Fractions

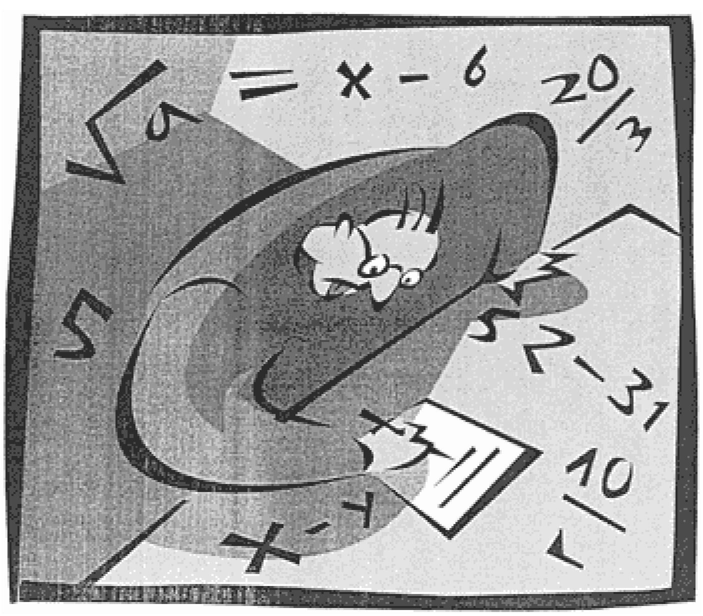
Fractions
Part 1

FRACTIONS, DECIMALS AND PERCENTAGES

Fractions, decimals and percentages are all ways of expressing parts of a whole. Each one of these forms can be renamed using the other two forms.

For example, the fraction $\frac{2}{5}$ can be represented as a different fraction ($\frac{4}{10}$), as a decimal (0.4) or as a percentage (40%). Consumers often see signs advertising $\frac{1}{2}$ price or 50% off. These two forms mean the same thing; they are simply different ways of expressing parts of a whole.

We use different forms because each form is more convenient to use in specific situations. For example, $\frac{1}{2}$ of 224 is easier to calculate than 0.5×224 or 50% of 224. As another example, it is easier to calculate $\frac{1}{3}$ of 360 than it is to calculate $360 \times 0.3333\ldots$, but it is easier to add $0.47 + 0.101$ than it is to add $\frac{47}{100} + \frac{101}{1000}$.

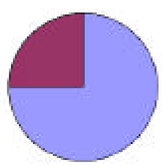


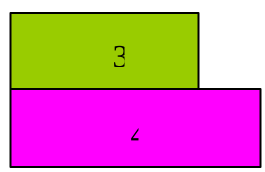
FRACTIONS

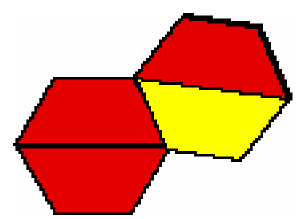
Preface

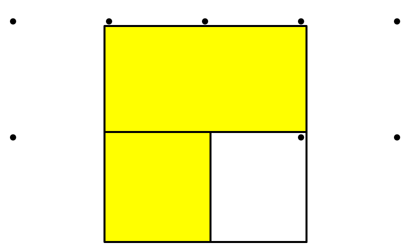
A knowledge of fractions is necessary to succeed in algebra, science and everyday life. Fractions are used daily at home and at work. For example, in cooking, recipes refer to $\frac{3}{4}$ cup or $\frac{1}{2}$ teaspoon, carpenters' measuring tools are labeled as $\frac{1}{16}$ of an inch, and sewing patterns refer to $\frac{5}{8}$ of an inch.

There are a variety of materials that can be used for modeling fractions. To develop the idea that a fraction is a portion of a quantity, geoboards, dot-paper, Cuisenaire rods and pattern blocks can be used. The following diagrams show different ways in which to represent the common fraction $\frac{3}{4}$ or three-quarters using some of these various models.









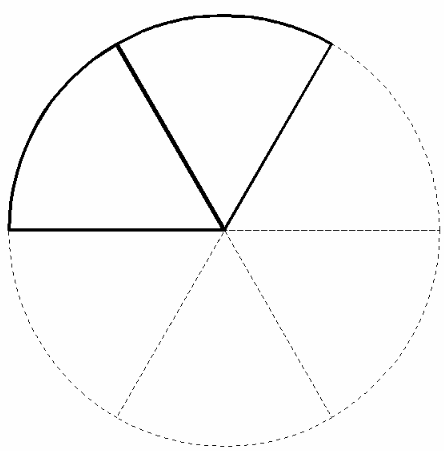
SEVERAL DIFFERENT MEANINGS OF FRACTIONS

Part-Whole

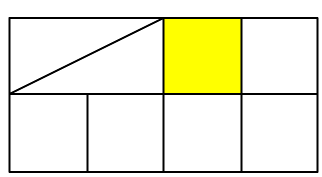
One of the common uses of fractions is to indicate part of a whole. If a whole is divided into equal parts, then you can use the fraction $\frac{a}{b}$ to describe a part of the whole, where the denominator b indicates the number of equal parts (equal in area) in the whole and the numerator a indicates the number of parts being considered.

Using $\frac{4}{6}$ as an example, the denominator 6 represents the number of equal parts that the whole has been divided into, that is, 6 equal parts. The numerator 4 represents the number of equal parts under consideration.

Ex.: $\frac{4}{6}$ of a pizza has been eaten.



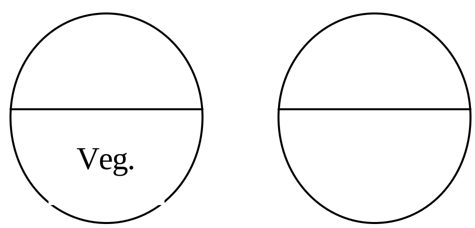
Be aware that the pieces need to have the same area, but need not look exactly the same. For example, each of the triangles and squares in the diagram below represents $\frac{1}{8}$.

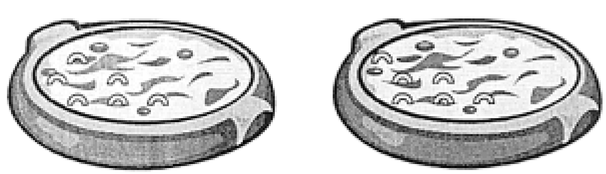


Notes:

1. The **numerator** of a fraction tells us the number of parts. If the **numerator** increases, there will be more parts, making the fraction greater.
2. The **denominator** of a fraction tells us how many parts there are in total, and, indirectly, the size of the parts. If the denominator increases, each part will become smaller, making the fraction less.
3. The **whole** or **unit** amount can represent one or more than one object. The unit can be thought of as a single whole, one hour, when you think of the fraction $\frac{3}{4}$ of an hour, but if you think of the hour as 60 minutes, you can still talk about $\frac{3}{4}$ of an hour. $\frac{1}{2}$ dozen eggs (of twelve items) is another example where a fraction is applied to a set of objects rather than a single region.

A fraction is a relative thing. For example, something can be $\frac{1}{4}$ of one thing, but $\frac{1}{2}$ of another. If two pizzas were ordered and $\frac{1}{2}$ of one pizza were vegetarian, then it means that $\frac{1}{4}$ of the whole order was vegetarian.





