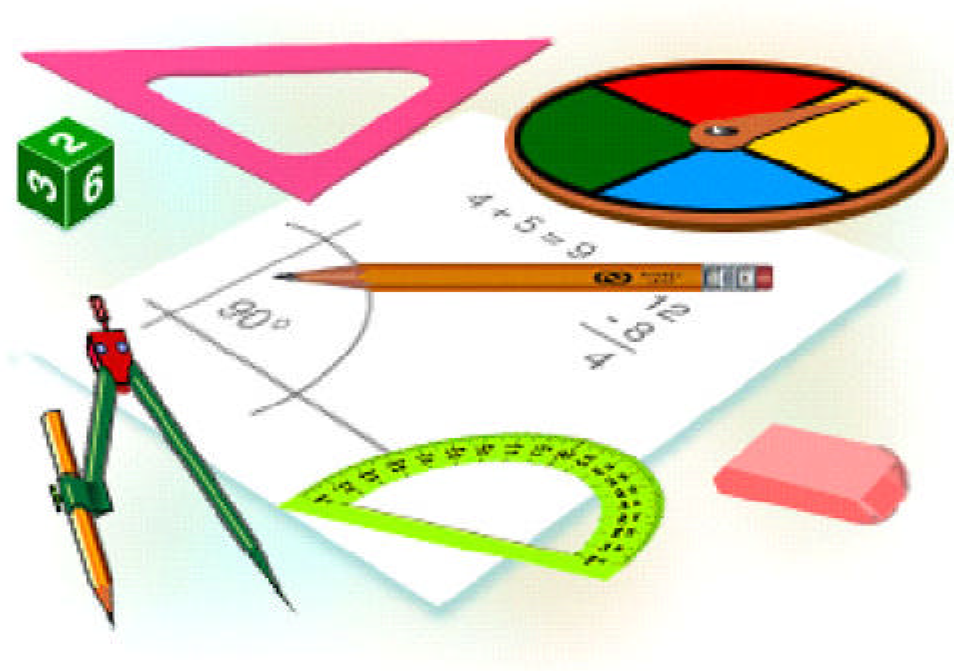
The Next Step

Mathematics Applications for Adults



Book 14019 – Fractions, Decimals and Percent

OUTLINE

Mathematics - Book 14019

Fractions, Decimals, and Percent
Fractions
perform the four mathematical operations with fractions.
write mixed fractions.
determine recipricol fractions.
write equivalent fractions.
determine the lowest term fraction.
Decimals
perform the four mathematical operations using decimals.
round off decimals to a given number of decimal points.
convert decimals to fractions.
convert fractions to decimals.
Percent
perform the four mathematical operations using percents.
write fractions or decimals as percents.
write percents as fractions or decimals.

THE NEXT STEP

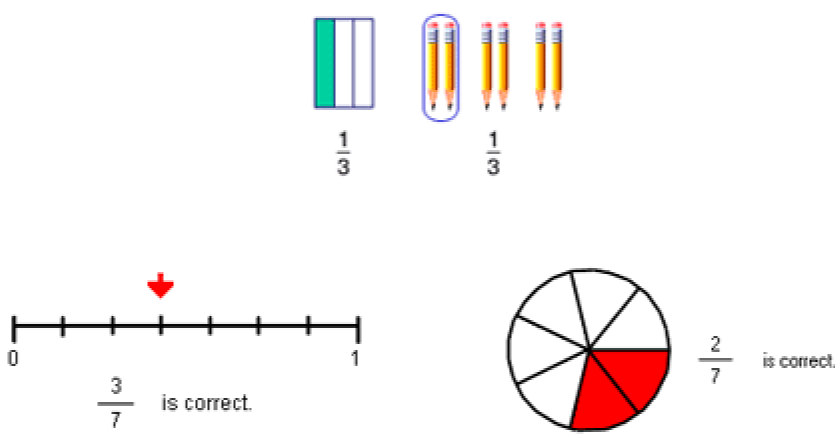
Book 14019

Fractions, Decimals, and Percent

Fractions

The word ***fraction*** means "part of a whole." The word comes from the Latin word ***fractio***, meaning "to break into pieces." In math, a fraction means one or more parts of a whole.

Example:



A fraction has two parts, a ***denominator*** and a ***numerator***. The denominator is the numeral written under the bar and tells the number of parts a whole is divided into. The numerator is the numeral written above the bar. The numerator tells the number of parts of the whole that are being counted. A ***proper fraction*** has a numerator that is smaller than its denominator.

numerator	**number of parts counted**	**1**
----------------------	**-----------------------------------**	**------**
denominator	**total parts of the whole**	**17**

Improper Fractions

When the numerator of a fraction is greater than or equal to the denominator, the fraction is called an ***improper fraction***.

$$\frac{3}{2} \quad \frac{4}{3} \quad \frac{5}{4} \quad \frac{6}{5} \quad \frac{7}{6} \quad \frac{8}{8}$$

Þ ***The value of an improper fraction is always greater than or equal to one.***

Mixed Numerals

Mixed numerals combine whole numbers and fractions. The values of mixed numerals can also be written as ***improper fractions***. To write a mixed numeral as an improper fraction, multiply the whole number by the denominator of the fraction, then add the numerator. Use your answer as the new numerator and keep the original denominator.

$$1\tfrac{1}{2} = \frac{(2 \times 1) + 1}{2} = \frac{3}{2} \qquad 2\tfrac{3}{4} = \frac{(2 \times 4) + 3}{4} = \frac{11}{4}$$

To change an improper fraction to a mixed numeral, divide the numerator by the denominator. Then place the remainder over the old denominator.

$$\frac{3}{2} = \begin{array}{r} 1 \\ 2\overline{)3} \\ -2 \\ \hline 1 \end{array} = 1\tfrac{1}{2} \qquad \frac{11}{4} = \begin{array}{r} 2 \\ 4\overline{)11} \\ -8 \\ \hline 3 \end{array} = 2\tfrac{3}{4}$$

Practice Exercise

Express each fraction as a whole number or as a mixed number.

1. $\frac{6}{3}$ =	2. $\frac{18}{9}$ =	3. $\frac{65}{10}$ =	4. $\frac{69}{8}$ =
5. $\frac{15}{2}$ =	6. $\frac{47}{4}$ =	7. $\frac{69}{7}$ =	8. $\frac{96}{11}$ =
9. $\frac{74}{6}$ =	10. $\frac{79}{7}$ =	11. $\frac{10}{4}$ =	12. $\frac{77}{11}$ =

13. $\frac{59}{8} =$ 14. $\frac{35}{10} =$ 15. $\frac{28}{3} =$ 16. $\frac{45}{6} =$

17. $\frac{51}{12} =$ 18. $\frac{31}{5} =$ 19. $\frac{21}{2} =$ 20. $\frac{78}{11} =$

21. $\frac{36}{6} =$ 22. $\frac{44}{5} =$ 23. $\frac{40}{7} =$ 24. $\frac{104}{10} =$

25. $\frac{17}{4} =$ 26. $\frac{114}{9} =$ 27. $\frac{38}{3} =$ 28. $\frac{89}{7} =$

29. $\frac{79}{11} =$ 30. $\frac{40}{9} =$ 31. $\frac{51}{6} =$ 32. $\frac{29}{5} =$

Common Denominators

Many fractions have ***common denominators***. That means that the numbers in their denominators are the same.

$$\frac{1}{2} \quad \frac{3}{2} \quad \frac{5}{2}$$

To find common denominators, (1) find the ***least common multiple*** for the denominators of the fractions you are comparing.

Compare:

$\frac{1}{2}$ and $\frac{2}{3}$ **Answer: least common multiple is 6**

(2) Divide the common multiple by the denominators.

$$2\overline{)6}^{\,3} \qquad 3\overline{)6}^{\,2}$$

(3) Multiply the quotients by the old numerators to calculate the new numerators.

$$\begin{array}{r} 3 \\ \underline{\times\ 1} \\ 3 \end{array} \qquad \begin{array}{r} 2 \\ \underline{\times\ 2} \\ 4 \end{array}$$

(4) Place the new numerators over the common denominator.

$$\frac{3}{6} \qquad \frac{4}{6}$$

Þ ***To reduce a fraction to its lowest terms, divide both the numerator and the denominator by their greatest common denominator.***

$$\frac{4}{8} \text{ ¸ } \frac{4}{4} = \frac{1}{2}$$

Equivalent Fractions

You know from experience that different fractions can have the same value.

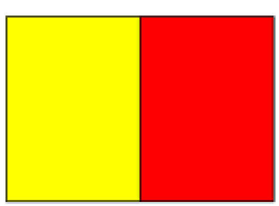
Since there are 100 pennies in a dollar, 25 pennies is equal to 25/100 of a dollar. The same amount also equals a quarter, or ¼ of a dollar.

On a measuring cup, ½ cup is the same amount as 2/4 cup.

On an odometer, 5/10 of a mile is the same as ½ mile.

Out of a dozen doughnuts, six doughnuts equal 6/12, or ½ dozen.

A napkin is folded into two parts. One part is yellow, the other red.



½ yellow

½ red

Then the napkin is folded again. Now there are two yellow parts and two red parts.

$\frac{2}{4}$ **yellow**

$\frac{2}{4}$ **red**

In this example, the red part of the napkin can be described as ½ red or 2/4 red. That makes ½ and 2/4 ***equivalent fractions***.

When solving math problems, reduce fractions to their lowest equivalent. Rather than describing the napkin as 2/4 yellow, call it ½ yellow.

