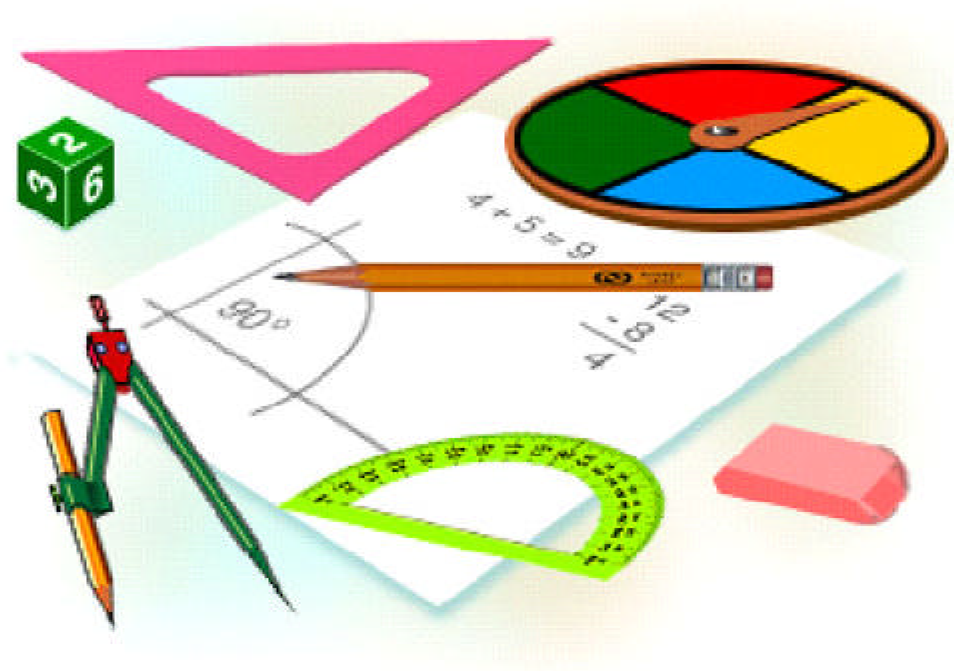
The Next Step

Mathematics Applications for Adults



Book 14019 – Equations: Equalities and Inequalities

OUTLINE

Mathematics - Book 14019

Equations: Equalities And Inequalities
Introduction to Equations: Equalities and Inequalities
rewrite English statements as math expressions.
solve equalities and inequalities.
simplify an expression using correct order of operations.
Problem Solving with Equations, Equalities, and Inequalities
solve multi-step problems requiring the performance of any combination of mathematical operations involving equalities and inequalities, with or without a calculator.

THE NEXT STEP

Book 14019

Equations: Equalities and Inequalities

Introduction to Equations: Equalities and Inequalities

Algebraic or number sentences use the symbols =, ≠, <, >, ≤, or ≥ to show the relationship between two quantities.

Any sentence using the symbol = is called an ***equality*** or ***equation.***

$$4 + 8 = 2 \times 6$$
$$3x \div 2 = 17$$

Any sentence using the symbol ≠, <, >, ≤, or ≥, is called an ***inequality***.

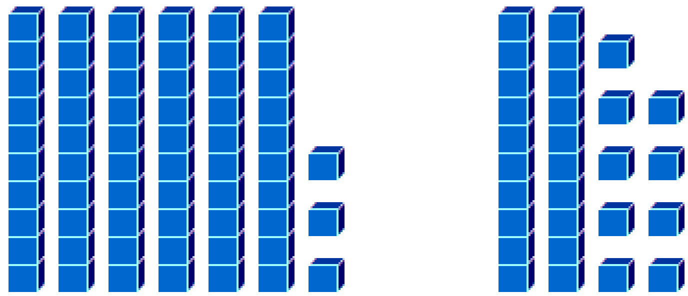
$$15 > 7$$
$$6 \neq 3 + 1$$
$$x + 2 \leq 12$$

SYMBOLS	
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$\sqrt{\ }$	positive square root
$\neq$	is not equal to
+	plus, add
-	minus, subtract
X	multiplied by, multiply
.	Multiplied by, multiply
$\div$	divided by, divide
=	equal to

Ordering numbers means listing numbers from least to greatest, or from greatest to least. Two symbols are used in ordering.

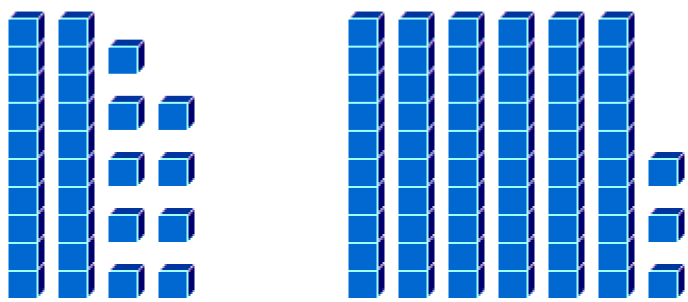
$<$	$>$
is less than	**is greater**
2 < 10	**10 > 2**

Greater Than >



63 is **greater than** 29.
63 > 29

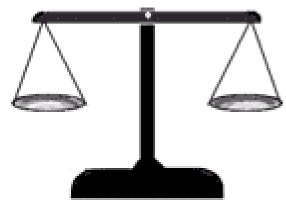
Less Than <



29 is **less than** 63.
29 < 63

Sometimes numbers in a set can be "***greater than or equal to***" members of another set. Likewise, members of a set are sometimes "***less than or equal to***" members of another set. A bar is added to ***less than*** and ***greater than*** symbols to show that they are also equal.

£	³
is less than or equal to	**is greater than or equal to**



Algebra is a division of mathematics designed to help solve certain types of problems quicker and easier. Algebra operates on the idea that an equation represents a scale such as the one shown above. Instead of keeping the scale balanced with weights, we use numbers, or constants. These numbers are called constants because they constantly have the same value. For example the number 47 always represents 47 units or 47 multiplied by an unknown number. It never represents another value.

In algebra, we often use letters to represent numbers. A letter that stands for a number is called a ***variable*** or ***unknown***.

A variable can be used to represent numbers in addition, subtraction, multiplication, or division problems. The symbols used in algebra are "+" for addition and "-" for subtraction. Multiplication is indicated by placing a number next to a variable; no multiplication sign is used. Division is indicated by placing a number or variable over the other.

An equation is made up of ***terms***. Each term is a number standing alone or an unknown multiplied by a ***coefficient*** (i.e. $7a$, $5x$, $3y$…).

For example, $3y + 5y = 32$ would be considered a three term equation. $12y – 11y – 9 = 17$ would be considered a four term equation.

Factors are numbers that are multiplied together. For instance, the factors of 12 are 3 and 4, 2 and 6, and 1 and 12. In the algebraic term, $7x$, 7 and x are factors.

An ***algebraic expression*** consists of two or more numbers or variables combined by one or more of the operations---addition, subtraction, multiplication, or division.

The following are examples of algebraic expressions:

Operation	**Algebraic Expression**	**Word Expression**
Addition	$x + 2$	x plus 2
Subtraction	$y - 3$	y minus 3
	$3 – y$	3 minus y
Multiplication	$4z$	4 times z
Division	$n/8$	n divided by 8
	$8/n$	8 divided by n

Many algebraic expressions contain more than one of the operations of addition, subtraction, multiplication, or division. Placing a number or variable outside of an expression in parentheses (brackets) means that the whole expression is to be multiplied by the term on the outside.