Ratio

This type of fraction is a means of comparison between two quantities. For example, $\frac{2}{1}$ or 2:1 represents a 2 to 1 ratio, meaning there is twice as much of one part as there is of the other part. To make orange juice from frozen concentrate, you need 3 cups of water to 1 can of frozen concentrate; this is a 3:1 ratio or $\frac{3}{1}$. The relationship can be reversed to a 1:3 or $\frac{1}{3}$ ratio as long as it is understood that you are referring to 1 can of frozen concentrate to 3 cups of water.

Sometimes, you compare a part to the whole to create a ratio. In this case, 1:3 could mean 1 part out of 3 of the mixture is concentrate. In this case, the ratio of concentrate to water is 1:2.

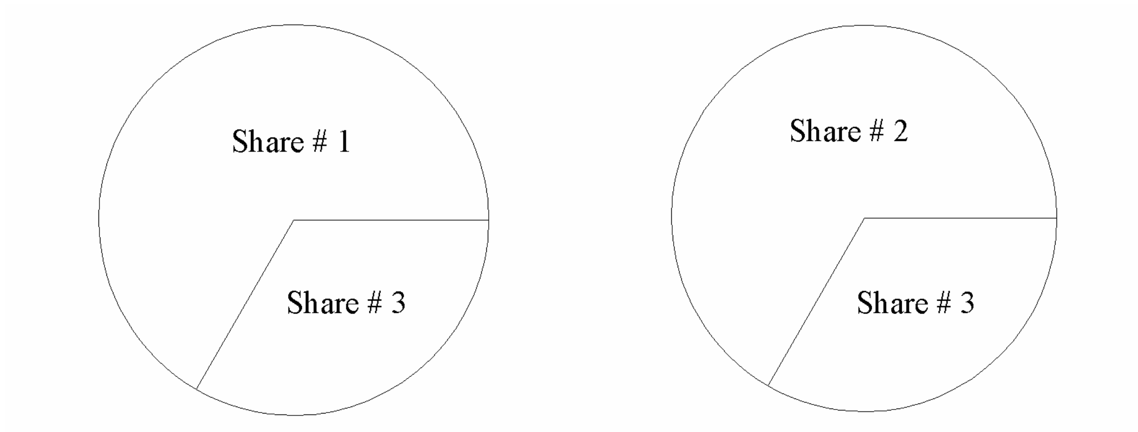
Ratios are often used to describe statistical information. For example, 1.1:1000 could represent a mortality rate for live births or it could represent betting odds.

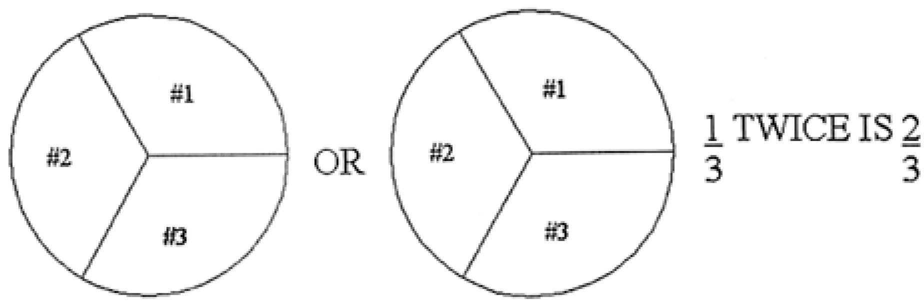
Rates are comparisons with each part of the comparison representing a different unit. Examples are 110 km/hr, 3 lemons for $.99 and $1.00US to $1.65Cdn. Rates are often used to compare prices.

Division

Division is used to determine the result of sharing. If, for example, 3 people share 12 eggs, each person would get 12 ÷ 3 eggs.

If two pizzas are shared equally by three people, this means that 2 wholes are divided into 3 parts. Each share is $\frac{2}{3}$ of a pizza.





Another way to look at this is that each of the 2 circles can be split into 3 equal pieces. Each share is one piece from each of the 2 circles, so the total share is 2 sets of $\frac{1}{3}$, or $\frac{2}{3}$. In general, if there are "*a*" objects to be shared among "*b*" people, each person gets $\frac{1}{b}$ from each objects, for a total of ""*a* pieces of size $\frac{1}{b}$, or $\frac{a}{b}$.

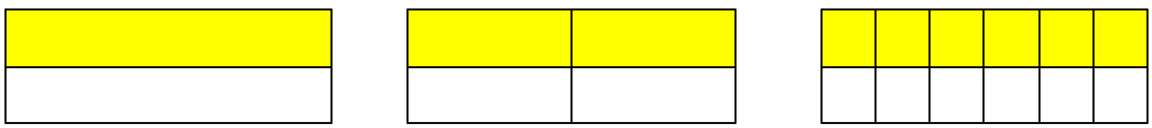
Because of this relationship between fractions and sharing, to find a decimal for $\frac{2}{3}$, you can use a calculator and divide 2 by 3. Similarly, to find a decimal for any fraction, you can divide the numerator by the denominator.

EQUIVALENT FRACTIONS

Equivalent fractions are fractions that are equal to each other even though they have different representations. Every number, even non-fractions, has other equivalent representations. For example, 7 + 1 = 6 + 2 = 8 – 0.

For every fraction, there are an infinite number of other fractions that represent the same value. By sorting fraction bars into piles of those with the same shaded length and naming the fractions they represent, one can visualize this concept. For example:

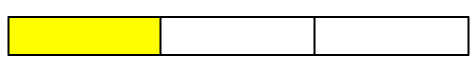
$$\frac{1}{2} = \frac{2}{4} = \frac{6}{12}$$



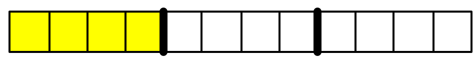
Each of the above fractions represents $\frac{1}{2}$ of the bar. The difference is that in each case, the bar is divided into different numbers of parts.

Generalization: Two fractions are equal if they represent the same portion of a whole. It turns out that one representation can be obtained from another by multiplying its numerator and denominator pair by the same non-zero integer. For example, if the numerator and denominator of $\frac{1}{3}$ are each multiplied by 4, you obtain the equivalent fraction $\frac{4}{12}$.

