Workplace Education Manitoba

NUMERACY:

The Basics Workbook

Set T: Geometry 2 Area

Companion Workbook to Numeracy: The Basics Video Series

Watch the video for this workbook at
www.wem.mb.ca/learning_on_demand.aspx

Workplace Education Manitoba would like to express appreciation to the following for supporting the development of this curriculum:

The Government of Canada
Human Resource Skills Development Canada (HRSDC)

and

The Manitoba Government
Industry Workforce Development (IWD), Entrepreneurship, Training and Trade (ETT)

Workplace Education Manitoba would also like to thank the individuals from across Manitoba who provided consultation, content, and feedback.

For more information,
visit www.wem.mb.ca
or contact the Project Coordinator
Lindsay Laidlaw at info@wem.mb.ca

Workplace Education Manitoba
1000 Waverley Street
Winnipeg, MB, R3T 0P3

ACKNOWLEDGMENTS

Workplace Education Manitoba

INTRODUCTION

What is Numeracy: The Basics Workbook?

This workbook is intended to accompany Workplace Education Manitoba's (WEM) Numeracy: The Basics Video Series, a set of 50 videos that explain essential numeracy concepts.

The refresher videos cover 25 critical numeracy topics, each broken into concept and practice.

The video series and accompanying downloadable workbooks can be found on the WEM website at http://www.wem.mb.ca/learning_on_demand.aspx

These Numeracy: The Basics workbooks provide an opportunity for additional skill-building practice.

Numeracy: The Basics topics are:

- Order of Operations 1
- Order of Operations 2
- Adding & Subtracting Fractions 1
- Adding & Subtracting Fractions 2
- Multiplying & Dividing Fractions
- Mixed & Improper Fractions
- Operations with Mixed Fractions 1
- Operations with Mixed Fractions 2
- Operations with Mixed Fractions 3
- Adding & Subtracting Decimals
- Multiplying Decimals
- Dividing Decimals
- Order of Operations & Decimals
- Decimals, Fractions & Percent 1
- Decimals, Fractions & Percent 2
- Imperial Conversions
- Metric Conversions
- Metric and Imperial Conversions
- Geometry 1 – Perimeter
- Geometry 2 – Area
- Geometry 3- Volume
- Solving Equations 1
- Solving Equations 2
- Ratio & Proportion
- Averages

GEOMETRY 2 AREA

This workbook contains five skill-building practice sections. Solutions can be found at the end of the workbook.

Practice Section A
Solve the following. Round each answer to two decimal places, if rounding is necessary. Note that diagrams are not drawn to scale.

1. Define and give an example of area.

____
____
____
____
____

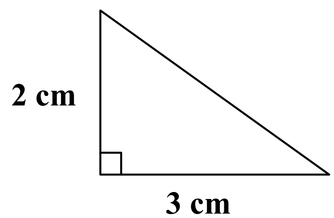
2. Find the area of a square with each side measuring 2 in.

3. Find the area of a rectangle with a length of 2 m and a width of 1 m.

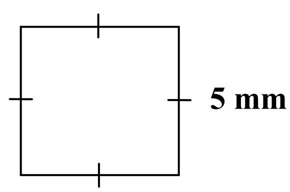
4. Find the area of a circle with a radius of 5 cm.

5. Find the area of a right-angled triangle with a base of 2 ft and a height of 5 ft.

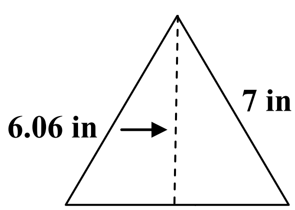
6. Find the area of the object in the diagram below.



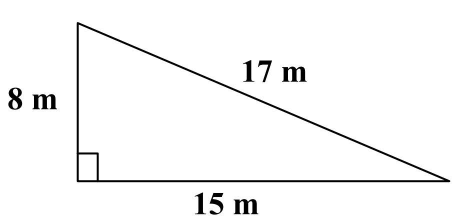
7. Find the area of the square drawn below.