For example, look at the difference in meaning between $3y + 7$ and $3(y + 7)$. If the number 2 were substituted for y in each expression, the following solutions would result:

$$3y + 7 = 3 \bullet 2 + 7 = 6 + 7 = \mathbf{13}$$

but

$$3(y + 7) = 3(2 + 7) = 3(9) = \mathbf{27}$$

Algebraic Expression	**Word Expression**
$3y - 7$	3 times *y* minus 7
$3(y - 7)$	3 times the quantity *y* minus 7
- $x + 5$	Negative *x* plus 5
- $(x + 5)$	Negative times the quantity *x* plus 5
$3x^2 + 2$	3 *x*-squared plus 2
$3(x^2 + 2)$	3 times the quantity *x*-squared plus 2

Practice Exercise

Express each of the following problems algebraically.
(Hint: Use n as the unknown number and create an equation from the problem)

1. 13 less than twice a number is 3 **2n - 13 = 3**	2. The sum of 57 and a number is 139
3. five times what number added to 5 is 65	4. 78 less than what number equals -46
5. One-twelfth of a number is 144	6. The product of 8 and a number is 48
7. six times what number equals 72	8. four times the sum of a number and 1, is 12
9. 10 less than the product of 7 and a number is 25	10. Twice the sum of a certain number and 85 is 176
11. one less than eight times what number is 7	12. two times a number, less 2, is 16
13. A number increased by 88 is 176	14. The sum of 9 and the product of 7 and a number is 51

15. The sum of what number and four times the same number is 45	16. 7 more than 2 times a number is 27
17. 26 less than a number equals 34	18. A number minus 16 is -14
19. A number minus 89 is -51	20. 9 more than 3 times a number is 18

Solving an algebraic equation means finding the value of the unknown or variable that makes the equation a true statement. The ***solution*** is the value of the unknown that solves the equation.

To check if a possible value for the unknown is the solution of an equation, follow these two steps:

Step 1 Substitute the value for the unknown into the original equation.

Step 2 Simplify (do the arithmetic) and compare each side of the equation.

Example 1 Is $y = 5$ the solution for $3y - 9 = 6$?

Step 1 Substitute 5 for y. $3(5) - 9 = 6$?

Step 2 Simplify and compare. $15 - 9 = 6?$

$6 = 6?$

Since **6 = 6, *y* = 5 is a solution** of the equation.

Example 2 Is $x = 23$ the solution for $x - 7 = 14$?

Step 1 Substitute 23 for *x*. $(23) - 7 = 14?$
Step 2 Simplify and compare. $16 = 14?$

Since 16 is not equal to 14, ***x* = 23 is not a solution** of the equation.

Sometimes the order in which you add, subtract, multiply, and divide is very important. For example, how would you solve the following problem?

2 x 3 + 6

Would you group

(2 x 3) + 6 or **2 x (3 + 6) ?**

Which comes first, addition or multiplication? Does it matter?
Yes. Mathematicians have written two simple steps:

1. ***All multiplication and division operations are carried out first, from left to right, in the order they occur.***

2. ***Then all addition and subtraction operations are carried out, from left to right, in the order they occur.***

For example:

8 ¸ 2 + 2 x 3 – 1 = 4 + 6 – 1 = 9

4 6 10

step 1 step 2

Þ ***Perform all operations with parentheses (brackets) and exponents before carrying out the remaining operations in an equation. Parentheses or brackets may come in these forms: ( ), { }, or [ ].***

8 ¸ (2 + 2) x 3 – 1 =

8 ¸ 4 x 3 – 1 =

2 x 3 – 1 =

6 – 1 = 5

To remember the order of operations, simply remember BEDMAS: Brackets, Exponents, Division, Multiplication, Addition, Subtraction.

Example:

10 ÷ (2 + 8) X 2^3 - 4 *Add inside parentheses.*
10 ÷ 10 X 2^3 - 4 *Clear exponent.*
10 ÷ 10 X 8 - 4 *Multiply and divide.*
8 - 4 *Subtract.*
4

Distributive Property

The distributive property says this: ***Multiplication and addition can be linked together by "distributing" the multiplier over the addends in an equation.***

3 x (1 + 4) = (3 x 1) + (3 x 4)
3 x 5 = 3 + 12
15 = 15

Associative Property of Addition

The property which states that for all real numbers *a*, *b*, and *c*, their sum is always the same, regardless of their grouping:

$$(a + b) + c = a + (b + c)$$

Example:

(2 + 3) + 4 = 2 + (3 + 4)

Associative Property of Multiplication

The property which states that for all real numbers *a*, *b*, and *c*, their product is always the same, regardless of their grouping:

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Example:

$$(5 \cdot 6) \cdot 7 = 5 \cdot (6 \cdot 7)$$

Simplify

To combine like terms

Example:

Simplify. $n^2 + 4n - 9n + 7 - 5$

$n^2 + 4n - 9n + 7 - 5$	*Collect like terms by using*
$n^2 + (4n - 9n) + (7 - 5)$	*the Associative Property.*
$n^2 + (^-5n) + 2$	*Combine like terms.*
$n^2 - 5n + 2$	

Solving Equations using Inverse Operations

Inverse operations are used in algebra to simplify an equation for solving.
One operation is involved with the unknown and the inverse operation is used to solve the equation.

Addition and subtraction are inverse operations.
Given an equation such as 7 + x = 10, the unknown x and

7 are *added*. Use the inverse operation subtraction. To solve for n, subtract 7 from 10. The unknown value is therefore 3.

Multiplication and division are inverse operations. Given an equation 7x = 21. x and 7 are multiplied to create a value of 21. To solve for x, divide 21 by 7 for an answer of 3.

<u>Examples for addition, subtraction, division, and multiplication.</u>

Addition Problem	Solution
$x + 15 = 20$	$x = 20 - 15 = 5$
Subtraction Problem	Solution
$x - 15 = 20$	$x = 20 + 15 = 35$
Multiplication Problem	Solution
$3x = 21$	$x = 21 \div 3 = 7$
Division Problem	Solution
$x \div 12 = 3$	$x = 3 \times 12 = 36$

Solving Equations using Rules of Equality

The Rule of Equality
The same fundamental operation can be made on

both sides of the equation, using the same number and the equation will remain equal to the original.
(This does not apply to division by 0)

This rules means that what you do on one side of an equation, has to be done on the other side. For example, it means you can add 5 to both the left and right side of the equations.
Given: x - 5 = 40
1. Add 5 to both sides of the equation which gives you: x - 5 + 5 = 40 + 5
2. Simplify for x - 5 + 5 and 40 + 5, to give you the equation x = 45

Solving Equations using Two or More Operations

Using your knowledge of applying the rule of equality and the inverse operations, you can solve more complicated equations that require more than one operation.

For example, 2x + 7 = 11, cannot be completed in one step. You need to use division and subtraction.

The easy approach for this equation is to subtract 7 on each side which gives you (2x + 7) - 7 = 11 - 7, or 2x = 4. You can then divide each side by 2 which gives you (2x) ÷ 2 = 4 ÷ 2 or x = 2!

Of course you can apply the division to 2x + 7 = 11 first, which would be (2x + 7) ÷ 2 = 11 ÷ 2. The problem with that is you have to divide the entire (2x + 7), which cannot be easily simplified. Using subtraction first makes solving the equation much easier.

Terms on Both Sides of an Equation

Terms that contain an unknown can appear on both sides of an equation. Usually, we move all terms containing an unknown to the left side of the equation. To move a term containing an unknown from the right side of the equation to the left side, change its sign and place it next to the unknown on the left side.

Rule 1: If the term is preceded by a positive sign, remove the term from the right side and subtract it from the left side.

Rule 2: If the term is preceded by a negative sign, remove the term from the right side and add it to the left side.

