more than two factors. In fact, every composite number can be written as the product of ***prime numbers***. You can see this on a ***factor tree***.



Prime numbers are counting numbers that can be divided by only two numbers---***1*** and themselves. A prime number can also be described as a counting number with only two factors, ***1*** and itself. The number ***1***, because it can be divided only by itself, is ***not*** a prime number.

Prime Numbers to 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,

53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Practice Exercise

Classify each number as prime or composite.

1. 9 ☐Prime ☐Composite	2. 52 ☐Prime ☐Composite	3. 20 ☐Prime ☐Composite	4. 97 ☐Prime ☐Composite
5. 8 ☐Prime ☐Composite	6. 45 ☐Prime ☐Composite	7. 21 ☐Prime ☐Composite	8. 60 ☐Prime ☐Composite
9. 5 ☐Prime ☐Composite	10. 42 ☐Prime ☐Composite	11. 19 ☐Prime ☐Composite	12. 96 ☐Prime ☐Composite
13. 67 ☐Prime ☐Composite	14. 17 ☐Prime ☐Composite	15. 47 ☐Prime ☐Composite	16. 69 ☐Prime ☐Composite
17. 10 ☐Prime ☐Composite	18. 43 ☐Prime ☐Composite	19. 59 ☐Prime ☐Composite	20. 44 ☐Prime ☐Composite
21. 81	22. 36	23. 51	24. 76

☐Prime ☐Composite	☐Prime ☐Composite	☐Prime ☐Composite	☐Prime ☐Composite

Find the prime factorization of each number.

1. 12 **2, 2, 3**
2. 10
3. 28
4. 30
5. 8
6. 4
7. 32
8. 96
9. 46
10. 36
11. 14
12. 26
13. 82
14. 48
15. 54
16. 40
17. 72
18. 84

The Greatest Common Factor

Common factors are numbers that are factors of two or more numbers. For example, ***2*** is a factor of ***12*** and ***36***, which makes ***2*** a common factor of ***12*** and ***36***. The common factor of two numbers with the greatest value is called the ***greatest common factor***. For example, ***2***, ***3***, ***4***, ***6***, and ***12*** are common factors of ***12*** and ***36***, but ***12*** is the greatest common factor.

Multiples

Find the ***multiples*** of a number by multiplying it by other whole numbers. The multiples of ***2***, for example, are:

0 x 2 = 0

1 x 2 = 2

2 x 2 = 4

2 x 3 = 6

2 x 4 = 8

2 x 5 = 10

…and so on.

As you can see, the multiples of ***2*** include ***0***, ***2***, ***4***, ***6***, ***8***, and ***10***. The list continues into infinity!

Some numbers share the same multiples. Those multiples are known as ***common multiples***.

Number **Multiples**

0	**0**	**0**	**0**	**0**	**0**	**0**
1	**0**	**1**	**2**	**3**	**4**	**5**
2	**0**	**2**	**4**	**6**	**8**	**10**
3	**0**	**3**	**6**	**9**	**12**	**15**
4	**0**	**4**	**8**	**12**	**16**	**20**
5	**0**	**5**	**10**	**15**	**20**	**25**
	0	**1**	**2**	**3**	**4**	**5**

The least multiple of two or more numbers is the least common multiple. For example, the least common multiple of ***2*** and ***3*** is ***6***.

2 x 1 = 2 **2 x 2 = 4** **2 x 3 = 6**
3 x 1 = 3 **3 x 2 = 6**

Practice Exercise

Find the greatest common factor (GCF) for the given numbers.

1. 5, 6 **1**
2. 12, 2

3. 12, 6
4. 4, 12
5. 10, 12
6. 9, 12
7. 6, 10
8. 8, 9
9. 3, 12
10. 14, 22
11. 12, 30
12. 8, 16
13. 12, 24
14. 30, 25
15. 13, 17
16. 6, 4
17. 16, 12
18. 9, 6
19. 45, 36
20. 36, 24
21. 48, 32
22. 20, 36
23. 40, 16
24. 24, 18
25. 17, 19
26. 45, 120
27. 32, 6
28. 110, 135
29. 6, 40

30. 25, 31

Find the least common multiple for the given numbers.

