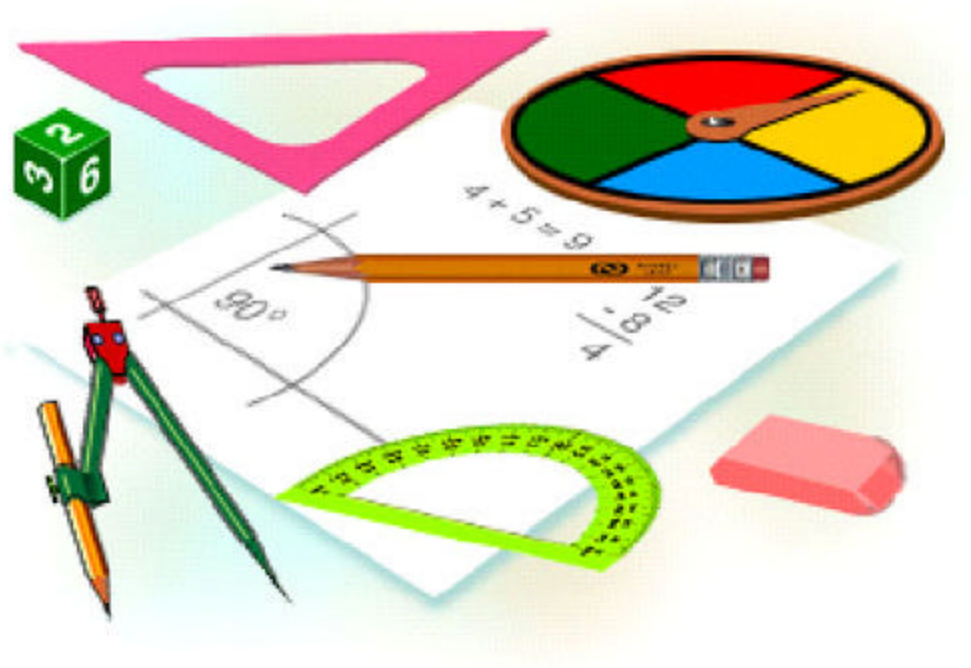
The Next Step

Mathematics Applications for Adults



Teacher's Manual:

Formulas

SECTION 5

Formulas

"Each problem that I solved became a rule, which served afterwards to solve other problems."
-*Rene Descartes*

OUTLINE

Mathematics – Teacher's Manual

Section 5
Formulas
Percent of a Number
Distance Formula
Distance Between Points on a Graph
Formula Quickguide

Percent of a Number

Some members of the focus group suggested an alternate way of teaching percent of a number. I find this strategy to be quite helpful, because it gives some sense of order and placement to the numbers and variables involved in the problem.

Let's say that you wanted to figure out what is 25% of 80.

Step 1 Write down that "n is n% of n". This simply means that some number is some percent of another number.

n is n% of n

Step 2 Insert the numbers stated in the original problem.

n is 25% of 80

Step 3 Change the "is" to an equal sign and the "of" to a multiplication sign.

n = 25% x 80

Step 4 Change the percent to a decimal or fraction.

n = .25 x 80

Step 5 Multiply.

$$n = 20$$

Answer: 25% of 80 is **20**.

We now know that 20 is 25% of 80, but let's say that we didn't know what percent 20 is of 80. The question would now be 20 is what percent of 80.

n is n% of n

$$20 \text{ is } n\% \text{ of } 80$$
$$20 = n\% \times 80$$

Okay, at this point, we have to change our variable percent to a fraction so that we can solve the problem. "n%" means that we have some part of a hundred. As a fraction, that would be represented as "n/100"

$$20 = n/100 \times 80$$
$$20 \times 100 = n/100 \times 80 \times 100$$
$$2000 = 80n$$
$$2000/80 = 80n/80$$
$$25 = n$$

The answer confirms what we already knew. 20 is **25%** of 80.