Some Equivalent Fractions

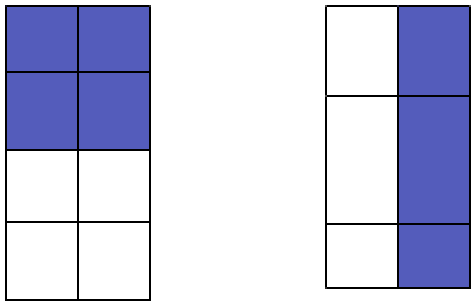
$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20}$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$$

You can tell if two fractions are equal by finding cross products.

Example Are 4/8 and 3/6 equal fractions?



Multiply diagonally as shown by the arrows below. If the cross products are equal, the fractions are equal.

$$\frac{4}{8} \times \frac{3}{6} \qquad \mathbf{4 \text{ x } 6 = 24} \quad \mathbf{8 \text{ x } 3 = 24}$$

Since the cross products are equal, **4/8 = 3/6.**

Sometimes you need to find an equal fraction with higher terms. You raise a fraction to higher terms by multiplying both the numerator and the denominator by the same number (except 0).

5/8 and 20/32 are equal fractions because $\frac{5 \text{ x } 4}{8 \text{ x } 4} = \frac{20}{32}$

Often you will need to find an equal fraction with a specific denominator. To do this, think, "What number multiplied by the original denominator will result in the new denominator?" Then multiply the original numerator by the same number.

Example ¾ = ?/24

Since 4 x 6 = 24, multiply the numerator 3 by 6. $\frac{3 \text{ x } 6}{4 \text{ x } 6} = \frac{18}{24}$

The fractions **¾ and 18/24 are equal fractions.**

Comparing Fractions

When two fractions have the same number as the denominator, they are said to have a common denominator, and the fractions are called like fractions. When you compare like fractions, the fraction with the greater numerator is the greater fraction.

Example 1 Which fraction is greater, 3/5 or 4/5?

The fractions 3/5 and 4/5 are like fractions because they have a common denominator, 5. Compare the numerators.

Since 4 is greater than 3, **4/5 is greater than 3/5.**

Fractions with different denominators are called unlike fractions. To compare unlike fractions, you must change them to fractions with a common denominator.

The common denominator will always be a multiple of both of the original denominators. The multiples of a number are found by going through the times tables for the number. For instance, the multiples of 3 are 3, 6, 9, 12, 15, 18, and so on.

You can often find a common denominator by using mental math. If not, try these methods:

1. See whether the larger denominator could be a common denominator. In other words, if the smaller denominator can divide into the larger denominator evenly, use the larger denominator as the common denominator.

2. Go through the multiples of the larger denominator. The first one that can be divided evenly by the smaller denominator is the lowest common denominator.

Example 2 Which is greater, 5/6 or ¾?

Go through the multiples of the larger denominator: 6, 12, 18, 24, 30…. Since 12 can be divided evenly by both 4 and 6, 12 is the lowest common denominator.

Build equal fractions, each with the denominator 12: $\frac{5 \times 2}{6 \times 2} = \frac{10}{12}$ $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

Compare the like fractions. Since 10/12 > 9/12, the fraction **5/6 > ¾.**

Practice Exercise

Fraction Comparison

1. $\frac{1}{12} < \frac{5}{7}$ 2. $\frac{1}{8}$ ___ $\frac{3}{7}$

3. $\frac{1}{3}$ ___ $\frac{1}{2}$ 4. $\frac{3}{5}$ ___ $\frac{1}{12}$

5. $\frac{1}{2}$ ___ $\frac{7}{15}$ 6. $\frac{3}{12}$ ___ $\frac{5}{15}$

7. $\frac{2}{5}$ — $\frac{1}{12}$ 8. $\frac{9}{14}$ — $\frac{6}{11}$

9. $\frac{11}{99}$ — $\frac{18}{36}$ 10. $\frac{12}{59}$ — $\frac{37}{15}$

11. $\frac{1}{2}$ — $\frac{3}{18}$ 12. $\frac{1}{2}$ — $\frac{11}{16}$

13. $\frac{7}{56}$ — $\frac{6}{17}$ 14. $\frac{7}{21}$ — $\frac{9}{21}$

15. $\frac{12}{48}$ — $\frac{4}{28}$ 16. $\frac{47}{62}$ — $\frac{2}{3}$

17. $\frac{5}{14}$ — $\frac{25}{57}$ 18. $\frac{30}{60}$ — $\frac{12}{16}$

Reduce each fraction to lowest terms.
(Hint: Divide its numerator and denominator by their Greatest Common Factor)

1. $\frac{6}{48} =$ 2. $\frac{20}{45} =$ 3. $\frac{12}{36} =$ 4. $\frac{12}{20} =$

5. $\frac{12}{132} =$ 6. $\frac{20}{20} =$ 7. $\frac{11}{77} =$ 8. $\frac{12}{48} =$

9. $\frac{33}{40} =$ 10. $\frac{9}{12} =$ 11. $\frac{8}{24} =$ 12. $\frac{50}{62} =$

13. $\frac{33}{20} =$ 14. $\frac{5}{10} =$ 15. $\frac{18}{54} =$ 16. $\frac{53}{63} =$

17. $\frac{24}{68} =$ 18. $\frac{10}{20} =$ 19. $\frac{25}{29} =$ 20. $\frac{18}{42} =$

21. $\frac{39}{66} =$ 22. $\frac{15}{19} =$ 23. $\frac{6}{54} =$ 24. $\frac{2}{6} =$

To add fractions, the fractions must have ***common denominators***. To add fractions with common denominators, simply add the numerators. The sum will become the numerator of your answer. The denominator will remain the same.

$$\mathbf{\frac{1}{3} + \frac{4}{3} = \frac{1+4}{3} = \frac{5}{3} = 1\,\frac{2}{3}}$$

Unlike fractions have different denominators. Use these steps to add unlike fractions.

Step 1 Find a common denominator and change one or both of the fractions to make like fractions.

$$\frac{1}{2} + \frac{3}{4} = ?$$

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

Step 2 Add the like fractions

$$2/4 + \frac{3}{4} = 5/4$$

Step 3 Reduce the answer if necessary. If the answer is an improper fraction, rewrite it as a whole or mixed number.

$$5/4 = 1\frac{1}{4}$$

A mixed number is a whole number and a proper fraction. To add mixed numbers, work with each part separately and then combine the results.

Þ ***Adding fractions is impossible without first writing the fractions with common denominators.***

Step 1 Write the fractions with common denominators.

$$6\,1/3 = 6\,\frac{1 \times 4}{3 \times 4} = 6\,\frac{4}{12}$$

$$\underline{+\,4\frac{3}{4}} = 4\,\frac{3 \times 3}{4 \times 3} = 4\,\frac{9}{12}$$

Step 2 Add the fractions first. Add the numerators and put the sum over the common denominator. Then add the whole numbers.

$$
\begin{array}{r}
6\ \frac{4}{12} \\
+\ 4\ \frac{9}{12} \\
\hline
\end{array}
$$

Step 3 Change the improper fraction to a mixed number. Add this to the whole number answer.

$$\frac{13}{12} = 1\ \frac{1}{12}$$

$$10 + 1\ \frac{1}{12} = 11\ \frac{1}{12}$$

Sometimes when you add the fraction parts, you get a whole number as an answer. If this happens, just add that whole number to the other one.

Example: $2\ \frac{3}{5} + 2\ \frac{2}{5}$

2 + 2 = 4

$\frac{3}{5} + \frac{2}{5} = \frac{5}{5} = 1$ **Remember that any number divided by itself is 1.**

4 + 1 = 5 **The answer is 5.**

Mixed numbers can be added to whole numbers by adding the whole numbers together and keeping the fraction. This makes sense because you are adding whole amounts plus another part of a whole.

Example: 3 + 2 ½ = 5 ½   3 + 2 = 5, 5 + ½ = 5 ½

To subtract fractions, the fractions must have ***common denominators***. To subtract fractions with common denominators, simply subtract the numerators. The difference will become the numerator of your answer. The denominator will remain the same.

$$\mathbf{\frac{7}{8} - \frac{5}{8} = \frac{7-5}{8} = \frac{2}{8} = \frac{1}{4}}$$

Unlike fractions have different denominators. Use these steps to subtract unlike fractions.

Step 1 Find a common denominator and change one or both of the fractions to make like fractions.

¾ - ½ = ?

$$\mathbf{½ = \frac{1 \text{ x } 2}{2 \text{ x } 2} = \frac{2}{4}}$$

Step 2 Subtract the like fractions.

¾ - 2/4 = ¼

Step 3 Reduce the answer if necessary. If the answer is an improper fraction, rewrite it as a whole or mixed number.

A mixed number is a whole number and a proper fraction. To subtract mixed numbers, work with each part separately and then combine the results.

Þ ***Subtracting fractions is impossible without first writing the fractions with common denominators.***

Step 1 Write the fractions with common denominators.

$$
\begin{array}{rl}
6\ 3/4 & = 6\,\dfrac{3 \times 3}{4 \times 3} = 6\,\dfrac{9}{12} \\[2ex]
\underline{-\ 4\ 1/3} & = 4\,\dfrac{1 \times 4}{3 \times 4} = 4\,\dfrac{4}{12}
\end{array}
$$

Step 2 Subtract the fractions first. Subtract the numerators and put the difference over the common denominator. Then subtract the whole numbers.

$$
\begin{array}{r}
6\,\dfrac{9}{12} \\[2ex]
\underline{-\ 4\,\dfrac{4}{12}} \\[2ex]
2\,\dfrac{5}{12}
\end{array}
$$

Step 3 If necessary, reduce to lowest terms.