EXAMPLE 1 Solve for x in $3x = 2x + 7$

Step 1 Write down the equation $3x = 2x + 7$. Subtract $2x$ from $3x$.

$$3x = 2x + 7$$
$$3x - 2x = 7$$

Step 2 Combine the x's.

$$\boldsymbol{x = 7}$$

Answer: $\boldsymbol{x = 7}$

EXAMPLE 2 Solve for y in $2y - 6 = -3y + 24$

Step 1 Write down the equation $2y - 6 = -3y + 24$. Add $3y$ to $2y - 6$.

$$2y - 6 = -3y + 24$$
$$2y + 3y - 6 = 24$$

Step 2 Combine the y's.

$$5y - 6 = 24$$

Step 3 Add 6 to 24.

$$5y = 24 + 6$$
$$5y = 30$$

Step 4 Divide 30 by 5.

$$y = 30 \div 5$$
$$\boldsymbol{y = 6}$$

Answer: $\boldsymbol{y = 6}$

Practice Exercise

Solve each equation.

1. $91 - \frac{5}{11}b = 41$	2. $\frac{b}{11} + 2.4 = 12.4$
3. $\frac{b}{4} - 7 = 3$	4. $14 + \frac{1}{5}b = 16$
5. $9x - 55 = 26$	6. $10x - 5.9 = 24.1$
7. $\frac{120}{x} = 12$	8. $11x + 20 = 42$
9. $8x + 31.66 = 55.66$	10. $5x + 53.71 = 113.71$
11. $4x - 116 + 6x = 4$	12. $15x + 20 - 4x = x + 130$
13. $\frac{b}{12} + 4 = 16$	14. $9n = 33 - 2n$
15. $\frac{b}{1} - 10 = 2$	16. $19x + 17 - 11x = 10x - 1$
17. $10x - 19 = 1$	18. $\frac{55}{x} = 11$

19. $7x + 99 = 141$	20. $5x - 14 + 3x = 34$
21. $3n = 33 - 8n$	22. $\frac{b}{7} + 5.1 = 15.1$

Solving an Equation With Parentheses

Parentheses or *brackets* are commonly used in algebraic equations. Parentheses are used to identify terms that are to be multiplied by another term, usually a number.

The first step in solving an equation is to remove the parentheses by multiplication. Then, combine separated unknowns and solve for the unknown.

Follow these four steps to solve an algebraic equation:

Step 1 Remove parentheses by multiplication.
Step 2 Combine separated unknowns.
Step 3 Do addition or subtraction first.
Step 4 Do multiplication or division last.

To remove parentheses, multiply each term inside the parentheses by the number outside the parentheses. If the parentheses are preceded by a negative sign or number, remove the parentheses by changing the sign of each term within the parentheses.

For example, $+4(x + 3)$ becomes $4 \bullet x + 4 \bullet 3 = 4x + 12$, and $-(3z + 2)$ becomes $-3z - 2$.

EXAMPLE 1 Solve for x in $4(x + 3) = 20$

Step 1 Write down the equation. Remove parentheses by multiplication.

$$4(x + 3) = 20$$
$$4x + 12 = 20$$

Step 2 Subtract 12 from 20.

$$4x = 20 - 12$$
$$4x = 8$$

Step 3 Divide 8 by 4.

$$x = 8 \div 4$$
$$\mathbf{x = 2}$$

Answer: ***x* = *2***

EXAMPLE 2 Solve for z in $4z - (3z + 2) = 5$

Step 1 Write down the equation. Remove parentheses by multiplication.

$$4z - (3z + 2) = 5$$
$$4z - 3z - 2 = 5$$

Step 2 Combine the z's.

$$z - 2 = 5$$

Step 3 Add 2 to 5.

$$z = 5 + 2$$
$$\mathbf{z = 7}$$

Answer: $\mathbf{z = 7}$

Parentheses may appear on both sides of an equation. Remove both sets of parentheses as your first step in solving for the unknown.

EXAMPLE Solve for x in $3(x - 6) = 2(x + 3)$

Step 1 Write down the equation. Remove both sets of parentheses by multiplication.

$$3(x - 6) = 2(x + 3)$$
$$3x - 18 = 2x + 6$$

Step 2 Subtract $2x$ from $3x - 18$.

$$3x - 2x - 18 = 6$$
$$x - 18 = 6$$

Step 3 Add 18 to 6.

$$x = 6 + 18$$
$$\mathbf{x = 24}$$

Answer: $\mathbf{x = 24}$

Practice Exercise

Solve each of the following equations.

1. $5(a + 1) = 75$
2. $5(b - 7) = 5$
3. $m + (m + 8) = 32$
4. $3y + 10 = 2(y + 10)$
5. $-2(3 + z) - 4 = -3z - 1$
6. $-6(a + 1) + 4 = 8a - 9$
7. $6(y + 4) = 12 - (y + 5)$
8. $8(z - 6) = -6(z - 9)$

Just like with equations, the solution to an inequality is a value that makes the inequality true. You can solve inequalities in the same way you can solve equations, by following these rules.

- You may add any positive or negative number to both sides of an inequality.

$$n + 5 > 10$$
$$n + 5 - 5 > 10 - 5$$
$$n > 5$$

Solution: all numbers greater than 5

- You may multiply or divide both sides of an inequality by any positive number.

$$6x \geq 18$$
$$6x \div 6 \geq 18 \div 6$$
$$x \geq 3$$

Solution: all numbers greater than or equal to 3

Watch out! If you multiply or divide both sides of an inequality by a negative number, reverse the direction of the inequality sign!

$-3n > 12$ if you divide or multiply by a negative number

$\frac{-3n}{-3} > \frac{12}{-3}$ reverse the inequality symbol

$n < -4$

Solution: all numbers less than -4

Solving an inequality is very similar to solving an equation. You follow the same steps, except for one very important difference. When you multiply or divide each side of the inequality by a negative number, you have to reverse the inequality symbol! Let's try an example:

$$-4x > 24$$

Since this inequality involves multiplication, we must use the inverse, or division, to solve it. We'll divide both sides by –4 in order to leave x alone on the left side.

$$\frac{-4x}{-4} > \frac{24}{-4}$$

When we simplify, because we're dividing by a negative number, we have to remember to reverse the symbol. This gives "x is less than –6," not "x is greater than –6."

$$x < -6$$

Why do we reverse the symbol? Let's see what happens if we don't. Think about the simple inequality $-3 < 9$. This is obviously a true statement.

$$-3 < 9$$

To demonstrate what happens when we divide by a negative number, let's divide both sides by –3. If we leave the inequality symbol the same, our answer is obviously not correct, since 1 is not less than –3.

$$\frac{-3}{-3} > \frac{9}{-3}$$

$1 < -3$ Not true!

$1 > -3$ Correct! You have to reverse the symbol.

We must reverse the symbol in order to find the correct answer, which is "1 is greater than –3."

Let's go back to the original problem and graph the solution x < –6 on a number line. To graph the solution for an inequality, you start at the defining point in the inequality. Here, it's –6. Then you graph all the points that are in the solution.

The red arrow shows that all the values on the number line less than –6 are in the solution. The open circle at –6 shows us that –6 is not in the solution. If the solution had been "x is less than or equal to –6," the circle would be a dark, or filled in, circle.

How can we check our answer? We can't use –6 to substitute in the inequality, because it lies outside our solution. To check, we can choose any value that lies in the solution. Let's use –7.

$$-4x > 24$$
$$-4(-7) > 24$$
$$28 > 24 \text{ Correct!}$$

Our substitution gave a true result, so the solution is correct.