To find a number when a percent of it is known, let's use 25% of what number is 20. We know that the answer should be 80, but let's see if we can prove it.

$$n \text{ is } n\% \text{ of } n$$

$$20 \text{ is } 25\% \text{ of } n$$
$$20 = 25\% \times n$$
$$20 = 25/100 \times n$$

If you choose to change the percent to a fraction instead of a decimal, you may reduce the fraction to lowest terms. 25/100 would reduce to ¼.

$$20 = \tfrac{1}{4} \times n$$
$$20 \times 4 = \tfrac{1}{4} \times n \times 4$$
$$80 = n$$

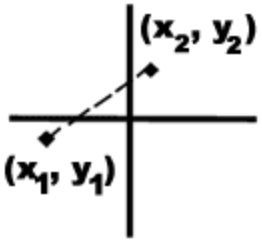
What a surprise!!! We can now say that we have proven that 20 is 25% of **80.**

Distance Formulas

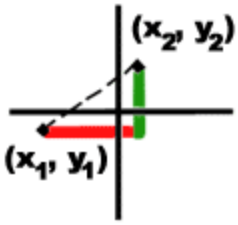
Finding the Distance Between Points on a Graph

In coordinate geometry, the formula for measuring the distance between points on a graph is $[(Y2-Y1)^2 + (X2-X1)^2]$.

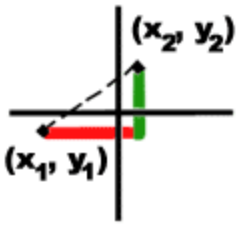
Let's suppose that you wanted to find the distance between the two points on the graph below. (represented by the dashed line)



Picture a right triangle. If we have a right triangle, it is possible to use the Pythagorean theorem to solve for the sides. Without the Pythagorean theorem, they probably wouldn't have discovered the distance formula.

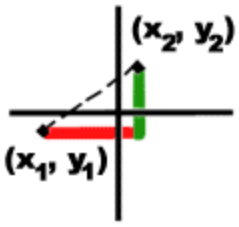


Now look at the green piece of the triangle. How long is it? If we could figure out how far apart the points are up-and-down, we would know. Remember that, on a graph, up-and-down is measured by the Y's. So to find out how far apart the Y's are, we would have to find the difference in the Y's. Difference means to subtract, hence (Y2-Y1)

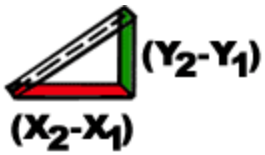


Now look at the red piece of the triangle. How long is it? If we could figure out how far apart the points are side-to-side, we would know. Remember that, on a graph, side-to-

side is measured by the X's. So to find out how far apart the X's are, we would have to find the difference in the X's. Difference means to subtract, hence (X2 -X1)



The picture below shows what we have so far.



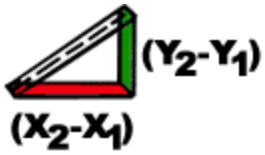
Now we are ready to use the magic of the Pythagorean theorem.

Pythagorean Theorem

$a^2+ b^2=c^2$

* The c in our problem is the distance between the two points.

So then:



$distance^2=(Y2-Y1)^2 + (X2-X1)^2$

To find distance, take the square root

$$\textbf{Distance} = \sqrt{\ [\ (Y2-Y1)^2 + (X2-X1)^2\ ]}$$

The advantage of the Distance Formula is that you do not need to draw a picture to find the answer. All you need to know are the coordinates of the endpoints of the segment.

It doesn't matter which point you start with. Just start with the same point for reading both the x and y coordinates.

Example Problem

Given the points **(1, -2)** and **(-3, 5)**, find the distance between them.

Solution

Label the points as follows
$(x_1, y_1) = (-1, -2)$ and $(x_2, y_2) = (-3, 5)$.
Therefore, $x_1 = -1$, $y_1 = -2$, $x_2 = -3$, and $y_2 = 5$.

To find the distance ***d*** between the points, use the distance formula :

$$\textbf{Distance} = \sqrt{\ [\ (Y2-Y1)^2 + (X2-X1)^2\ ]}$$
$$\textbf{Distance} = \sqrt{\ [\ (5 - (-2)^2 + (-3 - (-1))^2\ ]}$$

$$\textbf{Distance} = \sqrt{\ [\ (5 + 2)^2 + (-3 + 1)^2\ ]}$$

$$\textbf{Distance} = \sqrt{\ [\ (7)^2 + (-2)^2\ ]}$$

Distance = √ [49 + 4]

Distance = √ 53

By the way, it is almost always better to leave the answer in "exact" form (the square root "√ **53**" above). Rounding is usually reserved for the last step of word problems. If you're not sure which format is preferred, do both, like this:

d = √ 53 7.28

Now you try it.