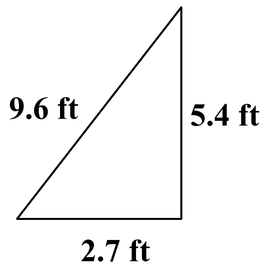
8. Find the area of the equilateral triangle (where all sides have the same measure) drawn below.



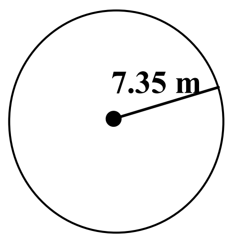
9. Find the area of the triangle in the diagram below.



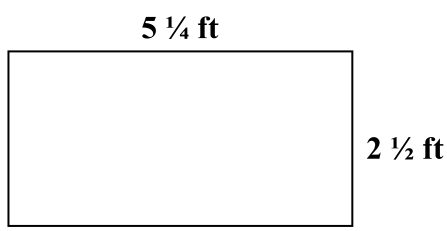
10. Find the area of the object in the diagram below.



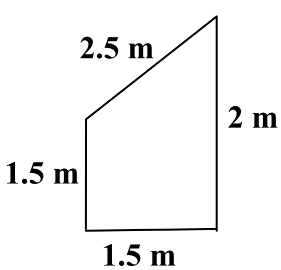
11. Find the area of the circle drawn in the diagram below.



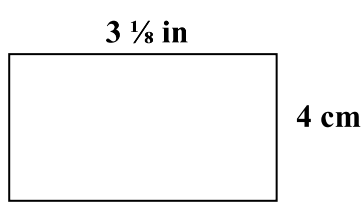
12. Find the area of the rectangle in the diagram below.



13. Find the area of the object drawn in the diagram below.



14. Find the area, in cm, of the rectangle drawn in the diagram below.



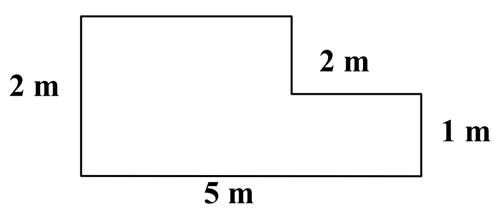
15. Find the area of a circle, in meters, that has a diameter of 6 in.

Practice Section B

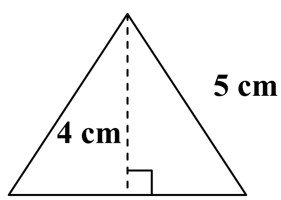
Solve the following. Round each answer to two decimal places, if rounding is necessary. Note that diagrams are not drawn to scale.

1. Find the area of a rectangle, in feet, that has a length of 3.5 ft and a width of 38 in.

2. Find the area of the figure drawn below.



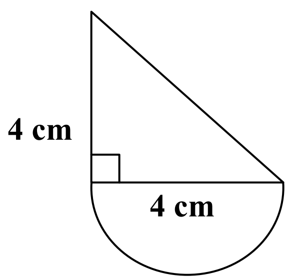
3. Find the area of an equilateral triangle having sides of 5 cm and a height of 4 cm.



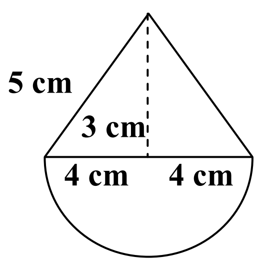
4. If the length of a rectangle is 4 m and the area is 12 m^2, what is the width of the rectangle?

5. The area of a right-angled isosceles triangle is 18 in^2. What is the measure of the equal sides?

6. A right-angled isosceles triangle, with side length 4 cm, is capped with a semi-circle having a diameter equal to the side length of the triangle, as in the diagram below. Find the area of the figure.

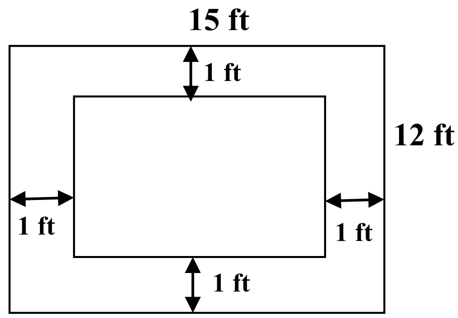


7. An isosceles triangle has equal sides measuring 5 m and a longer side of 8 m. This triangle is capped with a semi-circle that has a radius of 4 m, as in the diagram below. Find the area of the entire object.

