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'Rithmetic and readiness: Exploring meaningful professional development for three elementary mathematics teachers

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ABSTRACT

This study describes the barriers we encountered while engaging in professional development with three elementary mathematics teachers. We adjusted our work with the teachers by creating DVDs of their own students solving mathematics problems, which were then used to achieve the original goals of the professional development. Observational data and interviews with the teachers revealed several belief-oriented prerequisites for meaningful change in mathematics teaching, including that: (a) overarching principles of children's mathematical development can be used productively to assess even those students who struggle the most; and (b) instruction should be driven by the formative assessment of children's thinking in mathematics.

INTRODUCTION

The professional development of teachers at the K-12 level is widely researched and has been identified as the key to bringing about potentially meaningful and wide-spread adoption of current educational reforms (Borko, 2004; Sykes & Darling-Hammond, 1999). We propose that it is critical to explore the practices that would specifically address the predispositions of teachers and would likely lead to positive and lasting experiences in professional development activities. Our work with a group of elementary mathematics teachers in an inner-city school made it clear to us that unless certain cognitive dispositions are in place before teacher educators begin professional development initiatives, teachers will not be in a position to seriously challenge the ways they think and practice. This is particularly true in light of the goals of current reform-oriented professional development initiatives in which teachers are invited to uproot firmly-held beliefs, understandings, and longstanding habits of practice (Thompson & Zeuli, 1999).

For professional development to be successful, teachers must be ready for change; as such, more research should be devoted to understanding the cognitive characteristics of such "readiness." The purpose of this paper is to describe our experiences with three elementary mathematics teachers in an inner-city school in Canada as we attempted to launch the research-based professional development program Cognitively Guided Instruction (CGI; Carpenter, Fennema, & Franke, 1996; Carpenter, Fennema, Franke, Levi, & Empson, 1999). We began our work with the teachers by following the guidelines recommended by Fennema, Carpenter, Levi, Franke, and Empson (1999), with an eye to tailoring the professional development to the teachers' current needs and situations. Fennema *et al*. (1999) recommended four pedagogical guidelines for leaders of CGI professional development activities. The first is based on the notion of mutual respect—that both workshop leaders and teachers respect each other's knowledge and refrain from dictating the thoughts and practices of others. The second guideline is based on listening to teachers so that their development can build on their existing knowledge. The third recommendation for workshop leaders is that they encourage reflection and communication, which not only enhances teacher development, but also provides a model of good mathematics teaching. Finally, Fennema *et al*. recommend that any discussions with teachers about their beliefs be conducted within the context of children's mathematical thinking. This provides the foundation for future changes in beliefs about the teaching and learning of mathematics.

Through our professional development experiences at this school, we identified cognitive characteristics that were present in the teachers at the point at which they became ready and willing to embark on meaningful change as mathematics teachers. In this paper, we first describe the program we initially attempted to implement. Next, we describe the implementation process and the challenges we encountered. Then, we present what we learned through the experience, namely some characteristics of mathematics teachers who are ready and willing to embark on change. We close by suggesting how people designing and developing similar professional development programs might transfer some of the insights we learned.

PLACING TEACHERS' CONCERNS AT THE CENTER OF PROFESSIONAL DEVELOPMENT

Some scholars and teacher educators have noted that large differences exist among teachers in their willingness to participate in professional development programs, at least with respect to mathematics (Franke, Carpenter, Fennema, Ansell, & Behrend, 1998; Fennema & Nelson, 1997; Stein, Silver, & Smith, 1999). Their eagerness to embark on professional change appears to lie on a continuum. At one end are teachers who embrace new ideas and practices (Fennema & Nelson, 1997; Fosnot, 1996; Franke, Fennema, & Carpenter, 1997; Heaton, 2000; Kazemi & Franke, 2003; Knapp & Peterson, 1995; Steinberg, Carpenter, & Fennema, 1994). The notion that instruction should be driven by student thinking and learning is not at odds with such teachers' current pedagogical belief systems; through professional development such as CGI, they form deeper understandings of how mathematics can be viewed as a process of understanding patterns and concepts through problem solving, reasoning, and communication (National Council of Teachers of Mathematics, NCTM, 2000).

At the other end of the spectrum are those who forcibly resist change, often resulting in physical non-participation, psychological non-participation, or both (Janas, 1998). Researchers are only beginning to understand the reasons these teachers might resist change. At the least, this resistance might result from the fact that many professional development programs are only designed for the most motivated of teachers (Borko, 2004). More significant, however, is the human tendency to be more comfortable with the status quo than with the disequilibrium of change (Friend & Cook, 1996). Specifically, teachers who have seen more than one educational fad come and go without lasting effect are bound to become discouraged and less open to new ideas (Gitlin & Margonis, 1995; Janas, 1998). Other factors that may contribute to teachers losing interest in professional development initiatives include not feeling enough ownership in school-wide change and having personal experiences with their own students dismissed or not heard at all (Stein *et al*., 1999).

Change appears to be more likely and professional development more effective if teachers' needs and concerns are taken into account, particularly within the context of their everyday practice (Mewborn, 2003). Indeed, scholars have described the relatively recent evolution of professional development from isolated workshops or courses offered outside the context of the school to support for the teacher that is fully integrated into the curriculum, teacher knowledge and beliefs, instructional practice, and the institutional and political context (Anders & Bos, 1992; Fennema & Nelson, 1997; Loucks-Horsley, Hewson, Love, & Stiles, 1998; Stein *et al*., 1999). Such insights about the professional development of teachers mirror those experienced by people specializing in training of workers for work places other than schools. As the paradigm for teacher professional development has shifted toward a more situated perspective, so the general paradigm for training other types of workers has shifted from providing isolated training classes to supporting workers on the job (Stolovitch & Keeps, 1999, 2004, 2006).

One professional development program for mathematics teachers that has successfully brought change is Cognitively Guided Instruction[1] (CGI; Carpenter *et al*., 1996; Carpenter *et al*., 1999; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). CGI supports teachers in their efforts to raise the mathematics performance of their students, at least toward North American reform objectives (Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000). The current reform movement in mathematics in Canada and in the United States was in large part propelled by the *Curriculum and Evaluation Standards for School Mathematics* document by the NCTM that appeared originally in 1989, revised in 2000. This document outlines various content standards for students in K-12 mathematics, as well as process standards that include problem solving, reasoning and proof, connections, communication, and representation.

In a review of the literature in mathematics education, Kilpatrick *et al*. (2001) described the research rationale for these standards, concluding that effective teachers focus classroom interaction on high-level mathematical tasks and listen to their students' thinking as they grapple with key concepts and invent problem solving strategies. The curricular reforms in Quebec, where the present study took place, are largely influenced by the NCTM's position regarding teaching and learning in mathematics (see the *Quebec Education Program* [QEP], Gouvernement du Québec, 2001). In particular, the QEP highlights three specific competencies that are at the centre of student learning in mathematics and that align with the NCTM's reform principles: solving complex, authentic problems; using mathematical reasoning; and communicating by using mathematical language.

A typical CGI professional development initiative consists of two inter-related elements: ongoing workshops with teachers and sustained support in the classroom (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996). During the workshops, researchers show videotapes of children solving word problems using a variety of strategies and use these clips as anchors for discussion about children's underlying mathematical competencies. Teachers and researchers typically engage in discussions about the level of sophistication of children's strategies and the ways in which student thinking relates to the structure of word problems. Teachers are also invited to relate their evolving understanding of the model of children's thinking to their own curricular plans and classroom activities. The goals of the activities and discussions that take place in CGI professional development initiatives are to support teachers in reform-oriented instruction: teachers reflect on how to engage children in meaningful problem solving and mathematical reasoning as well as on how to create classroom environments where students communicate about the mathematical understanding they are developing.

In a typical CGI professional development context, workshop leaders also offer support to teachers in the school setting; researchers and support staff visit the teachers' classrooms, often weekly or bi-weekly, to engage in impromptu individual or group meetings before or after mathematics lessons. One of the primary goals of the in-class support is to assist teachers in incorporating children's thinking strategies in their short- and long-term instructional decisions.

Although CGI initiatives have been shown to be effective in changing teachers' beliefs and practices in mathematics, more research is needed to clarify the processes and approaches to professional development that are most likely to ensure successful teacher change (Borko & Putnam, 1996; Franke *et al*., 1997; Guskey, 1994). In the case of Steinberg *et al*. (1994), for example, the authors described the changes of a practicing teacher who, at the start of professional development, already embraced many key principles of reform mathematics. Their experiences with this teacher would in all likelihood have been different had she not been as prepared or eager for change. We argue that little is known about the cognitive predispositions that must be in place so that professional development is taken seriously by teachers who are resistant to change, particularly in mathematics. Our study was an attempt to fill this gap.