When subtracting mixed numbers, sometimes the fraction you are subtracting from will be smaller than the fraction you are taking away. In this situation, you will need to regroup, or borrow, 1 from the whole number and rewrite it as a fraction.

Remember, a fraction with the same numerator and denominator equals 1.

Example $5\frac{1}{8}$

$$\begin{array}{r} 5\frac{1}{8} \\ -3\frac{3}{4} \\ \hline \end{array}$$

Step 1 Write the fractions with common denominators. The lowest common denominator is 8.

$$5\frac{1}{8} = 5\frac{1 \times 1}{8 \times 1} = 5\frac{1}{8}$$

$$-3\frac{3}{4} = 3\frac{3 \times 2}{4 \times 2} = 3\frac{6}{8}$$

Step 2 Because 1/8 is less than 6/8, you need to regroup, or borrow. Borrow 1 from the whole number 5, rewriting 5 as 4 8/8. Then add the fractional parts 1/8 and 8/8.

$$5\frac{1}{8} = 4\frac{8}{8} + \frac{1}{8} = 4\frac{9}{8}$$

$$\begin{array}{r} 5\frac{1}{8} \\ -3\frac{6}{8} \\ \hline \end{array} \qquad \begin{array}{r} 4\frac{9}{8} \\ -3\frac{6}{8} \\ \hline 1\frac{3}{8} \end{array}$$

Step 3 Subtract. If necessary, reduce the fraction to lowest terms

Sometimes when you subtract the fraction parts, you get a whole number as an answer. If this happens, just subtract that whole number from the other one.

Example: $4\frac{7}{5} - 2\frac{2}{5}$

4 - 2 = 2

$\frac{7}{5} - \frac{2}{5} = \frac{5}{5} = 1$ **Remember that any number divided by itself is 1.**

2 - 1 = 1 **The answer is 1.**

Mixed numbers can be subtracted from whole numbers by subtracting the whole numbers and keeping the fraction.

Example: 3 - 2 ½ = 1 ½ 3 - 2 = 1, 1 + ½ = 1 ½

Practice Exercise

Solve for each of the given problems.
Write the answer in lowest terms.

1. $\frac{8}{10} + 7\frac{1}{3}$ = ______

2. $\frac{1}{2} + 6\frac{4}{11}$ = ______

3. $\frac{6}{7} - \frac{1}{4}$ = ______

4. $7\frac{1}{3} - 3\frac{2}{6}$ = ______

5. $\frac{1}{5} + 7\frac{7}{11} =$ ______

6. $11\frac{3}{10} - \frac{7}{8} =$ ______

7. $12\frac{3}{4} - 11\frac{5}{11} =$ ______

8. $\frac{6}{8} + 2\frac{1}{2} =$ ______

9. $11\frac{4}{5} - 11\frac{5}{12} =$ ______

10. $\frac{4}{7} + 10\frac{5}{9} =$ ______

11. $1\frac{2}{3} + 11\frac{6}{10} =$ ______

12. $4\frac{9}{10} - 3\frac{1}{5} =$ ______

13. $11\frac{10}{15} - \frac{1}{5} =$ ______

14. $7\frac{1}{8} + 4\frac{15}{16} =$ ______

15. $11\frac{7}{12} - \frac{1}{6} =$ ______

16. $9\frac{2}{10} - 3\frac{3}{5} =$ ______

To multiply a fraction by a whole number, change the whole number to a fraction by placing it over a denominator of one.

(This does not change the value of the whole number.) Multiply the numerators then multiply the denominators to get the product.

$$\frac{1}{2} \text{ x } 1 = \frac{1}{2} \text{ x } \frac{1}{1} = \frac{1 \text{ x } 1}{2 \text{ x } 1} = \frac{1}{2}$$

$$\frac{2}{7} \text{ x } 3 = \frac{2}{7} \text{ x } \frac{3}{1} = \frac{2 \text{ x } 3}{7 \text{ x } 1} = \frac{6}{7}$$

$$\frac{8}{9} \text{ x } 6 = \frac{8}{9} \text{ x } \frac{6}{1} = \frac{8 \text{ x } 6}{9 \text{ x } 1} = \frac{48}{9} = 5\frac{3}{9} = 5\frac{1}{3}$$

Þ ***Change improper fractions to mixed numerals. Be sure the fraction part of the mixed numeral is written in the lowest possible terms.***

To multiply one fraction by another fraction, multiply the numerators. Their product will become the new numerator. Next, multiply the denominators. Their product will become the new denominator.

multiply the numerators

$$\frac{1}{2} \text{ x } \frac{1}{3} = \frac{1 \text{ x } 1}{2 \text{ x } 3} = \frac{1}{6}$$

multiply the denominators

$$\frac{7}{8} \text{ x } \frac{1}{3} = \frac{7}{24}$$

$$\frac{4}{3} \times \frac{1}{10} = \frac{4 \times 1}{3 \times 10} = \frac{4}{30} = \frac{2}{15}$$

To multiply mixed numerals by fractions, change the mixed numerals to improper fractions. Then multiply the fractions.

change the mixed numeral to an improper fraction

$$1\frac{6}{7} \times \frac{2}{3} = \frac{13}{7} \times \frac{2}{3} = \frac{13 \times 2}{7 \times 3} = \frac{26}{21} = 1\frac{5}{21}$$

$$2\frac{1}{8} \times 3\frac{1}{2} = \frac{17}{8} \times \frac{7}{2} = \frac{17 \times 7}{8 \times 2} = \frac{119}{16} = 7\frac{7}{16}$$

As you know, reducing a fraction means to divide the numerator and the denominator by the same number. You can use this principle to simplify before you work the problem. This process is called canceling.

Example Find 1/6 of 2/3.

Both the numerator of one fraction and the denominator of the other fraction can be divided by 2. Since 2 ¸ 2 = 1, draw a slash through the numerator 2 and write 1. Since 6 ¸ 2 = 3, draw a slash through the denominator 6 and write 3. Then multiply the simplified fractions.

$$\frac{1}{6} \times \frac{2}{3} = \frac{1 \times \overset{1}{\cancel{2}}}{\underset{3}{\cancel{6}} \times 3} = \frac{1}{9}$$

Since you used canceling before multiplying, there is no need to reduce the answer: 1/6 of 2/3 is **1/9.**

When you cancel, make sure you divide a numerator and a denominator by the same number. The canceling shown in the following example is **incorrect**.

$$\frac{1}{6} \times \frac{2}{3} = \frac{1 \times 2}{\underset{2}{\cancel{6}} \times \underset{1}{\cancel{3}}}$$

Although 6 and 3 can both be divided by 3, both numbers are in the denominator.

To multiply with mixed numbers, change the mixed numbers to improper fractions before you multiply.

Example Multiply 1 2/3 by 7 ½.

Step 1 Change to improper fractions.

$$1\frac{2}{3} \times 7\frac{1}{2} = \frac{5}{3} \times \frac{15}{2}$$

Step 2 Cancel and multiply.

$$\frac{5}{\underset{1}{\cancel{3}}} \times \frac{\overset{5}{\cancel{15}}}{2} =$$

Step 3 Write as a mixed number.

$$\frac{25}{2} = 12\frac{1}{2}$$

The product of 1 2/3 and 7 ½ is **12 ½.**

To divide a fraction by a whole number, change the whole number to an improper fraction with a denominator of one. Invert the divisor fraction. Then multiply the fractions.

$$\frac{1}{2} \div 2 = \frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\frac{2}{7} \div 3 = \frac{2}{7} \div \frac{3}{1} = \frac{2}{7} \times \frac{1}{3} = \frac{2}{21}$$

To divide a whole number by a fraction or to divide a fraction by another fraction, ***invert*** the divisor fraction. Then multiply the fractions.

$$\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times \frac{3}{1} = \frac{1 \times 3}{2 \times 1} = \frac{3}{2} = 1\frac{1}{2}$$

Invert the divisor fraction and multiply

$$7 \div \frac{6}{8} = \frac{7}{1} \times \frac{8}{6} = \frac{7 \times 8}{1 \times 6} = \frac{56}{6} = 9\frac{2}{6} = 9\frac{1}{3}$$

To divide a mixed numeral by another mixed numeral, first change the mixed numerals to improper fractions. Then invert the divisor fraction and multiply.

$$4\frac{1}{2} \div 2\frac{1}{3} = \frac{9}{2} \div \frac{7}{3} = \frac{9}{2} \times \frac{3}{7} = \frac{27}{14} = 1\frac{13}{14}$$

$$7\frac{6}{8} \div 6\frac{1}{3} = \frac{62}{8} \div \frac{19}{3} = \frac{62}{8} \times \frac{3}{19} = \frac{186}{152} = 1\frac{34}{152} = 1\frac{17}{76}$$

Turn it Upside Down: Inverting

Inverting a fraction means turning it upside down, or reversing the numerator and the denominator.

$\frac{1}{3}$ **inverted is** $\frac{3}{1}$ $\frac{6}{8}$ **inverted is** $\frac{8}{6}$

Inverting a whole number means to make it the denominator of a fraction with 1 as the numerator. 3 inverted is 1/3, 7 inverted is 1/7.