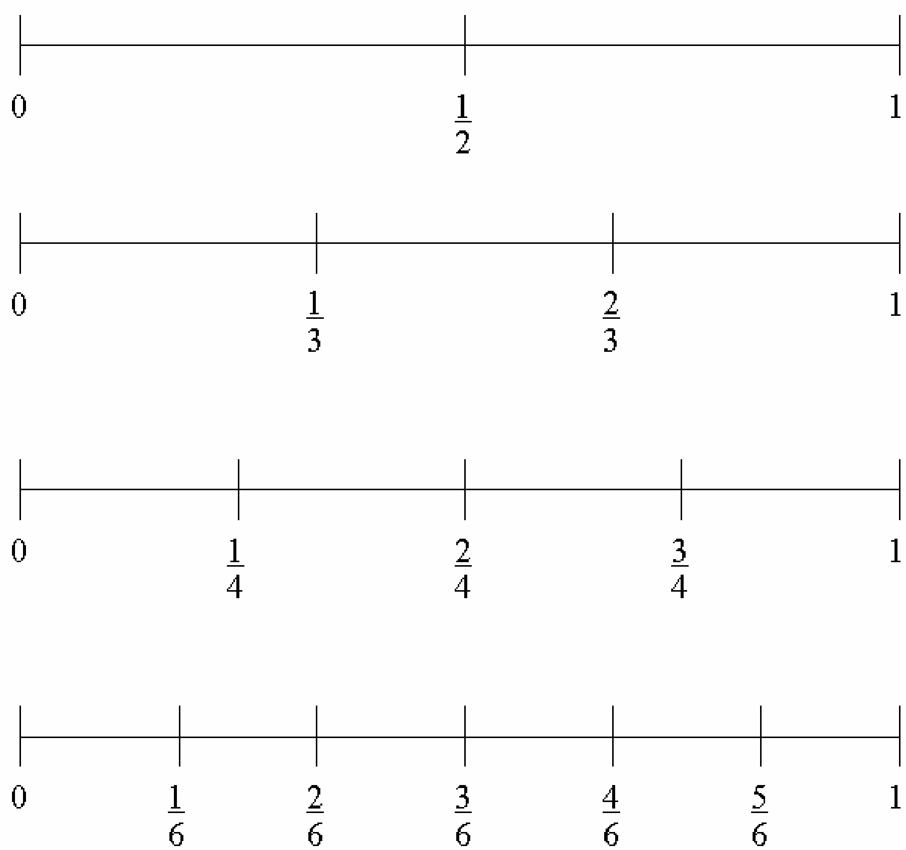
The following fraction bar illustrates this rule. The first fraction bar is divided into 3 equal parts. The shaded section represents $\frac{1}{3}$ of the bar.



In the illustration below, the shaded area is now showing 4 parts, as are each of the other non-shaded sections of the bar. The bar has remained the same, but the number of equal pieces has been multiplied by 4. That is, there are 4 times as many pieces in each part of the bar. This results in 4 times the total number of pieces in the entire bar.



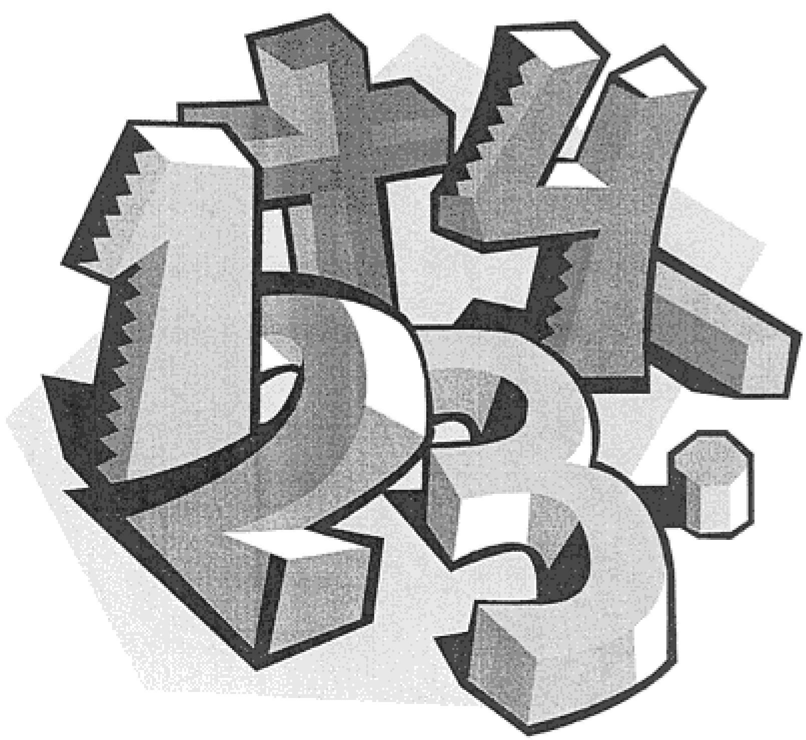
The following number line also illustrates equivalent fractions:



FRACTIONS BETWEEN ZERO AND ONE

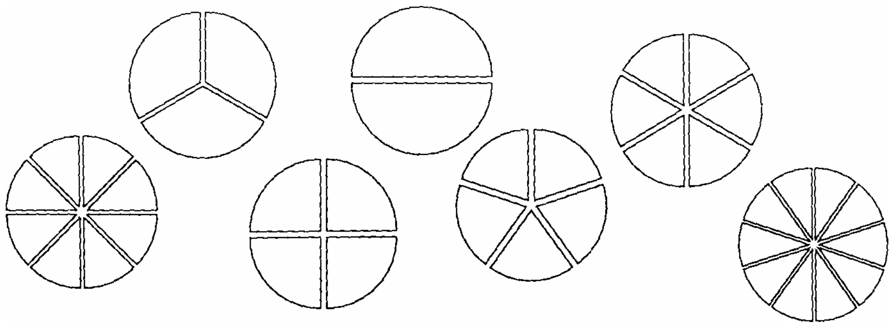
It is helpful to relate other fractions to the fractions 0, $\frac{1}{2}$ and 1. Try the following exercises.

1. Name a fraction between $\frac{1}{2}$ and 1.
2. Name a fraction between $\frac{1}{4}$ and $\frac{3}{4}$, other than $\frac{1}{2}$.
3. Name a fraction between $\frac{1}{4}$ and $\frac{1}{2}$ with a denominator of 10.
4. Name a fraction between $\frac{7}{8}$ and 1. How many can you name?
5. Name a fraction between 0 and $\frac{1}{10}$ with a numerator that is not 1.



IDENTIFYING FRACTIONS NEAR 0, $\frac{1}{2}$ AND 1

Materials: Using a different colour for each circle, make a set of fraction pieces that represent halves, thirds, fourths, fifths, sixths, eighths, and tenths.



1. Use one group of fraction pieces (eighths, for example) at a time to represent fractions near 0, $\frac{1}{2}$, and 1. Record the answers in a chart:

FRACTIONS NEAR 0	**FRACTIONS NEAR $\frac{1}{2}$**	**FRACTIONS NEAR 1**
1/8	3/8, 4/8, 5/8	7/8

   Continue with other sets of fraction pieces.

2. After completing the chart, look for patterns within the column that describes the group. What's alike about all fractions close to 1?

3. Sort these fractions into three groups (close to 0, close to $\frac{1}{2}$, close to 1) using the generalizations made earlier.

4. Look at the fractions listed in the column "Fractions Near $\frac{1}{2}$." Sort these into three groups (using the models if necessary): those less than $\frac{1}{2}$, those equal to $\frac{1}{2}$, and those greater than $\frac{1}{2}$.

COMPARING FRACTIONS

Fractions are easily compared when each fraction has the same denominator or the same numerator.