METHOD

In this paper, we describe our experiences with participants who were not initially ready to jump aboard the CGI program we had planned. Our work with these teachers allowed us to document (a) the beliefs of the teachers as we began our work at the school, (b) the evolving conceptions and beliefs of the teachers during the modified professional development intervention we designed and implemented, and (c) the cognitive characteristics and beliefs of the teachers at the end of the eight-week intervention period. In so doing, we geared our research toward uncovering the belief-oriented dispositions that are possibly prerequisite for change to occur in mathematics teaching. According to Guskey (2002), change can be measured by investigating shifts in teachers' practice, attitudes and beliefs, and the learning outcomes of their students. In this study, we focused primarily on teachers' developing beliefs and attitudes, but we also measured change by examining their growing *knowledge* about children's thinking, a necessary condition for effective instructional practice in the mathematics classroom (Ball, Thames, & Phelps, 2008; Carpenter *et al*., 1996).

At the start, our goals at Vaughn Elementary[2] seemed simple: we planned a professional development program based on CGI, with the intention of documenting this professional development experience as a case study for research purposes. Specifically, we had planned to use CGI to assist three teachers in appropriating conceptual tools for assessing student thinking within the developmental framework described by Carpenter *et al*. (1996) and in using this knowledge to guide their classroom practice. (More information on Carpenter *et al*.'s developmental framework for children's thinking in mathematics can be found in the textbox.)

The content of the CGI program is based on several decades of research on the development of children's thinking in mathematics. In CGI workshops, teachers are given readings about children's problem solving strategies and are shown videoclips of children solving a variety of mathematical problems. The developmental model described to teachers entails learning as a process of progressive abstraction: when children are encouraged to use personally meaningful strategies to solve problems, they often begin by using concrete methods, which, with time and practice, are then replaced by increasingly abstract strategies. The ultimate objective for students in CGI classrooms is that they use entirely abstract methods to solve problems that rely their growing understanding of how numbers work.

At first, children's strategies are governed by the structure of the problem and involve concretely modeling the objects and actions in it. For example, in solving the problem, "I have 8 clay animals and Diana has 12 clay animals. How many more clay animals does Diana have than I?" a child using a *Direct Modelling* strategy might place a row of 8 blocks next to a row of 12 blocks, matching up the blocks one-to-one until the first set is used up. The child would then count the 4 unmatched blocks.

Solving a variety of problems using Direct Modelling, children gradually adopt strategies that are partially abstract, called *Counting* strategies (Carpenter *et al.*, 1999). In solving the problem above, a child using a Counting strategy would not need to physically represent both sets. She might start with the number 8, for example, and count up to 12, keeping track of the number of counts using tallies or her fingers, resulting in the answer of 4 in this case.

Finally, with experience using Counting strategies, children appropriate problem solving methods that capitalize on their growing conceptual understanding. Such abstract strategies, called *Derived Fact* by Carpenter *et al.* (1999), entail using known facts (such as doubles or sums of ten) as the basis for thinking about the problem. Again in the context of the problem above, a child using a Derived Fact strategy might reason, "I know that 8 and 2 are ten, and I'd need another 2 to make 12, so Diana has 4 more clay animals."

Carpenter *et al*. (1999) maintain that this developmental progression is not linear or tidy. The development of mathematical strategies depends on a variety of factors, including the difficulty of the problem and the level of conceptual proficiency of the student. In some cases, the student moves directly from Direct Modelling to Derived Fact strategies, in other cases, the student can solve a novel problem by using a Counting strategy, skipping Direct Modelling altogether. Also, when a student is faced with a problem that is difficult for him, he may revert to less sophisticated strategies to solve it, even if he has used semi-abstract and abstract strategies for other problems in the past.

We intended to offer one workshop a week for eight weeks, and to visit each classroom once or twice a week to provide the teachers with support as they worked with children solving problems. The support was designed to assist the teachers in communicating about mathematics with their students and to help them, sometimes before each class and sometimes afterward, to reflect on their lessons within the context of the CGI framework. The workshops were designed to last one hour each and we had hoped, depending on the teachers' schedules, to offer out-of-class support for about 20 minutes after each classroom visit. While there is no prescribed length for CGI workshops, Fennema *et al*. (1999) maintained that 40 to 50 workshop hours are required for teachers to benefit from them. Because we had a small number of teachers in our sample, we presumed that about half that amount of time would be sufficient to achieve our professional development goals.

In this section, we first describe how we selected and entered the environment and how we selected participants. Next, we describe the process we followed to implement professional development with the three teachers—our initial attempt at doing so and how we revised the program in response to some of the realities of the Vaughn Elementary environment. We then outline how we documented the case study: (a) how the data were recorded and coded, and (b) how the data were analyzed within a theoretical framework to assess teacher change. Finally, we explain how we assured the credibility and trustworthiness of the data.

Selecting and Entering the Environment

We spent some time searching for one site in which to conduct our research on professional development. Given that our approach to professional development is to work with teachers over an extended period of time, lack of resources prevented us from collecting data at more than one school. As we were searching for a site, a researcher on another project informed us about the teachers at Vaughn, who were seeking assistance in teaching mathematics. Because the interest level appeared high among these teachers and because professional development is an important foundation for change, we took the opportunity to conduct our research at Vaughn.

Vaughn is a severely socioeconomically (SES) depressed urban elementary school covering Kindergarten through grade 6. During the year the study was conducted, there were 238 students in the school. The student body at Vaughn has similar characteristics to inner-city schools in the United States: at the time of data collection, the school ranked in the bottom 10% on a government index of income level and in the bottom 50% on an index of socioeconomic status, measured in part by the mother's level of education and in part by the percent of families in the school in which no parent is employed[3].

Selecting the Participants

We began the project by contacting the principal at Vaughn, who in turn presented information about our professional development initiative to the teachers at a staff meeting. Initially, only four teachers expressed interest, and we held our first meeting with them a few weeks later. Shortly thereafter, we held our first "formal" CGI workshop, during which one of the teachers withdrew her participation in an unusually dramatic way. The three teachers who remained part of the project until its completion were Gabby, a first-grade teacher, Belinda, a second-grade teacher, and Jora, who taught third-grade remedial mathematics to a class of fifth-grade students. Both Belinda and Jora were veteran teachers; Belinda had 17-years' experience at the elementary level, and Jora had taught at both the elementary and secondary levels for a combined 30 years of teaching experience. Jora had also acted as a special education consultant during her career. Gabby was in her first year of teaching; after obtaining an undergraduate degree in Sociology, she attended an accelerated teacher education program that earned her a Bachelor's degree in Kindergarten and Elementary Education.

Conducting the Workshops: The First Try

The research team consisted of the principal investigator (first author) and four research assistants, whose main responsibility was to assist in the implementation of the CGI initiative and in the coding of the collected data. The two undergraduate research assistants were both enrolled in the teacher training program at our institution, which covers CGI as part of its mathematics methods curriculum. The two graduate assistants were actively involved in research in mathematics education, and as such were well versed in CGI and children's thinking. In addition, the team held weekly research meetings in which critical components of CGI were reviewed and applied to Vaughn. Our initial plan when we began the project was to conduct the CGI program largely as described in the literature (e.g., Fennema *et al*., 1999; Hiebert *et al*., 1997). Specifically, we planned to (a) conduct formal workshops covering problem types and children's strategies, and (b) provide sustained support in and out of the classroom.

CGI workshops. In the first workshop, we showed the teachers a video from the CGI collection, featuring children's solution strategies to various word problems and individual interviews with CGI teachers. This viewing was then followed by discussions about children's problem solving and its relation to the reform. In the next part of the workshop, the participants examined different addition and subtraction problem types (as described in *Carpenter et al*., 1996) and were encouraged to explore the relative difficulty of several such problems. Following a brief description on problem types presented by the researchers, the teachers attempted to identify and sort problems that were written on cards. The teachers discussed their thoughts about the card sort with the researchers.

The second and third workshops were conducted more informally. In them, addition and subtraction problem types were reviewed, and the development of children's thinking from direct modelling to using derived number facts was introduced (see textbox). The teachers were also shown different videoclips of CGI teachers in the classroom; the techniques and strategies used by these teachers to assess student thinking formed part of the ensuing discussion. During both the second and third workshops, issues related to curriculum, special needs, and district and governmental constraints surfaced and formed the basis of discussion. The content of our interactions with the teachers was centered primarily on their perception that too many obstacles stood in their way to adopt CGI in their respective classrooms.

Sustained support. During this effort, our research team visited the teachers' classrooms weekly or bi-weekly. Each class period lasted 50 minutes, and the research team was present for the duration of the class. During each visit, some of the assistants provided hands-on support to the teachers by working with the students either one-on-one or in small groups as they solved mathematics problems, and other assistants took detailed field notes based on their observations of the classroom activity. The observational data were intended to gather more information about the individual teachers' needs in their mathematics classrooms. In addition, the team engaged the teachers in impromptu individual or group meetings before or after mathematics lessons. These meetings, which lasted anywhere from 20 to 45 minutes, were designed to assist the teachers to reflect on their students' thinking during previous lessons and on their goals for upcoming lessons. Our objective in providing the sustained support was to assist teachers in incorporating children's thinking strategies in their short- and long-term instructional decisions.

