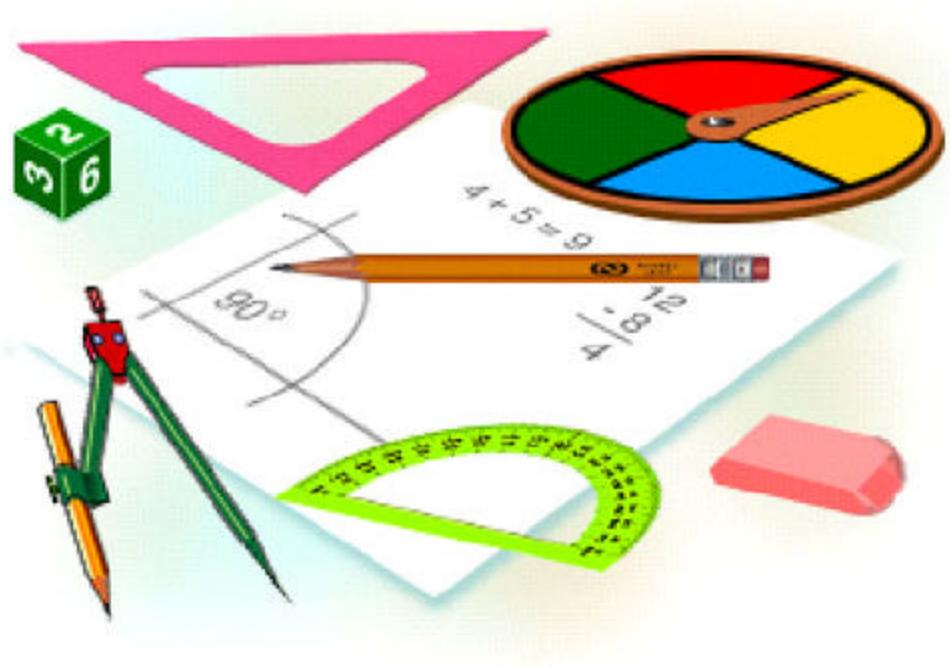
The Next Step

Mathematics Applications for Adults



Book 14019 – Ratio, Proportion and Percent

OUTLINE

Mathematics - Book 14019

Ratio, Proportion and Percent
Introduction to Ratio, Proportion, and Percent
find the percentage that one number is of another number.
find the number when a percentage is given.
percent of a given number.
use the formula $r/100 = P/W$ and cross-multiplication.
determine which ratio in a given list is equal to given ratio.
determine which of a given list of compared ratios are proportions and which are false statements.

THE NEXT STEP

Book 14019

Ratio, Proportion and Percent

Introduction to Ratio, Proportion, and Percent

To find a percentage of a number, multiply the number by the percentage written in its decimal fraction form. Find 25% of 12.

.25 x 12 = 3

To find what percentage one number is of another, write the numbers as a fraction. Divide the fraction into its decimal form. Then change the decimal into its percentage form. ***12*** is what percent of ***48***?

$\frac{12}{48}$ or $48\overline{)12.00}$ **.25 = 25%**

To find a number when a percentage of it is known, try this:

Nine is ***25%*** of what number?

$$\frac{25}{100} = \frac{9}{?}$$

$$25 \times ? = 100 \times 9$$
$$25 \times ? = 900$$
$$? = 900 \div 25$$
$$? = 36$$

Nine is ***25%*** of 36.

Some people like to use a formula to find the percent of a number, what percent one number is of another, or a number when a percent is given. The formula looks like this:

$$\frac{r}{100} = \frac{P}{W}$$

***r* = percent rate**
***P* = part of the number**
***W* = the whole (entire) number**

So, to solve the problem, nine is ***25%*** of what number, we would follow these steps.

Step 1 Write down the formula.

$$\frac{r}{100} = \frac{P}{W}$$

Step 2 Insert the necessary information in the correct places.

$$\frac{25}{100} = \frac{9}{?}$$

Step 3 Cross multiply.

$$25 \times ? = 9 \times 100$$
$$25 \times ? = 900$$

Step 4 Divide and solve.

$$? = 900 \div 25$$
$$? = 36$$

Therefore, nine is ***25%*** of 36.