1. 2, 5 **10**
2. 12, 9
3. 10, 7
4. 3, 5
5. 12, 6
6. 4, 10
7. 7, 5
8. 6, 4
9. 8, 10
10. 18, 12
11. 5, 27
12. 24, 15
13. 16, 24
14. 18, 2
15. 6, 9
16. 18, 5
17. 19, 24
18. 18, 24
19. 24, 36
20. 27, 14

21. 25, 16
22. 9, 44
23. 12, 5
24. 36, 18
25. 6, 120
26. 6, 45
27. 14, 10
28. 42, 28
29. 36, 30
30. 16, 8

Exponents

Powers and Exponents

To find the ***powers*** of a number, multiply the number over and over by itself. The ***first power*** is the number. The ***second power*** is the product of the number multiplied once by itself or ***squared***. The ***third power*** is the number multiplied twice by itself or ***cubed***, and so on. For example:

2^1 = 2 x 1 **2^2 = 2 x 2** **2^3 = 2 x 2 x 2**

10^1 = 10 x 1 **10^2 = 10 x 10** **10^3 = 10 x 10 x 10**

The numbers above are written in expanded form.

5^2 can be read as "five to the second power" or "five squared".

5^3 can be read as "five to the third power" or "five cubed".

5^4 can be read as "five to the fourth power".

Þ ***There is a special way of writing the power of a number called an exponent. It's the tiny number written above and to the right of the number.***

Þ ***Sometimes you may see an exponent expressed like this, 2^3. This would be the same as 2^3. Both are examples of writing the power of a number in exponential form.***

Base

A number used as a repeated factor

Example:

8^3 = 8 X 8 X 8

The base is 8. It is used as a factor three times.

The exponent is 3.

Converting Numbers to Scientific (Exponential) Notation

Scientific notation is used to express very large or very small numbers. A number in scientific notation is written as the product of a number (integer or decimal) and a power of 10. This number is always 1 or more and less than 10.

For example, there are approximately 6,000,000,000 humans on earth. This number could be written in scientific notation as $6x10^9$. The number 6,000,000,000 is equivalent to 6x1,000,000,000. The number 1,000,000,000 is equivalent to 10^9 or 10x10x10x10x10x10x10x10x10.

Number	Scientific Notation?	Product of	Places after 1st Digit
1	$1.0x10^0$	1	0 places
10	$1.0x10^1$	1x10	1 places
100	$1.0x10^2$	1x10x10	2 places
1,000	$1.0x10^3$	1x10x10x10	3 places
10,000	$1.0x10^4$	1x10x10x10x10	4 places
100,000	$1.0x10^5$	1x10x10x10x10x10	5 places
1,000,000	$1.0x10^6$	1x10x10x10x10x10x10	6 places

A number can be converted to scientific notation by increasing the power of ten by one for each place the decimal point is moved to the left. In the example above, the decimal point was moved 9 places to the left to form a number more than 1 and less than 10.

Scientific notation numbers may be written in different forms. The number 6x10^{9} could also be written as 6e+9. The +9 indicates that the decimal point would be moved 9 places to the right to write the number in standard form.

Negative powers of 10 are useful for writing very small numbers. Any number to a negative power represents a fraction or decimal.

$$10^{-1} = \frac{1}{10} = 0.1$$

$$10^{-2} = \frac{1}{10} \text{ x } \frac{1}{10} = 0.01$$

$$10^{-3} = \frac{1}{10} \text{ x } \frac{1}{10} \text{ x } \frac{1}{10} = 0.001$$

Example In a scientific experiment, the mass of a sample is 2^{-5} x 10 kilogram. Write the mass in standard notation.

Step 1 Write the given number with a string of zeros in front of it. You haven’t changed the value.

0000002.

Step 2 Move the decimal point to the left by the number of places shown in the exponent. Discard extra zeros.


00.00002.

$$\mathbf{2 \text{ x } 10^{-5} = 0.00002}$$

As you know a negative is the opposite of positive. You also know that multiplication is the opposite of division. Positive exponents show how many times the number 1 is multiplied by a number. Negative exponents show how many times 1 is divided by a number.

Take a look at the example below:

5^2
$1 \cdot 5 \cdot 5$
25

To find what 5 to the 2nd power is you simply multiply 1 by the number 5 two times. To find what 5 to the negative 2nd power is (as in the example below) you would do the same thing but only dividing instead of multiplying. Divide 1 by 5 two times:

5^{-2}
1 / 5 / 5   (The / symbol is used to represent

division)

0.04

This method works well, but division is not always the easiest operation to do over and over again.

Take a look at the problem on the next page. This example is not important to commit to long-term memory, but it will help you understand how to avoid division.

$1 / 2 / 2$

$.25$

$1 / (2 \cdot 2)$

$.25$

As you can see, the result of dividing 1 by 2, then by 2 again is the same as 1 divided by the product of 2 and 2. This rule can now be applied to negative exponents:

5^{-2}

$1 / (5 \cdot 5)$

$1 / 25$

$.04$

If you are able to do the multiplication mentally, then there is no need to write out the $(5 \cdot 5)$ multiplication, you can simply work the problem like below.