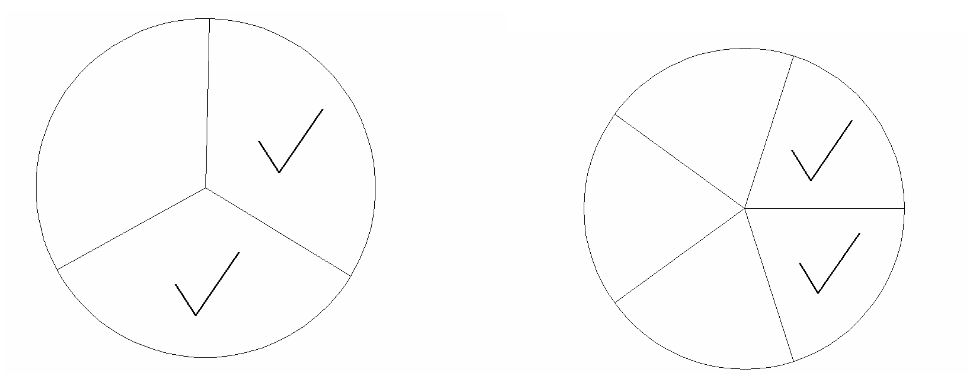
Same Denominator: $\frac{1}{5}$ compared to $\frac{3}{5}$.

Would a child rather have $\frac{1}{5}$ of a chocolate bar or $\frac{3}{5}$ of the bar?

Same Numerator: $\frac{1}{2}$ compared to $\frac{1}{3}$.

When items go on sale in a department store, which is the better bargain, $\frac{1}{2}$ off the regular price or $\frac{1}{3}$ off the regular price? Since $\frac{1}{2}$ is the greater fraction, this is the better bargain. In this example, the numerator 1 is common to both fractions.

Compare $\frac{2}{3}$ and $\frac{2}{5}$. Which is greater? Notice that the numerators are the same and both fractions represent a situation where 2 parts have been selected. What needs to be considered is which set of two pieces is larger, the two pieces where the whole is divided into 3 parts or the two pieces where the whole is divided into 5 parts.



Note: Both of these types of comparison had one thing that was in common, either the whole represented the same number of pieces (as in the common denominator) or the number of parts being considered were the same (as in the common numerator).

USING BENCHMARKS

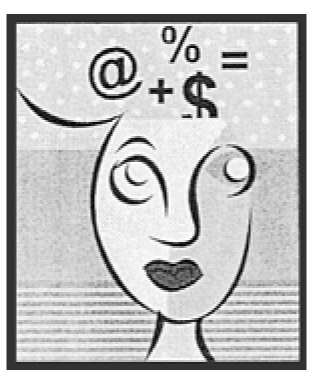
When comparing $\frac{9}{10}$ to $1\frac{1}{3}$, it should be clear that $1\frac{1}{3} > 1$, whereas $\frac{9}{10} < 1$.

To compare $\frac{7}{8}$ to $\frac{15}{16}$, you can compare both fractions to one. The fraction $\frac{15}{16}$ is closer to 1, since it is only $\frac{1}{16}$ away from one compared to $\frac{1}{8}$ away from 1 for the fraction $\frac{7}{8}$.

When comparing the following fractions, visualize which fraction is closer to representing the whole. This will be the greater fraction since both fractions are less than 1.

(a) $\frac{1}{8}$ compared to $\frac{4}{7}$
(b) $\frac{2}{3}$ compared to $\frac{5}{6}$
(c) $\frac{2}{3}$ compared to $\frac{4}{7}$

In Example **(c),** if the parts of the whole are split into 2 equal sections, then $\frac{2}{3}$ will be renamed as $\frac{4}{6}$ and there will be a common numerator that makes the fractions easier to compare. Remember that in order to apply this procedure, both the numerator and the denominator of the fraction were doubled. (See previous work on equivalent fractions.)



MIXED NUMBERS

A mixed number contains both a whole number part and a fraction part.

Improper Fractions

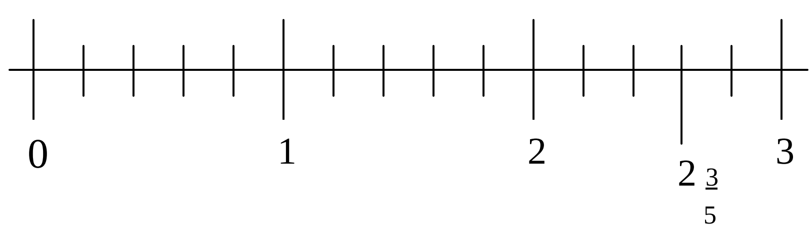
Historically, fractions stood for part of a whole and represented numbers less than one. The idea of fractions, such as $\frac{7}{4}$ or $\frac{5}{5}$, with a numerator greater than or equal to the denominator, was uncommon as late as the sixteenth century. Such fractions are often called improper fractions. Improper fractions are also known as top-heavy fractions. Note that the numerator is greater than or equal to the denominator. Examples are $\frac{10}{4}$, $\frac{7}{3}$ and $\frac{15}{2}$.

Mixed Numbers

When a quantity is symbolized with a combination of an integer and a fraction, it is called a mixed number. Improper fractions can easily be converted into whole or mixed numbers by dividing the numerator by the denominator. This will tell us how many whole numbers can be made up out of the parts.

For example, $\frac{12}{3}$ is equivalent to asking how many threes there are in 12. Since there are 4 threes in 12, 12 ÷ 3 = 4. As another example, $\frac{7}{4}$ is equivalent to asking how many fours there are in 7. Since 4 fours make a whole ($\frac{4}{4}$ = 1), then there is 1 whole and $\frac{3}{4}$ left over. Therefore, the answer is 1 + $\frac{3}{4}$ or $1\frac{3}{4}$.

Mixed numbers can be written as improper fractions. The following number line illustrates this. The line shows the mixed number $2\frac{3}{5}$. To write this number in fraction form, the number of spaces between each whole unit must be 5, giving a denominator of 5, illustrating the fraction $\frac{13}{5}$.



You can represent a mixed number in another way. When you write a mixed number, such as $2\frac{3}{5}$, you omit the "+" sign. So, $2\frac{3}{5}$ means $2 + \frac{3}{5}$. As an improper fraction, this example is equivalent to $\frac{13}{5}$, as shown here: $2 + \frac{3}{5} = 1 + 1 + \frac{3}{5}$ or $\frac{5}{5} + \frac{5}{5} + \frac{3}{5} = \frac{13}{5}$.

You have a better sense of the size of a number when you use it in mixed number form. For example it is clear that $2\frac{1}{10}$ is just a bit more than 2. But it is sometimes easier to calculate with the improper fraction form of the number.

ADDITION AND SUBTRACTION

Estimation

Estimation is a good way to begin the actual process of adding and subtracting fractions.

Ex. 1: $\frac{7}{8} + \frac{8}{9}$

$\frac{7}{8}$ is close to 1

$\frac{8}{9}$ is close to 1 — Answer is close to 2.

Ex. 2: $\frac{4}{7} + \frac{9}{10}$

$\frac{4}{7}$ is close to $\frac{1}{2}$

$\frac{9}{10}$ is close to 1 — Answer is close to $1\frac{1}{2}$.

Ex. 3: $\frac{6}{7} - \frac{1}{9}$

$\frac{6}{7}$ is close to 1

$\frac{1}{9}$ is close to 0 — Answer is close to 1.

Estimation gives you a good idea about the approximate answer one should expect.