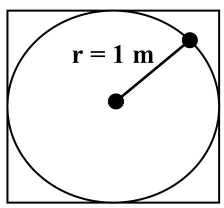


8. Find the area of a circle that has a circumference of 56.52 m.

9. A concrete path 12 ft wide and 15 ft long borders a rectangular pool as shown in the diagram below. Find the area of the pool if the path is 1 ft wide.



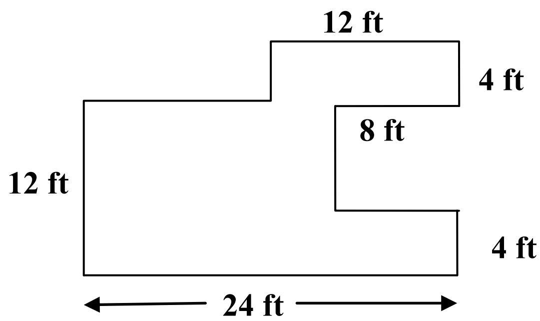
10. A circle having a radius of 1 m is drawn inside a square as in the diagram below.



What area of the square is not covered by the circle?

11. A rectangle has a length of 9 cm and a width of 5 cm. What is the area of the largest circle that has the same perimeter as the rectangle?

12. Calculate the area of the figure drawn below.

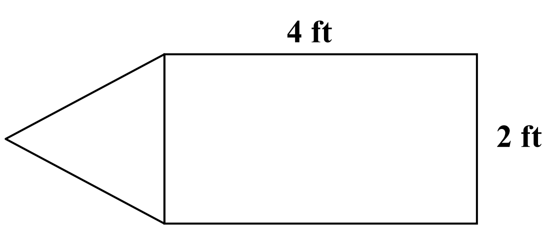


13. How many 2 cm by 5 cm rectangles will exactly fit into a square that has the smallest possible area?

Practice Section C

Solve the following. Round each answer to two decimal places, if rounding is necessary. Note that diagrams are not drawn to scale.

1. An isosceles triangle, with a height of 1 m, is placed at one end of a rectangle as in the diagram below.

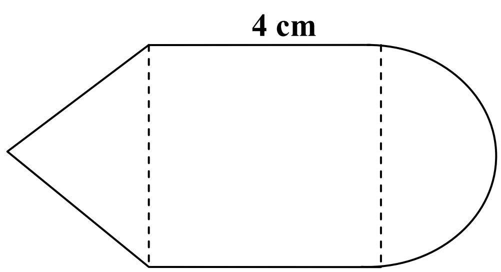


Find the area, in meters, of the figure.

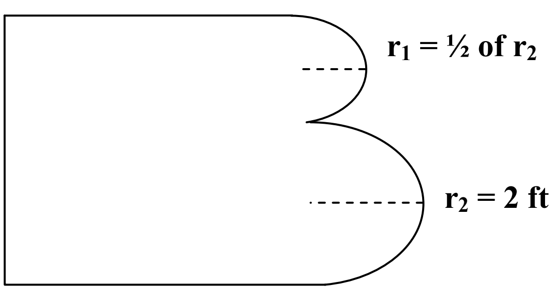
2. If an equilateral triangle has an area of 16.2 cm^2 and a height of 6 cm, calculate the side length of the triangle.

3. If 'b' represents the base of the triangle in question 2, how many equilateral triangles with the same height and with a base of $25\% \times b$ will have an area equal to the larger equilateral triangle's area in question 2?

4. A 4 cm square is capped by an equilateral triangle at one end and a semi-circle at the other as in the diagram below. If the height of the triangle is 3.4641 cm, what is the area of the entire figure?