Practice Exercise

Solve each equation.
(Hint: Use inverse operation rules to solve)

1.	$29 + y < 74$	$y < 45$	2.	$x - 26 > 42$	______
3.	$x + -50 > -72$	______	4.	$36 < a - 54$	______
5.	$x + -86 > -14$	______	6.	$47 > a - 32$	______
7.	$x - 40 < 10$	______	8.	$-87 + y < -6$	______
9.	$53 < a - 19$	______	10.	$x - 73 < 3$	______
11.	$x - 23 > 69$	______	12.	$45 + y > 76$	______
13.	$11.87 < a - 29.76$	______	14.	$-45 + y < 15$	______
15.	$x + -86.5 > -16.9$	______	16.	$x + -94 > -127$	______
17.	$x + 59 > 15$	______	18.	$-65 + y < -114$	______

Solve each equation.
(Hint: Use inverse operation rules to solve)

1.	$-21 < 7a$	$a > -3$	2.	$9b < 63$	______
3.	$n \div -13 > 4$	______	4.	$\frac{b}{1} > 5$	______
5.	$-36 < -4a$	______	6.	$n \div -25 < 8$	______
7.	$6b > 6$	______	8.	$3b < 24$	______

9. $\frac{b}{-5} < 3$ ______ **10.** **12b > 132** ______

11. **23.4 < -2.6a** ______ **12.** $\frac{b}{7} < 6$ ______

13. **146.4 > 12.2a** ______ **14.** **n ÷ -31 < 1** ______

15. **8b > 75.2** ______ **16.** **-24 < -8a** ______

17. **n ÷ -37 > 2.75** ______ **18.** $\frac{b}{-7} < 9$ ______

19. 3a – 7 < 2a + 1
20. b + 12 < 5(b + 8)
21. 6c – 4 < 3c + 2
22. 7d – 3d – d < 3d + 2d + 10

Quadratic Expression

A quadratic expression is one which contains the variable raised to the second power, or "squared," as in $x^2 + 2x$. Quadratic expressions will always have the variable (x) in both factors---in this case, x and $x + 2$ are the factors.

You may be asked to factor a quadratic expression with three terms: for example, $x^2 - 3x – 4$. Both factors for this kind of expression will have two terms: the variable and an integer.

Example Factor $x^2 - 3x - 4$.

Step 1 Find all possible factors of the whole number third term.

The possible factors for –4 are: -4 x 1, 4 x –1, -2 x 2, and 2 x –2.

Step 2 Find the integer factors from Step 1 that can be combined to make the number part of the middle term.

Combining –4 and 1 will give you the number part of the middle term: -4 + 1 = -3

Step 3 Write the two factors with the variable as the first term in each factor.

Write the factors. Put the variable as the first term in both factors: $(x\ \ )\ (x\ \ )$. Then add the integers from Step 2, -4 and 1: $(x - 4)\ (x + 1)$.

Step 4 Check your work. Multiply both terms of the second factor by each term in the first factor.

Check: $(x - 4)(x + 1) = x^2 + x - 4x - 4 = x^2 - 3x - 4$.

Practice Exercise

1. $x^2 + 9x + 20$
2. $x^2 - x - 12$
3. $x^2 + 5x - 6$
4. $x^2 - 11x + 18$
5. $x^2 - 2x - 3$
6. $x^2 + 16x + 48$
7. $x^2 - 7x + 12$
8. $x^2 - 10x + 24$
9. $x^2 + 3x - 10$
10. $x^2 - 6x - 7$

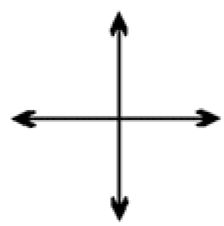
A ***graph*** is a kind of drawing or diagram that shows ***data***, or information, usually in numbers. In order to make a graph, you must first have data.

Making a Coordinate Graph

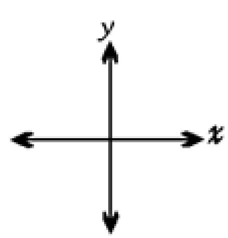
Many graphs show information on a ***grid***. The grid is made up of lines that intersect to create a screen pattern. The bottom line of the grid is called the ***horizontal axis*** and the vertical line on the left or right is called the ***vertical axis***.

The Plane

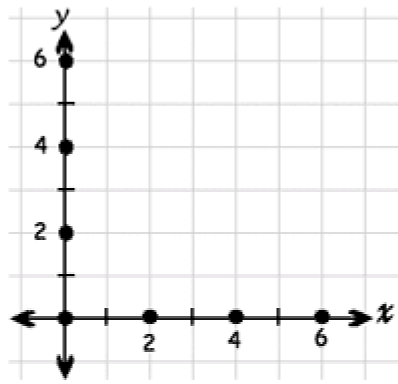
Here is a picture of a plane. Two lines are drawn inside the plane. Each of these lines is an axis. (Together they are called axes.) The axes are like landmarks that we can use to find different places in the plane.



We can label the axes to make them easier to tell apart. The axis that goes from side to side is the x-axis, and the axis that goes straight up and down is the y-axis.



Let's zoom in on one corner of the plane. (This corner is called the first quadrant.)



We have marked some of the points on each axis to make them easier to find. The point where the two axes cross has a special name: it is called the origin.

The gray lines will help us find points. When you make your own graphs, you can use the lines on your graph paper to help you.

Finding Points in the Plane

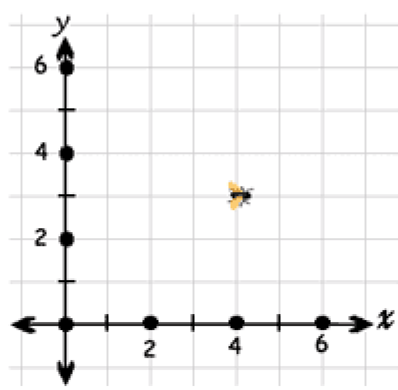
We can find every point in the plane using two numbers. These numbers are called coordinates. We write a point's coordinates inside parentheses, separated by a comma, like this: (5, 6). Sometimes coordinates written this way are called an ordered pair.

The first number in an ordered pair is called the x-coordinate**. The x-coordinate tells us how far the point is along the x-axis.**

The second number is called the y-coordinate**. The y-coordinate tells us how far the point is along the y-axis.**

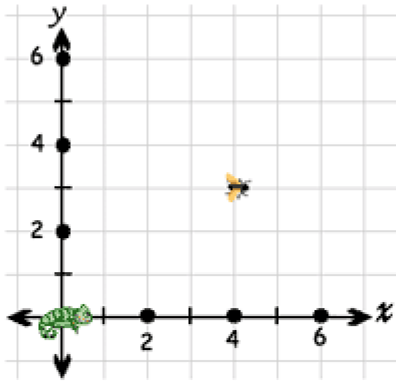
Let's try an example.

A fly is sitting in the plane.

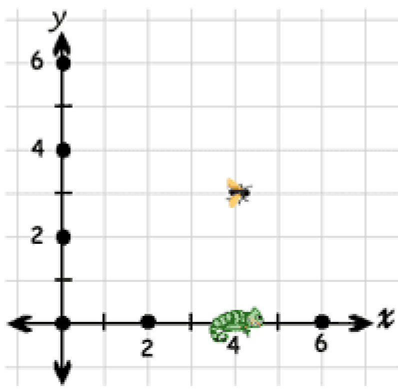


Sam knows that the fly is at point (4, 3). What should he do?

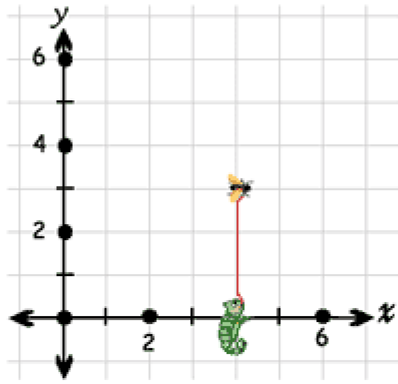
Sam starts at the origin. So far, he has not moved along the x-axis or the y-axis, so he is at point (0, 0).



Because he wants to find (4, 3), Sam moves four units along the x-axis.