Although we expected our plans to be implemented as originally articulated, we nevertheless entered this study anticipating some resistance from teachers. From descriptions of previous efforts to implement CGI (e.g., Fennema *et al*., 1996), we recognized that teachers who begin CGI often express concerns about several issues, including finding the techniques that are most effective in making children's thinking explicit, organizing the classroom so that there is enough time to listen to students, and covering all the topics in the mandated curriculum. More specifically, because almost half of the students at Vaughn were coded with some form of cognitive or behavioural disorder, we found from our previous efforts that some teachers at the school were historically resistant to outside interventions that did not directly account for the special needs present there.

We thus proceeded cautiously with our project. One way we did so was to plan for a certain amount of action research to uncover possible ways that CGI could be tailored to the specific needs of the teachers. The resistance we experienced, however, was considerably stronger than we had anticipated.

For example, at the first workshop session, one of the original participants proclaimed her disinterest in any and all of the material presented, announced her withdrawal from the project publicly, and left the room. In later weeks, as we progressed through the video clips, classroom visits, and informal one-on-one and group discussions, we encountered mounting skepticism from the remaining three teachers. They were reticent to try the CGI approach because of the perceived loss of control and order in the classroom they thought it might bring. The teachers also expressed concern about the expectations of other teachers in the school, particularly those who were to receive their students the following year. Finally, the teachers were also concerned about whether students would meet the stringent competency-based expectations in mathematics, which had been established by the school board for grades two, four, and six. These expectations were driven, in large part, by the recently-mandated curricular reforms set by the provincial government (Gouvernement du Québec, 2001).

The most significant concern for the participating teachers, however, was that they did not perceive the approach as useful or applicable for students at Vaughn. The teachers believed that their students' special needs prevented them from generating the meaningful and appropriate strategies seen on the CGI videotapes. Although the teachers found the videotapes of other children solving problems impressive, they did not believe the clips were relevant to their current situations because of the large proportion of students with special needs at Vaughn and in their respective classrooms. Not surprisingly, therefore, the teachers told us that they believed that the theoretical knowledge presented in CGI workshops was not useful for teaching mathematics to their students.

We were not the first to experience resistance in implementing CGI. Encountering similar concerns from teachers, Franke *et al*. (1998) were able to steer the conversations back to student strategies. We were not; the teachers at Vaughn did not believe that such "generic" student thinking was applicable to them. These beliefs, although not measured directly, led us to recognize that the teachers were not ready for the CGI program we had planned. As a result, their strongly-held opinions about the perceived usefulness of the program were the catalyst for our change in plans.

Conducting the Workshops: The Second Try

Instead of abandoning our work with the teachers at Vaughn, we implemented a swift change in direction with respect to the implementation of the same professional development goals. More specifically, after approximately two months of working with the teachers inside and outside of their classrooms, it became obvious to us that a different approach to professional development was necessary, at least with these teachers. Supplementing CGI with principles of adult learning (Knowles, Holton, & Swanson, 1998), we predicted that making the professional development more relevant and immediate would more likely have the desired impact on the teachers' beliefs about children's thinking and their understanding of the current reform climate in Québec. Called andragogy, these principles of adult learning suggest that adults have different learning needs and approaches than children. Among the specific and practical implications of andragogy are that adult learners are pressed for time, goaloriented (which includes the need to see how a given learning program helps them achieve their goals), and experienced—they bring a lifetime of previous knowledge and practical skill to any learning situation (Knowles *et al*., 1998).

One key adjustment made so that the content would seem more relevant within the teachers' own context involved dispensing with the CGI videotapes and substituting them with clips we made ourselves of students from the teachers' own classrooms solving mathematics problems. Over eight weeks, we implemented what we call here the video-viewing intervention, where we met with the teachers individually to view and discuss these clips. We believed this would help the teachers see more directly how the program might be used in their own environment to help their students achieve the learning objectives of the CGI program. We selected the students for these interviews by asking the teachers to identify a small number from their classrooms who represented a wide range of mathematical performance (low, middle, and high performance). Three students from Belinda's class, three from Gabby's class, and five from Jora's class were selected. Permission was sought, and granted, from the students themselves and their parents.

All student interviews were conducted within a two-week period before the video-viewing intervention began. The student interviews lasted anywhere from 20 to 45 minutes, and were conducted by a member of the research team during the school day in a quiet area, such as the library. All interviews were digitally videotaped and burned, unedited, onto separate DVDs that were later segmented into chapters for ease of viewing and reviewing. The DVDs were later shown to the teachers as a basis for discussion of children's thinking in whole number operations. Because of the resistance we experienced from the teachers in our initial work with them, we assume that few changes were made early on to the mathematics instruction in their classrooms. We thus argue that it is highly unlikely that our early professional development activities with the teachers had any effect of the children's problem solving documented in the videoclips.

The student interviews began with a number of preliminary counting tasks aimed at putting the children at ease; these tasks also helped us to determine whether the students possessed the prerequisite counting skills to continue with the remainder of the interview, which comprised a variety of word problems (Baroody & Wilkins, 1999). The interview protocol consisted of two tracks, only one of which was used during the interview depending on the student's competencies with whole number operations. Both tracks consisted of one part-part-whole, two multiplication, one measurement division, and one partitive division word problem. Track 1 contained single digit operations and Track 2 contained double digit, but the corresponding problems were isomorphic in structure.

During an eight-week period, we held four one-on-one video-viewing sessions with each teacher. The four sessions were used to view and discuss the DVDs on which were recorded the problem solving strategies of the three students from the teacher's class (five in the case of Jora). Each session lasted between 60 and 90 minutes and together comprised the bulk of the modified professional development we conducted with the teachers. In these sessions, the teacher and one of the researchers on the team viewed the DVDs together and discussed the unique characteristics of her students' strategies. This approach provided opportunities for the teachers and researchers to work collaboratively on understanding the mathematics in their own students' strategies and on ways to incorporate their observations into their teaching.

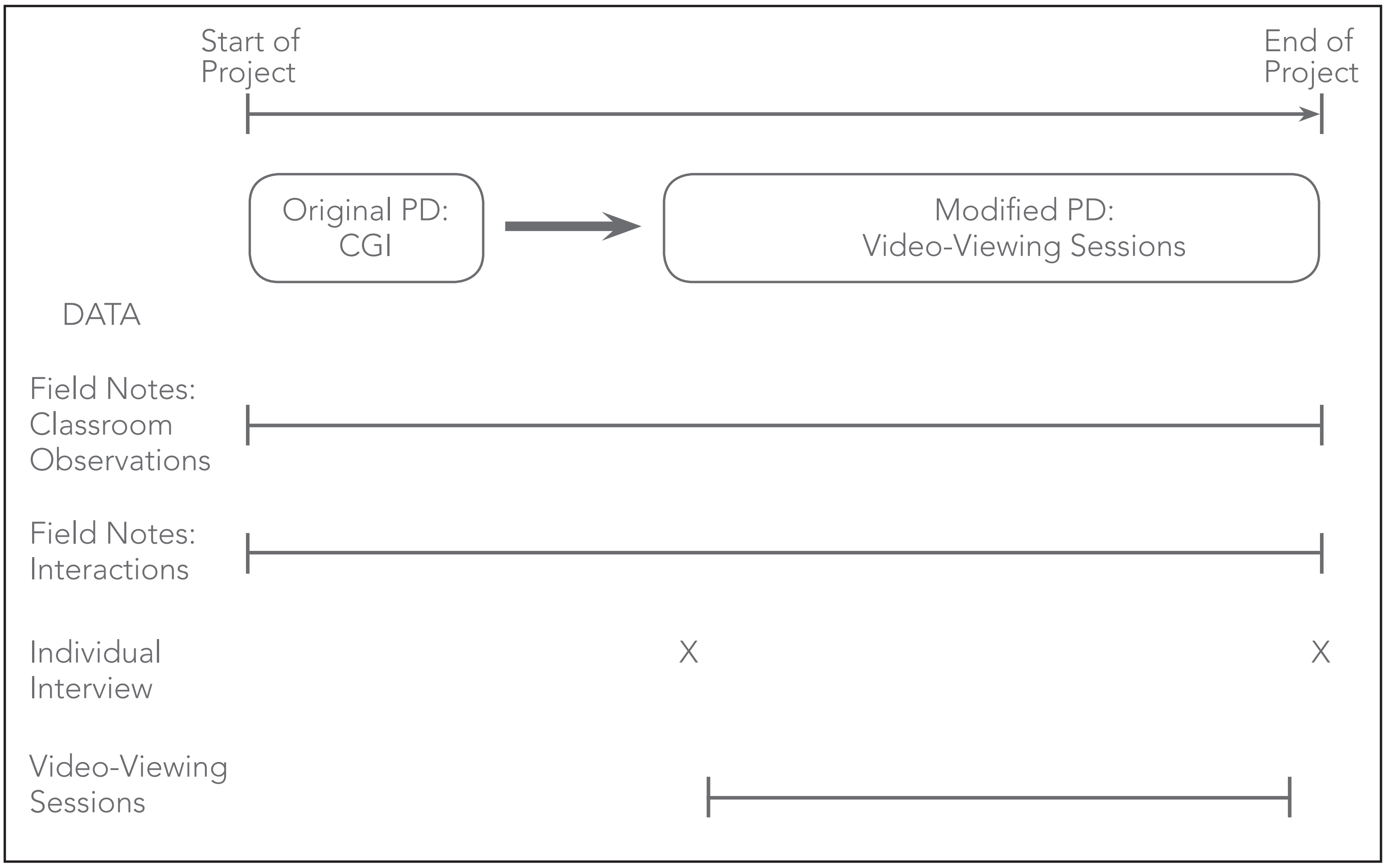
The researchers previewed each video before each session and prepared a number of key points to discuss with the teachers, such as strategies used by the child, competencies and difficulties, possible explanations for observed results, and implications for instruction. During each video-viewing session, the main goal for the researcher was to use the teacher's own student as an anchor for the introduction and subsequent reflection on the CGI framework. The strategies in this framework were presented to the teachers on an as-needed basis and as such constituted "just-in-time" scaffolds as they reflected on their students' specific solution strategies. The use of these school-specific videos situated this professional development initiative in a context that was more personally relevant to the teachers, and offered tools for growth and reflection, which included the videoclips themselves as well as expert guidance, articles, and diagrams (Lave & Wenger, 1991; Rogoff, 1990).