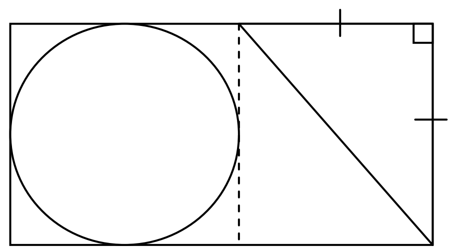
5. Two semi-circles cap a square at one end as in the diagram below. The larger semi-circle has a diameter of 4 ft, which is twice the diameter of the smaller semi-circle. Find the area of the object, in square meters.



Practice Section D

In this section, solutions for the practice questions contain commonly-made errors. For each question, circle the error(s) and give a correct solution.

1. A rectangle with a perimeter of 54 cm is half as wide as it is long. The rectangle has a circle and a right-angled isosceles triangle inscribed in it, as in the diagram below.



If the diameter of the circle and the side length of the isosceles triangle are equal, what area of the rectangle is not covered by the circle and the triangle?

Solution:

If the rectangle is twice as long as it is wide, then there are 6 'widths' in one perimeter:

$$Perimeter = l + l + w + w$$
$$= (w + w) + (w + w) + w + w$$
$$54 = (w + w) + (w + w) + w + w$$
$$54 = 6w$$
$$\frac{54}{6} = w$$
$$w = 9\,cm$$

If the width is 9 cm, the length must be 18 cm. The right-angled isosceles triangle has two equal sides of 9 cm giving it an area of:

$$A = \frac{b \times h}{2}$$
$$= \frac{9 \times 9}{2}$$
$$= 40.5\,ft^2$$

The area of the circle is:

$$A = \pi \times r^2$$
$$= 3.14 \times (9)^2$$
$$= 3.14 \times 81$$
$$= 254.34\ ft^2$$

The area of the rectangle is:

$$A = l \times w$$
$$= 18 \times 9$$
$$= 162\ ft^2$$

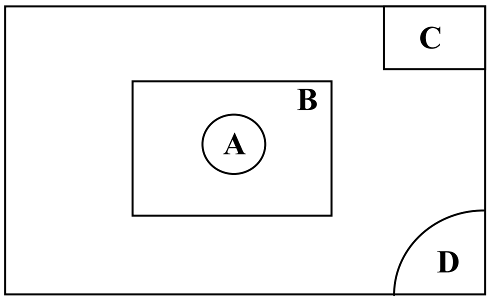
The area of the rectangle not covered by the circle and the triangle is:

$$A_{not\ covered} = A_{rectangle} - A_{circle} - A_{triangle}$$
$$= 254.34 - 162 - 40.5$$
$$= 51.84\ ft^2$$

Practice Section E

Challenge Question. If you can do this one, then you get an A^+. ☺

The diagram of a backyard of a house is given below followed by a description. Calculate the cost of sod for the required area of this backyard.



A – Fire Pit
B – Patio
C – Shed
D – Flowers

- The cost of sod is \$3.50 per roll and each roll covers 2.54 m^2.
- The shed is 3 m long and 1 yd wide.
- The patio is a rectangle that is twice as long as it is wide and has a perimeter of 21 m.
- The flower garden has a radius of 67 in.
- The yard measures 17 yd by 30 yd.
- The fire pit has a diameter of 1 m.

SOLUTIONS

Set T

Geometry 2 Area

GEOMETRY 2 AREA

Practice Section A

1. Solution:
Area is the amount of surface something covers.
For example, if a room measures 3 m by 5 m, then the area is $3m \times 5m = 15m^2$.
Notice that the area has 'square' units because we are measuring two dimensions, length and width. Perimeter had units like cm or in, because the measurements are in one dimension.

2. Solution:
$A = l \times w$
$= 2in \times 2in$
$= 4in^2$

3. Solution:
$A = l \times w$
$= 2m \times 1m$
$= 2m^2$

4. Solution:
$A = \pi \times r^2$
$= 3.14 \times (5)^2$
$= 3.14 \times 25$
$= 78.5cm^2$

5. Solution:
$A = \frac{b \times h}{2}$
$= \frac{\not{2} \times 5}{\not{2}}$
$= 5ft^2$

6. Solution:
$A = \frac{b \times h}{2}$
$= \frac{\not{2} \times 3}{\not{2}}$
$= 3cm^2$

7. Solution:
$A = l \times w$
$= 5 \times 5$
$= 25mm^2$

8. Solution:
$A = \frac{b \times h}{2}$
$= \frac{7 \times 6.06}{2}$
$= \frac{42.42}{2}$
$= 21.21in^2$

9. Solution:

$$A = \frac{b \times h}{2}$$
$$= \frac{15 \times 8}{2}$$
$$= \frac{120}{2}$$
$$= 60\,m^2$$