Answer the following questions pertaining to distance (length) and coordinate geometry.

1. Find the length of the line segment whose endpoints are (-3, 4) and (5,4).

2. Find the distance between the points (-4,-5) and (1,-2).

3. The point (5,4) lies on a circle. What is the length of the radius of this circle if the center is located at (3,2)?

Answers: 1. **8** 2. **√34** 3. **√8**

The Distance Formula in Algebra

The formula Distance = Rate x Time expresses one of the most frequently used relations in algebra.

Since an equation remains true as long as you divide through by the same non-zero element on each side, this formula can be written in different ways:

To find **rate**, divide through on both sides by *time:*

$$\text{Rate} = \frac{\text{Distance}}{\text{Time}}$$

Rate is distance (given in units such as miles, feet, kilometers, meters, etc.) divided by time (hours, minutes, seconds, etc.). Rate can always be written as a fraction that has distance units in the numerator and time units in the denominator, e.g., 25 miles/l hour.

To find **time**, divide through on both sides by *rate:*

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

When using this equation, it's important to keep the units straight. For instance, if the rate the problem gives is in miles per hour (mph), then the time needs to be in hours, and the distance in miles. If the time is given in minutes,

you will need to divide by 60 to convert it to hours before you can use the equation to find the distance in miles. Always make your units match: if the time is given in years and the distance in meters, then the rate should be given in meters per year.

You can see why this is true if you look carefully at how the units are expressed. Say a car is traveling at 30 mph and you want to figure out how far it will go in 2 hours. You can use the formula:

Rate x Time = Distance

$$30 \frac{\text{miles}}{\text{hour}} \times 2 \text{ hours} = 60 \text{ miles}$$

The hours cancel, leaving only miles.

What if you want to calculate the number of miles a car traveling 30 mph goes in 120 minutes?

Since 120 minutes is equal to two hours (60 minutes in one hour x 2 hours = 120 minutes), we should get the same distance of 60 miles, but we will *not* get the answer this way:

$$30 \frac{\text{miles}}{\text{hour}} \times 120 \text{ minutes} = 3600 \frac{\text{mile minutes}}{\text{hour}}$$

Now, 3600 mile minutes per hour isn't very helpful, since we'd like our answer in miles. We need to divide by 60 minutes per hour:

$$3600 \frac{\text{mile minutes}}{\text{hour}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 60 \text{ miles}$$

The hours and the minutes cancel, leaving only miles.

Although we can find an answer this way in the correct units, a better method would be to convert minutes to hours *before* using the formula.

Remembering to be careful about units, let's look at a problem.

> **Superheroes Wonder Woman and Batman leave the same camp and run in opposite directions. Wonder Woman runs 2 miles per second (mps) and Batman runs 3 mps. How far apart are they in miles after 1 hour?**

To begin, we can either convert rates to miles per hour, or we can convert the time to seconds. Let's convert from miles per second to miles per hour.

There are 3600 seconds in an hour, so if Wonder Woman runs 2 miles in a second, then she will run at 3600 x 2 = 7200 mph. Similarly, Batman will run at 3600 x 3 = 10,800 mph.

$$2\ \frac{\text{miles}}{\text{second}} \times \frac{3600\ \text{seconds}}{1\ \text{hour}} = 7200\ \frac{\text{miles}}{\text{hour}}$$

The seconds cancel, leaving miles per hour.

Back to the problem. How far does Wonder Woman run in one hour? We know her rate (7200 mph) and the time that she runs (one hour), so we can use the formula:

$$7200\ \frac{\text{miles}}{\text{hour}} \times 1\ \text{hour} = 7200\ \text{miles}$$

This makes sense because, by definition, if Wonder Woman's speed is 7200 miles per hour, then she runs 7200 miles in an hour.

Batman, whose speed is 10,800 miles per hour, will run 10,800 miles in an hour.

How far apart will the two runners be after an hour? The answer is simply the sum of the distance each runs in an hour: 7200 + 10,800 = 18,000 miles apart.