5^{-2}

$1 / 5^{2}$

$1 / 25$

$.04$

As you can see above, the five is simplified to 1 divided by 5 to the same exponent [2]. But in this case the exponent's sign is changed to positive. Often, the work for simplifying negative exponents is shown with fractions in place of the / or division symbol.

Since a fraction is really just a division problem shown in a different way, the work to our problem [5^{-2}] can be shown in a fraction as below:

$$5^{-2}$$

$$\frac{1}{5^{2}}$$

$$\frac{1}{5 \cdot 5}$$

$$\frac{1}{25}$$

$$.04$$

The answer can be left as a simplified fraction (in the above problem this would be 1/25) or as a decimal (in the above problem .04).

Laws of Exponents

To multiply powers of the same base, *add their exponents.*

Example **$2^{2\ \text{x}}\ 2^{3\ =}\ 2^5$**

This is true because $2^2 = 2$ x $2 = 4$; $2^3 = 2$ x 2 x $2 = 8$; $2^5 = 2$ x 2 x 2 x 2 x $2 = 32$

4 x $8 = 32$

To divide powers of the same base, *subtract the exponent of the divisor from the exponent of the dividend.*

Example **$3^5 \cdot 3^3 = 3^2$**

This is true because $3^5 = 3$ x 3 x 3 x 3 x $3 = 243$; $3^3 = 3$ x 3 x $3 = 27$; $3^2 = 3$ x $3 = 9$

$243 \div 27 = 9$

You may need to find the value of a product or a quotient of powers that do not share the same base. In either case, first find the value of each power and then multiply or divide as indicated.

Example Find the value of the product $3^2 \bullet 2^4$.

Step 1 Find the value of each power.

$3^2 = 9$ and $2^4 = 16$

Step 2 Multiply the values found in Step 1.

$3^2 \bullet 2^4 = 9 \bullet 16 = 144$

Answer: 144

Example 2 Find the value of the quotient $4^3/2^2$.

Step 1 Find the value of each power.
$4^3 = 64$ and $2^2 = 4$

Step 2 Divide 64 by 4.
$64/4 = 16$

Answer: 16

You may need to find the value of a sum or a difference of powers. In either case, first find the value of each power and then add or subtract as indicated.

Example Find the value of the sum $7^2 + 3^3$.

Step 1 Find the value of each power.
$7^2 = 49$ and $3^3 = 27$

Step 2 Add the values found in Step 1.
$7^2 + 3^3 = 49 + 27 = 76$

Answer: 76

Example 2 Find the value of the difference $8^2 - 3^2$.

Step 1 Find the value of each power.
$8^2 = 64$ and $3^2 = 9$

Step 2 Subtract 9 from 64.
64 - 9 = 55

Answer: 55

Practice Exercise

Express each of the following numbers in scientific notation:

1. 60,000
2. 308,000,000,000
3. 520,000
4. .0018
3. .00000079
4. .06356

Use exponential form to write:

1. 2 x 2 x 2 x 2
2. 5 x 5 x 5 x 5 x 5 x 5 x 5 x 5 x 5 x 5
3. Express 36 as a power of 6.
4. Express 32 as a power of 2.

Solve each problem. The first one has been done for you.

1. 3^2 $3 \text{ x } 3 = 9$	2. 11^2
3. 5^{-3}	4. 8^4
5. 10^{-2}	6. 9^3
7. $3^4 + 3^5$	8. $10^3 - 8^5 + 2^4$
9. $7 + 7^5 - 7^3 + 7^3$	10. $6^4 - 6^3 + 6^4 - 6^2$
11. $3^4 + 7^5 + 6^5$	12. $4^5 \div 8^2$
13. $4^2 \times 4 \times 4^{-5} \times 4$	14. $10^2 + 7^4 + 6$
15. $9^5 \div 9^4$	16. $8 + 6^2 + 9^4 + 7^6$
17. $4^2 \times 4^5$	18. $6^3 + 6^4$

19. $5^4 - 5$	**20. $3 \div 3^4$**
21. $4^5 + 8^2 + 10^2 + 12^4$	**22. $5 + 5^4 - 5^2$**
23. $9 + 8^4$	**24. 10×10**
25. $8^5 - 8^5$	**26. $6 + 6^4 + 6^6 + 6^2$**

Squares and Square Roots

Raising a number to the second power is also called ***squaring*** a number.

For example, ***3 squared*** (3^2) is equal to ***9***, and the ***square root*** of ***9*** is ***3***.