10. Solution:

$$A = \frac{b \times h}{2}$$
$$= \frac{5.4 \times 2.7}{2}$$
$$= \frac{14.58}{2}$$
$$= 7.29\,ft^2$$

11. Solution:

$$A = \pi \times r^2$$
$$= 3.14 \times (7.35)^2$$
$$= 3.14 \times 54.0225$$
$$= 169.6306$$
$$= 169.63\,m^2$$

12. Solution:

$$A = l \times w$$
$$= 5\frac{1}{4} \times 2\frac{1}{2}$$
$$= \frac{21}{4} \times \frac{5}{2}$$
$$= \frac{105}{8} = 13\frac{1}{8} = 13.125 = 13.13\,ft^2$$

13. Solution:
This area can be found by adding the area of the right triangle and the rectangle.

$$A_\Delta = \frac{b \times h}{2}$$
$$= \frac{1.5 \times 0.5}{2}$$
$$= \frac{0.75}{2}$$
$$= 0.375\,m^2$$

$$A_\square = l \times w$$
$$= 1.5 \times 1.5$$
$$= 2.25\,m^2$$

The total area is:

$$A_{total} = A_\Delta + A_\square$$
$$= 0.375 + 2.25$$
$$= 2.625$$
$$= 2.63\,m^2$$

14. Solution:

$$A = l \times w$$
$$= 3\frac{1}{8}\,in \times 4\,cm$$
$$= \frac{25}{8}\,\cancel{in} \times \frac{2.54\,cm}{1\,\cancel{in}} \times 4\,cm$$
$$= \frac{25 \times 2.54}{8 \times 1}\,cm \times 4\,cm$$
$$= 7.9375\,cm \times 4\,cm$$
$$= 31.75\,cm^2$$

15. Solution:
If the diameter of the circle is 6 in, then its radius is 3 in. It is easier to convert the units and then find the area, rather than finding the area first and then converting.

$$3\,\cancel{in} \times \frac{2.54\,\cancel{cm}}{1\,\cancel{in}} \times \frac{1\,m}{100\,\cancel{cm}} = 0.0762\,m$$

The area is:

$$\begin{aligned} A &= \pi \times r^2 \\ &= 3.14 \times (0.0762)^2 \\ &= 3.14 \times 0.0058 \\ &= 0.0182 \\ &= 0.02\,m^2 \end{aligned}$$

Practice Section B

1. Solution:

$$\begin{aligned} A &= l \times w \\ &= 3.5\,ft \times 38\,in \\ &= 3.5\,ft \times 38\,\cancel{in} \times \frac{1\,ft}{12\,\cancel{in}} \\ &= 3.5\,ft \times 3.1\overline{6}\,ft \\ &= 11.0833\,ft^2 \\ &= 11.08\,ft^2 \end{aligned}$$

2. Solution:
This composite object (made up of two or more smaller objects) is really two rectangles measuring 2 m by 1 m and 3 m by 2 m each. The area will be 2 m^2 (for the smaller rectangle) and 6 m^2 (for the larger rectangle) so the total area is
2 + 6 = 8 m^2.