Since the earth has a circumference of about 24,000 miles at its equator, that's about 3/4 of the way around the world!

As is frequently the case with word problems, setting up the equations is the hardest part. Once that's done, the rest is relatively easy. Remember always to answer what the

question asks - don't stop once you've solved for x, because that may be only part of what the question asked - and *always* check your answer.

Now you try it.

In the following problems, decide whether you are looking for the distance, rate---or speed, or time of travel. Then use the appropriate formula to solve.

1. What distance can Bert drive in 12 hours if he averages 50 kilometers per hour?

2. On the second day of his trip, George drove 440 miles in 8 hours. Find his average speed during the second day.

3. Jennifer drove from Fredericton to Toronto. On the first day of her trip, she averaged 100 kilometers per hour for 6 hours. How far did she drive the first day?

4. On the second day of her trip, Joan drove 364 miles in 7 hours. Find her average speed during the second day.

5. On the last day of his trip, Sam averaged 90 kilometers per hour as he drove 360 kilometers. How many hours did Sam drive on his last day?

Answers: 1. **600 kilometers** 2. **55 mph**
3. **600 kilometers** 4. **52 mph** 5. **4 hours**

Formula QuickGuide

This guide contains a list of the formulas given you on the mathematics section of the GED Tests, complete with examples. It has been revised to reflect the requirements of the latest series of tests. The guide is not intended to be studied in detail or memorized. It is only meant to be a quick reference guide. Use it if you need more information when you are working on mini-tests or completing exercises in a GED Preparation book.

---

1. Area of a square. $A=s^2$

John wants to carpet a square family room in his basement. If the family room is 3.5 meters wide, how many square meters of carpet will he need?

$A=s^2$

$A= 3.5^2$

A= (35 x 3.5) A= 12.25 square meters

---

2. Area of a rectangle A=lw (l=height w=width)

Sally has a rectangular driveway that needs to be paved. If the paving company charges $100 per square meter, how much will it cost Sally to have her driveway paved?

A= lw

A= 5 x 3

A= 15 square meters

Cost of paving = 15 x $100 = $1500

---

3. Area of a parallelogram A=bh (b=base h=height)

Tara bought a house in a new subdivision. The back yard, which is an odd shape, has no grass. How many square meters of sod will Tara have to buy to cover the yard?

A=bh

A=4 x 7.5

A=30 square meters

---

4. Area of a Triangle A=1/2bh (b=base h=height)

If Hugo ate a slice of pizza that is 35 centimeters high and 15 centimeters across the bottom, how many square centimeters of pizza did he eat?

A=1/2bh

A=1/2 (15 x 35)

A=1/2 (525)

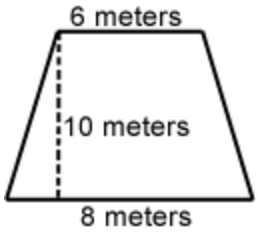
A= 525 ÷ 2

A= 262.5 square centimeters

---

5. Area of a Trapezoid A=1/2 x (base$_1$ + base$_2$) x height

Julie rents a store space in a local mall. She pays $32 per square meter each month in rent. The store is a trapezoid in shape. The front of the store is 8 meters wide. The rear of the store is 6 meters wide. The store is 10 meters deep. What is the area of the store in square meters?



A=1/2 x (base$_1$ + base$_2$) x height

A=1/2 x (6 + 8) x 10

A= 1/2 x (14) x 10

A = 7 x 10

A = 70 meters^2

---

6. Area of a circle A= πr^2 (π=3.14 r=radius)

A dog is tied in the center of a large back yard with a 4 meter rope. How many square meters of space does the dog have to run?

A=πr^2

A= 3.14 x $(4)^2$

A= 3.14 x (16)

A= 50.24 square meters

---

7. Perimeter of a square P=4s (s=length of a side)

Looked at from an angle, the diamond shaped infield (from home to first base to second, third, and home again) forms a square. If it is 30 meters from home to first base, how far would a runner have to run to go all the way around the bases?

P= 4s

P= 4(30)

P= 120 meters

---

8. Perimeter of a Rectangle P= 2l + 2w (l=length w=width)

Whitney wants to put a wallpaper border at the top of the wall all the way around her living room. If the living room is 4.8 meters long and 3.7 meters wide, how many meters of border will she need?