So, to solve he problem1/3 ÷ 3,

invert 3 **or** $\frac{3}{1}$ **to** $\frac{1}{3}$

then $\frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9}$

Practice Exercise

Solve for each of the given problems.
Write the answer in lowest terms.

1. $3\frac{1}{2} \times 1\frac{1}{2}$	2. $\frac{4}{6} \times 3\frac{2}{7}$
3. $2\frac{5}{8} \times \frac{5}{7}$	4. $\frac{1}{3} \times 3\frac{6}{7}$
5. $3\frac{2}{4} \times 2\frac{6}{9}$	6. $\frac{3}{4} \times 3\frac{3}{4}$
7. $13\frac{1}{8} \times 12\frac{6}{1}$	8. $10\frac{1}{5} \times 10\frac{1}{9}$
9. $11\frac{8}{9} \times 8\frac{5}{9}$	10. $4\frac{6}{10} \times 5\frac{1}{7}$
11. $7\frac{3}{6} \times 12\frac{2}{3}$	12. $6\frac{3}{11} \times 12\frac{6}{9}$
13. $7\frac{2}{3} \times 14\frac{1}{2}$	14. $4\frac{3}{10} \times 4\frac{1}{2}$

15. $11\frac{9}{13} \times 1\frac{8}{11}$	16. $12\frac{5}{6} \times 1\frac{4}{9}$
17. $12\frac{4}{11} \times 17\frac{4}{6}$	18. $5\frac{6}{14} \times 3\frac{1}{2}$

Solve for each of the given problems.
Write the answer in lowest terms.

1. $2\frac{1}{6} \div \frac{1}{3}$	2. $1\frac{2}{4} \div \frac{1}{3}$
3. $3\frac{2}{3} \div 3\frac{1}{4}$	4. $\frac{4}{6} \div \frac{6}{7}$
5. $\frac{1}{2} \div 2\frac{5}{7}$	6. $3\frac{7}{9} \div 3\frac{1}{7}$
7. $13\frac{1}{3} \div 10\frac{1}{10}$	8. $11\frac{8}{9} \div 13\frac{1}{8}$
9. $10\frac{3}{10} \div \frac{2}{3}$	10. $6\frac{3}{5} \div 4\frac{1}{5}$

11. $9\frac{4}{7} \div 7\frac{8}{9}$	**12.** $15\frac{1}{4} \div 7\frac{5}{8}$
13. $1\frac{1}{9} \div 3\frac{1}{3}$	**14.** $9\frac{9}{10} \div 4\frac{1}{5}$
15. $\frac{4}{8} \div 6\frac{6}{13}$	**16.** $13\frac{1}{7} \div 1\frac{4}{5}$
17. $7\frac{3}{8} \div \frac{1}{2}$	**18.** $3\frac{4}{11} \div 1\frac{2}{3}$

Decimals

The numerals we use today are called ***decimal*** numerals. These numerals stand for the numbers in the decimal system. The decimal system is also known as the Arabic system. The decimal system was first created by Hindu astronomers in India over a thousand years ago. It spread into Europe around 700 years ago.

The ***decimal system*** uses ten symbols: ***0, 1, 2, 3, 4, 5, 6, 7, 8,*** and ***9***. The word "decimal" comes from the Latin root ***decem***, meaning "ten."

Comparing Decimals

Comparing decimals uses an important mathematical concept. You can add zeros to the right of the last decimal digit without changing the value of the number. Study these examples.

RULE When comparing decimals with the same number of decimal places, compare them as though they were whole numbers.

Example Which is greater, 0.364 or 0.329?
Both numbers have three decimal places. Since 364 is greater than 329, the decimal **0.364 > 0.329.**

The rule for comparing whole numbers in which the number with more digits is greater does not hold true for decimals. The decimal number with more decimal places is not necessarily the greater number.

RULE When decimals have a different number of digits, write zeros to the right of the decimal with fewer digits so the numbers have the same number of decimal places. Then compare.

Example Which is greater, 0.518 or 0.52?
Add a zero to 0.52.
Since 520 > 518, the decimal **0.52 > 0.518.**

RULE When numbers have both whole number and decimal parts, compare the whole numbers first.

Example 1 Compare 32.001 and 31.999.

Since 32 is greater than 31, the number **32.001 is greater than 31.999.** It does not matter that 0.999 is greater than 0.001.

Using the same rules, you can put several numbers in order according to value. When you have several numbers to compare, write the numbers in a column and line up the decimal points. Then add zeros to the right until all the decimals have the same number of decimal digits.

Example 2 A digital scale displays weight to thousandths of a pound.
Three packages weigh 0.094, 0.91, and 0.1 of a pound. Arrange the weights in order from greatest to least.

Step 1	Write the weights in a column, aligning the decimal point.	0.094 0.91**0**
Step 2	Add zeros to fill out the columns.	0.1**00**
Step 3	Compare as you would whole numbers.	

In order from greatest to least, the weights are **0.91, 0.1,** and **0.094 of a pound.**

Equivalent Decimals

Decimals that name the same number or amount

Example:
0.5 = 0.50 = 0.500

Practice Exercise

Compare the given decimals.

1.	0.902	<	9.02
2.	1168.2	________	116.62
3.	0.950	________	0.85
4.	.910	________	0.091
5.	902842.032	________	902842.039
6.	4535.3	________	4531.6
7.	428871.038	________	428871.059
8.	3382.292	________	3328.292
9.	0.979	________	0.055
10.	647467.088	________	647467.068
11.	0.259	________	25.9
12.	854869.085	________	854869.04
13.	0.76	________	0.83
14.	0.464	________	46.4

15.	9.400	________	0.49
16.	0.96	________	0.02
17.	0.93	________	9.3
18.	0.936	________	9.36
19.	0.19	________	0.91

Decimals and Place Value

Decimal

A number that uses place value and a decimal point to show values less than one, such as tenths and hundredths

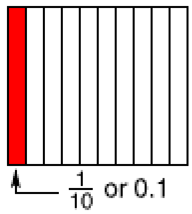
Example:
3.47

	hundreds	tens	ones	Decimal point	tenths	hundredths	thousandths
$10\frac{1}{10}$		1	0	.	1		
$205\frac{3}{100}$	2	0	5	.	0	3	
$4\frac{9}{1000}$			4	.	0	0	9

Tenth

One of ten equal parts

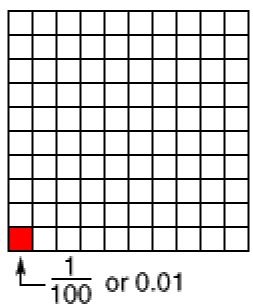
***Example*:**



Hundredth

One of one hundred equal parts

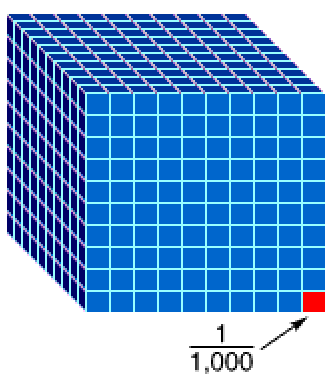
***Example*:**



Thousandth

One part of 1,000 equal parts

***Example*:**



How do you write 16.034 in words?

Read the whole number part of the number. Say *and* to represent the decimal point. Read the digits to the right of the decimal point, and say the place name of the last digit on the right. Note that there are no commas setting off groups of three digits in the decimal part of the number to the right of the decimal point.

The number 16.034 is read ***sixteen and thirty-four thousandths.***

Þ **Be careful!!! Although most Canadians and Americans recognize the "." as a decimal point, the decimal point is expressed as a comma in many countries. Most French Canadians use the comma to represent the decimal point.**

Decimal Fractions and Decimal Numbers

Decimal fractions or ***decimals*** are fractions with denominators of ***10***, ***100***, ***1,000***, ***10,000***, and so on.

Decimal fractions are written using a decimal point:

$$\frac{1}{10} = .1 \qquad \frac{1}{100} = .01 \qquad \frac{1}{1000} = .001$$

Changing a Fraction to a Decimal

Any fraction can be written as a decimal by dividing the numerator by the denominator, and adding a decimal point in the correct place.

$$\frac{1}{10} = 10\overline{)1.0}^{\;.1} \qquad \frac{3}{5} = 5\overline{)3.0}^{\;.6} \qquad \frac{1}{4} = 4\overline{)1.00}^{\;.25}$$

Þ ***In decimal notation, a decimal point distinguishes whole numbers from decimal fractions:***

$$1 = 1.0$$

$$\frac{1}{10} = 0.1$$

$$1\frac{1}{10} = 1.1$$

Changing Decimals to Fractions

Both decimals and fractions can be used to show part of a whole. Sometimes it is easier to calculate using fractions. At other times, decimals are more useful. If you know how to change from one form to the other, you can solve any problem using the form that is best for the situation.

Example Change 0.375 to a fraction.

Step 1 Write the number without the decimal point as the numerator of the fraction.

$$0.375 = \frac{375}{?}$$

Step 2 Write the place value for the last decimal digit as the denominator.

$$0.375 = \frac{375}{1000}$$

Step 3 Reduce the fraction to lowest terms.

$$\frac{375 \div 125}{1000 \div 125} = \frac{3}{8}$$

The decimal 0.375 is equal to the fraction **3/8.**

Practice Exercise

Write each fraction in decimal format.