3. Solution:
If the triangle has sides 5 cm long and a height of 4 cm, it would have a base equal to its side length of 5 cm and its area would be:

$$A = \frac{b \times h}{2}$$
$$= \frac{5 \times \cancel{4}(2)}{\cancel{2}}$$
$$= 10\,cm^2$$

4. Solution:

$$A = l \times w$$
$$12\,m^2 = 4\,m \times w$$
$$\frac{12(m)\,\cancel{m^2}}{4\,\cancel{m}} = w$$
$$w = 3\,m$$

5. Solution:

$$18 = \frac{b \times h}{2} = \frac{b \times b}{2}$$
$$18 = \frac{b^2}{2}$$
$$b^2 = 36$$
$$b = \sqrt{36}$$
$$b = 6\,in$$

6. Solution:
The area of the semi-circle is:

$$A = \frac{\pi \times r^2}{2}$$
$$= \frac{3.14 \times (2)^2}{2}$$
$$= \frac{12.56}{2}$$
$$= 6.28\,cm^2$$

The triangle's area is:

$$A = \frac{b \times h}{2}$$
$$= \frac{(2)\not{4} \times 4}{\not{2}}$$
$$= 8\,cm^2$$

The total area is $6.28 + 8 = 14.28\,cm^2$.

7. Solution:

$$A_{total} = A_{triangle} + A_{semi-circle}$$
$$= \frac{b \times h}{2} + \frac{\pi \times r^2}{2}$$
$$= \frac{8 \times 3}{2} + \frac{3.14 \times (4)^2}{2}$$
$$= \frac{24}{2} + \frac{50.24}{2}$$
$$= 12 + 25.12$$
$$= 37.12\,cm^2$$

8. Solution:

First we need to use the circumference to find the radius of the circle.

$$C = 2 \times \pi \times r$$
$$56.52 = 2 \times 3.14 \times r$$
$$56.52 = 6.28 \times r$$
$$\frac{56.52}{6.28} = r$$
$$r = 9\,m$$

Using this radius, the area of the circle is:

$$A = \pi \times r^2$$
$$= 3.14 \times (9)^2$$
$$= 3.14 \times 81$$
$$= 254.34\,m^2$$

9. Solution:
The length and the width of the pool will each be 2 ft shorter. Therefore, if the concrete path is 15 ft by 12 ft, then the pool must be $15-2=13$ ft long and $12-2=10$ ft wide, giving it an area of $13\times10=130\,ft^2$.

10. Solution:
The radius of the circle is 1 m, giving it an area of:

$$\begin{aligned}A &= \pi\times r^2\\ &= 3.14\times(1)^2\\ &= 3.14\times1\\ &= 3.14\,m^2\end{aligned}$$

The side length of the square will be twice the radius, so it will be 2 m.
The area of the square is:

$$\begin{aligned}A &= l\times w\\ &= 2\times2\\ &= 4\,m^2\end{aligned}$$

The total area of the square not covered by the circle is:

$$\begin{aligned}A_{not\,covered} &= A_{square} - A_{circle}\\ &= 4-3.14\\ &= 0.86\,m^2\end{aligned}$$

11. Solution:
The perimeter of the rectangle will be $9+9+5+5=28$ cm.
The circumference of the circle must be equal to 28 cm, so its radius is:

$$\begin{aligned}C &= 2\times\pi\times r\\ 28 &= 2\times3.14\times r\\ 28 &= 6.28\times r\\ \frac{28}{6.28} &= r\\ r &= 4.4585\,cm\end{aligned}$$

The area of the circle is:

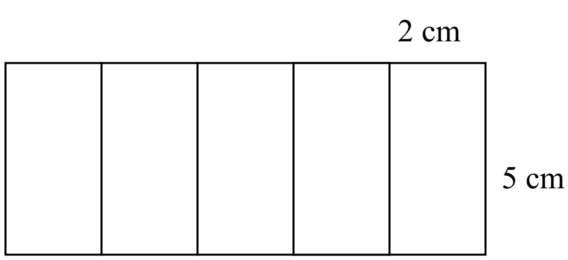
$$\begin{aligned}A &= \pi\times r^2\\ &= 3.14\times(4.4585)^2\\ &= 62.4176\\ &= 62.42\,cm^2\end{aligned}$$

12. Solution:
This figure can be divided into three different sections. The first, #1, is a 12 ft by 4 ft rectangle at the top right of the figure. The second, #2, is a 12 ft by 16 ft rectangle in the middle. The third, #3, is the 8 ft by 4 ft rectangle at the bottom right.