⟹ **There is a shortcut for squaring any number ending in 5.**

Example **Evaluate 15^2**

15 lies between 10 and 20

$$15^2 = 10 \times 20 + 5^2$$
$$= 200 + 25$$
$$= 225$$

To find the square root of a number ask yourself, “What number times itself equals this number?”

There is a special way to write the symbol for a square root called a ***radical sign*** $\sqrt{\quad}$.

2 squared: 2^2 = 2 x 2 = 4

3 squared: 3^2 = 3 x 3 = 9

4 squared: 4^2 = 4 x 4 = 16

Square root of 16: $\sqrt{16}$ = 4

Square root of 9: $\sqrt{9}$ = 3

Square root of 4: $\sqrt{4}$ = 2

Table of Square Roots to 25

$\sqrt{1}$	1	$\sqrt{100}$	10	$\sqrt{361}$	19
$\sqrt{4}$	2	$\sqrt{121}$	11	$\sqrt{400}$	20
$\sqrt{9}$	3	$\sqrt{144}$	12	$\sqrt{441}$	21
$\sqrt{16}$	4	$\sqrt{169}$	13	$\sqrt{484}$	22
$\sqrt{25}$	5	$\sqrt{196}$	14	$\sqrt{529}$	23
$\sqrt{36}$	6	$\sqrt{225}$	15	$\sqrt{576}$	24
$\sqrt{49}$	7	$\sqrt{256}$	16	$\sqrt{625}$	25
$\sqrt{64}$	8	$\sqrt{289}$	17		
$\sqrt{81}$	9	$\sqrt{324}$	18		

Perfect Square

A number that has positive or negative whole numbers (integers) as its square roots

Example:

16 is a perfect square.

$\sqrt{16} = 4 \qquad {}^{-}\sqrt{16} = {}^{-}4$

Practice Exercise

Solve the problems below.

1. 6^2 = **36**	2. 2^2 =	3. 3^2 =	4. 11^2 =
5. 4^2 =	6. 7^2 =	7. 12^2 =	8. 13^2 =
9. 8^2 =	10. 9^2 =	11. 1^2 =	12. 10^2 =
13. 18^2 =	14. 23^2 =	15. 21^2 =	16. 25^2 =
17. 20^2 =	18. 24^2 =	19. 17^2 =	20. 22^2 =
21. $\sqrt{64}$ =	22. $\sqrt{81}$ =	23. $\sqrt{196}$ =	24. $\sqrt{169}$ =

25. $\sqrt{1}$ =	26. $\sqrt{25}$ =	27. $\sqrt{121}$ =	28. $\sqrt{36}$ =
29. $\sqrt{16}$ =	30. $\sqrt{4}$ =	31. $\sqrt{100}$ =	32. $\sqrt{49}$ =
33. $\sqrt{484}$ =	34. $\sqrt{144}$ =	35. $\sqrt{529}$ =	36. $\sqrt{256}$ =

Finding an Approximate Square Root

A number that is not a perfect square does not have a whole number square root. For example, the square root of 30 is between the whole numbers 5 and 6:

$\sqrt{30}$ is larger than 5, since $5^2 = 25$
$\sqrt{30}$ is smaller than 6, since $6^2 = 36$

To find the approximate square root of a number that is not a perfect square, follow the three steps of the Method of Averaging:

Example Find the approximate square root of 30.

Step 1 Choose a number that is close to the correct square root.

$$5 \approx \sqrt{30}$$

since $5 = \sqrt{25}$

Step 2 Divide this chosen number into the number you're trying to find the square root of.

$$30 \div 5 = 6$$

Step 3 Average your choice from Step 1 with the answer from Step 2. The average of these two numbers is the approximate square root.

$$5 + 6 = 11$$
$$11 \div 2 = 5\ 1/2$$

Answer: 5 ½ = **5.5**
Check: $(5.5)^2 = 30.25$
Answer: 5.5 » $\sqrt{30}$
Note: The approximation sign "≈" means "is approximately equal to."

Square Root Algorithm

An algorithm is a special method that can be used to add subtract, or complete some mathematical operation. There is an algorithm for finding the square root.

Follow the steps below to find the square root of a number.

Example Find the square root of 676.

Step 1 Separate the numeral into groups of two figures each, starting at the decimal point. Attach zeros if needed.

$$\sqrt{6\ \ 76}$$

Step 2 Place the largest possible square under the first group at the left.

$$\begin{array}{l}\sqrt{6\ \ 76}\\ \ \ 4\end{array}$$

Step 3 Write the square root of the number in Step 2 above the first group.

$$\begin{array}{l}\ \ 2\\ \sqrt{6\ \ 76}\\ \ \ 4\end{array}$$

Step 4 Subtract the square from the first group. Attach the next group to the remainder.

$$\begin{array}{l}\ \ 2\\ \sqrt{6\ \ 76}\\ \ \ 4\\ \sqrt{2\ \ 76}\end{array}$$

Step 5 Form the trial divisor by doubling the root already found and leaving a space.