P= 2l + 2w

P= 2(4.8) + 2(3.7)

P= 9.6 + 7.4

P= 17 meters

---

9. Perimeter of a Triangle P= a + b + c (a,b,c = length of sides)

Alex has a loft with a triangular window. The window needs a new frame. What length of framing in centimeters does Alex need to buy to go around the window?

P = a + b + c

P = 90 + 90 + 50

P = 230 centimeters

---

10. Circumference of a Circle C = πd (π=3.14 d=diameter)

A circular fountain is 9 meters in diameter. How many meters of rope would be needed to rope off the fountain around its edge?

C= πd

C = 3.14 x 9

C= 28.26 meters

---

11. Volume of a Cube V= s^3 (s=length of one side)

George wants to freeze gravy in a cube shaped plastic container. If the container is 15 centimeters deep, how much gravy will it hold?

$V= s^3$

V= s x s x s

V= 15 x 15 x 15

V= 3375 cubic centimeters

---

12. Volume of a Rectangular Container V= lwh (l=length w=width h=height)

An aquarium measures 1 meter in length 0.5 meters in width and 0.8 meters in height. What volume of water will it hold?

V= lwh

V= 1 x .5 x .8

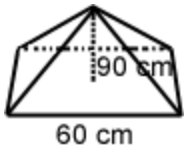
V= .4

---

13. Volume of a square pyramid 1/3 x (base edge)^2 x height

A caterer plans to make an ice sculpture in the shape of a square pyramid for a banquet. The base of the sculpture will be 60 centimeters long. The height of the sculpture will

be 90 centimeters. How many cubic centimeters of ice will be in the sculpture?



1/3 x (base edge)^2 x height

1/3 x $(60)^2$ x 90

1/3 x (3600) x 90

3600/3 x 90

1200 x 90

108,000 cm^3

---

14. Volume of a Cylinder V= $\pi r^2 h$ (π= 3.14 r=radius h=height)

If a farmer builds the silo shown below, how many cubic meters of grain will he be able to store in it?

V= $\pi r^2 h$

V= 3.14 x $(5)^2$ x 12

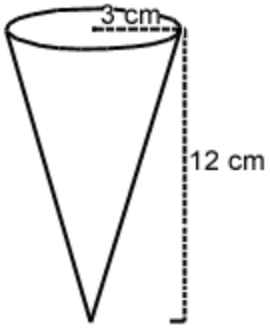
V= 3.14 x 25 x 12

V = 942 cubic meters

---

15. Volume of a Cone V= 1/3 x π x radius^2 x height (π= 3.14)

Kim filled an ice cream cone with soft ice cream. If the cone had a radius of 3 cm and was 12 cm high, how much ice cream was inside the cone?



V= 1/3 x π x radius^2 x height

V= 1/3 x 3.14 x $(3)^2$ x 12

V= 1/3 x 3.14 x 9 x 12

V = 1/3 x 3.14 x 108

V = 1/3 x 339.12

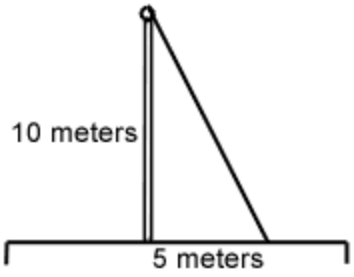
V = 339.12/3

V = 113.04 cm^3

---

16. Pythagorean Relationship $c^2= a^2 + b^2$

A 10-meter flagpole is erected straight up on the top of a flat-roofed building. If support wires are run from the roof of the building, 5 meters from the base of the pole, to the top of the pole, how long must the support wires be?