Next, Sam turns around and shoots his tongue three units. Sam's tongue goes straight up, in the same direction that the y-axis travels.



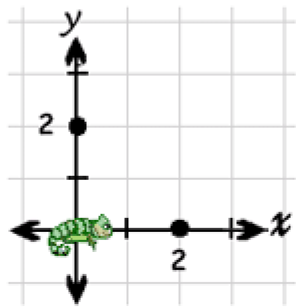
Sam has found point (4, 3). **He eats the fly happily**.

Graphing Points in the Plane

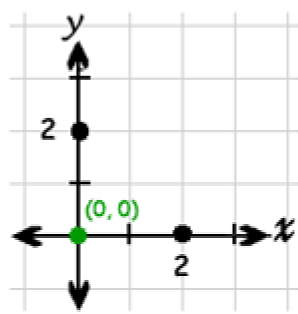
You can graph points the same way that Sam found the fly. Let's practice graphing different points in the plane.

We'll begin by graphing point (0, 0).

Sam starts at the origin and moves 0 units along the x-axis, then 0 units up. He has found (0,0) without going anywhere!



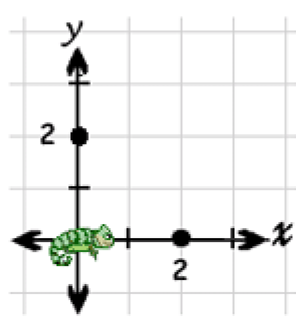
Sam marks the point with a green dot, and labels it with its coordinates.



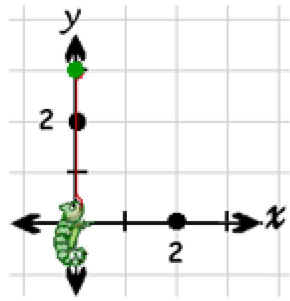
Sam has finished graphing point (0, 0).

Next, let's graph point (0, 3).

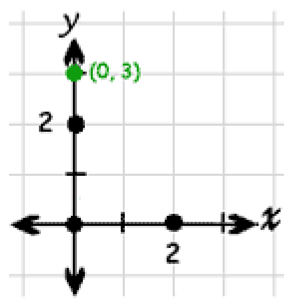
Sam starts at the origin, just like always. He moves 0 units along the x-axis, because the x-coordinate of the point he is trying to graph is 0.



Sam uses his tongue to move a green dot 3 units straight up.



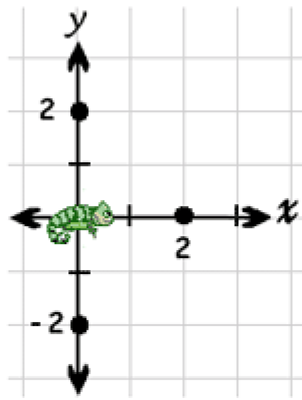
The final step is labeling the point.



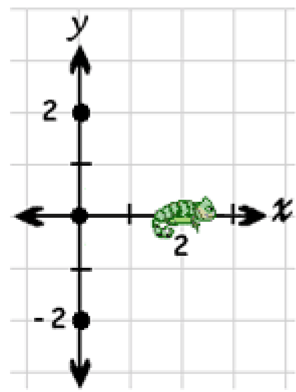
Notice that point (0, 3) is *on the y-axis* and its x-coordinate is 0. Every point on the y-axis has an x-coordinate of 0, because you don't need to move sideways to reach these points. Similarly, every point on the x-axis has a y-coordinate of 0.

Let's end with a more complicated example: graphing point (2, -2).

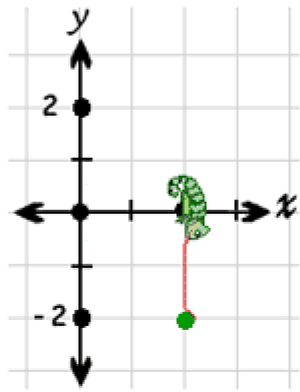
Sam begins at point (0, 0).



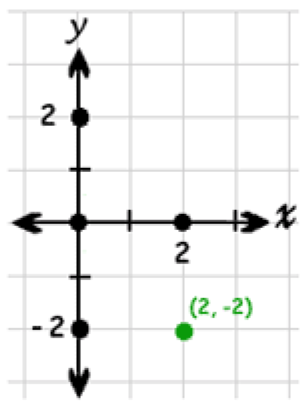
He moves 2 units along the x-axis.



The y-coordinate of the point Sam wants to graph is -2. Because the number is negative, Sam sticks his tongue down two units. This makes sense, because negative numbers are the opposite of positive numbers, and down is the opposite of up.



Before he leaves, Sam labels the point he graphed.



Graphing Equations

Often, you must use an equation to graph a line. For example, somebody might ask you to graph the line 4x + 2y = 8. When this happens, you have to find some points on the line before you can graph it.

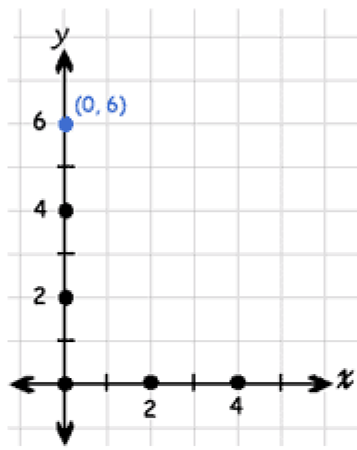
Let's ask Joan to **graph 2x + y = 6.**

First, Joan needs to find values of x and y that make this equation true. She decides to let x = 0.

Now that Joan has chosen a value for x, she needs to find y, so she substitutes 0 into the original equation:

$2x + y = 6$
$2 \cdot 0 + y = 6$
$0 + y = 6$
$y = 6.$

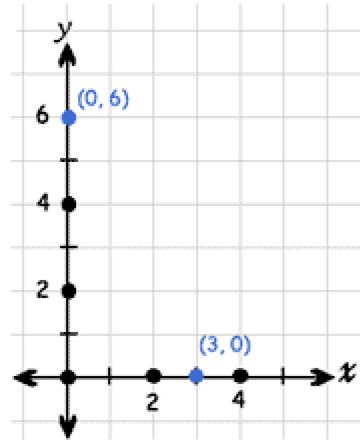
Joan has found that when x = 0, y = 6. She can graph this information as the point (0, 6):



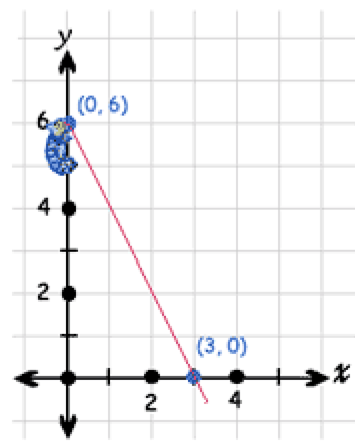
Joan needs two points before she can graph a line, so she has to find another set of values for x and y. She decides to try making y = 0. This time, she substitutes 0 into her equation for y:

$2x + y = 6$
$2x + 0 = 6$
$2x = 6$
$x = 3$

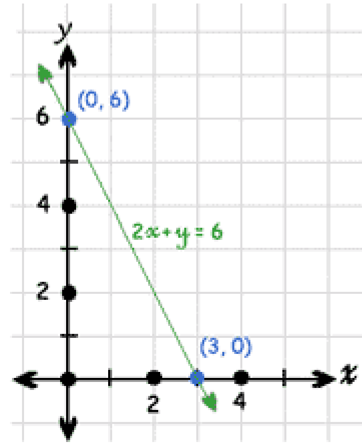
Joan has found that (x = 3, y = 0) makes her equation true, so she graphs the point (3, 0).