2
√6 76
4
4 √2 76

Step 6 Divide the dividend from Step 4 by the divisor in Step 5. Join the quotient to the root already found and put the same number in the space that you left next to the trial divisor to form the complete divisor.

2 6
√6 76
4
46 √2 76

Step 7 Multiply the complete divisor by the new figure of the root.

2 6
√6 76
4
46 √2 76
$\underline{2\ 76}$

Step 8 Subtract this product from the dividend.

$$\begin{array}{rr} & 2\quad 6 \\ & \sqrt{6\;76} \\ & 4 \\ 46 & \sqrt{2\;76} \\ & \underline{2\;76} \\ & 0 \end{array}$$

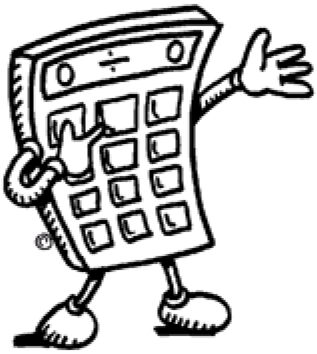
Step 9 Continue until all groups are used or the answer is of the desired accuracy.

Step 10 If necessary, place a decimal point in the root directly above the one in the given number.

Step 11 Check by squaring the square root to obtain the given number.

$$\begin{array}{r} 26 \\ \underline{26} \\ 156 \\ \underline{52\_} \\ 676 \end{array}$$

Answer: The square root of 676 is **26**.



To find the square of a number, you multiply the number by itself. For example, $6^2 = 6 \times 6 = 36$. You can square numbers quickly using the {x^2} key on your calculator. You can also perform operations using squares. You will find this feature useful when solving problems involving the Pythagorean Theorem.

Examples

Solve: $8^2 = ?$ Enter: 8 {x^2} The answer is 64.
Solve: $12^2 - 7^2 = ?$ Enter: 12{x^2}{–}{7}{x^2}{=}95.

The square root function is the second operation assigned to the square key {x^2}. To find the square root of a number, enter the number, then press SHIFT and the square key.

Examples

Solve: What is the square root of 225?
Enter: 225{SHIFT}{x^2} The answer is 15.

Solve: $\sqrt{256} + \sqrt{81} = ?$
Enter: 256{SHIFT}{x^2}{+}81{SHIFT}{x^2}{=}25

Word Problems with Whole Numbers

Within every story (word) problem are several ***clue words***. These words tell you the kind of math sentence (equation) to write to solve the problem.

Addition Clue Words

add
sum
total
plus
in all
both
together
increased by
all together
combined

Subtraction Clue Words

subtract
difference
take away
less than
are not
remain
decreased by
have or are left
change (money problems)
more
fewer

Multiplication Clue Words

times
product of

Division Clue Words

quotient of
divided by

multiplied by
by (dimension)

half [or a fraction]
split
separated
cut up
parts
shared equally

⇒ ***Division clue words are often the same as subtraction clue words. Divide when you know the total and are asked to find the size or number of "one part" or "each part."***

Following a system of steps can increase your ability to accurately solve problems. Use these steps to solve word problems.

1. Read the problem carefully. Look up the meanings of unfamiliar words.

2. Organize or restate the given information.

3. State what is to be found.

4. Select a strategy (such as making a chart of working backward) and plan the steps to solve the problem.

5. Decide on an approximate answer before solving the problem.

6. Work the steps to solve the problem.

7. Check the final result. Does your answer seem reasonable?

The Problem Solving System was used to solve the following problem:

Mary has ten marbles. Lennie has thirteen. How many marbles do they have in all?

1. **Mary has ten marbles. Lennie has thirteen. How many marbles do they have in all?**

2. **Mary – 10 marbles**
   Lennie – 13 marbles

3. **How many marbles in all?**

4. **Add**

5. **A little over 20 marbles (10 + 10 = 20)**

6. $$\begin{array}{l} \ \ \ \mathbf{10} \\ \mathbf{+13} \\ \hline \ \ \ \mathbf{23 \text{ marbles}} \end{array}$$

7. **The final sum of 23 marbles is close to the estimated answer of 20 marbles. The final result is reasonable.**

Þ ***Be sure to label answers whenever possible. For example: marbles, grams, pounds, feet, dogs, etc.***

Þ ***Some problems may require several steps to solve. Some may have more than one correct answer. And some problems may not have a solution.***

Have you ever tried to help someone else work out a word problem? Think about what you do. Often, you read the problem with the person, then discuss it or put it in your own words to help the person see what is happening. You can use this method---restating the problem---on your own as a form of "talking to yourself."