1. $\frac{8}{10}$ = **0.8**

2. $\frac{1}{100}$ = ________

3. $\frac{7}{50}$ = ________

4. $\frac{5}{25}$ = ________

5. $\frac{65}{20}$ = ______

6. $\frac{3}{4}$ = ______

7. $\frac{12}{50}$ = ______

8. $\frac{13}{20}$ = ______

9. $\frac{12}{20}$ = ______

10. $\frac{12}{100}$ = ______

11. $\frac{106}{20}$ = ______

12. $\frac{35}{100}$ = ______

13. $\frac{27}{50}$ = ______

14. $\frac{15}{20}$ = ______

15. $\frac{9}{10}$ = ______

16. $\frac{235}{50}$ = ______

17. $\frac{1}{5}$ = ______

18. $\frac{238}{50}$ = ______

19. $\frac{204}{40}$ = ______

20. $\frac{5}{50}$ = ______

21. $\frac{2}{5}$ = ______

Write each decimal as a fraction in lowest terms.

1.	0.8	=	$\frac{4}{5}$	2.	0.12	=	________
3.	0.16	=	________	4.	0.35	=	________
5.	0.41	=	________	6.	0.84	=	________
7.	0.37	=	________	8.	0.04	=	________
9.	0.32	=	________	10.	0.81	=	________
11.	0.09	=	________	12.	0.75	=	________
13.	0.49	=	________	14.	0.55	=	________
15.	0.7	=	________	16.	0.58	=	________
17.	0.65	=	________	18.	0.76	=	________
19.	0.21	=	________	20.	0.95	=	________

Repeating Decimals

When ¼ is written as a decimal, the process is exact. The quotient is exactly .25 and the remainder is 0. This type of decimal is called a ***terminating decimal***.

Some fractions, when written as a division sentence, never reach a final digit. For example:

$$\frac{1}{3} = \quad 3\overline{)1.000}$$

```
      .333
3 ) 1.000
   - 9
     10
   -  9
      10
```

Since the pattern in the quotient repeats, we write 1/3 as $.\overline{3}$ or .3…to show that the pattern continues forever.

To convert a repeating decimal to a fraction, use the following example.

Example Change $0.\overline{3}$ to a fraction.
Note that $0.\overline{3}$ is a repeating decimal. (The bar indicates which number is repeated.) It can also be written 0.333333….
To change this type of decimal to a fraction, follow these steps:

Step 1 Let F represent the repeating decimal 0.333333….

So 10F (10 x F) represents the repeating decimal
3.333333….

Step 2 Subtract F from 10F.

10F = 3.333333…
F = 0.333333…
9F = 3

This means F = 3/9, which reduces to 1/3.
The decimal **0.3 = 1/3**

To convert repeating decimals that have two or three numbers that repeat, use the following examples.

Example Change .2424… to a fraction.

Step 1 Let F represent the repeating decimal .2424….
So 100F (100 x F) represents the repeating decimal
24.2424….

Step 2 Subtract F from 100F.

100F = 24.2424
F = 00.2424
99F = 24

This means F = 24/99, which reduces to 8/33.
The decimal **$.\overline{24}$ = 8/33**

Example Change .833… to a fraction.

Step 1 Let F represent the repeating decimal .833….
So 100F (100 x F) represents the repeating decimal 83.333… and 10F (10 x F) represents the repeating decimal 8.333….

Step 2 Subtract 10F from 100F.

$$\begin{array}{r} 100F = 83.333 \\ \underline{\;10F = \;\;8.333} \\ 90F = 75 \end{array}$$

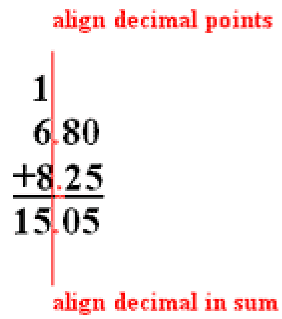
This means F = 75/90, which reduces to 5/6.
The decimal **.83 = 5/6**

Practice Exercise

1. Convert 0.083333… to a fraction.
2. Convert 0.444… to a fraction.
3. Convert 0.466666… to a fraction.
4. Convert 0.1083333… to a fraction.
5. Convert 0.9166666… to a fraction.
6. Convert 0.5454… to a fraction.
7. Convert 0.148148… to a fraction.
8. Convert 0.066666… to a fraction.

Adding decimals is easy.

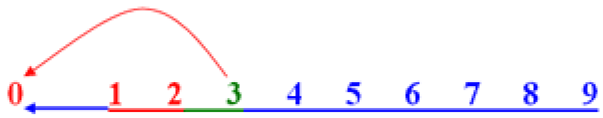
First, align the decimal points of the decimals. Then treat decimal fractions like whole numbers, aligning the decimal point in the sum. Adding decimals may look familiar---it's just like adding money.



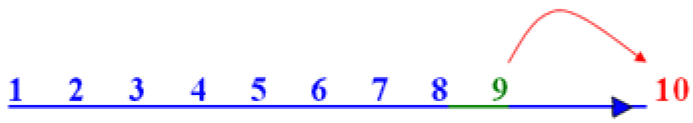
Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. You can estimate when you want to know if you have enough cash to pick up the three things you want at the grocery store or about how much each person should contribute to split the cost of lunch. In such cases, you can use amounts rounded to the nearest dollar (the ones place).

Rounding means to express a number to the nearest given place. The number in the given place is increased by one if the digit to its right is 5 or greater. The number in the given place remains the same if the digit to its right is less than 5. When rounding whole numbers, the digits to the right of the given place become zeros (digits to the left remain the same). When rounding decimal numbers, the digits to the right of the given place are dropped (digits to the left remain the same).

If you are rounding 3 to the nearest tens place, you would round down to 0, because 3 is closer to 0 than it is to 10.



If you were rounding 9, you would round up to 10.



General Rule for Rounding to the Nearest 10, 100, 1,000, and Higher!

Round down from numbers under 5 and round up from numbers 5 and greater.

The same holds true for multiples of 10. Round to the nearest 100 by rounding down from 49 or less and up from 50 or greater. Round to the nearest 1,000 by rounding down from 499 or less and up from 500 or greater.

Example Using the following price list, about how much would Pat pay for a steering wheel cover, a wide-angle mirror, and an oil drip pan?

Auto Parts Price List	
Outside Wide-Angle Mirror	$13.45
Steering Wheel Cover	$15.95
Oil Drip Pan	$ 8.73
Windshield Washer Fluid	$ 2.85
Brake Fluid	$ 6.35

Round the cost of each item to the nearest dollar and find the total of the estimates.

Item	**Cost**	**Estimate**
Steering wheel cover	$15.95	$16
Wide-angle mirror	13.45	13
Oil drip pan	+ 8.73	+ 9
Total:	$38.13	$38

The best estimate is **$38** which is close to the actual cost of **$38.13.**

The steps for rounding decimals are similar to those you use for rounding whole numbers. The most important difference is that once you have rounded off your number, you must ***drop the remaining digits.***

Example Round 5.362 to the nearest tenth.

Step 1 Find the digit you want to round to.

It may help to circle, underline, or highlight it.

5.362

Step 2 Look at the digit immediately to the right of the highlighted digit.

5.362

Step 3 If the digit to the right is 5 or more, add 1 to the highlighted digit. If the digit to the right is less than 5, do not change the highlighted digit. ***Drop the remaining digits.***

5.4

Examples Round 1.832 to the nearest hundredth.

1.832 rounds to 1.83

Round 16.95 to the nearest tenth.

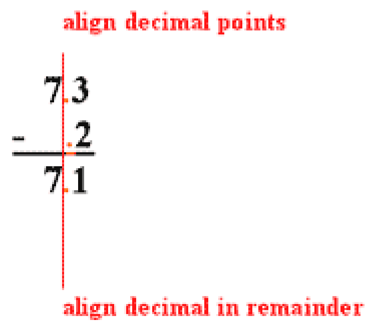
16.95 rounds to 17.0

Round 3.972 to the ones place.

3.972 rounds to 4

Subtracting decimals is easy.

First, align the decimal points of the decimals. Then treat decimal fractions like whole numbers, aligning the decimal point in the remainder. Subtracting decimals may look familiar---it's just like subtracting money.



To subtract decimals, if necessary, use place-holding zeros. Note: Whole numbers are understood to have a decimal point to the right of the ones place.

$$
12 - 4.08 = \quad \begin{array}{r} 12.00 \\ \underline{-4.08} \\ 7.92 \end{array}
$$

Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. In such cases, you can use amounts rounded to the nearest dollar (the ones place).

Example Susan has $213 in a checking account. If she writes a check for $32.60, <u>about</u> how much will be left in the account?

Round the amount of the check off to the nearest dollar and find the difference.