Despite the adjustments to the program, our overarching goals remained unchanged. Our primary objectives were still to provide the teachers with information about children's strategies and to assist the teachers in moving from reflecting on their own specific students to reflecting on the thinking of hypothetical students, such as those presented on the CGI videotapes. Our challenge was to find the most effective balance between providing the resources and support that the teachers needed for their students and achieving our own goals as teacher educators to using empirically-established approaches for affecting change in elementary mathematics classrooms. In doing so, we needed to be sensitive to the development of teachers' reflections from the particular (analyzing their own students' strategies) to the hypothetical (thinking about other students' strategies or situations involving hypothetical children).

Documenting the Case Study: Sources of Data

Overview. We documented our experience in a variety of ways. We collected (a) field notes from classroom observations, (b) field notes from informal interactions with the teachers during and after the school day, (c) digital audio-recordings of individual interviews with the teachers, and (d) digital audio-recordings from the video-viewing sessions. These data were collected at different points during the study. Figure 1 illustrates when each of these four data sources were collected between the start and end of our work with the teachers at Vaughn.

Figure 1. Timing of the data collection from the start to the end of the project



The observational field notes were used at the start of the project primarily to assess the teachers' specific individual needs and to evaluate how CGI might be blended with their individual instructional styles. Because observational data are not appropriately sensitive to teachers' beliefs (Pajares, 1992), we used the data from the two individual interviews and video-viewing sessions as the primary indicators of belief change. Nevertheless, the field notes compiled from the informal interactions with the teachers as well as the observational records were valuable data as well and were used in two ways. First, we used these field notes to make reasoned inferences about the teachers' beliefs and conceptions. Second, these data served as context within which to interpret specific comments made by the teachers in the interviews and video-viewing sessions.

Our observations in the teachers' classrooms revealed a variety of instructional activities and approaches, but within the first month of the classroom visits, we realized that their practice was not organized in a way that allowed for the appropriate accessing and evaluation of students' underlying conceptual competencies in mathematics (Ginsburg, 1996). As a result, we adopted an additional goal in our work with the teachers: to encourage them to reflect more deeply about the characteristics of effective mathematics learning environments. These characteristics are based on comprehensive reviews of effective mathematics teaching (e.g., Fuson, 2004; Hiebert *et al*., 1997; Kilpatrick *et al*., 2001; Kilpatrick, Martin, & Schifter, 2003) and are listed in Table 1. Effective mathematics instruction involves centering activity on problem solving and on the conceptual underpinnings of studentinvented and standard procedures. Instruction and assessment are based on how children progress in their thinking about important mathematical concepts and ideally should be characterized by asking open-ended questions, providing appropriate models and scaffolds, and giving students time to think about problem solving. Effective teachers recognize the variety of strategies children bring to the classroom and that they learn best by concretely modelling problem structure in meaningful ways before making connections to more abstract or symbolic representations of rule-based solutions (see also Osana, Pitsolantis, & Zimmerman, 2006).

Table 1: Characteristics of Effective Elementary Mathematics Learning Environments

I. Mathematics Teaching
- Teachers should provide time in class for students to think about mathematics.
- Mathematics teaching should be centered around problem-solving (problems not exercises).
- Teachers should spend time asking open-ended questions.
- What a teacher does in class is dependent on what the students think and do.
- Guidance is given when student is stuck. The teacher does not always solve or model the answer when a student is at an impasse.
- Students are allowed to concretely model problem or structure.
- Students should compare strategies or solutions.
- Teacher values all ideas and methods.
- Teacher places emphasis on the construction of meaningful strategies and - conceptual understanding (versus procedural competence and final product).

II. Student Learning
- Students learn by explaining/sharing their reasoning (communication).
- Students do not always need direct instructions to solve problems. They learn by being given the opportunity to explore the mathematics underlying the problem.
- Students can use different strategies to solve problems.
- All students are capable of thinking mathematically when solving problems, even though they progress at their own pace.
- Students can learn by examining their mistakes.
- Learning mathematics implies moving from the concrete towards more abstract/ efficient strategies.

While the teachers were not familiar with the scholarly literature describing these characteristics, we found that they were nonetheless knowledgeable of many of the general issues surrounding them. For example, the teachers were aware of the importance of problem solving in the mathematics classroom and that students can learn by examining their mistakes. Because of their knowledge, we were able to explore the teachers' beliefs about many of these characteristics in our discussions and reflections during the professional development activities. Other characteristics were admittedly less apparent to the teachers. The notion that students do not always need direct instruction to solve problems, for instance, was novel to all the participating teachers. Nevertheless, we encouraged the teachers to explore these less intuitive ideas by illustrating them with specific instances on the student DVDs and in their own classrooms. We also asked straightforward questions about these characteristics in all our discussion with the teachers, including during impromptu conversations, video-viewing sessions, and individual interviews.

Primary data sets. The primary data sets used to document teachers' beliefs were generated from the individual interviews and video-viewing sessions. With respect to the individual interviews, we conducted these with each teacher twice: (1) at the moment when we shifted our approach to professional development but yet before the video-viewing sessions began, and (2) after the video-viewing sessions were completed. The first author individually interviewed each teacher approximately three to four days before the first video-viewing session took place and again within four days following the final video-viewing session. We shall refer to these teacher interviews as the pre video-viewing and post video-viewing interviews.

Questions on the pre and post video-viewing interview protocols were general in nature and related to teaching experience, beliefs about mathematics teaching and students' mathematical ability, current mathematics reform, instructional techniques and activities, curriculum, classroom materials, and assessment. The protocol was semi-structured. The research team specified the questions ahead of time, but the interviewer took the opportunity to deviate slightly from the pre-assigned protocol to further explore particular issues or concerns raised by the teacher during the pre and post video-viewing interviews. All interviews were recorded and the transcriptions were coded using a rubric that will be described in the next section.

The video-viewing data were collected by digitally recording the conversations held between a member of the research team and the teacher as they discussed the problem solving strategies used by the students on the DVD clips. There was no pre-established protocol for these conversations, although the discussions were loosely directed by a number of key points specified ahead of time by the research team. These points were specific to the mathematical understanding and skill of the particular student featured in the DVD. The interviewer's main objective was still to expose the teachers to the model of children's mathematical development central to the CGI program, but this time by using the videoclips to offer illustrations in the context of the teachers' own specific classrooms. Because we were committed to making the experience meaningful for each teacher, we allowed many of the discussions surrounding students' problem solving to emerge naturally.

Recording and Analyzing Video-Viewing Data

All video-viewing sessions with the individual teachers were digitally audio-recorded and imported into HyperResearch®, software for qualitative data management and coding. Only the useable portions of each session were coded and analyzed. Useable portions were those discussions between researcher and teacher about a student strategy that was clearly visible to all researchers on the project. For example, a student who used a discernable direct modelling strategy, such as Joining All for a Join Result Unknown problem, was considered usable; even strategies that did not correspond to the problem structure (such as a Joining All strategy for a partitive division problem) or those that involved the standard algorithm were included for coding, on the condition that they were discernable from the videoclip. Discussions about problems for which the student did not provide any visible strategy or simply stated, "I just know" or "My brain told me" when asked to solve a problem or justify a solution were not coded.