Restating a problem can be especially helpful when the word problem contains no key words. Look at the following example:

Example: Susan has already driven her car 2,700 miles since its last oil change. She still plans to drive 600 miles before changing the oil. How many miles does she plan to drive between oil changes?

Step 1: *question:* How many miles does she plan to drive between oil changes?

Step 2: *necessary information:* 2,700 miles, 600 miles

Step 3: *decide what arithmetic to use:* Restate the problem in your own words: "You are given the number of miles Susan has already driven and the number of miles more that she plans to drive. You need to add these together to find the total number of miles between oil changes."

Step 4: 2,700 miles + 600 miles = **3,300 miles** between oil changes.

Step 5: It makes sense that she will drive 3,300 miles between oil changes, since you are looking for a number larger than the 2,700 miles that she has already driven.

For some problems, you have to write two or three equations to solve the problem. For others, you may need to make charts or lists of information, draw pictures, find a pattern, or even guess and check. Sometimes you have to work backwards from a sum, product, difference, or quotient, or simply use your best logical thinking.

List/Chart

Marty's library book was six days overdue. The fine is $.05 the first day, $.10, the second, $.20 the third day, and so on. How much does Marty owe?

Marty's library book was six days overdue. The fine is $.05 the first day, $.10, the second, $.20 the third day, and so on. How much does Marty owe?

Days	**1**	**2**	**3**	**4**	**5**	**6**
Fine	**$.05**	**$.10**	**$.20**	**$.40**	**$.80**	**$1.60**

Answer: $1.60

Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?

Veronica, Archie, and Betty are standing in line to buy tickets to a concert. How many different ways can they order themselves in line?

Veronica
Archie
Betty

Veronica
Betty
Archie

Archie
Veronica
Betty

Archie
Betty
Veronica

Betty
Veronica
Archie

Betty
Archie
Veronica

Answer: 6 ways

Find a Pattern

Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X_______. How did Jenny fill in the blanks?

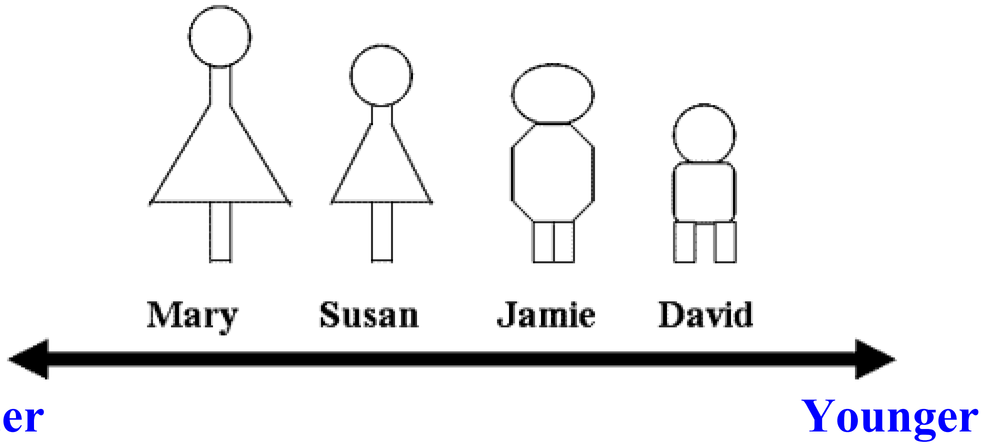
Jenny's friend handed her a code and asked her to complete it. The code read 1, 2, 3 Z 4, 5, 6 Y 7, 8, 9 X_______. How did Jenny fill in the blanks?

Answer: 10, 11, 12 W

Draw a Picture

Mary is older than Jamie. Susan is older than Jamie, but younger than Mary. David is younger than Jamie. Who is oldest?

Mary is older than Jamie. Susan is older than Jamie, but younger than Mary. David is younger than Jamie. Who is oldest?



Answer: Mary is oldest.

Guess and Check

Farmer Joe keeps cows and chickens in the farmyard. All together, Joe can count 14 heads and 42 legs. How many cows and how many chickens does Joe have in the farmyard?

Farmer Joe keeps cows and chickens in the farmyard. All together, Joe can count 14 heads and 42 legs. How many cows and how many chickens does Joe have in the farmyard?