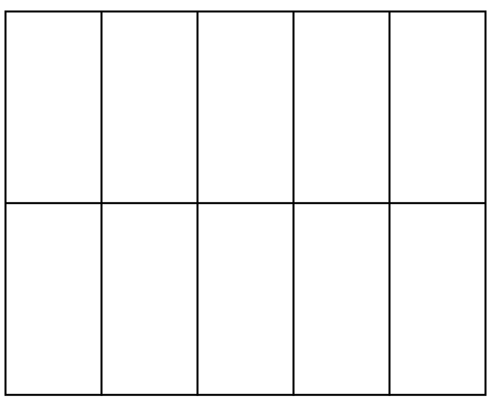
The total area will be:

$$A = A_1 + A_2 + A_3$$
$$= (12 \times 4) + (12 \times 16) + (8 \times 4)$$
$$= 48 + 192 + 32$$
$$= 272\, ft^2$$

13. Solution:
The smallest possible area will be the lowest common multiple (LCM) between 2 and 5. If we place five 2 cm by 5 cm rectangles beside one another, we get a rectangle such as the one below.



To make a square, another layer of rectangles needs to be added.



The top length of the rectangle is $2\,cm \times 5 = 10$ cm long and the side length is $5\,cm \times 2 = 10$ cm long. This creates a square. There would be 10 such rectangles that would fit inside a square with the smallest possible area.

Practice Section C

1. Solution:

Adding the area of the triangle with the area of the rectangle will result in:

$A_{total} = A_{\Delta} + A_{\square}$

$= \frac{b \times h}{2} + l \times w$

$= \frac{2 \times 1}{2} + 4 \times 2$

$= 1 + 8$

$= 9\,ft^2$

2. Solution:

The side length of the triangle is equal to the base of the triangle.

$A_{\Delta} = \frac{b \times h}{2}$

$16.2 = \frac{b \times (3)\cancel{6}}{\cancel{2}}$

$16.2 = b \times 3$

$\frac{16.2}{3} = b$

$b = 5.4\,cm$

3. Solution:

If the base is $25\% \times b$, then the base is:

$25\% \times b = 0.25 \times b$

$= 0.25 \times 5.4$

$= 1.35\,cm$

The area of each smaller equilateral triangle is:

$A = \frac{b \times h}{2}$

$= \frac{1.35 \times 6}{2}$

$= 4.05\,cm^2$

Dividing the larger area by the smaller area gives a value of:

$$=\frac{16.2}{4.05}$$
$$=4$$

Therefore, 4 smaller equilateral triangles will exactly cover one larger one.

4. Solution:
The radius of the semi-circle is 2 cm. Adding the area of the triangle, the square and the semi-circle gives the total area of the figure.

$$A_{total} = A_{triangle} + A_{square} + A_{circle}$$
$$=\frac{b\times h}{2}+s\times s+\frac{\pi\times r^2}{2}$$
$$=\frac{4\times 3.4641}{2}+4\times 4+\frac{3.14\times(2)^2}{2}$$
$$=6.9282+16+6.28$$
$$=29.2082$$
$$=29.21cm^2$$

5. Solution:
The diameter of the larger semi-circle is 4 ft, so the smaller one has a diameter of 2 ft. Their respective radii are 2 ft and 1 ft. Therefore, the square has a side length of 6 ft (by adding the two diameters).

The area is:

$$A_{total} = A_{semi-circle} + A_{semi-circle} + A_{square}$$
$$=\frac{\pi\times r^2}{2}+\frac{\pi\times r^2}{2}+s\times s$$
$$=\frac{3.14\times(2)^2}{2}+\frac{3.14\times(1)^2}{2}+6\times 6$$
$$=6.28+1.57+36$$
$$=43.85\,ft^2$$

Practice Section D

1. Solution:

There are three errors made in this solution. The first error was made when the diameter of the circle was used in the area calculation instead of the length of the radius. The second error was changing the units from cm^2 to ft^2 halfway through the solution. The third error was made when the area of the circle and area of the rectangle were switched. The larger number was subtracted from the smaller number (so not to get a negative value), but this meant that the values of the areas had to be switched.