All useable portions were subsequently coded using the rubric found in Table 2. Two main codes were used to categorize each utterance made by the teacher during the session: Particular and Hypothetical. Particular utterances were straightforward observations or descriptions of how the student solved the problem, and included statements such as, "He used blocks here" or "He counted out loud." We further divided the Particular category into two subcategories that increased in level of abstraction. We used the Particular 1 (P1) code for any descriptive or observational statement about the specific student in the video clip. We used the Particular 2 (P2) code for similar descriptive statements of other students in the teacher's current classroom. For example, we coded statements such as, "I've seen other students in my class count by tens like that" or "Some of the students in my class this year use blocks to solve these types of problems" as P2.

Table 2: Rubric for Coding of Particular and Hypothetical Utterances During Video-Viewing Sessions

Particular Utterances: Focus on Description Only

P1. Focus on specific child in video by describing actions or statements.

P2. Focus on other children in the current mathematics classroom or the group of students as a whole. Emphasis on description of actions or statements.

Hypothetical Utterances: Connections Made to Research or Theory

H1. Use of research-related information or theory to interpret specific child's actions or statements.

H2. Use of research-related information or theory to interpret actions or statements of other children in the current mathematics classroom or the group of students as a whole.

H3. Utterance based on research or theory as a general statement about children's thinking.

The Hypothetical category included any statements that made connections to research or theory about children's thinking. Through further subdivisions, we identified additional levels of abstraction, which we called Hypothetical 1, Hypothetical 2, and Hypothetical 3. We coded statements that linked the child's strategy in the video to general principles about children's thinking as Hypothetical 1. For instance, we placed a statement such as, "I see a Counting On To strategy here" in this category. We coded statements that related a strategy seen on the video to other students in the teacher's classroom, such as, "Many of the students in my class started off as direct modellers at the beginning of the year" as Hypothetical 2. Finally, we coded any utterance made about general principles of children's thinking as Hypothetical 3. Examples of Hypothetical 3 statements might include, "With practice, students will move from derived number facts to recall," or "counting is more abstract than direct modelling." We also accepted statements about research or theory that did not use precise CGI terminology, such as "direct modelling" and "counting down to." For example, we would place a statement such as, "I would like to get my students to the point where they don't need to physically represent both numbers in the problem" in Hypothetical 2.

All useable portions of the video-viewing sessions were independently coded by two trained raters. The raters subsequently met to compare their codes and resolve any discrepancies through discussion. The frequency of each subcode (such as Particular 1, Particular 2, Hypothetical 1, Hypothetical 2, and Hypothetical 3) was measured by calculating the proportion of the number of teacher utterances of the type in question to the total number of teacher utterances coded. Multiplying the ratio by 100 yielded a percent, which we used as a measure of the amount of time each type of utterance occurred over the course of the video-viewing session. The total number of utterances used in this analysis included only those made by the teacher, and not the researcher, and also included an "Other" category, in which utterances that were not directly related to children's thinking and problem solving were placed. An example of an utterance place in the Other category would be, "He was always better at language arts than math."

Recording and Analyzing the Pre and Post Video-Viewing Interview Data

The transcripts of the pre and post video-viewing interviews with teachers formed a distinct set of data from those collected during the video-viewing sessions. We coded these data using a rubric that was based on the list of features found in Table 1. The rubric was based on key reform principles in mathematics education (e.g., Fuson, 2004; Hiebert *et al*., 1997; Kilpatrick *et al*., 2001; 2003; NCTM, 2000) and was constructed before any coding was performed. We subdivided the codes into two categories: the first group of codes related to teaching mathematics and the second to aspects of student learning in mathematics.

Our coding of the pre and post video-viewing interview transcripts entailed examining phrases, sentences, or groups of sentences, and attaching one or more codes to each meaningful chunk of data (Bereiter & Bird, 1985). Any statement or groups of statements that were inconsistent with or directly contradicted the code in question were labeled with the same code label, but with a prime [‘] attached (such as A’). Two trained raters independently coded all interview transcripts and subsequently met to resolve all discrepancies.

Our analysis of the interview data involved measuring growth from the pre video-viewing interview to the post video-viewing interview. For each of the interviews, the frequency of each code was calculated by subtracting the number of the prime [‘] codes from the number of non-prime codes. The net growth frequency for each code was calculated by subtracting the pre video-viewing interview frequency from the post video-viewing interview frequency. To illustrate, consider code A: Teachers should provide time in class for students to think about mathematics. We calculated the net growth for code A using the following formula:

> Net Growth for Code A=
>
> Post video-viewing interview [Number of instances of A - Number of instances of A’] – Pre video-viewing interview [Number of instances of A - Number of instances of A’]

Thus, net growth for each of the codes for each teacher incorporated not only the statements that were consistent with reform principles, but also took into account the statements that either contradicted or were not in line with reform principles.

Model Used to Assess Change

The theoretical framework used to assess the teachers’ change was proposed by Franke, Fennema, and Carpenter (1997), who created a model for documenting teacher change in the context of CGI professional development. This model emerged from the research the authors conducted with practicing teachers who demonstrated considerable growth, both in their practice and in their beliefs, about how children learn mathematics (see also Carpenter, Franke, Jacobs, & Fennema, 1998). Franke *et al*. (1997) proposed that teachers undergo belief development in the following stages, which are consistent with the maturation scale of adult learners proposed by Knowles, Swanson, and Holton (1998). Level 1 teachers believe that children cannot solve problems without being told explicitly how to do so. Teachers at this level look for outside help when seeking guidance on their instructional practice. At Level 2, teachers begin to understand that children can use their own strategies for solving some mathematics problems, but that there are many conditions, such as lack of mathematical ability, under which children still need to be shown how to carry out standard procedures. At Level 3, teachers believe that children solve problems on their own in a variety of ways and that the problems they tackle govern the types of strategies children use. Such teachers are beginning to understand how their students’ thinking can effectively guide their instructional decisions. Finally, Level 4 teachers believe that their practice should be governed by what their students think and do as well as how their competencies relate to the underlying mathematics.

Ensuring Trustworthiness and Credibility

To ensure that our conclusions were ultimately trustworthy, we triangulated our data, relying on the large number of data sources described in this section (i.e., field notes on interactions and observations, pre and post video-viewing interviews, and video-viewing data). To verify that the interpretations of the source data are credible, the transcripts of all pre and post video-viewing interviews and video-viewing sessions were coded by two individuals, and discrepancies were resolved through discussion. In addition, all conclusions were reviewed and approved at each stage by all members of the research team, lending a reasonable degree of credibility to our claims. Finally, the second author, who was not a researcher on the team, served as an auditor of the final report.

RESULTS AND CONCLUSIONS

This section, divided into three parts, addresses what we learned and how we interpreted our findings. First, we describe the belief characteristics of the teachers as they began their work with us. Second, we describe evidence of change as the teachers participated in the eight-week video-viewing intervention. The analyses are based on the coding of the video-viewing transcripts as well as the pre and post video-viewing interviews. Finally, we conclude with a discussion on how our findings assisted us to speculate about the belief-oriented dispositions that appear to be necessary for professional development to make meaningful change in teachers' practices.

Teachers' Starting Belief Characteristics

Gabby. Being a first-year teacher, Gabby was open to many new ideas and willing to explore alternative ways to teach mathematics. Shortly after she joined the school mid-way through the year, she came to the conclusion that her first-grade students were afraid of mathematics. Because of this, her main goal was affective in nature: to allay her students' fears and to create a welcoming classroom environment. Two of the adjectives she used to describe herself as a mathematics teacher were "hands-on" and "interactive."

Our informal observations of her classroom, however, revealed that many of her activities were geared toward low-level factual and procedural knowledge. For instance, one activity involved counting to 20 in chorus and subsequently repeating the same exercise several times. Another activity involved working on addition and subtraction number sentences. Gabby would allow students to use manipulatives, such as blocks, but would provide very little time, if any, for students to model solutions with them. She often required her students to supply quick answers to the exercises she placed on the board or on worksheet pages. She rarely prompted her students to explain their thinking, and we did not observe her modelling alternative solutions to problems. Indeed, when asked how she decided what should be taught in her mathematics class, she cited *Interactions* (Hope & Small, 1994) as her guide, a structured and sequenced curriculum that was strongly recommended for the lower grades at Vaughn. This evidence points to Gabby being a Level 1 teacher (within the Franke *et al*. framework) at the start of the intervention.