$213.00	$213
- $ 32.60	- $ 33
$180.40	$180

The best estimate is **$180** which is close to the actual amount of **$180.40.**

Practice Exercise

1. 1.297 + 1.534	**2.** 780.44 - 476.938
3. 9580.58 - 4227.53	**4.** 9.992 - 9.783
5. 8879.82 - 1675.13	**6.** 796.939 + 531.87
7. 552 + 320.583	**8.** 110.206 + 668.37
9. 7.47 + 43.41 =	**10.** 88.49 - 0.081 =
11. .861 + 0.081 =	**12.** 601.2 + 24.11 =
13. 0.378 - .103 =	**14.** 8 - 4.535 =

15. **9.5 + 83.17 =**	**16.** **56.89 - 6.86 =**
17. **0.9 + 44.82 =**	**18.** **656.1 + 16.75 =**

Round to the nearest dollar:

(1) \$28.54 (2) \$2.19 (3) \$15.03

(4) \$9.31 (5) \$23.11 (6) \$15.25

Round to tenths:

(7) 4.29 (8) 31.87 (9) 321.33

(10) 5.72 (11) 49.28 (12) 869.43

Round to hundredths:

(13) 0.483 (14) 8.881 (15) 31.737

(16) 0.157 (17) 2.09 (18) 64.257

Round to thousandths:

(19) 0.2006 (20) 0.9373 (21) 0.7708

(22) 0.0773 (23) 0.5167 (24) 5.7805

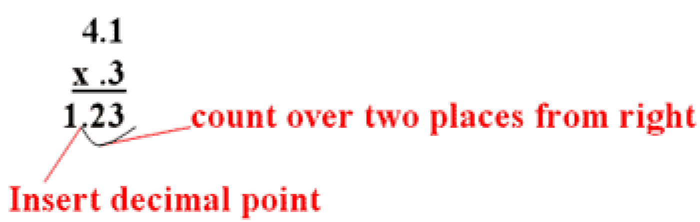
To multiply decimals, treat them as if they were whole numbers, at first ignoring the decimal point.

4.1
x .3
123

Next, count the number of places to the right of the decimal point in the multiplicand. Add this to the number of places to the right of the decimal point in the multiplier.

4.1 multiplicand ------------ one place
x .3 multiplier --------------- +one place
two places

Last, insert the decimal point in the product by counting over from the right the appropriate number of places.



Here are two other examples:

```
   8.9          65.003
 x 1.0          x .025
   0 0          325015
 8 9 0         1300060
 8. 9 0        1.625075
```

Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. In such cases, round each factor to its greatest place. Then multiply.

Example Richard earns $7.90 per hour and works 38.5 hours each week. How much are his total earnings per week?

Round each factor to its greatest place and multiply.

```
    38.5          40 hours
   $7.90          $8 per hour
    0000        $320 weekly wages, estimate
   3465
  2695
$304.150
```

The best estimate is **$320** which is close to the actual solution of **$304.15.**

Multiplying Decimals by 10, 100, and 1,000

There are shortcuts you can use when multiplying decimals by 10, 100, and 1,000.

To multiply a decimal by 10, move the decimal point **one place to the right.**

Example .26 x 10

.26 x 10 = 2.6 = 2.6

To multiply a decimal by 100, move the decimal point **two places to the right.**

Example 3.7 x 100

3.7 x 100 = 3 70 = 370

To multiply a decimal by 1,000, move the decimal point **three places to the right.**

Example 1.4 x 1,000

1.4 x 1,000 = 1 400 = 1,400

Begin dividing decimals the same way you would divide whole numbers.

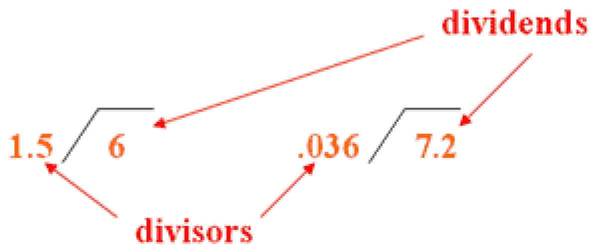
If the number in a division box (the dividend) has a decimal, but the number outside of the division box (the divisor) does not have a decimal, place the decimal point in the quotient (the answer) directly above the decimal point in the division box.

$$\begin{array}{r} 0.002 \\ 5\overline{)0.010} \end{array}$$

If both the numbers inside and outside of the division box have decimals, count how many places are needed to move the decimal point outside of the division box (the divisor) to make it a whole number. Move the decimal point in the number inside of the division box (the dividend) the same number of places. Place the decimal point in the quotient (the answer) directly above the new decimal point.

$$0.05\overline{)0.01} = 5\overline{)1} = \begin{array}{r} 0.2 \\ 5\overline{)1.0} \end{array}$$

If the number outside of the division box has a decimal, but the number inside of the division box does not, move the decimal place on the outside number however many places needed to make it a whole number. Then to the right of the number in the division box (a whole number with an "understood decimal" at the end) add as many zeros to match the number of places the decimal was moved on the outside number. Place the decimal point in the quotient directly above the new decimal place in the division box.



(Note that 6 = 6.0.)

Estimating can be a very useful skill. In many everyday situations involving money, for example, you do not need exact amounts. In such cases, round the divisor to its greatest place, and round the dividend so that it can be divided exactly by the rounded divisor. Then divide.

Example If a plane flew 2,419.2 miles in 6.3 hours, what was its average speed in miles per hour?

Round the divisor to its greatest place, round the dividend so that it can be divided exactly by the rounded divisor, and divide.

6.3	6 hours
2,419.2	2,400 miles

2,400 ÷ 6 = 400 miles per hour, estimate
2,419.2 ÷ 6.3 = 384 miles per hour

The best estimate is **400 miles per hour** which is close to the actual answer of **384 miles per hour.**

Dividing Decimals by 10, 100, and 1,000

There are shortcuts you can use when dividing decimals by 10, 100, and 1,000.

To divide a decimal by 10, move the decimal point **one place to the left.**

Example 7.2 ÷ 10

$$7.2 \div 10 = .7\,2 = .72$$

To divide a decimal by 100, move the decimal point **two places to the left.**

Example 364 ÷ 100

$$364 \div 100 = 3.64 = 3.64$$

To divide a decimal by 1,000, move the decimal point **three places to the left.**

Example 25.3 ÷ 1,000

$$25.3 \div 1{,}000 = .025\,3 = .0253$$

Practice Exercise

Solve each problem.

1. 0.9×3
2. 8.2×0.6
3. 0.2×5
4. 6.1×0.4
5. 5.8×0.49
6. 2.5×0.16
7. 4.9×0.33
8. 4.5×0.96
9. 1.7×0.25
10. 7.3×0.65
11. 5.6×0.5
12. 9.4×0.08
13. 19.44×20.887
14. 22.66×24
15. 49.33×27.154
16. 31.42×18.867
17. 26.05×16.98
18. 34.8×21.013
19. 14.58×40.286
20. 49.46×23.1

Solve each problem.

1. $0.74\overline{)5.032}$
2. $5.4\overline{)444.42}$
3. $5.49\overline{)9168.3}$
4. $9.6\overline{)478.08}$
5. $5.6\overline{)193.424}$
6. $0.81\overline{)20922.3}$

7. $8.9\overline{)5503.76}$	**8.** $1.14\overline{)55.119}$	**9.** $4.75\overline{)12.825}$
10. $3.2\overline{)24.128}$	**11.** $0.39\overline{)160.563}$	**12.** $1.3\overline{)4651.4}$
13. $7.8\overline{)4606.68}$	**14.** $7.1\overline{)112.748}$	**15.** $7.33\overline{)491.843}$
16. $0.2\overline{)9.132}$	**17.** $2.4\overline{)187.68}$	**18.** $0.81\overline{)51151.5}$
19. $6.88\overline{)13436.64}$	**20.** $3.4\overline{)12.274}$	**21.** $6.2\overline{)507.532}$
22. $0.63\overline{)208.215}$	**23.** $1.95\overline{)141.96}$	**24.** $5.3\overline{)335.49}$

Money

The word ***dollar*** comes from the German word for a large silver coin, the ***Thaler***. In 1781, ***cent*** was suggested as a name for the smallest division of the dollar. Thomas Jefferson, third President of the United States and an amateur scientist, thought that the dollar should be divided into 100 parts. The word ***cent*** comes from the Latin ***centum***, which means one hundred.

Canadian currency was first proposed in 1850, but the first coins were not released for circulation until December 12, 1858.

1 penny = 1 cent (¢)
1 nickel = 5 cents
1 dime = 10 cents
1 quarter = 25 cents
1 dollar ($) = 100 cents

Penny (Cent)

Nickel

Dime

Quarter

Dollar (Loonie)

Toonie

Canadian money is created in decimal-based currency. That means we can add, subtract, divide, and multiply money the same way we do any decimal numbers.

The basic unit of Canadian currency is the “loonie” or dollar. The dollar has the value of one on a place value chart. The decimal point separates dollars from cents, which are counted as tenths and hundredths in a place value chart.

$1.11	**ones = dollars**	**.**	**tenths = dimes**	**hundredths = pennies**
one cent				**1**
ten cents		**.**	**1**	**0**
one dollar	**1**	**.**	**0**	**0**

$4.63	**ones = dollars**	**.**	**tenths = dimes**	**hundredths = pennies**
three cents				**3**
sixty cents		**.**	**6**	**0**
four dollars	**4**	**.**	**0**	**0**

$1.11 = $1.00 + 10Ë + 1Ë is read as 1 dollar and 11 cents
$4.63 = $4.00 + 60Ë + 3Ë is read as 4 dollars and 63 cents

When you write down amounts of money using the dollar sign, *$*, you write the amounts the same way as you write decimal numbers—in decimal notation. There is a separate cents sign, ¢. The cents sign does not use decimal notation. So if you have to add cents to dollars, you have to change cents to dollar notation.