(1) What % of 3100 is 2511?
(2) What is 13 ½ % of 39?
(3) What is 3 ½ % of 90?
(4) What % of 27000 is 202.5?
(5) 170.4 is 0.71% of what number?
(6) What is 40% of 4000?
(7) What is 0.41% of 20000?
(8) 1292 is 68% of what number?
(9) What is 0.68% of 9000?
(10) What is 51% of 3400?
(11) What % of 34000 is 170?
(12) What % of 3800 is 2698?
(13) 924 is 77% of what number?
(14) What is 8 ½ % of 97?

(15) What is 7 ½ % of 64?
(16) 98.4 is 0.82% of what number?
(17) What is 3 ½ % of 75?
(18) What is 3% of 3900?
(19) What % of 47000 is 432.4?
(20) What is 0.06% of 12000?
(21) 500 is 20% of what number?
(22) 61.6 is 0.28% of what number?
(23) What is 19% of 29?
(24) What % of 700 is 350?
(25) What % of 1700 is 306?
(26) 70 is 0.25% of what number?
(27) 2310 is 77% of what number?
(28) What is 68% of 1300?

Percent increase and decrease

Percent increase and decrease of a value measure how that value changes, as a percentage of its original value.

Example: A collectors' comic book is worth $120 in 1994, and in 1995 its value is $132. The change is $132 - $120 = $12, an increase in price of $12; since $12 is 10% of $120, we say its value increased by 10% from 1994 to 1995.

Example: A bakery makes a chocolate cake that has 8 grams of fat per slice. A new change in the recipe lowers the fat to 6 grams of fat per slice. The change is 8g - 6g = 2g, a decrease of 2 grams; since 2 grams is 25% of 8, we say that the new cake recipe has 25% less fat, or a 25% decrease in fat.

Example: Amy is training for the 1500 meter run. When she started training she could run 1500 meters in 5 minutes and 50 seconds. After a year of practice her time decreased by 8%. How fast can she run the race now? Her old time was $5 \times 60 + 50 = 350$ seconds, and 8% of 350 is 28, so she can run the race in 350 - 28 = 322 seconds (5 minutes and 22 seconds).

Example: A fishing magazine sells 110000 copies each month. The company's president wants to increase the sales by 6%. How many extra magazines would they have to sell to reach this goal? This problem is easy, since it only asks for the change in sales: 6% of 110000 equals 6600 more magazines.

Practice Exercise

Solve the problems below.

1. By changing jobs, Dave increased his monthly salary by 7%. What does Dave earn now if his previous job paid him $1230 per month?

2. To lose weight, Greg has been advised to cut his daily calorie intake by 30%. Before starting this diet, Greg consumed about 2700 calories each day. While on the diet, what target level (to the nearest 100 calories per day) should Greg try to attain?

3. Central Hardware places a 30% markup on each item it sells. If Central pays $13.50 for StrongArm Hammers, what price will Central charge its own customers for these hammers?

4. During the July 4th sale, Valley Appliances is marking down its appliances by 20%. What will be the sale price of a washer that normally sells for $289?

Ratios describe the size of things in comparison to each other. Ratios are sometimes written in the form of fractions. More often, the symbol : is used to separate the numerator and the denominator.

For example, if you ate ***2*** parts of a pie that had been cut into ***5*** parts, the ratio of pieces of pie you ate to the

uneaten pieces of pie is ***2 to 3***. The ratio may be written as ***2:3*** or $\frac{2}{3}$.

Writing the ratio in words will help you keep the numbers in the correct order. The words will also help you remember the meaning of the numbers. Including labels in your final ratio is also helpful.

Example If Barbara earns $180 in 15 hours, how much does she earn per hour?