6 cows +8 chickens 14 heads	Guess a number of cows. Then add the number of chickens to arrive at the sum of 14 heads. Then check the total legs.	6 cows = 24 legs +8 chickens = 16 legs 40 legs
7 cows +7 chickens 14 heads	Adjust your guesses. Then check again until you solve the problem.	7 cows = 28 legs +7 chickens = 14 legs 42 legs

Answer: 7 cows and 7 chickens

Work Backwards

Marsha was banker for the school play. She took in $175 in ticket sales. She gave Wendy $75 for sets and costumes and Paul $17.75 for advertising and publicity. After paying for the props, Marsha had $32.25 left. How much did the props cost?

Marsha was banker for the school play. She took in $175 in ticket sales. She gave Wendy $75 for sets and

costumes and Paul $17.75 for advertising and publicity. After paying for the props, Marsha had $32.25 left. How much did the props cost?

$$\begin{array}{rl} \$\ 175.00 & \text{tickets} \\ -\quad 75.00 & \text{costumes} \\ \hline \$\ 100.00 & \\ -\quad \underline{17.75} & \text{advertising} \\ \hline \$\quad 82.25 & \end{array} \qquad \begin{array}{rl} \$\ 82.25 & \\ -\ \ 32.25 & \\ \hline \$\ 50.00 & \text{cost of props} \end{array}$$

Logical Reasoning

Jim challenged Sheila to guess his grandmother's age in ten questions or less. It took her six. Here's what Sheila asked:

Jim challenged Sheila to guess his grandmother's age in ten questions or less. It took her six. Here's what Sheila asked:

"Is she less than fifty?" "No."	**50+ years old**
"Less than seventy-five?" "Yes."	**50 to 74 years old**
"Is her age an odd or even number?" "Odd."	**ends in 1, 3, 5, 7 or 9**
"Is the last number less than or equal to five?" "No."	**ends in 7 or 9**

"Is it nine?" "No."	**ends in 7 –**
	57 or 67
"Is she in her sixties?" "No."	**57 years old**

Not Enough Information

Now that you know how to decide whether to add, subtract, multiply, or divide to solve a word problem, you should be able to recognize a word problem that cannot be solved because not enough information is given.

Look at the following example:

Problem: At her waitress job, Sheila earns $4.50 an hour plus tips. Last week she got $65.40 in tips. How much did she earn last week?
Step 1: *question:* How much did she earn last week?
Step 2: *necessary information:* $4.50/hour, $65.40
Step 3: *decide what arithmetic to use:*

tips + (pay per hour **x** hours worked) = total earned

missing information: hours worked

At first glance, you might think that you have enough information since there are 2 numbers. But when the solution is set up, you can see that you need to know the number of hours Sheila worked to find out what she earned. **(Be Careful!!!)**

1. Sharon’s rent has been increased $65 a month to $390 a month. What had she been paying?

2. Lily’s allergy pills come in a 250 tablet bottle. She takes 4 tablets a day. How many tablets did she have left after taking the tablets for 30 days?

3. An oil truck carried 9,008 litres of oil. After making 7 deliveries averaging 364 litres each, how much oil was left in the truck?

4. Mary and Lucy disagree on the meaning of the expression $3^2 – 2^3$. Mary says it means 9 – 6, or 3. Lucy says it means 6 – 6, or 0. Who is right?

5. What is the average temperature for the full day if the daytime temperature is 23 degrees Celsius and the nighttime temperature is 15 degrees Celsius?

6. Joe scored 78 on his math test. The rest of Joe’s math group scored 81, 85, 83, 92, 86, and 90. What was the median of the scores in Joe’s math group?

7. After two strings of bowling, Sally’s team had scores of 80, 75, 93, 81, 98, 93, 57, and 62. What is the mode of the scores for Sally’s bowling team?

8. Which of the following numbers is a prime number: 14, 23, 39, 51, or 85?

9. Many of the things we take for granted were invented not so long ago. For example, pencils were first used in 1565. They were square until Joseph Dixon made them round in 1876. Round ones are easier to hold. For how many years did people use square pencils?

10. Ben’s trip to work is longer now that he has moved. He used to drive 5 kilometers to get to work. Now he drives 9 kilometers twice a day, 5 days a week, 4 weeks a month. How many extra kilometers does Ben now travel in a month?