Common Denominators

In order to add or subtract fractions, it is important to note that the whole must be divided into the same number of equal parts. When working with fractions, it is only the numerators in fractions that are added or subtracted. That is because it is only the parts of the whole that are being considered.

Ex.: $\frac{4}{6} - \frac{3}{6}$

4 sixths – 3 sixths = 1 sixth, just like

4 tens – 3 tens = 1 ten (or 40 – 30 = 10)

If the wholes are not divided into equal parts, it is harder to immediately calculate the difference.

Ex.: $\frac{4}{6} - \frac{3}{12}$ 4 sixths – 3 twelfths

If each fraction is changed to an equivalent fraction with the same denominator, the calculation becomes much simpler. Since 6 pieces can be made into 12 pieces by doubling the number of pieces, the fraction $\frac{4}{6}$ can be changed to the equivalent fraction $\frac{8}{12}$, as follows: $\frac{4 \cdot 2}{6 \cdot 2} = \frac{8}{12}$. Now the two fractions can be easily subtracted: 8 twelfths – 3 twelfths = 5 twelfths, so $\frac{4}{6} - \frac{3}{12} = \frac{5}{12}$.

A common denominator can always by found by multiplying the two denominators together.

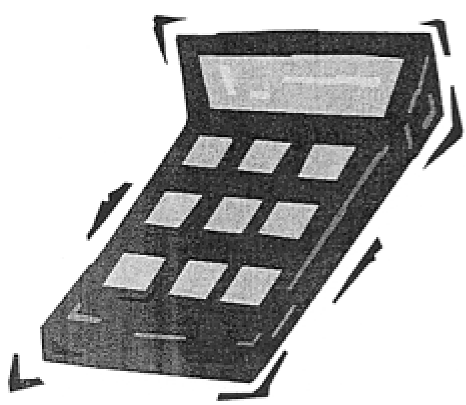
Ex.: $\frac{3}{8} + \frac{2}{7}$

Multiply $8 \times 7 = 56$, which gives a common denominator of 56.

So, $\frac{3 \cdot 7}{8 \cdot 7} + \frac{2 \cdot 8}{7 \cdot 8} = \frac{21}{56} + \frac{16}{56} = \frac{37}{56}$.

Adding Mixed Numbers

Ex.: $1\frac{5}{6} + 1\frac{3}{4} = 1\frac{10}{12} + 1\frac{9}{12} = (1+1) + (\frac{10}{12} + \frac{9}{12}) = 2\frac{19}{12} = 3\frac{7}{12}$



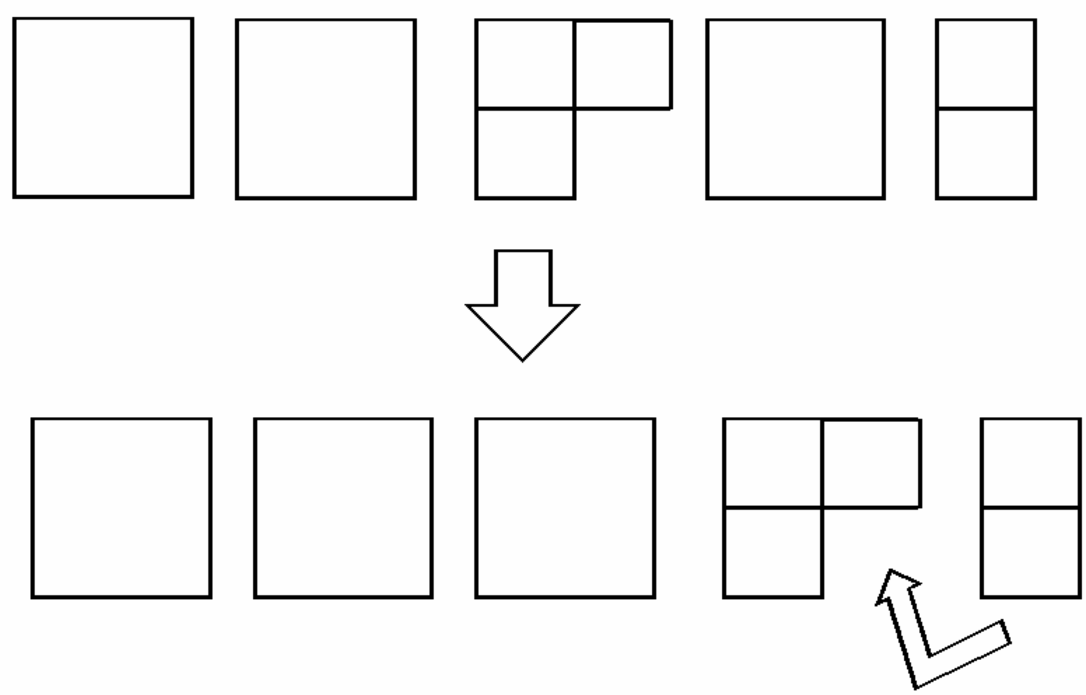
ADDING AND SUBTRACTING MIXED NUMBERS

Adding

Ex.: $2\frac{3}{4} + 1\frac{2}{4}$

Adding the whole numbers (2 + 1 = 3) and the fractions ($\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$) gives an answer of $3\frac{5}{4}$. The improper fraction $\frac{5}{4}$ represents a whole ($\frac{4}{4} = 1$) and $\frac{1}{4}$. The new answer then becomes $3 + 1 + \frac{1}{4} = 4\frac{1}{4}$.

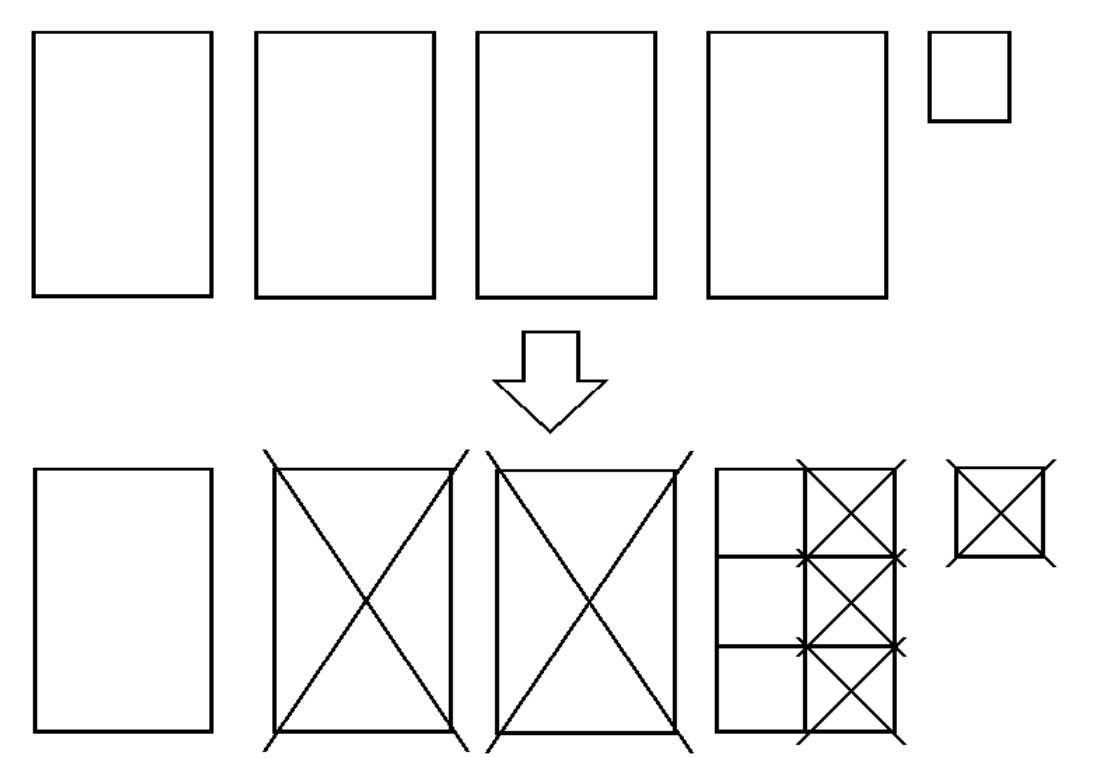
Adding the above mixed numbers using fraction pieces looks like this:



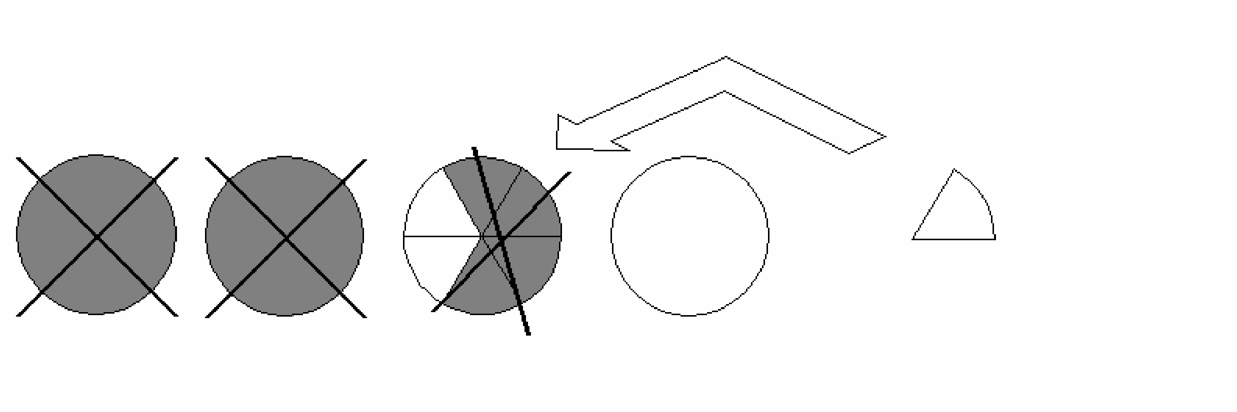
Subtracting

Ex.: $4\frac{1}{6} - 2\frac{4}{6}$

To do this, partition one of the wholes into 6 parts. The question then becomes, $3\frac{7}{6} - 2\frac{4}{6}$, giving an answer of $1\frac{3}{6}$ or $1\frac{1}{2}$.



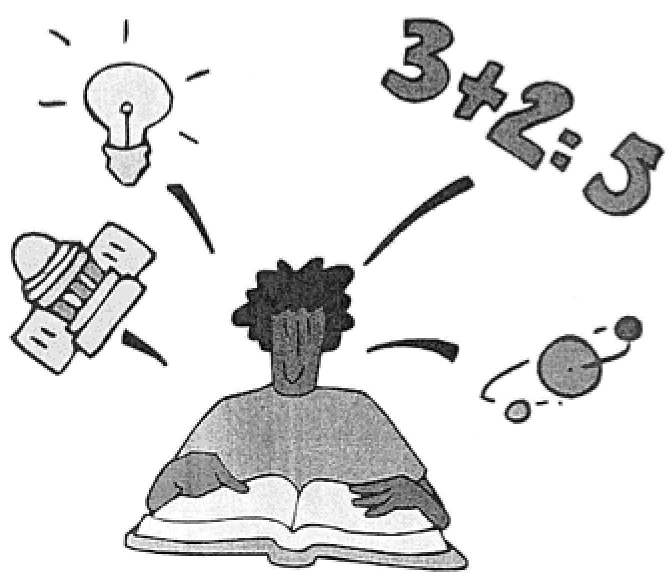
Using another approach, first take the 2 wholes from the 4 wholes. Then take $\frac{4}{6}$ out of one of the wholes that is left, and don't forget to add in the original $\frac{1}{6}$ from $4\frac{1}{6}$.



Hint: Check your answer to this problem by using estimation:

$4\frac{1}{6}$ is close to 4.

$2\frac{4}{6}$ is close to $2\frac{1}{2}$. => Estimate: $4 - 2\frac{1}{2} = 1\frac{1}{2}$



Fractions

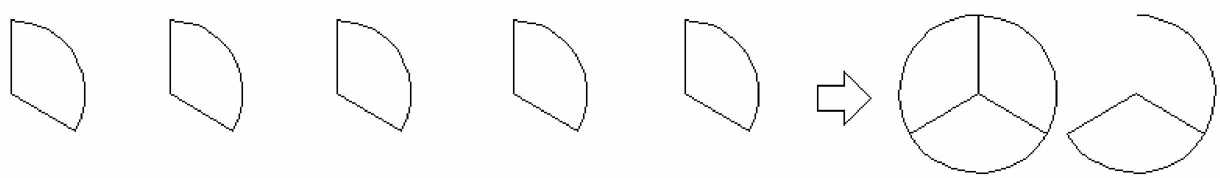
Part 2

MULTIPLICATION OF FRACTIONS

Multiplying a Fraction by a Whole Number

Ex.: Five boys each ate $\frac{1}{3}$ of a pizza. How many pizzas were eaten?

Using fraction circles marked off in thirds, the problem can be visualized,



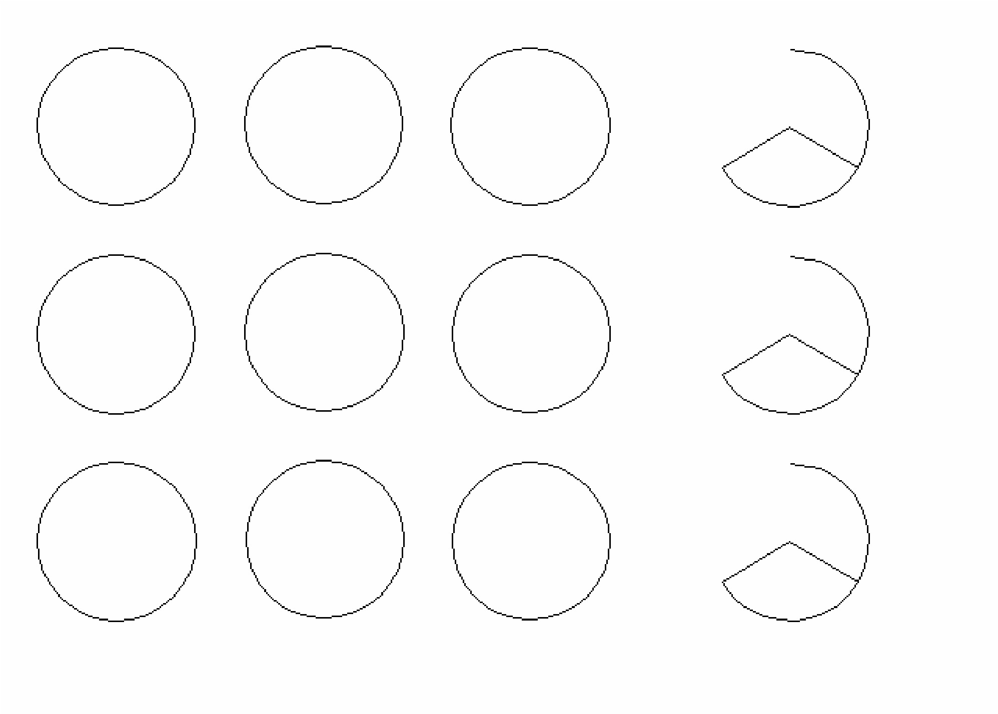
or, you can repeatedly add the fractions five times.