8Ë = $.08

36Ë = $.36

100Ë = $1.00

Practice Exercise

(1) Express 10 dollars, 1 nickel, and 14 dimes in dollars. ____________________

(2) Express 4 dollars, 14 pennies, and 7 dimes in dollars. ____________________

(3) Express 7 dollars, 4 dimes, 4 nickels, 11 quarters, and 9 pennies in dollars. ________________________

(4) Express 11 pennies, 3 nickels, and 9 quarters in dollars. ________________________

(5) Express 1 dollar, 14 pennies, 10 nickels, and 5 dimes in dollars. ________________________

(6) Express 13 dimes, 9 dollars, 15 pennies, 5 quarters, and 15 nickels in dollars. ________________________

(7) Express 10 pennies, 15 nickels, 9 dimes, 15 dollars, and 9 quarters in dollars. ________________________

(8) Express 5 dimes, 6 nickels, 7 pennies, 15 quarters, and 3 dollars in dollars. ________________________

(9) Express 4 nickels in dollars. ________________________

Percent

The term ***percent*** means ***parts per hundred***. Any fraction with a denominator of ***100*** can be written as a percentage, using a percent sign, ***%***. So, if you ate ½ of a pie, you ate **50/100** or ***.50*** or ***50%*** of the pie.

If you ate $\frac{1}{8}$ of the pie, you ate 12.5%

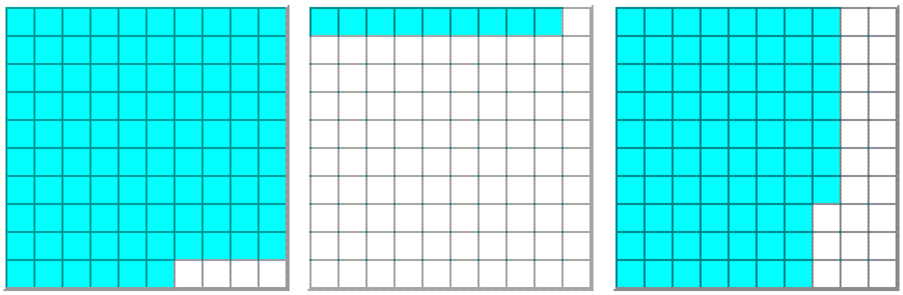
If you ate $\frac{3}{4}$ of the pie, you ate 75%

If you ate $\frac{1}{5}$ of the pie, you ate 20%

Þ **If you ate 1 whole pie, you ate 1.00 or 100% of the pie. You ate the whole thing!!!**

What fraction of each grid is shaded?

Grid 1 Grid 2 Grid 3



Each grid above has 100 boxes. For each grid, the ratio of the number of shaded boxes to the total number of boxes can be represented as a fraction.

Comparing Shaded Boxes to Total Boxes		
Grid	**Ratio**	**Fraction**
1	96 to 100	$\frac{96}{100}$
2	9 to 100	$\frac{9}{100}$
3	77 to 100	$\frac{77}{100}$

We can represent each of these fractions as a percent using the symbol %.

$\frac{96}{100}$ = 96% $\frac{9}{100}$ = 9% $\frac{77}{100}$ = 77%

Let's look at our comparison table again. This time the table includes percents.

Comparing Shaded Boxes to Total Boxes			
Grid	**Ratio**	**Fraction**	**Percent**
1	96 to 100	$\frac{96}{100}$	96%
2	9 to 100	$\frac{9}{100}$	9%
3	77 to 100	$\frac{77}{100}$	77%

It is easy to convert a fraction to a percent when its denominator is 100. If a fraction does not have a denominator of 100, you can convert it to an equivalent fraction with a denominator of 100, and then write the equivalent fraction as a percent.

Example 1: Write each fraction as a percent:
$\frac{1}{2}, \frac{18}{20}, \frac{4}{5}$

Solution		
Fraction	**Equivalent Fraction**	**Percent**
$\frac{1}{2}$	$\frac{1 \times 50}{2 \times 50} = \frac{50}{100}$	50%
$\frac{18}{20}$	$\frac{18 \times 5}{20 \times 5} = \frac{90}{100}$	90%
$\frac{4}{5}$	$\frac{4 \times 20}{5 \times 20} = \frac{80}{100}$	80%

You may also change a fraction to a percentage by dividing the fraction.

$$\frac{2}{5} = 5\overline{)2.00} = .40$$

Then change the decimal to a fraction with ***100*** in the denominator.

$$.40 = \frac{40}{100} = 40\%$$

To change a mixed number to a percent, change the mixed number to an improper fraction and multiply by 100.

Example Change 3 ¼ to a percent.

Step 1 Change the mixed number to an improper fraction.

$$3\ \tfrac{1}{4} = \frac{13}{4}$$

Step 2 Multiply by 100 and add the percent sign.

$$\frac{13}{4} \times 100$$

$$\frac{13}{\cancelto{1}{4}} \times \frac{\cancelto{25}{100}}{1} = 325\%$$

To change a percentage to a fraction or mixed number, write the percent as a fraction with a denominator of 100. Be sure to write the fraction in its lowest possible terms.

$$4\% = \frac{4}{100} = \frac{1}{25} \qquad 13\% = \frac{13}{100}$$

$$150\% = \frac{150}{100} = \frac{3}{2} = 1\ \tfrac{1}{2}$$

Converting percents with fraction parts requires extra steps.

Example Change 41   % to a fraction.
Write 41   over 100 and divide.

$$\frac{41\frac{2}{3}}{100} = 41\tfrac{2}{3} \div 100 = \frac{\overset{5}{\cancel{125}}}{3} \times \frac{1}{\underset{4}{\cancel{100}}} = \frac{5}{12}$$

The percent 41 2/3% is equal to the fraction 5/12.

Writing Decimals as Percents

Problem: What percent of a dollar is 76 cents?

76 cents = .76

.76 = 76%

Solution: 76 cents is 76% of a dollar.

The solution to the above problem should not be surprising, since both dollars and percents are based on the number 100. As a result, there is nothing complicated about converting a decimal to a percent. **To convert a decimal to a percent, move the decimal point two places to the right.** Look at the example on the next page:

Example 1 Write each decimal as a percent:
.93, .08, .67, .41

Solution	
Decimal	**Percent**
.93	93%
.08	8%
.67	67%
.41	41%

Each of the decimals in Example 1 has two places to the right of the decimal point. However, a decimal can have any number of places to the right of the decimal point. Look at Example 2 and Example 3:

Example 2 Write each decimal as a percent:
.786, .002, .059, .8719

Solution	
Decimal	**Percent**
.786	78.6%
.002	.2%
0.59	5.9%
.8719	87.19%

Example 3 Write each decimal as a percent:
.1958, .007, .05623, .071362

Solution	
Decimal	**Percent**
.1958	19.58%
.007	.7%
.05623	5.623%
.071362	7.1362%

Writing Percents as Decimals

Problem: What is 35 percent of one dollar?

We know from the previous lesson that .35 = 35%. The word "of" means multiply. So we get the following:

35% x $1.00 = .35 x $1.00

.35 x $1.00 = .35 x 1 = .35

Solution: 35% of one dollar is $.35, or 35 cents.

The solution to the problem on page 86 should not be surprising, since percents, dollars and cents are all based on the number 100. **To convert a percent to a decimal, move the decimal point two places to the left.** Look at the example below:

Example 1 Write each percent as a decimal:
18%, 7%, 82%, 55%

Solution	
Percent	**Decimal**
18%	.18
7%	.07
82%	.82
55%	.55

In Example 1, note that for 7%, we needed to add in a zero. **To write a percent as a decimal, follow these steps:**

- Drop the percent symbol.
- Move the decimal point two places to the left, adding in zeros as needed.

Why do we move the decimal point 2 places to the left? Remember that percent means parts per hundred, so 18% equals $\frac{18}{100}$. From your knowledge of decimal place value, you know that $\frac{18}{100}$ equals eighteen hundredths (.18). So 18% must also equal eighteen hundredths (.18). In Example 2 below, we take another look at Example 1, this time including the fractional equivalents.

Example 2 Write each percent as a decimal:
18%, 7%, 82%, 55%

Solution		
Percent	**Fraction**	**Decimal**
18%	$\frac{18}{100}$	.18
7%	$\frac{7}{100}$	.07
82%	$\frac{82}{100}$	.82
55%	$\frac{55}{100}$	.55

Let's look at some more examples of writing percents as decimals.

Example 3 Write each percent as a decimal:
12.5%, 89.19%, 39.2%, 71.935%

Solution	
Percent	**Decimal**
12.5%	.125
89.19%	.8919
39.2%	.392
71.935%	.71935

Þ **To remember which way to move the decimal point when changing from a decimal to a percent or vice versa, think of your alphabet. Think of the decimal as "d" and the percent as "p". To change from a decimal to a percent, move two places up your alphabet. Move two places down your alphabet to go from a percent to a decimal.**

Converting percents with fraction parts to decimals requires extra steps.

Example Change 15 ¼% to a decimal.
Change ¼ to a decimal.

$$\frac{1 \times 25}{4 \times 25} = \frac{25}{100}$$

$$\frac{25}{100} = .25$$

Combine the decimal with the original whole umber part of the percent and then convert the percent to a decimal by moving the decimal point two places to the left.

15 ¼% = 15.25% = .1525

The percent 15 ¼% is equal to the decimal **.1525.**

Practice Exercise

Write each fraction as a percent.