Jora. Jora's fifth-grade remedial mathematics class was focused on rote reproductions of algorithms and the memorization of procedural rules; her class was not centered on problem solving. Students were rarely asked to communicate with each other or with her about their mathematical thinking. Classroom observations revealed an emphasis on progressing rapidly through exercises in the context of either individual seatwork or choral recitation and providing immediate feedback on the accuracy of students' calculations.

During the pre video-viewing interview, Jora described herself as "teacher-directed," "structured," and "non-democratic." She argued that her structure and teacher-governed approach was necessary because of the nature of the special needs in her class, some of which were behavioural. She stated, "It's got to be structured in that fashion for them because they are the types who would be wanting to just get out of control in no time whatsoever."

Jora attended professional development workshops as often as she could to "pick up" ideas and techniques that might be beneficial for the students with special needs in her classes. She stated, "There's so much I don't know… I really believe that, and if I can pick up one little idea from you, or from anything that I attend that's going to make a difference in here, then I want to do it. I don't want to miss it." The pre video-viewing interview and classroom observations with Jora revealed that, like Gabby, Jora was a Level 1 teacher as described by Franke et al. (1997).

Belinda. In the pre video-viewing interview, Belinda described herself as continually searching ("groping" in her words) for the types of activities that would help her second-grade students become excited about mathematics. She also described herself as insecure because she was never certain about whether she was "getting the results that [her students] need to go on to next year." Her purpose for participating in professional development projects was to try different activities year after year because "it's much more interesting, much more fun for me to present something fresh."

Informal observations of Belinda's teaching and conversations with her revealed that one of her primary goals was to provide students with the real-world skills that would enable them to use mathematics in practical contexts outside school. One example she provided was to help students know that they were receiving the correct change when purchasing an item in a store. Although manipulatives were present in her classroom, they were rarely taken out of the cupboard for the children to work with. When introducing new material in class, Belinda almost immediately presented her students with the standard symbolic representations of the target concepts and students often practiced procedures individually on collections of exercises in their notebooks. She also cited *Interactions* as her primary guide in terms of the content taught in her mathematics classes. Although it is possible that Belinda was a Level 2 teacher at the start of the intervention, it is clear that she looked to outside help, and not her students, for guidance on instructional decisions. Because this is a key characteristic of teaching at Level 1, that is the level at which we placed her at the start of this study.

Evidence of Change

In this section, we present evidence on the possible impact of the CGI training, in its adjusted form, on the beliefs of Gabby, Jora, and Belinda. The first section describes discussions of student problem solving as explored in the video-viewing sessions. The second examines changes in teachers' beliefs about effective mathematics teaching and learning as gleaned from the interview data.

Video-viewing sessions. One of the significant findings is that overall, the teachers' observations, comments, and reactions during the video-viewing sessions became more hypothetical over time, as evidenced by an analysis of their comments. The percentage of each type of Particular and Hypothetical code made by the teachers during the first and last video-viewing sessions respectively can be found in Table 3.

Table 3: Percentage of Utterances During the Video-Viewing Sessions by Code and Teacher

	Code											
	Particular				Hypothetical						Other	
	P1		P2		H1		H2		H3			
	VS_Start	VS_End	VS_Start	VS_End	VS_Start	VS_End	VS_Start	VS_End	VS_Start	VS_End	VS_Start	VS_End
Gaby	38.52%	60.18%	12%	1.35%	10.07%	21.37%	1.98%	2.14%	3.17%	3.52%	35.25%	11.45%
Jora	85.45%	44.56%	1.15%	0%	9.59%	42.79%	0%	0.70%	0%	0%	3.82%	11.94%
Belinda	27.98%	54.19%	6.48%	0%	15.65%	21.76%	0%	0%	0%	0%	49.88%	24.05%

Note. VS_Start = First viewing session; VS_End = Final viewing session. The transcripts of the first and fourth video-viewing sessions were used for Jora and Belinda in this analysis. The transcripts of the first and third video-viewing sessions were used for the analysis of Gabby's growth because very little useable data were available in the fourth session.

Gabby showed a general move from Particular to Hypothetical utterances from the first to the third video-viewing session. Although we observed an increase in the first level of Particular utterances (P1) from the first to the third viewing sessions, the data revealed a decrease in Particular utterances of the P2 variety. That is, during the first session, 12% of Gabby's statements were descriptions of the types of strategies other students generally use in her classroom. During the third session, less than 2% of her utterances were at this Particular level. At the same time, we observed a total increase of 11.81% in Gabby's Hypothetical utterances; in particular, she spent more of the viewing discussion time interpreting what the student on the video was saying and doing in relation to theoretical principles of student cognition in mathematics. As can be seen in Table 3, Gabby's utterances became more Hypothetical at the other levels (H2 and H3) during the third session, but the increase was not as impressive as at the H1 level.

Jora showed the most growth of all three teachers. Over 85% of her utterances during the first video-viewing session were focused on the student depicted in the video and were characterized as simple descriptions of the student's actions and statements with no interpretation. During the final session, such descriptive comments were reduced to less than half of her total utterances (44.56%). While few of her comments focused on the other students in her class, even at a particular level, during the first viewing session (1.15%), none of her comments was at this Particular level during the final session. Finally, we also observed a large jump in the number of Hypothetical comments made between the first and final viewing sessions. Although fewer than 10% of Jora's comments during the first session incorporated theoretical interpretations of the students' problem solving, almost 43% of her utterances were of this variety during the final session.

Although the change was not as marked, Belinda's growth from Particular to Hypothetical mirrored the pattern found in the transcripts for Gabby's sessions. Although the number of particular utterances at the P1 level increased for Belinda as well, the number of Particular utterances at the P2 level decreased from 6.48% to 0%, and the proportion of Hypothetical utterances increased at level H1 increased from 10.07% to 21.37%. That is, Belinda made substantially more connections to theoretical principles when interpreting the student's problem solving seen in the video in the fourth session compared to the first. Belinda made no hypothetical statements at levels H2 and H3 either before or after the video-viewing sessions.

Teacher interviews. For each of the pre and post video-viewing interviews, the frequencies and calculation of net growth for each code is listed by teacher in Table 4.

Table 4: Video-Viewing Interview Growth Scores for Three Teachers

	Beliefs	
	Mathematics Teaching	Student Learning
Gaby	17	14
Jora	14	11
Belinda	9	3

Note. Growth scores were calculated by subtracting the net number of reform-oriented principles mentioned on the pre video-viewing interview from the net number of reform-oriented principles mentioned on the post video-viewing interview.

Gabby's overall growth in the area of mathematics teaching was about equivalent to her growth in the area of student learning, with a net total growth of 17 and 14 reform-oriented statements respectively. In the area of mathematics teaching, we observed substantial growth in her belief that mathematics teaching should be based on student thinking as opposed to a prescriptive curriculum or textbook. The following excerpt from Gabby's post video-viewing interview describes this change:

I: What things, if any, did you get out of our discussions about mathematics teaching and learning?

G: ...seeing how I could take the CGI problem solving and apply it to the curriculum to the best of my ability. I saw the…benefits of…giving [the students] the power to figure out how they can solve it.

I: What do you think some of the benefits are that you let them solve it?

G: ...It showed me more than say filling out p. 50 in the book would show me, way more. And I taught from that some of the time. Like I would do my next lesson from what that showed me.

In addition, Gabby grew in her understanding that students use different strategies to solve problems and should be given the opportunity to explore the meaning of strategies during mathematics class as opposed to being told which procedures or rules to use. An excerpt from the post video-viewing interview illustrates her change in thinking:

(Continued from the excerpt above):

G: …when [my students] went around the groups showing, "I did this," "I solved it," that showed them different ways to solve it. And so first of all it's giving them, "Ok, I'm not that stupid, we all solve it in different ways," which is very good. They need to know that we're not all the same, we're not going to think about math the same way.

(Later in the interview):

I: Has your curriculum changed? The activities you do in class?

G: Definitely. I don't do as much in the book as I did. … And [my students] are getting more out of the activities I do in the classroom than in the book…. And more of them exploring things like exploring the shapes and we were doing the geoboard…just exploring on their own. I think that's really important. I think it gives them responsibility and independence.

We concluded that, by the end of the study, Gabby had progressed to Level 2 in the model of development proposed by Franke *et al*. (1997).

As evidenced by her statements on the individual interviews, Jora also showed substantial growth from the beginning to the completion of the intervention. The total number of instances that indicated growth in her beliefs about the teaching of mathematics was 14 and about student learning was 11. We observed that during the pre video-viewing interview, Jora mentioned several times that her students needed explicit instruction on how to perform standard procedures, such as representing and operating on mixed numerals and performing the long division algorithm. Her main goal in this context was to expose students to mathematical terminology and to boost their self-esteem, even if genuine understanding is sacrificed:

I: Think about the students you have now and I'd like you to tell me what your current goals are for your students.