Write the ratio of earnings to hours. Then divide to simplify.

$$\frac{\text{dollars earned}}{\text{hours}} = \frac{\$180}{15} = \frac{180 \div 15}{15 \div 15} = \frac{\$12}{1 \text{ hr}}$$

Barbara earns $12 for every 1 hour she works. In other words, she earns **$12 per hour.**

Reducing Ratios

Reducing a ratio means finding an equal, simplified version of the original. The ratio is reduced to lowest terms when there is no number other than 1 that will divide evenly into both of the numbers that make up the ratio.

To simplify a ratio, divide both of the numbers that make up the ratio by the same number, and write the new ratio.

Example In one hour, 10 customers visited Stuart's newsstand. Of those, 6 bought a magazine. What is the ratio of those who bought a magazine to those that did not?

Write a ratio: bought magazines:didn't buy magazines

6:4

Simplify the ratio by dividing both of the numbers in the ratio by 2.

6:4 = 3:2

You may also represent the ratios as fractions when simplifying.

$$\frac{6}{4} = \frac{6 \div 2}{4 \div 2} = \frac{3}{2}$$

$$\frac{3}{2} = 3:2$$

When using ratios, it is often necessary to change from one unit of measure to another.

Example The ratio of 59 minutes to 4 hours.

Step 1 Use the appropriate conversion factor.

1 hr = 60 min
4 hr = 60 x 4
4 hr = 240 min

Step 2 Substitute the converted units into the original ratio.

The ratio of 59 minutes to 4 hours
becomes
the ratio of 59 minutes to 240 minutes
or
59:240

Step 3 Reduce if necessary.

Answer: The ratio 59:240 cannot be reduced; therefore your final answer is **59:240**

If decimals are being used in a ratio, convert them to whole numbers and then reduce if necessary.

Example The ratio of .3 to 21

Step 1 Multiply both of the numbers in the ratio by the same number so that the decimal can be converted to a whole number.

.3 to 21
10 x .3 to 21 x 10
3 to 210
3:210

Step 2 Reduce if necessary.

3:210
3 ÷ 3:210 ÷ 3
1:70

Answer: The ratio of .3 to 21 when reduced is **1:70**

1. The ratio of 16 to 35 **16:35 or 16 to 35**	2. The ratio of 22 to 21 _____
3. The ratio of 19 to 6 _____	4. The ratio of 9 to 35 _____
5. The ratio of 15 to 20 _____	6. The ratio of 17 mins to 85 mins _____
7. The ratio of 128 secs to 32 secs _____	8. The ratio of 66 yr to 22 yr _____

9. The ratio of 39 yr to 156 yr _____	10. The ratio of 25 secs to 25 secs _____
11. The ratio of 8 secs to 16 secs _____	12. The ratio of 23 mins to 23 mins _____
13. The ratio of 8 mins to 6 hours _____	14. The ratio of 2 hours to 1 day _____
15. The ratio of 0.5 to 6 _____	16. The ratio of 0.38 to 44% _____
17. The ratio of 28% to 85% _____	18. The ratio of 0.41 to 0.2 _____
19. The ratio of 3 oz to 30 oz _____	20. The ratio of 9 dimes to 8 dimes _____

Calculating Equal Ratios

If one cherry pie is baked for every 4 apple pies, the ratio is 1:4, or ¼.

If the number of apple pies is increased to 12, how many cherry pies are needed to keep the same ratio?

To find the solution, write the ratios as an equation.

$$\frac{1}{4} = \frac{?}{12}$$

To solve, multiply (or divide) each term of the first ratio by the same number to make a true statement.

$$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$

1:4 and 3:12 are equal ratios.

You can also find the missing term by cross-multiplying and then dividing.

$$\frac{1}{4} \times \frac{?}{12}$$

$$1 \times 12 = 4 \times ?$$

$$\frac{12}{4} = \frac{4}{4} \times ?$$

$$3 = ?$$

When ratios are equal or equivalent, they are said to be **proportional** and can be referred to as **true proportions**. When ratios are not equal, they are said to be non-equivalent or disproportionate.