Answer Key

Book 14018 – Number Operations

Page 28 **1.** 321 **2.** 96 **3.** 434 **4.** 7590
5. 56316 **6.** 840 **7.** 319 **8.** 86 **9.** 195
10. 458 **11.** 645 **12.** 22025 **13.** 10
14. 39375 **15.** 24 **16.** 789 **17.** 368
18. 53 **19.** 94 **20.** 547 **21.** 26404
22. 346 **23.** 561 **24.** 23184

Page 31 **1.** 27 **2.** 4 **3.** 54 **4.** 338 **5.** 6
6. 12863 **7.** 183 **8.** 47 **9.** 517 **10.** 2
11. 140 **12.** 52 **13.** 104 **14.** 206
15. 4945 **16.** 50 **17.** 13 **18.** 146
19. 57 **20.** 1717

Page 36

	Median	**Mean**	**Mode**
1.	5	7.6	5
2.	15	13.2	6, 15
3.	15	14.8	15
4.	16	15.2	24
5.	2.7	2.7	2.2, 2.9
6.	5	15.2	5
7.	76	74.5	63

Page 39 **1.** Composite **2.** Composite **3.** Composite
4. Prime **5.** Composite **6.** Composite

7. Composite **8.** Composite **9.** Prime
10. Composite **11.** Prime **12.** Composite
13. Prime **14.** Prime **15.** Prime
16. Composite **17.** Composite **18.** Prime
19. Prime **20.** Composite **21.** Composite
22. Composite **23.** Composite
24. Composite

Page 40 **2.** 2, 5 **3.** 2, 2, 7 **4.** 2, 3, 5 **5.** 2, 2, 2
6. 2, 2 **7.** 2, 2, 2, 2, 2 **8.** 2, 2, 2, 2, 2, 3
9. 2, 23 **10.** 2, 2, 3, 3 **11.** 2, 7 **12.** 2, 13
13. 2, 41 **14.** 2, 2, 2, 2, 3 **15.** 2, 3, 3, 3
16. 2, 2, 2, 5 **17.** 2, 2, 2, 3, 3 **18.** 2, 2, 3, 7

Page 42 **2.** 2 **3.** 6 **4.** 4 **5.** 2 **6.** 3 **7.** 2
8. 1 **9.** 3 **10.** 2 **11.** 6 **12.** 8 **13.** 12
14. 5 **15.** 1 **16.** 2 **17.** 4 **18.** 3 **19.** 9
20. 12 **21.** 16 **22.** 4 **23.** 8 **24.** 6
25. 1 **26.** 15 **27.** 2 **28.** 5 **29.** 2
30. 1

Page 44 **2.** 36 **3.** 70 **4.** 15 **5.** 12 **6.** 20
7. 35 **8.** 12 **9.** 40 **10.** 36 **11.** 135
12. 120 **13.** 48 **14.** 18 **15.** 18 **16.** 90
17. 456 **18.** 72 **19.** 72 **20.** 378
21. 400 **22.** 396 **23.** 60 **24.** 36
25. 120 **26.** 90 **27.** 70 **28.** 84 **29.** 180
30. 16

Page 54 **1.** 6 x 10 to the fourth power **2.** 3.08 x 10 to the eleventh power **3.** 5.2 x 10 to the fifth power **4.** 1.8 x 10 to the negative third power **5.** 7.9 x 10 to the negative seventh power **6.** 356 x 10 to the negative second power

Page 54 **(exponential form)** **1.** 2 to the fourth power **2.** 5 to the tenth power **3.** 6 to the second power **4.** 2 to the fifth power

Page 55 **(solve each problem)** **2.** 121 **3.** .008 **4.** 4096 **5.** .01 **6.** 729 **7.** 324 **8.** –31752 **9.** 16814 **10.** 2340 **11.** 24664 **12.** 16 **13.** .25 **14.** 2507 **15.** 9 **16.** 124254 **17.** 16384 **18.** 1512 **19.** 620 **20.** 1/27 **21.** 21924 **22.** 605 **23.** 4105 **24.** 100 **25.** 0 **26.** 47994

Page 58 **2.** 4 **3.** 9 **4.** 121 **5.** 16 **6.** 49 **7.** 144 **8.** 169 **9.** 64 **10.** 81 **11.** 1 **12.** 100 **13.** 324 **14.** 529 **15.** 441 **16.** 625 **17.** 400 **18.** 576 **19.** 289 **20.** 484 **21.** 8 **22.** 9 **23.** 14 **24.** 13 **25.** 1 **26.** 5 **27.** 11 **28.** 6 **29.** 4 **30.** 2 **31.** 10 **32.** 7 **33.** 22 **34.** 12 **35.** 23 **36.** 16

Page 75 **1.** $325 a month **2.** 130 tablets **3.** 6460 liters **4.** Neither one **5.** 19 degrees Celsius **6.** 85 **7.** 93 **8.** 23 **9.** 311 years **10.** 355 kilometers