$c^2 = a^2 + b^2$

$c^2 = (10)^2 + (5)^2$

$c^2 = 100 + 25$

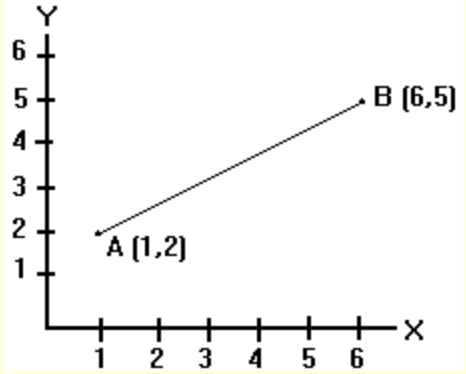
$c^2 = 125$

$c = 11.18$ meters

---

17. Distance between two points in a plane. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(d = distance and (x_1, y_1) a nd (x_2, y_2) are two points in a plane)



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

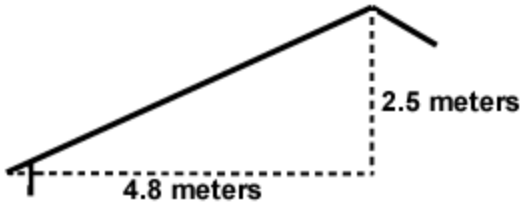
$d = \sqrt{(\quad)^2 + (\quad)^2}$

---

18. Slope of a Line

(m=slope $(x_2 - x_1)$ and $(y_2 - y_1)$ are two points on a plane)

The vertical length from the eves to the peak of a roof is 4.8 meters. The height of the roof at the peak is 2.5 meters higher than at the eves. What is the slope of the roof?



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{(2.5) - (0)}{(4.8) - (0)}$$

$$m = \frac{2.5}{4.8}$$

m= .52 meters

(which means that the roof rises .52 metes (y)for every meter of vertical length(x))

---

19. Mean

(The x's are the values for which you are looking for the mean (average) n=the number of values)

The class recently wrote a math test. Out of a possible score of 50, the marks for the class were 22, 27, 31, 32, 38, 40, 44. What is the mean (average) of the marks?

22 + 27 + 31 + 32 + 38 + 40 + 44 = 234

234 ÷ 7 = 33

The mean = 33

---

20. Median (the number in the middle of an ordered set of numbers)

What is the median of the class scores on the test in item 17?

22, 27, 33, 32, 38, 40, 44

Median = 32 (because there are 3 scores below this number and 3 above it)

---

21. Simple Interest I= prt (i=interest p=principal r=rate t=time)

John borrowed $5,000 at an interest rate of 12%/a (per annum-year) to buy a car. If he paid off the loan in 1 year and 7 months, how much interest did he pay?

I = prt

I = 5000 x 12% x 1 7/12

I = 5000 x 12/100 x 19/12

I = 5000 x ~~12~~/~~100~~ x 19/~~12~~

I = 50 x 19

I = $950

---

22. Distance d= rt (r=rate t=time)

Angel boarded a train at 11:00 a.m. and traveled at an average speed of 90 kilometers an hour until 2:00 p.m. How far did she travel?

d= rt

d= 90 kilometers x 3 hours

d= 270 kilometers

---

23. Total Cost c= nr (n=number of units r=cost per unit)

Ann bought 24 packets of noodles at a cost of $.29 each. What was the total cost of the noodles?

c= nr

c= 24 x $.29

c= $6.96

The Last Word

"A successful person is one who can lay a firm foundation with the bricks that others throw at him."
-*David Brink*

"Good questions outrank easy answers."
-*Paul A. Samuelson*

"You can tell whether a man is clever by his answers. You can tell whether a man is wise by his questions."
-*Naguib, Mahfouz*

"Courage is not simply one of the virtues, but the form of every virtue at the testing point."
-*Clive Staples Lewis*

"The true test of character is not how much we know how to do, but how we behave when we don't know what to do."
-*John Holt*

"Opportunities are usually disguised as hard work, so most people don’t recognize them."
-*Ann Landers*

"The greatest thing a man can do in this world is to make the most possible out of the stuff that has been given him. This is success, and there is no other."
-*Orison Swett Marden*

"If you want to test your memory, try to recall what you were worrying about one year ago today."
-*Rotarian*

"Our greatest glory is not in never falling, but in rising every time we fall."
-*Confucius*

"A journey of a thousand miles begins with a single **step**."
-*Confucius*

"If a man does not keep pace with his companions, perhaps it is because he hears a different drummer. Let him **step** to the music which he hears, however measured or far away."
-*Henry David Thoreau*, *Walden*

"The whole world **steps** aside for the man who knows where he is going."
-*Anon.*

"Don't be afraid to take a big **step**. You can't cross a chasm in two small jumps."
-*David Lloyd George*