Now that Joan knows where two points are, she can graph her line. She crawls up the y-axis until she reaches the first point, then sticks her tongue straight into the second point.



The straight part of Joan's tongue leaves a sticky green mark behind. She adds arrows to this line and labels it with her equation, then goes looking for a fly.



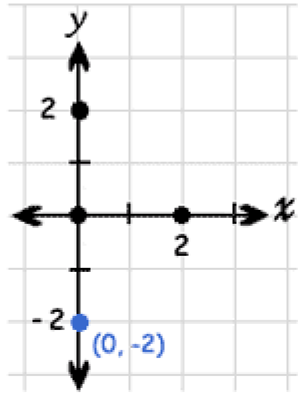
Joan has graphed 2x + y = 6.

Let's ask Joan to graph a different line: **y = 4x - 2**.

Joan needs to find values for x and y that make the equation true. She decides to start by substituting 0 for x, just as before.

> y = 4x - 2
> y = 4 • 0 -2
> y = 0 - 2
> y = -2

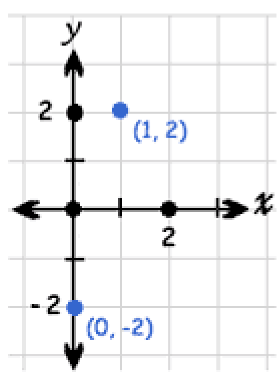
Joan's substitution tells her the equation is true when x = 0 and y = -2, so she graphs the point (0, -2).



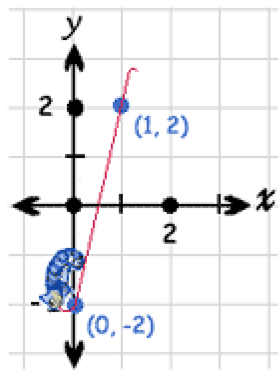
When an equation is written like this one, it's fairly simple to find y by substituting different values for x. Joan decides to let x = 1:

$y = 4x - 2$
$y = 4 \cdot 1 - 2$
$y = 4 - 2$
$y = 2$

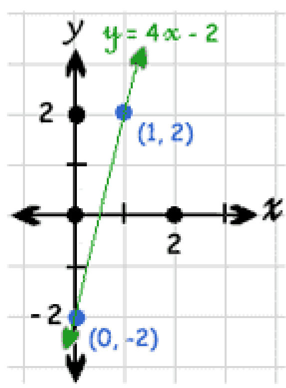
Now Joan knows that x = 1, y = 2 also works, so she graphs (1, 2).



Next, Joan crawls down the y-axis and sticks her tongue straight through the two points. The straight part of her tongue will make a green trace.



Before she leaves, Joan draws arrows to show that her line continues forever. She also labels the line with the equation it came from.



Joan just graphed y = 4x - 2.

Graphing an Inequality

Graph the following inequality

$$y \leq 3x - 1$$

1. Solve the equation for y (if necessary).

(2, 5): $5 \leq 3(2) - 1$

$5 \leq 6 - 1$

$5 \leq 5$

(0, -1): $-1 \leq 3(0) - 1$

$-1 \leq 0 - 1$

$-1 \leq -1$

2. Graph the equation as if it contained an = sign.
3. Draw the line solid if the inequality is $\leq$ or $\geq$
4. Draw the line dashed if the inequality is $<$ or $>$
5. Pick a point not on the line to use as a test point. The point (0,0) is a good test point if it is not on the line.
6. If the point makes the inequality true, shade that side of the line. If the point does not make the inequality true, shade the opposite side of the line. Any point on the shaded side of the line would make the inequality true.

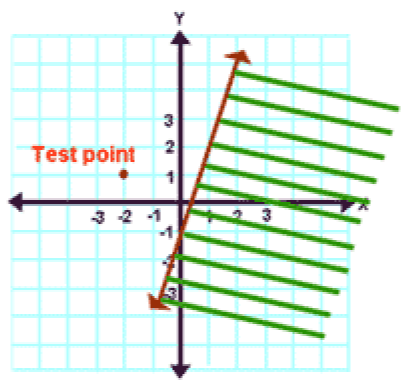
Graph the inequality

$y \leq 3x - 1$

The test point in this problem was (-2,1).
[The easiest test point is usually (0,0)]

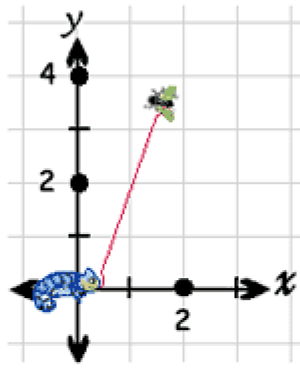
$1 \leq 3(-2) - 1$
$1 \leq -6 - 1$
$1 \leq -7$
false

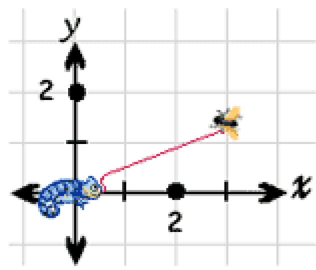
(shade the opposite side of the line)



Slope

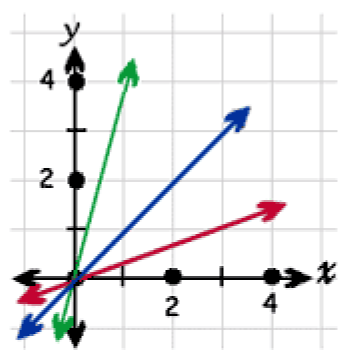
Here are two snapshots of Joan catching bugs:





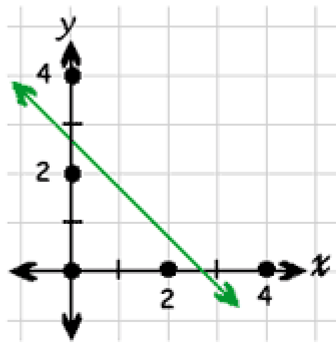
Notice that when Joan caught the green fly, she had to stick her tongue almost straight up in the air. When Joan caught the yellow fly, her tongue was close to flat.

Lines can be slanted different ways, just like Joan's tongue:



We use slope to measure a line's slant. The green line above has a big slope, because it is slanted so sharply. Because the red line is close to flat, it has a small slope.

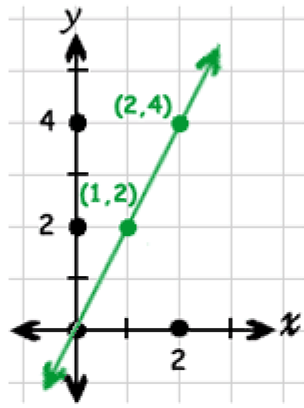
Lines with negative slope point down instead of up. Here is a line with a negative slope:



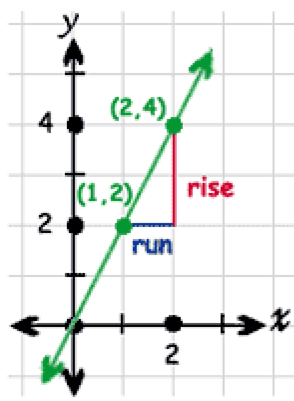
Calculating Slope

We can tell whether a line's slope is big or small, and whether the slope is positive or negative. But what if we want to compare two big slopes, or two small slopes? We need a more exact definition of slope.

Let's start by drawing a line and picking two points on the line. (There's nothing special about this line or these points -- we just want an example.)