J: …in terms of understanding, possibly. Computation? They're probably at a 1st-grade level, yet, they can be proud of…they have a certain pride at that age, too — they're pre-teens. They were all thrilled because we were doing long division. Long division is what you do when you are in 5th- and 6th-grade. So, here we are, "Oh, is this long division." "Yes, guess what, this is long division." It's also so that they don't look and feel so different from other kids. Understanding of it — it's a bonus. But it's for them not to be doing 6 + 1 forever and be 11 years old. You don't want to walk out here and say, "Yeah, we did 6 + 1 again." You want to walk out of here and say, "Yeah, we're doing equivalent fractions. We did long division." So it's a social thing. So, when their friends talk about math, they've got at least something that they can talk about. Affective rather than the actual understanding of it.

We found that on the post video-viewing interview, however, Jora placed emphasis on the importance of teachers asking open-ended questions in mathematics class which, in turn, would allow for dialogue, sharing, and reflection on the part of the students:

I: What would be your best guess about what would help [your students in mathematics]?

J: …I hate to say it, but it's one-to-one help, half an hour a day, everyday, but that's impossible, so … really somebody there to ask them those questions, those questions to force them to reflect.

I: So that reflection is key in your mind…

J: Yeah. "Why?" Why did you do that? What's that?" Yes, definitely.

Later in the interview, Jora expanded on her views about the importance of communication among students in mathematics class, but qualified her responses with pessimism; because of the students with special needs in her classes, she believed she would never have the opportunity to allow her students to engage in mathematical discourse:

I: If you were to think of one single thing from our … math partnership … that changed your view of mathematics teaching and learning, what would it be?

J: Just that there's a lot of value in reflection.

I: In the students' reflection?

J: Yeah, the students' reflection.

I: What was it that helped you to come to that conclusion or that realization?

J: I would say it's all. And right from the beginning of the sessions. More and more and more seeing evidence of it. Being interested in it and being interested in how the kids on the video saw the value of it, too. It sort of helped them to sort it out as they verbalized it.

(Later in the interview):

I: How is your idea or understanding of the reform … different now than it was [before the partnership began]?

J: Just due to this project?

I: Yes.

J: …I happened to have a sense of curriculum reform. I did, but I don't know if everybody had a sense of it. I knew that we would have to be doing reflection and things because we did a thing on portfolios last year; we were at workshops for portfolios…where portfolios involve reflection. "Look at your work. What do you think you've done? Where do you think you want to be?" So it was sort of part of that. But it's just pushed it a little more. Pushed the envelope a little more in terms of the math. Like just not giving up. In my mind it's not giving up, if I ever had the opportunity. But on a kid who gives you an explanation, if you're not completely clear on the explanation…you keep pushing them to make themselves clear. Not that I'll ever be able to do that.

We concluded that, by the end of the study, Jora had progressed to Level 2.

From the interview data, we observed that Belinda underwent less change than the other two teachers. Nevertheless, her net growth as presented in Table 4 shows positive change in a variety of areas, constituting a total of 9 positive instances of net change in her beliefs about mathematics teaching and 3 instances in her beliefs about student learning. With respect to mathematics teaching, Belinda's development centered on structuring classroom activity around problem solving as opposed to performing mathematical exercises. An example from the post video-viewing interview illustrates:

> I: I'd like for you to think about this math project, this partnership that we've had. What things, if any, did you get out of our discussions about mathematics, teaching, and learning?
>
> B: Well, I learned that even in grade 2 they can do a lot more sophisticated work than I imagined. That's one thing for sure. I learned to trust them [the students] more. About teaching? There was teaching incidentally in groups. Basically I had to look at teaching like, "How do I present problems?" rather than actual teaching. You know, to the group. Give them the problem. And the teaching was very individualized and it ran the gamut from very simple things to very complex.

In addition, Belinda grew in her appreciation for the variety of strategies children bring with them to the classroom and that this inevitably led to a "letting go" or a loss of mathematical authority on the part of the teacher:

> B: Well, there were so many discussions, so many things that I had to question and think about. To let go, to have the courage to let go.
>
> I: What do you mean by letting go? Letting go of what, do you think?
>
> B: Letting go of the traditional way, routine teaching a lesson. You're taking a concept and you're presenting it. And here, you're just giving them a problem and they're doing it on their own, in their own way. So I didn't teach multiplication really.
>
> I: Instead you…
>
> B: Instead, you know, I followed them. The tables were reversed (laugh).

We concluded that, by the end of the study, Belinda had progressed to Level 3 or at the very least was in transition to Level 3.

CONCLUSIONS ABOUT THE COGNITIVE PREDISPOSITIONS FOR TRANSFORMING MATHEMATICS TEACHING

We claim that these data of the teachers' change can be used to describe the possible cognitive predispositions that we propose should be in place before a professional development program such as CGI begins. By the end of the eight-week period, the teachers with whom we worked were considerably more open to new ways of thinking about the variety of strategies children bring to the classroom and ways to build on student thinking in instruction. Furthermore, the teachers were more willing to consider and reflect upon general principles of student thinking instead of remaining mired in the particulars of the specific students in their charge.

At the same time, however, our data show that by the end of the intervention, the teachers were clearly not at Level 4 as described by Franke *et al*. (1997). At best, Gabby and Jora moved up one level (to Level 2) and at best, Belinda moved up two levels (to Level 3). The main conclusion we derive from the data is that demonstrating movement in levels does not say enough about the readiness of teachers to embark on genuine change in their beliefs and implied actions related to mathematics teaching. As such, our work allows us to present an emerging framework with which to conceive of readiness with respect to CGI professional development. More specifically, we propose the following three cognitive characteristics that may very well be necessary before any reform-oriented professional development takes place. Because our study was limited to a single location and a small number of teachers, however, additional research would be needed to validate this framework and determine the extent to which the our suggestions transfer.

First, we argue that before professional development begins, at least in any formal sense, teachers should be ready to think beyond the confines of their own classroom or school situations. For example, perhaps as an opening activity in a professional development initiative, teachers might be encouraged to brainstorm about the commonalities among their students, both former students as well as those in their current classrooms, in the context of mathematics learning. This might activate appropriate schemas related to the notion that general principles can be used to describe many students in their cognitive development. We are not suggesting that teachers' specific classroom needs do not form an important component of change, but we are suggesting that being open to the existence of general principles of teaching and learning can provide fruitful avenues for reflection about fostering children's mathematical development. As a result, effective professional development might include advance preparation that addresses this aspect of readiness.

Second, teachers should be ready to accept the notion that effective mathematics instruction cannot be fully scripted in a set of curricular materials. Instead of being directed solely by textbooks and curriculum guides, teachers need to accept that instruction should be driven primarily by student thinking and the mathematical understanding that underlie their performance in the classroom (Vacc & Bright, 1999). Although professional development programs such as CGI often result in teachers accepting this notion, we argue that perhaps before any formal workshops begin, teacher educators should assist teachers in conceiving of their practice as emanating from their formative assessment of student thinking. Another helpful notion with which to enter professional development is that students come to the classroom with a great deal of informal knowledge about mathematics (Carpenter *et al*., 1996; Clements & Sarama, 2004) and many of their successful problem solving efforts are not a result of formal instruction (Franke *et al*., 1997; Kilpatrick *et al*., 2001). We suggest that an informal focus on children's competencies, perhaps even the students in the teachers' own classrooms, should permeate the initial stages of professional development in mathematics.

To address both of these issues, professional development specialists might need to survey teachers in advance to find out about their entering beliefs and approaches because the content of professional development might contradict them. An awareness of teachers' beliefs and possible discrepancies with professional development needs to be directly addressed. For example, an advance survey might present teachers with different scenarios that embody various pedagogical approaches to teaching mathematics. Asking teachers to select the one that best represents their own interests and views may serve to reveal their entering beliefs about mathematics teaching and learning. The advance survey might also aim to assess teachers' feelings about teaching without the use of a scripted curriculum.

Finally, our data point to the importance of teachers being willing to shift the authority in the classroom, both from curricular and management perspectives, from themselves to the locus of understanding that occurs as children communicate to each other about the mathematics they are learning (Hiebert *et al*., 1997). Although Jora was not shy about admitting that she was not ready to "let go" in this sense, even after the intervention, she realized, perhaps for the first time, that discourse and communication form a basis for the development of genuine understanding in mathematics. Belinda discussed the "courage to let go," or the courage to reduce the amount of teaching in the traditional sense of telling students how to solve problems. This relinquishing of mathematical authority is a particularly important challenge for teachers and teacher educators in schools where a large portion of the student body has learning difficulties or behavioural disorders. Indeed, we learned from our work at Vaughn that the exceptionalities of many of its students formed the primary barrier for the teachers' change.