1 third + 1 third + 1 third + 1 third + 1 third = 5 thirds or $\frac{5}{3}$, so $\frac{1}{3} \times 5 = \frac{5}{3}$.

Another example:

$3\frac{2}{3} \times 3$

The first factor is shown by the number of circles in each row, $3\frac{2}{3}$. The second factor is shown by the number of rows, 3.



From the illustration, it is easy to see that there are nine complete circles as well as three partial circles. These parts can be combined to form two more complete circles, for a total of 11 circles. Expressed as an improper fraction the answer would be $\frac{33}{3}$.

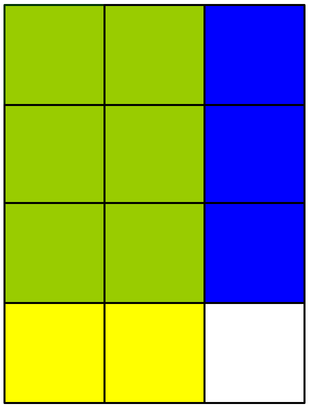
$$3\frac{2}{3} \times 3 = 1\frac{1}{3} \times 3 = \frac{33}{3} = 11$$

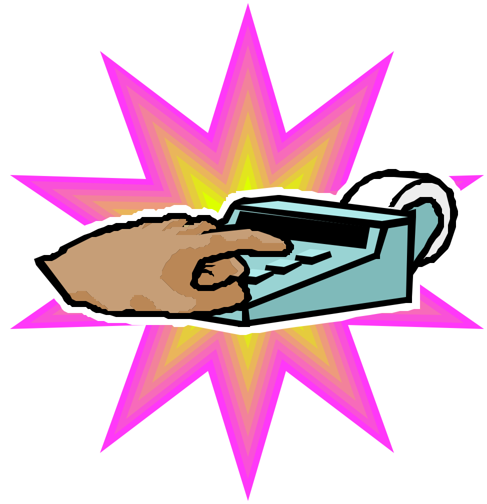
Multiplying a Fraction By a Fraction

Multiplication can be visualized by considering area (The area of a rectangle represents the product of the two numbers being multiplied).

Ex. 1: $\frac{2}{3} \times \frac{3}{4}$

This means $\frac{2}{3}$ of $\frac{3}{4}$, just like $2 \times \frac{3}{4}$ means 2 of $\frac{3}{4}$. So you need to show $\frac{3}{4}$ and then $\frac{2}{3}$ of it. Start by sketching a square. Mark off one side of the square in thirds and the adjacent side in fourths. The area has been divided into 12 equal parts. Shade in 2 columns of the thirds (to show $\frac{2}{3}$) and 3 rows of the fourths (to take $\frac{3}{4}$ of it). The common area shaded in consists of 6 parts or $\frac{6}{12}$ of the square.



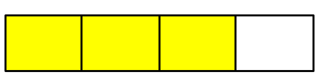


This same answer can be arrived at by multiplying together the numerators and by multiplying together the denominators. Notice that in the square, there are 3×2 boxes in the common shaded area (the product of the numerators) and there are 4×3 parts in the total area (the product of the denominators).

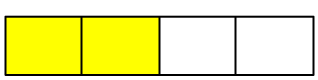
Ex. 2: $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

In this particular example, you could also use the following process:

1. Show $\frac{3}{4}$.
2. $\frac{2}{3}$ of it means 2 out of the 3 parts, so you would use the following models:
   Begin with:



Then:



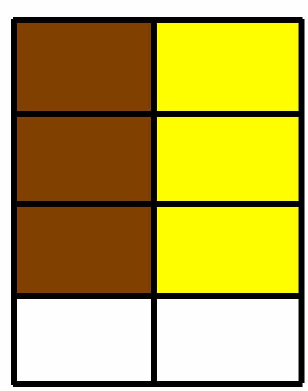
which is $\frac{2}{4}$ or $\frac{1}{2}$.

As another example, a strawberry grower harvests $\frac{3}{4}$ of his field for commercial purposes (saving $\frac{1}{4}$ for family use). To date, berry pickers have only picked $\frac{1}{2}$ of $\frac{3}{4}$ of the field. What part of the entire field has been picked?

This problem can be solved by folding paper. First fold the paper into 4 equal parts (lengthwise).

Colour $\frac{3}{4}$ to represent the part of the field available for commercial picking. Now divide the paper in half, to represent the part of the field that has been picked. Count the parts that have been picked (3) and compare them to the parts that represent the entire field (8). That is $\frac{3}{8}$ of the entire field has been picked.

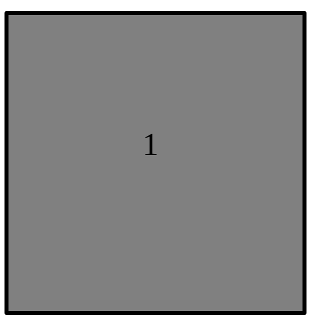
This can be described by $\frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8}$.