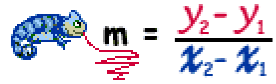
Slope is defined as **the change in the y-coordinates** divided by **the change in the x-coordinates**. People often remember this definition as "rise/run." In this picture, the change in y-coordinates (rise) is red, and the change in x-coordinates (run) is blue:



Writing "change in x-coordinates" and "change in y-coordinates" many times is a lot of work, so let's use the Greek letter delta, Δ, as an abbreviation for change. The traditional abbreviation for slope is m. Now we can write a formula for slope:

$$m = \frac{\Delta y}{\Delta x}$$

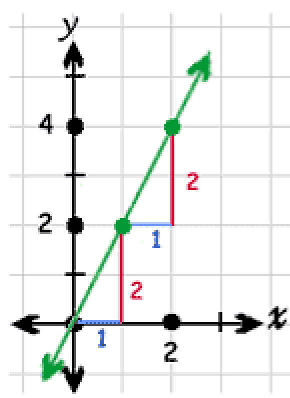
If we name our first point (x_1, y_1) and our second point (x_2, y_2), we can rewrite our formula to get rid of the delta:



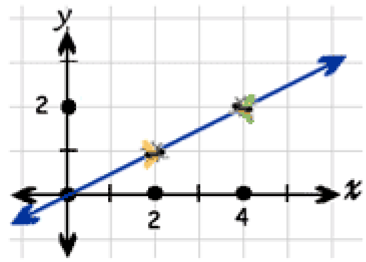
We can use this formula to find the slope of our example line. Our first point was (1, 2), so $x_1 = 1$ and $y_1 = 2$. Similarly, $x_2 = 2$ and $y_2 = 4$, because our second point was (2, 4).

$m = (y_2 - y_1) / (x_2 - x_1)$
$= (4 - 2) / (2 - 1)$
$= 2/1$
$= 2.$

Now we know that the slope of our line is 2. You can see from the graph that the line moves up two spaces for every space that it moves to the right:



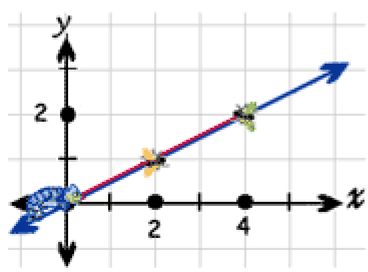
Let's try finding the slope of the line between these two flies:



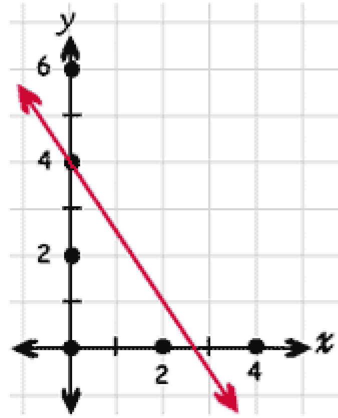
The first fly is at point (2, 1), and the second fly is at point (4, 2). We can substitute this into our slope equation to find the slope of the line.

$$m = (y_2 - y_1) / (x_2 - x_1)$$
$$= (2 - 1) / (4 - 2)$$
$$= 1/2.$$

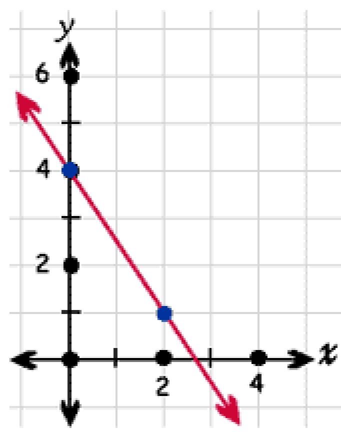
The line's slope is 1/2. If Joan finds a point on the line and then gives her tongue the same slope, she should be able to catch both flies:



Now let's find the slope of this line. Notice that the line slants down instead of up. Because the line is slanting down, its slope should be negative.



Before we can find the line's slope, we need to locate two points on the line. We can see that the line intersects the y-axis at the point (0, 4). Finding a second point is more difficult. We can try to be exact by finding a point on the line where two gridlines cross. One point like this is (2, 1). In this picture, the points we have chosen are colored blue:



Now we can use our slope equation to find the line's slope.

$m = (y_2 - y_1) / (x_2 - x_1)$
$= (1 - 4) / (2 - 0)$
$= -3/2$, or -1.5.

Our line's slope is a negative number, just as we predicted.

Practice Exercise

Solve and graph the following equations and inequalities.

1. $2x - y = 4$
2. $x + y > 5$
3. $-2x + 3y = 6$
4. $2y - x = 0$
5. $3x - 4y = 10$
6. $-10x + 15y = -5$
7. $2x - 3y < 5$
8. $x + y \leq -4$

Find the slope of a line passing through each of the following pairs of points.
State the answer in simplest form.

1. (9, -4) and (-9, -0)	2. (-8, 5) and (-2, 9)
3. (-9, 6) and (-3, -4)	4. (-7, 7) and (6, 1)
5. (-1, -0) and (5, -4)	6. (3, 1) and (8, -7)
7. (3, -7) and (-4, 2)	8. (-2, -8) and (-6, 6)
9. (-4, 3) and (2, 6)	10. (-3, -2) and (7, 9)
11. (10, -4) and (-6, 12)	12. (0, -6) and (-10, -0)

13. (3, -18) and (17, -14)	14. (-16, -14) and (-14, -2)
15. (18, 15) and (-19, -2)	16. (13, 6) and (-4, -1)
17. (83, -82) and (-98, 92	18. (-67, -79) and (39, -24)

Problem Solving with Equations, Equalities and Inequalities

Equations may be used to solve word problems. To solve a word problem, read the whole problem carefully and then follow these three steps:

Step 1 Represent the unknown with a letter.
Step 2 Write an equation that represents the problem.
Step 3 Solve the equation for the unknown.

EXAMPLE 1 Seven times a number is equal to 147. What is the number?

Step 1 Let x equal the unknown number.
Step 2 Write an equation for the problem.

$$7x = 147$$

Step 3 Solve the equation. Divide each side by 7.

$$\frac{7x}{7} = \frac{147}{7}$$

$$\boldsymbol{x = 21}$$

Answer: $x = 21$

The unknown number is 21.

EXAMPLE 2 **Six times a number plus 7 is equal to 55. What is the number?**

Step 1 Let x equal the unknown number.
Step 2 Write an equation for the problem.

$$6x + 7 = 55$$

Step 3 Solve the equation.
a) Subtract 7 from 55.
b) Divide 48 by 6.

$$6x = 55 - 7$$
$$6x = 48$$
$$x = 48 \div 6$$
$$\boldsymbol{x = 8}$$

Answer: The number is 8.

EXAMPLE 3 **Three times the quantity a number minus 4 is equal to two times the sum of the number plus 3. What is the number?**

Step 1 Let x equal the unknown number.
$3(x – 4)$ is three times the quantity x minus 4
$2(x + 3)$ is two times the sum of x plus 3
Step 2 Write an equation for the problem.

$$3(x - 4) = 2(x + 3)$$

Step 3 Solve the equation.
a) Remove parentheses.
b) Subtract $2x$ from $3x – 12$.
c) Add 12 to 6.

$$3x - 12 = 2x + 6$$
$$3x - 2x - 12 = 6$$
$$x - 12 = 6$$
$$x = 6 + 12$$
$$\mathbf{x = 18}$$

Answer: The number is 18.

Examples 4, 5 and 6 show how to set up a word or story problem in algebra. Study these carefully.