The **terms** of a proportion are:

$$\textbf{first} \rightarrow \frac{a}{b} = \frac{c}{d} \leftarrow \textbf{third}$$
$$\textbf{second} \rightarrow b \quad d \leftarrow \textbf{fourth}$$

The **extremes** are the first and fourth terms of a proportion.
The **means** are the second and third terms of a proportion.

Proportions are often written using a fraction bar that stands for "is compared to". For example, 2/3 = 6/9 means 2 compared to 3 is the same as 6 compared to 9.

A **direct proportion** is indicated when two quantities are so related that an increase in one causes a corresponding increase in the other or when a decrease in one causes a corresponding decrease in the other.

The following is a list of directly proportional relationships.

a. The faster the speed, the greater the distance covered.
b. The more people working, the greater amount of work done.
c. The slower the speed, the lower the number of revolutions.
d. The shorter the object, the shorter the shadow.

Practice Exercise

In each of the following proportions solve for the unknown value.

1. $\frac{3}{36} = \frac{?}{12}$
2. $\frac{?}{36} = \frac{7}{6}$
3. $\frac{?}{28} = \frac{10}{7}$
4. $\frac{12}{?} = \frac{36}{27}$
5. $\frac{60}{?} = \frac{10}{8}$
6. $\frac{33}{?} = \frac{11}{5}$
7. $\frac{20}{30} = \frac{?}{39}$
8. $\frac{98}{35} = \frac{?}{10}$
9. $\frac{?}{8} = \frac{91}{28}$
10. $\frac{?}{130} = \frac{42}{28}$
11. $\frac{?}{37} = \frac{108}{148}$
12. $\frac{36}{38} = \frac{90}{?}$
13. $\frac{52}{88} = \frac{?}{44}$
14. $\frac{?}{54} = \frac{40}{15}$
15. $\frac{36}{?} = \frac{99}{11}$
16. $\frac{?}{13.7} = \frac{130.4}{54.8}$
17. $\frac{75.6}{80.6} = \frac{37.8}{?}$
18. $\frac{48}{4.3} = \frac{96}{?}$
19. $\frac{60}{63} = \frac{40}{?}$
20. $\frac{10}{2} = \frac{?}{42}$
21. $\frac{18}{54} = \frac{?}{60}$

Mark True or False for Each of the Following

1. 2:3 = 10:15 T F
2. 14:9 = 7:11 T F
3. 9:20 = 12:40 T F
4. 28:72 = 70:180 T F
5. 16.8:12 = 14:10 T F

Understanding Probability

What is an event?

An event is an experiment or collection of experiments.

Examples:

The following are examples of events.

1) A coin toss.
2) Rolling a die.
3) Rolling 5 dice.
4) Drawing a card from a deck of cards.
5) Drawing 3 cards from a deck.
6) Drawing a marble from a bag of different colored marbles.
7) Spinning a spinner in a board game.
8) Tossing a coin and rolling a die.

Possible Outcomes of an Event

Possible outcomes of an event are the results that may occur from any event. (Remember, they may not occur.)

Examples:

The following are possible outcomes of events.

1) A coin toss has two possible outcomes. The outcomes are "heads" and "tails".
2) Rolling a regular six-sided die has six possible outcomes. You may get a side with 1, 2, 3, 4, 5, or 6 dots.
3) Drawing a card from a regular deck of 52 playing cards has 52 possible outcomes. Each of the 52 playing cards is different, so there are 52 possible outcomes for drawing a card.
4) How many different outcomes are there for the color of marble that may be drawn from a bag containing 3 red, 4 green, and 5 blue marbles? This event has 3 possible outcomes. You may get a red marble, a green marble, or a blue marble. Even if the marbles are different sizes, the outcome we are considering is the *color* of the marble that is drawn.
5) How many different outcomes are there for the colors of two marbles that may be drawn from a bag containing 3 red, 4 green, and 5 blue marble? This event has 6 possible outcomes: you may get two reds, two greens, two blues, a red and blue, a red and green, or a blue and green.
6) Rolling two regular dice, one of them red and one of them blue, has 36 possible outcomes. The outcomes are listed in the table below.