1. $\frac{2}{100}$ = **2%**	2. $\frac{1}{10}$ = ____	3. $\frac{47}{100}$ = ____
4. $\frac{15}{25}$ = ____	5. $\frac{32}{100}$ = ____	6. $\frac{3}{4}$ = ____
7. $\frac{17}{50}$ = ____	8. $\frac{63}{90}$ = ____	9. $\frac{13}{20}$ = ____
10. $\frac{4}{5}$ = ____	11. $\frac{6}{60}$ = ____	12. $\frac{34}{50}$ = ____
13. $\frac{7}{20}$ = ____	14. $\frac{24}{75}$ = ____	15. $\frac{4}{20}$ = ____
16. $\frac{9}{30}$ = ____	17. $\frac{16}{40}$ = ____	18. $\frac{14}{50}$ = ____

19. $\frac{8}{50}$ = _____

20. $\frac{39}{75}$ = _____

21. $\frac{86}{100}$ = _____

22. Write each decimal as a percent

a) 0.26 ____________________
b) 0.94 ____________________
c) 0.71 ____________________
d) 0.35 ____________________

e) 0.01 ____________________
f) 0.13 ____________________
g) 0.36 ____________________
h) 0.07 ____________________

23. Write each percent as a fraction

a) 16% ____________________
b) 54% ____________________
c) 59% ____________________
d) 45% ____________________

e) 58% ____________________
f) 86% ____________________
g) 47% ____________________
h) 8% ____________________

24. Write each percent as a decimal

a) 95% ____________________

b) 59% ____________________

c) 24% ____________________

d) 35% ____________________

e) 22% ____________________

f) 55% ____________________

g) 73% ____________________

h) 38% ____________________

Adding, subtracting, multiplying, and dividing percents follows the same procedure as whole numbers.

Examples 10% + 20% = 30%

40% - 30% = 10%

50% x 60% = 3000%

80% ¸ 20% = 4%

Þ **You should know that, although it is possible to add and subtract percents, an initial discount of 15%, for example, followed by an additional discount of 25% does NOT create a 40% reduction in price but rather a discount of 36.25%.**

Example June was about to purchase a $250 coat at a 15% discount when the cashier told her that the store was having a special promotion. All June had to do was stick her hand into a box and pull out a circular disc. Whatever number was on the disc would be discounted from the price of the coat that she was going to buy. June put her hand in and pulled out a circle with "25%" marked on it. How much did June end up paying for the coat?

Step 1 Calculate the original amount to be discounted off the original price.

$250 x 15% =
$250 x .15 = $37.50

Step 2 Subtract the original amount to be discounted from the original price.

$250.00
- $ 37.50
$212.50

Step 3 Take the new discounted price and calculate how much more is to be taken off the discounted price of the coat.

$212.50 x 25% =
$212.50 x .25 = $53.125 = $53.13 (rounded to the nearest cent)

Step 4 Subtract the new amount from the discounted price.

$$\begin{array}{r} \$212.50 \\ - \ \$\ \ 53.13 \\ \hline \$159.37 \end{array}$$

June may now purchase the $250 coat for **$159.37.**

If the original discount had been 40% (15% + 25%), we could have multiplied $250 by .40 and the price would have been brought down by $100 to $150. At least, June can feel fortunate that she received her coat at a discount.

Answer Key

Book 14019 – Fractions, Decimals and Percent

Page 5

1. 2 **2.** 2 **3.** 6 ½ **4.** 8 5/8 **5.** 7 ½
6. 11 ¾ **7.** 9 6/7 **8.** 8 8/11 **9.** 12 1/3
10. 11 2/7 **11.** 2 ½ **12.** 7 **13.** 7 3/8
14. 3 ½ **15.** 9 1/3 **16.** 7 ½ **17.** 4 ¼
18. 6 1/5 **19.** 10 ½ **20.** 7 1/11 **21.** 6
22. 8 4/5 **23.** 5 5/7 **24.** 10 2/5 **25.** 4 ¼
26. 12 2/3 **27.** 12 2/3 **28.** 12 5/7
29. 7 2/11 **30.** 4 4/9 **31.** 8 ½ **32.** 5 4/5

Page 12

2. < **3.** < **4.** > **5.** > **6.** < **7.** >
8. > **9.** < **10.** < **11.** > **12.** < **13.** <
14. < **15.** > **16.** > **17.** < **18.** <

Page 13

1. 1/8 **2.** 4/9 **3.** 1/3 **4.** 3/5 **5.** 1/11
6. 4/5 **7.** 1/7 **8.** ¼ **9.** 33/40 **10.** ¾
11. 1/3 **12.** 25/31 **13.** 1 13/20 **14.** ½
15. 1/3 **16.** 53/63 **17.** 6/17 **18.** ½
19. 25/29 **20.** 3/7 **21.** 13/22 **22.** 15/19
23. 1/9 **24.** 1/3

Page 20

1. 8 2/15 **2.** 6 15/22 **3.** 17/28 **4.** 4
5. 7 46/55 **6.** 10 17/40 **7.** 1 13/44
8. 3 ¼ **9.** 23/60 **10.** 11 8/63
11. 13 4/15 **12.** 1 7/10 **13.** 11 7/15
14. 12 1/16 **15.** 11 5/12 **16.** 5 3/5

Page 27 **1.** 5 ¼ **2.** 2 4/21 **3.** 1 7/8 **4.** 1 2/7
5. 9 1/3 **6.** 2 13/16 **7.** 165 3/8
8. 103 2/15 **9.** 101 58/81 **10.** 23 23/35
11. 95 **12.** 79 5/11 **13.** 111 1/6
14. 19 7/20 **15.** 20 28/143 **16.** 18 29/54
17. 218 14/33 **18.** 19

Page 28 **1.** 6 ½ **2.** 4 ½ **3.** 1 5/39 **4.** 7/9
5. 7/38 **6.** 1 20/99 **7.** 1 97/303
8. 856/945 **9.** 15 9/20 **10.** 1 4/7
11. 1 106/497 **12.** 2 **13.** 1/3 **14.** 2 5/14
15. 13/168 **16.** 7 19/63 **17.** 14 ¾
18. 2 1/55

Page 32 **2.** > **3.** > **4.** > **5.** < **6.** > **7.** <
8. > **9.** > **10.** > **11.** < **12.** > **13.** <
14. < **15.** > **16.** > **17.** < **18.** <
19. <

Page 37 **2.** .01 **3.** .14 **4.** .2 **5.** 3.25 **6.** .75
7. .24 **8.** .65 **9.** .6 **10.** .12 **11.** 5.3
12. .35 **13.** .54 **14.** .75 **15.** .9 **16.** 4.7
17. .2 **18.** 4.76 **19.** 5.1 **20.** .1 **21.** .4

Page 39 **2.** 3/25 **3.** 4/25 **4.** 7/20 **5.** 41/100
6. 21/25 **7.** 37/100 **8.** 1/25 **9.** 8/25
10. 81/100 **11.** 9/100 **12.** ¾ **13.** 49/100
14. 11/20 **15.** 7/10 **16.** 29/50 **17.** 13/20
18. 19/25 **19.** 21/100 **20.** 19/20

Page 42 **1.** 1/12 **2.** 1/60 **3.** 7/15 **4.** 13/120

5. 11/12 **6.** 1/6 **7.** 4/75 **8.** 1/15

Page 48 **1.** 2.831 **2.** 303.502 **3.** 5353.05 **4.** .209
5. 7204.69 **6.** 1328.809 **7.** 872.583
8. 778.576 **9.** 50.88 **10.** 88.409
11. .942 **12.** 625.31 **13.** .275 **14.** 3.465
15. 92.67 **16.** 50.03 **17.** 45.72
18. 672.85

Page 49 **1.** $29 **2.** $2 **3.** $15 **4.** $9 **5.** $23
6. $15 **7.** 4.3 **8.** 31.9 **9.** 321.3
10. 5.7 **11.** 49.3 **12.** 869.4 **13.** .48
14. 8.88 **15.** 31.74 **16.** .16 **17.** 2.09
18. 64.26 **19.** .201 **20.** .937 **21.** .771
22. .077 **23.** .517 **24.** 5.781

Page 56 **1.** 2.7 **2.** 4.92 **3.** 1 **4.** 2.44 **5.** 2.842
6. .4 **7.** 1.617 **8.** 4.32 **9.** .425
10. 4.745 **11.** 2.8 **12.** .752
13. 406.04328 **14.** 543.84 **15.** 1339.50682
16. 592.80114 **17.** 442.329 **18.** 731.2524
19. 587.36988 **20.** 1142.526

Page 56 **1.** 6.8 **2.** 82.3 **3.** 1670 **4.** 49.8
5. 34.54 **6.** 25830 **7.** 618.4 **8.** 48.35
9. 2.7 **10.** 7.54 **11.** 411.7 **12.** 3578
13. 590.6 **14.** 15.88 **15.** 67.1 **16.** 45.66
17. 78.2 **18.** 63150 **19.** 1953 **20.** 3.61
21. 81.86 **22.** 330.5 **23.** 72.8 **24.** 63.3

Page 60 **1.** $11.45 **2.** $4.84 **3.** $10.44 **4.** $2.51

5. $2.14 **6.** $12.45 **7.** $19.00 **8.** $7.62
9. $.20

Page 73 **2.** 10% **3.** 47% **4.** 60% **5.** 32%
6. 75% **7.** 34% **8.** 70% **9.** 65%
10. 80% **11.** 10% **12.** 68% **13.** 35%
14. 32% **15.** 20% **16.** 30% **17.** 40%
18. 28% **19.** 16% **20.** 52% **21.** 86%
22. **a.** 26% **b.** 94% **c.** 71% **d.** 35%
e. 1% **f.** 13% **g.** 36% **h.** 7%
23. **a.** 4/25 **b.** 27/50 **c.** 59/100 **d.** 9/20
e. 29/50 **f.** 43/50 **g.** 47/100 **h.** 2/25
24. **a.** .95 **b.** .59 **c.** .24 **d.** .35 **e.** .22
f. .55 **g.** .73 **h.** .38