EXAMPLE 4 **Bill saves 1/2 of his monthly paycheck. If his monthly savings is $92, how much does he earn each month?**

Step 1 **Let x = monthly income because this is the unknown quantity that you must find.**
$92 = monthly savings

1/2 = fraction saved

Step 2 **Write an equation for the problem.**
Fraction saved times income = savings

$$1/2x = \$92$$

Step 3 **Solve the equation. Multiply each side by 8.**

$$(8)1/2x = \$92(8)$$
$$x = \$736$$

Answer: x = $736
Bill earns $736 each month

EXAMPLE 5 **Jack and Steve do yard work. Because Jack provides the truck, gas, and yard equipment, he receives twice the money that Steve does. If they collect $540, how much does each receive?**

Step 1 **Let** x **= Steve's share**
2x **= Jack's share (We know that Jack receives twice Steve's share.)**

Step 2 **Write an equation for the problem.**
Jack's share plus Steve's share = $540

$$2x + x = 540$$

Step 3 **Solve the equation.**
a) Combine the x**'s.**

b) Divide 540 by 3.

$$3x = 540$$
$$x = 540 \div 3$$
$$x = \$180$$
$$2x = 2(180) = \$360$$

Answer: $x = \$180$ Steve's share

$2x = \$360$ Jack's share

EXAMPLE 6 **Mary, Anne, and Sally share living expenses. Anne pays $25 less rent than Mary. Sally pays twice as much rent as Anne. If the total rent is $365, how much rent does each pay?**

Step 1 ***Hint:*** **Since you know nothing about how much Mary pays for rent, let Mary's rent equal *x*.**
Let *x* = Mary's rent
***x* - 25 = Anne's rent**
2(*x* – 25) = Sally's rent

Step 2 **Write an equation for the problem.**
Mary's + Anne's + Sally's = total rent.

$$x + (x - 25) + 2(x - 25) = 365$$

Step 3 **Solve the equation.**

a) Remove parentheses.
b) Combine the *x*'s and the numbers.
c) Add 75 to 365.
d) Divide 440 by 4.

$$\boldsymbol{x + x - 25 + 2x - 50 = 365}$$

$$\boldsymbol{4x - 75 = 365}$$

$$\boldsymbol{4x = 365 + 75}$$

$$\boldsymbol{4x = 440}$$

$$\boldsymbol{x = 440 \div 4}$$

$$x = 110$$

Answer: x = \$110, Mary's rent

$x - 25$ = \$85, Anne's rent

$2(x - 25)$ = \$170, Sally's rent

Word problems are sometimes solved by using an inequality.

Example Sandy has three tasks to complete. The second will take three times as long as the first, and the third will take 30 minutes. If Sandy wants to be finished with all three tasks in less than 90 minutes, what is the longest amount of time the first task can take?

Let x stand for the time of the first task. Then $3x$ stands for the time of the second task. Since all three tasks must take less than 90 minutes, the inequality can be written:

$$x + 3x + 30 < 90$$

Solve:

$$4x + 30 < 90$$

$$4x < 60$$

$$x < 15$$

As long as the first task takes **less than 15 minutes,** Sandy will finish the three tasks in less than 90 minutes.

Practice Exercise

Solve for the number.

1. The sum of a number and 66 increased by 45 is 116. What is the number?

2. The sum of eleven times a number and 45 is 122. What is the number?

3. The quotient of a number and 9 decreased by 5 is 19. What is the number?

4. One number is 7 more than another. If the sum of the two numbers is 55, what is the smaller of the two numbers?

5. How many boys are in the club of 32 members, if girls outnumber boys by 8?

6. If five times a number is added to 6, the result is less than 4 times that same number added to 10. What is the solution?

7. The product of what number and 8 is 40?

8. The product of 9 and a number, minus 24 is 8 less than the difference of 57 and 19. What is the number?

9. In an election, the winning candidate had 2400 more votes than the loser. If the total number of votes cast was 9800, how many votes did the winner receive?

10. 7 times a number, decreased by 20 is 1. What is the number?

11. A number is decreased by 6 and the result is multiplied by 3 to get an answer of 9. What is the number?

12. The perimeter of a triangle must be less than or equal to 65cm. One side is 21cm. the second side is 18 cm. What is the longest the third side can be (in centimeters)?

13. The product of what number and 4 is 8?

14. The product of 8 and a number, minus 20 is 18 less than the difference of 85 and 15. What is the number?

15. What is the solution when 5 times a number plus 2 is less than 6 times that same number added to 3 times the number plus 10?

Answer Key

Book 14019 – Equations: Equalities and Inequalities

Page 9

2. 57 + n = 139 **3.** 5n + 5 = 65
4. n – 78 = -46 **5.** n ÷ 12 = 144
6. 8n = 48 **7.** 6n = 72 **8.** 4 (n + 1) = 12
9. 7n – 10 = 25 **10.** 2 (n + 85) = 176
11. 8n – 1 = 7 **12.** 2n –2 = 16
13. n + 88 = 176 **14.** 9 + 7n = 51
15. n + 4n = 45 **16.** 2n + 7 = 27
17. n – 26 = 34 **18.** n – 16 = -14
19. n – 89 = -51 **20.** 3n + 9 = 18

Page 19

1. b = 110 **2.** b = 110 **3.** b = 40
4. b = 10 **5.** x = 9 **6.** x = 3 **7.** x = 10
8. x = 2 **9.** x = 3 **10.** x = 12
11. x = 12 **12.** x = 11 **13.** b = 144
14. n = 3 **15.** b = 12 **16.** x = 9
17. x = 2 **18.** x = 5 **19.** x = 6 **20.** x = 6
21. n = 3 **22.** b = 70

Page 23

1. a = 14 **2.** b = 8 **3.** m = 12 **4.** y = 10
5. z = 9 **6.** a = .5 **7.** y = -2 3/7
8. z = 7 2/7

Page 27

2. $x > 68$ **3.** $x > -22$ **4.** $90 < a$
5. $x > 72$ **6.** $79 > a$ **7.** $x < 50$ **8.** $y < 81$
9. $72 < a$ **10.** $x < 76$ **11.** $x > 92$
12. $y > 31$ **13.** $41.63 < a$ **14.** $y < 60$

15. x > 69.6 **16.** x > -33 **17.** x > -44
18. y < -49

Page 27 **(solve each equation. multiplication and divison)** **2.** b < 7 **3.** n < -52 **4.** b > 5
5. 9 > a **6.** n > -200 **7.** b > 1 **8.** b < 8
9. b > -15 **10.** b > 11 **11.** –9 > a
12. b < 42 **13.** 12 > a **14.** n > -31
15. b > 9.4 **16.** 3 > a **17.** n < -101.75
18. b > -63 **19.** a < 8 **20.** –7 < b
21. c < 2 **22.** –5 < d

Page 30 **1.** (x + 4) (x + 5) **2.** (x + 3) (x – 4)
3. (x + 6) (x – 1) **4.** (x – 2) (x – 9)
5. (x – 3) (x + 1) **6.** (x + 12) (x + 4)
7. (x – 3) (x – 4) **8.** (x – 6) (x – 4)
9. (x – 2) (x + 5) **10.** (x – 7) (x + 1)

Page 53 **Make sure that the graphs are constructed according to the instructions.**
(Sample Points)
1. (0, -4); (2, 0) **2.** (0, 5); (5,0)
3. (0, 2); (-3, 0) **4.** (0, 0); (2, 1)
5. (6, 2); (10, 5) **6.** (2, 1); (5, 3)
7. (4, 1); (10, 5) **8.** (0, -4); (-4, 0)

Page 53 **(find the slope)** **1.** –2/9 **2.** 2/3 **3.** –1 2/3
4. –6/13 **5.** –2/3 **6.** –1 3/5 **7.** –1 2/7
8. –3 ½ **9.** ½ **10.** 1 1/10 **11.** –1
12. –3/5 **13.** 2/7 **14.** 6 **15.** 17/37
16. 7/17 **17.** –174/181 **18.** 55/106

Page 60 **1.** n = 5 **2.** n = 7 **3.** n = 216 **4.** 24
5. 12 boys **6.** n < 4 **7.** 5 **8.** 6
9. 6100 **10.** 3 **11.** 9 **12.** 26 cm
13. 2 **14.** 9 **15.** –2 < n