		Red Die					
	Result:	Red1	Red2	Red3	Red4	Red5	Red6
Blue Die	Blue1	Blue1, Red1	Blue1, Red2	Blue1, Red3	Blue1, Red4	Blue1, Red5	Blue1, Red6
	Blue2	Blue2, Red1	Blue2, Red2	Blue2, Red3	Blue2, Red4	Blue2, Red5	Blue2, Red6
	Blue3	Blue3, Red1	Blue3, Red2	Blue3, Red3	Blue3, Red4	Blue3, Red5	Blue3, Red6
	Blue4	Blue4, Red1	Blue4, Red2	Blue4, Red3	Blue4, Red4	Blue4, Red5	Blue4, Red6
	Blue5	Blue5, Red1	Blue5, Red2	Blue5, Red3	Blue5, Red4	Blue5, Red5	Blue5, Red6
	Blue6	Blue6, Red1	Blue6, Red2	Blue6, Red3	Blue6, Red4	Blue6, Red5	Blue6, Red6

Note that the event tells us how to think of the outcomes. Even though there are 12 different marbles in example 4, the event tells us to count only the color of the die, so there are three outcomes. In example 6, the two dice are different, and there are 36 possible outcomes. Suppose we don't care about the color of the dice in example 6. Then we would only see 21 different outcomes: 1-1, 1-2, 1-3, 1-4, 1-5, 1-6, 2-2, 2-3, 2-4, 2-5, 2-6, 3-3, 3-4, 3-5, 3-6, 4-4, 4-5, 4-6, 5-5, 5-6, and 6-6. (We think of a 1 and a 2, a 1-2, as being the same as a 2 and a 1.)

Probability of an Outcome

The probability of an outcome for a particular event is a number telling us how likely a particular outcome is to

occur. This number is the ratio of the number of ways the outcome may occur to the number of total possible outcomes for the event. Probability is usually expressed as a fraction or decimal. Since the number of ways a certain outcome may occur is always smaller or equal to the total number of outcomes, the probability of an event is some number from 0 through 1.

Example:

Suppose there are 10 balls in a bucket numbered as follows: 1, 1, 2, 3, 4, 4, 4, 5, 6, and 6. A single ball is randomly chosen from the bucket. What is the probability of drawing a ball numbered 1? There are 2 ways to draw a 1, since there are two balls numbered 1. The total possible number of outcomes is 10, since there are 10 balls.

The probability of drawing a 1 is the ratio 2/10 = 1/5.

Example:

Suppose there are 10 balls in a bucket numbered as follows: 1, 1, 2, 3, 4, 4, 4, 5, 6, and 6. A single ball is randomly chosen from the bucket. What is the probability of drawing a ball with a number greater than 4? There are 3 ways this may happen, since 3 of the balls are numbered greater than 4. The total possible number of outcomes is 10, since there are 10 balls. The probability of drawing a number greater than 4 is the ratio 3/10. Since this ratio is larger than the one in the previous example, we say that this event has a greater chance of occurring than drawing a 1.

Example:

Suppose there are 10 balls in a bucket numbered as follows: 1, 1, 2, 3, 4, 4, 4, 5, 6, and 6. A single ball is randomly chosen from the bucket. What is the probability of drawing a ball with a number greater than 6? Since none of the balls are numbered greater than 6, this can occur in 0 ways. The total possible number of outcomes is 10, since there are 10 balls. The probability of drawing a number greater than 6 is the ratio 0/10 = 0.

Example:

Suppose there are 10 balls in a bucket numbered as follows: 1, 1, 2, 3, 4, 4, 4, 5, 6, and 6. A single ball is randomly chosen from the bucket. What is the probability of drawing a ball with a number less than 7? Since all of the balls are numbered greater than 7, this can occur in 10 ways. The total possible number of outcomes is 10, since there are 10 balls. The probability of drawing a number less than 7 is the ratio 10/10 = 1.
Note in the last two examples that a probability of 0 meant that the event would not occur, and a probability of 1 meant the event definitely would occur.