This being said, however, researchers are now beginning to understand that expert teaching incorporates more than being able to "let go" and have the students teach themselves (Chazan & Ball, 1999). Good teaching in mathematics entails finding the right balance between encouraging the construction and communication of meaningful strategies and scaffolding the learning that takes place in this context. This finding serves to confirm our speculation that the teachers in our study had not come far enough along the trajectory toward expert teaching or appropriating the set of beliefs necessary for meeting the goals of mathematics reform. Nevertheless, we are proposing that simple realizations, such as the notion that mathematics teaching necessitates more than modelling standard rules and procedures and that psychological principles of student learning can serve to describe even the most difficult student, are potentially necessary prerequisites for change in reform-oriented professional development contexts.

Our error as we began our work at Vaughn was assuming that the prerequisites we are proposing were already in place or worse, being unaware that any cognitive predispositions were even necessary. We assumed that the program could be implemented as described in the literature without adjusting it for the particular environment in which we were working. Perhaps this was naïve. Adult learning theory suggests that learners in different cultures — in this case psychological, political, and organizational — have different needs. As a result, instructional design practice (e.g., Dick, Carey, & Carey, 2004; Reigeluth, 1999; Smith & Ragan, 2004) suggests that even off-the-shelf interventions need to be customized to address these unique needs. In addition, teachers—like learners—might need to be assessed in advance to determine their readiness for the professional development initiative in question. Additional research might explore ways to measure predispositions and investigate ways to use the results to assist teacher educators adapt complex and resource-intensive projects aimed at transforming instructional practice and student outcomes in mathematics.

Footnotes

1. For more information on CGI, please see Carpenter, Fennema, and Franke (1996). There is no official website for the program, but the interested reader can read a description at http://www.promisingpractices.net/program.asp?programid=114#overview.
2. To protect the confidentiality of our participants, pseudonyms are used for the school and participants.
3. Statistics retrieved from http://www.mels.gouv.qc.ca/stat/indice_defav/index_ind_def.htm

Authors' note

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References

Anders, P. L., & Bos, C. S. (1992). Dimensions of professional development: Weaving teacher beliefs and strategic content. In M. Pressley, K. R. Harris, & J. T. Guthrie (Eds.), *Promoting academic competence and literacy in schools* (pp. 457-476). New York: Academic Press.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Baroody, A. J., & Wilkins, J. L. M. (1999). The development of informal counting, number, and arithmetic skills and concepts. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 48-65). Reston, VA: National Council of Teachers of Mathematics.

Bereiter, C., & Bird, M. (1985). Use of thinking aloud in identification and teaching of reading comprehension strategies. *Cognition and Instruction, 2*, 131-156.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3-15.

Borko, H., & Putnam, R. (1996). Learning to teach. In D. Berliner & R. Calfee (Eds.), *Handbook of educational psychology* (pp. 673-708). New York, Macmillan.

Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively Guided Instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97*, 3-20.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499-531.

Carpenter, T. P., Franke, M. L., Jacobs, V., & Fennema, E. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education, 29*, 3-20.

Chazan, D., & Ball, D. L. (1999). Beyond being told not to tell. *For the Learning of Mathematics, 9*, 2-10.

Clements, D. H., & Sarama, J. (Eds.). (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Erlbaum.

Dick, W. O., Carey, L., & Carey, J. O. (2004). *The systematic design of instruction* (6th ed.). Boston, MA: Allyn & Bacon.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V., & Empson, S. B. (1996). Learning to use children's thinking in mathematics instruction: A longitudinal study. *Journal for Research in Mathematics Education, 27*, 403-434.

Fennema, E., Carpenter, T. P., Levi, L., Franke, M. L., & Empson, S. B. (1999). *Children's mathematics Cognitively Guided Instruction: A guide for workshop leaders.* Portsmouth, NH: Heinemann.

Fennema, E., & Nelson, B. S. (Eds.). (1997). *Mathematics teachers in transition.* Mahwah, NJ: Erlbaum.

Fosnot, C. T. (Ed.). (1996). *Constructivism: Theory, perspectives, and practice.* New York: Teachers College Press.

Franke, M. L., Carpenter, T., Fennema, E., Ansell, E., & Behrend, J. (1998). Understanding teachers' self-sustaining, generative change in the context of professional development. *Teacher and Teacher Education, 14*(1), 67-80.

Franke, M. L., Fennema, E., & Carpenter, T. (1997). Teachers creating change: Examining evolving beliefs and classroom practice. In E. Fennema & B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 255-282). Mahwah, NJ: Erlbaum.

Friend, M., & Cook, L. (1996). *Interactions: Collaborative skills for school professionals* (2nd ed.). White Plains, NY: Longman.

Fuson, K. C. (2004). Pre-K to grade 2 goals and standards: Achieving 21st-century mastery for all. In D. H. Clements & J. Sarama (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 105-148). Mahwah, NJ: Erlbaum.

Ginsburg, H. (1996). Toby's math. In R. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 175-202). Mahwah, NJ: Lawrence Erlbaum.

Gitlin, A., & Margonis, F. (1995). The political aspect of reform: Teacher resistance as good sense. *American Journal of Education, 103*(4), 377-405.

Gouvernement du Québec. (2001). *The Québec education program: Approved version.* Québec, QC: Author.

Guskey, T. R. (1994). Results-oriented professional development: In search of an optimal mix of effective practices. *Journal of Staff Development, 15*, 42-50.

Guskey, T. R. (2002). Professional development and teacher change. *Teachers and Teaching: Theory and Practice, 8*, 381-391.

Heaton, R. M. (2000). *Teaching mathematics to the new standards: Relearning the dance.* New York: Teachers College Press.

Hiebert, J., Carpenter, T. R., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Hope, J., & Small, M. (1994). *Interactions 2.* Scarborough, Canada: Ginn.

Janas, M. (1998). Shhhhh, the dragon is asleep and its name is resistance. *Journal of Staff Development, 19*(3), 13-16.

Kazemi, E., & Franke, M. L. (2003). *Using student work to support professional development in elementary mathematics* (Working paper No. W-03-1). Seattle, WA: Center for the Study of Teaching and Policy.

Kilpatrick, J., Martin, W. G., & Schifter, D. (Eds.). (2003). *A research companion to principles and standards for school mathematics.* Reston, VA: NCTM.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington: National Academy Press.

Knapp, N. F., & Peterson, P. L. (1995). Teachers' interpretations of "CGI" after four years: Meanings and practices. *Journal for Research in Mathematics Education, 26*(1), 40-65.

Knowles, M. S., Holton, E. F., & Swanson, R. A. (1998). *The adult learner* (6th ed.). San Francisco: Butterworth-Heinemann.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* New York: Cambridge University Press.

Loucks-Horsley, S., Hewson, P., Love, N., & Stiles, K. E. (1998). *Designing professional development for teachers of science and mathematics.* Thousand Oaks, CA: Corwin Press.

Mewborn, D. S. (2003). Teaching, teachers' knowledge, and their professional development. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics* (pp. 45-52). Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Osana, H. P., Pitsolantis, N., & Zimmerman, C. (2006). *Capturing the complexity of the elementary mathematics classroom.* Paper presented at the annual meeting of the Canadian Society for the Study of Education, Toronto, ON, Canada.

Pajares, M. (1992). Teacher beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307-332.

Quebec Ministry of Education. (2003). *Indices de défavorisation.* Retrieved August 14, 2004, from Quebec Ministry of Education Web site: http://www.meq.gouv.qc.ca/stat/Indice_defav/index_ind_def.htm

Reigeluth, C. (1999). *Instructional design theories and models: A new paradigm of instructional theory*, Vol. 2. Mahwah, NJ: Erlbaum.

Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context.* New York: Oxford University Press.

Smith, P., & Ragan, T. J. (2004). *Instructional design* (3rd ed.). New York: Wiley.

Stein, M. K., Silver, E. A., & Smith, M. S. (1999). The development of professional developers: Learning to assist teachers in new settings in new ways. *Harvard Educational Review, 69*(3), 237-269.

Steinberg, R. M., Carpenter, T. P., & Fennema, E. (1994). *Toward instructional reform in the math classroom: A teacher's process of change.* Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Stolovitch, H. L., & Keeps, E. J. (Eds.) (1999). *Handbook of human performance technology* (2nd ed.). San Francisco: Jossey-Bass.

Stolovitch, H. L., & Keeps, E. J. (2004). *Training ain't performance.* Alexandria, VA: American Society for Training and Development Press.

Stolovitch, H. L., & Keeps, E. J. (2006). *Beyond training ain't performance fieldbook.* Alexandria, VA: American Society for Training and Development Press.

Sykes, G., & Darling-Hammond, L. (Eds.). (1999). *Teaching as the learning profession: Handbook of policy and practice.* San Francisco: Jossey-Bass.

Thompson, C. L., & Zeuli, J. S. (1999). The frame and the tapestry: Standards-based reform and professional development. In G. Sykes & L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 341-375). San Francisco: Jossey-Bass.

Vacc, N., & Bright, G. W. (1999). Elementary pre-service teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education, 30*, 89-110.

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