The correct solution is:

If the rectangle is twice as long as it is wide, then there are 6 'widths' in one perimeter:

$$Perimeter = l + l + w + w$$
$$= (w + w) + (w + w) + w + w$$
$$54 = (w + w) + (w + w) + w + w$$
$$54 = 6w$$
$$\frac{54}{6} = w$$
$$w = 9\,cm$$

If the width is 9 cm, the length must be 18 cm. The right-angled isosceles triangle has two equal sides of 9 cm giving it an area of:

$$A = \frac{b \times h}{2}$$
$$= \frac{9 \times 9}{2}$$
$$= 40.5\,cm^2$$

The area of the circle is:

$$A = \pi \times r^2$$
$$= 3.14 \times (4.5)^2$$
$$= 3.14 \times 20.25$$
$$= 63.585\,cm^2$$

The area of the rectangle is:

$$A = l \times w$$
$$= 18 \times 9$$
$$= 162\,cm^2$$

The area of the rectangle not covered by the circle and the triangle is:

$$A_{not\,covered} = A_{rectangle} - A_{circle} - A_{triangle}$$
$$= 162 - 63.585 - 40.5$$
$$= 57.915$$
$$= 57.92\,cm^2$$

Practice Section E

Solution:

The area of the entire yard is:

$$A_{total} = 17\,\cancel{yd} \times \frac{3\,\cancel{ft}}{1\,\cancel{yd}} \times \frac{12\,\cancel{in}}{1\,\cancel{ft}} \times \frac{2.54\,\cancel{cm}}{1\,\cancel{in}} \times \frac{1m}{100\,\cancel{cm}} \times 30\,\cancel{yd} \times \frac{3\,\cancel{ft}}{1\,\cancel{yd}} \times \frac{12\,\cancel{in}}{1\,\cancel{ft}} \times \frac{2.54\,\cancel{cm}}{1\,\cancel{in}} \times \frac{1m}{100\,\cancel{cm}}$$
$$= 15.5448\,m \times 27.432\,m$$
$$= 426.4249\,m^2$$
$$= 426.42\,m^2$$

The area of the shed is:

$$A_{total} = l \times w$$
$$= 3m \times 1\,\cancel{yd} \times \frac{3\,\cancel{ft}}{1\,\cancel{yd}} \times \frac{12\,\cancel{in}}{1\,\cancel{ft}} \times \frac{2.54\,\cancel{cm}}{1\,\cancel{in}} \times \frac{1m}{100\,\cancel{cm}}$$
$$= 3\,m \times 0.9144\,m$$
$$= 2.7423\,m^2$$
$$= 2.74\,m^2$$

The area of the flower garden is:

$$
\begin{aligned}
A_{total} &= \frac{\pi \times r^2}{4} \\
&= \frac{3.14 \times \left(67\,\cancel{in} \times \frac{2.54\,\cancel{cm}}{1\,\cancel{in}} \times \frac{1\,m}{100\,\cancel{cm}} \right)^2}{4} \\
&= \frac{3.14 \times (1.7018)^2}{4} m^2 \\
&= 2.2734\,m^2 \\
&= 2.27\,m^2
\end{aligned}
$$

The width of the patio is:

$$
\begin{aligned}
&P = 2w + 2w + w + w \\
&21 = 6w \\
&\frac{21}{6} = w \\
&w = 3.5\,m
\end{aligned}
$$

The length is then double the width or $3.5 \times 2 = 7$ m.

The area of the patio is therefore:

$$
\begin{aligned}
A &= l \times w \\
&= 7 \times 3.5 \\
&= 24.5\,m^2
\end{aligned}
$$

The total area that needs sod is:

$$
\begin{aligned}
A_{sod} &= A_{total} - A_{shed} - A_{flower\ garden} - A_{patio} \\
&= 426.42 - 2.74 - 2.27 - 24.5 \\
&= 396.91\,m^2
\end{aligned}
$$

The cost of the sod will be:

$$
\begin{aligned}
Cost &= Area \times Price \\
&= 396.91\,\cancel{m^2} \times \frac{\$3.50}{2.54\,\cancel{m^2}} \\
&= \$546.92
\end{aligned}
$$