Example:

Suppose a card is drawn at random from a regular deck of 52 cards. What is the probability that the card is an ace? There are 4 different ways that the card can be an ace, since 4 of the 52 cards are aces. There are 52 different total

outcomes, one for each card in the deck. The probability of drawing an ace is the ratio 4/52 = 1/13.

Example:

Suppose a regular die is rolled. What is the probability of getting a 3 or a 6? There are a total of 6 possible outcomes. Rolling a 3 or a 6 are two of them, so the probability is the ratio of 2/6 = 1/3.

Determine the probability for each.

1. Drawing a heart from a deck of cards? _____
2. Drawing a green ball from a box containing 9 white balls and 3 green balls? _____
3. Drawing a black card from a deck of cards? _____
4. Rolling an even number on a die? _____
5. Drawing a queen from a deck of cards? _____
6. Rolling an odd number on a die? _____
7. Drawing a club from a deck of cards? _____
8. Drawing a black checker from a box containing 14 checkers, 6 of which are black? _____
9. Drawing a 7 from a deck of cards? _____
10. Drawing a red card from a deck of cards? _____
11. Drawing a black sock from a drawer containing 3 white socks, 6 brown socks, 2 black socks, and 7 blue socks? _____

12. Drawing an 8 from a deck of cards? _____
13. Drawing a diamond from a deck of cards? _____
14. A box contains 4 pennies, 12 nickels, and 20 dimes. Suppose you reach into the box and randomly take out 1 coin. What is the probability that the coin you choose will be a penny? _____
15. At his diner, Joe serves orange, cola, and root beer soft drinks. He also serves hamburgers, fish sandwiches, and chicken sandwiches. How many soft drink-sandwich combinations are possible at Joe's?

    _____

Answer Key

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Page 5

1. 81% **2.** 5.265 **3.** 3.15 **4.** .75%
5. 24000 **6.** 1600 **7.** 82 **8.** 1900
9. 61.2 **10.** 1734 **11.** .5% **12.** 71%
13. 1200 **14.** 8.245 **15.** 4.8 **16.** 12000
17. 2.625 **18.** 117 **19.** .92% **20.** 7.2
21. 2500 **22.** 22000 **23.** 5.51 **24.** 50%
25. 18% **26.** 28000 **27.** 3000 **28.** 884

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1. $1316.10 **2.** 1900 calories **3.** $17.55
4. $231.20

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2. 22:21 or 22 to 21 **3.** 19:6 or 19 to 6
4. 9:35 or 9 to 35 **5.** 3:4 or 3 to 4
6. 1:5 or 1 to 5 **7.** 4:1 or 4 to 1
8. 3:1 or 3 to 1 **9.** 1:4 or 1 to 4
10. 1:1 or 1 to 1 **11.** 1:2 or 1 to 2
12. 1:1 or 1 to 1 **13.** 1:45 or 1 to 45
14. 1:12 or 1 to 12 **15.** 1:12 or 1 to 12
16. 19:22 or 19 to 22 **17.** 28:85 or 28 to 85
18. 41:20 or 41 to 20 **19.** 1:10 or 1 to 10
20. 9:8 or 9 to 8

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1. 1 **2.** 42 **3.** 40 **4.** 9 **5.** 48 **6.** 15
7. 26 **8.** 28 **9.** 26 **10.** 195 **11.** 27
12. 95 **13.** 26 **14.** 144 **15.** 4 **16.** 32.6
17. 40.3 **18.** 8.6 **19.** 42 **20.** 70

21. 20

Page 17 **1.** True **2.** False **3.** False **4.** True
5. True

Page 22 **1.** ¼ **2.** ¼ **3.** ½ **4.** ½ **5.** 1/13
6. ½ **7.** ¼ **8.** 3/10 **9.** 1/13 **10.** ½
11. 1/9 **12.** 1/13 **13.** ¼ **14.** 1/9 15. 9