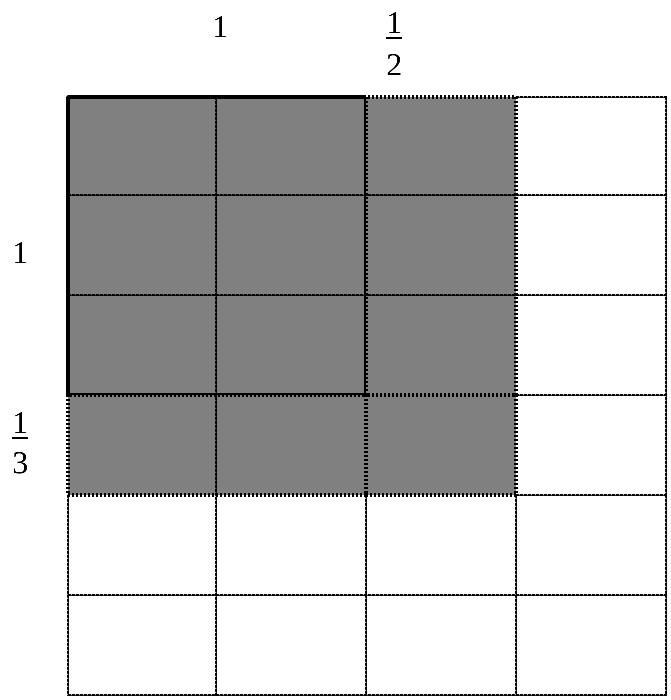


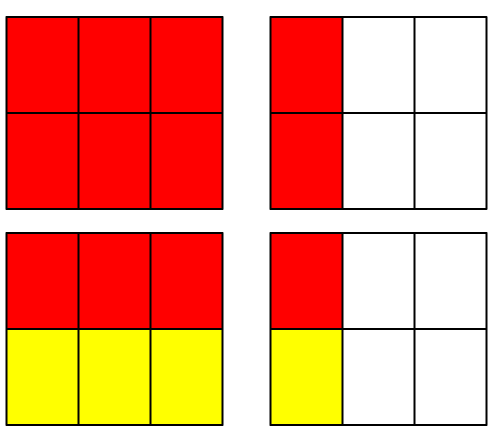
Multiplication of Improper Fractions

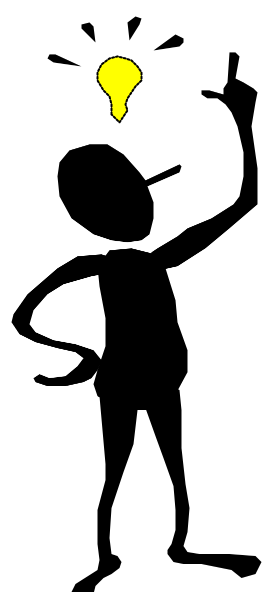
Ex.: $\frac{4}{3} \cdot \frac{3}{2}$

This , too, can be modelled using area. Draw one square and mark off three equal parts; this allows you to see 3 thirds. To show a fourth third, another square will need to be sketched below the first one. Notice that the first square also allows you to se 2 halves. To show the third half, squares will need to be sketched to the right. The area in question is made up of 12 small rectangles. But each rectangle is $\frac{1}{6}$ of 1 whole, so $\frac{4}{3} \cdot \frac{3}{2} = \frac{12}{6}$ = 2.









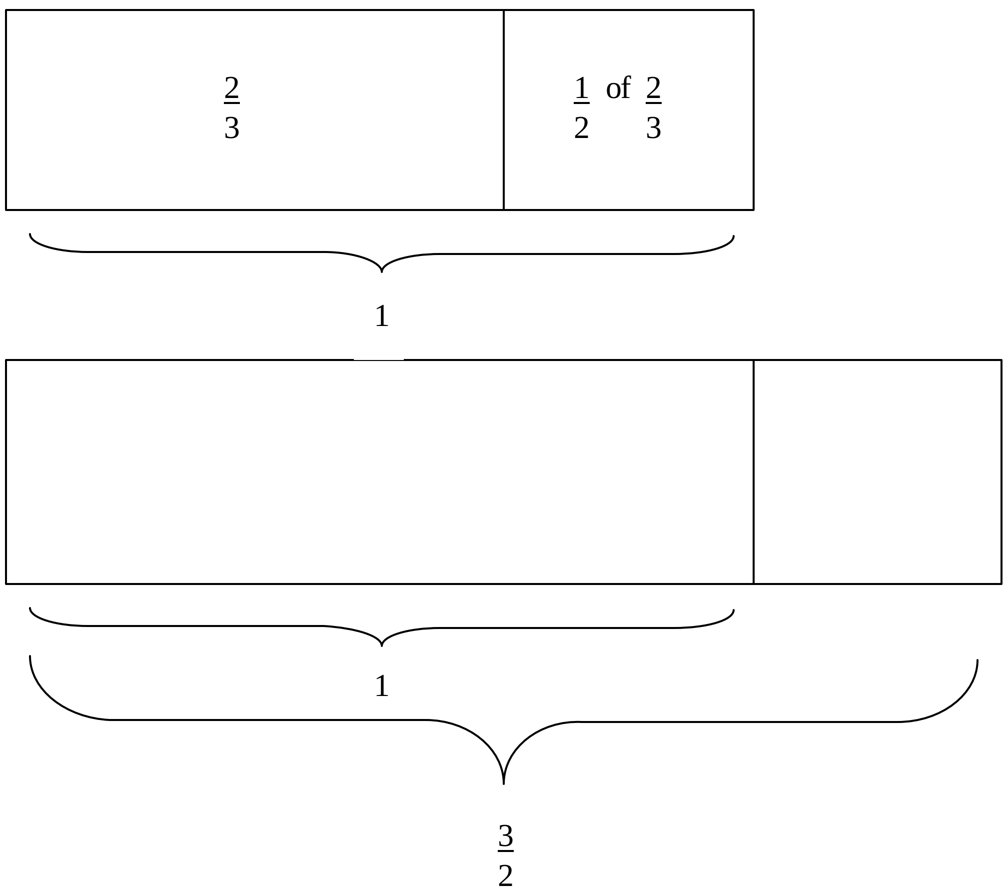
RECIPROCALS

Two numbers are reciprocals or multiplicative inverses of each other if the product of the numbers is one.

Ex. 1: 2 and $\frac{1}{2}$ are reciprocals of each other, because their product is 1.

Ex. 2: $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals of each other, since $\frac{3}{2} \times \frac{2}{3} = \frac{6}{6} = 1$.

It is also interesting to observe that the number of pieces of size $\frac{2}{3}$ within 1 is $\frac{3}{2}$ and the number of pieces of size $\frac{3}{2}$ within 1 is $\frac{2}{3}$.



In general, the numerator and denominator are exchanged to find a reciprocal.

DIVISION OF FRACTIONS

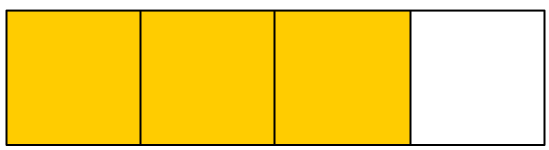
Dividing either tells how much a share is or how many groups of a certain size can be created. For example, 12 ÷ 3 tells, **(a)** what the size of each share is, if 12 is divided into 3 shares; or **(b)** how many threes there are in 12.

The same meanings apply when dividing fractions.

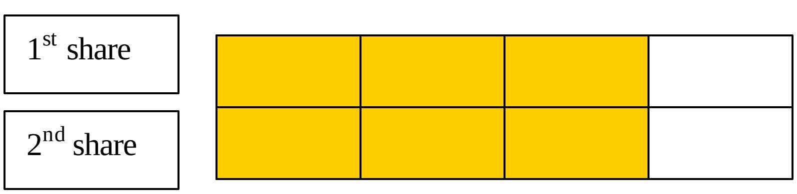
Dividing a Fraction by a Whole Number

Two children want to share $\frac{3}{4}$ of a chocolate bar equally. What fraction of the original bar will each child receive, that is, what is $\frac{3}{4} \div 2$?

The shaded portion represents the $\frac{3}{4}$ of the bar that is to be shared.



Each piece that is shaded represents $\frac{1}{4}$ of the bar. Each piece can be split into two equal parts. Then each child will receive 3 of these smaller pieces. If the whole bar had been divided in the same way, there would have been 8 pieces.



Therefore, $\frac{3}{4} \div 2 = \frac{3}{8}$ of the original bar.

Note: Thinking of division as sharing works well when the item being shared is divided by a whole number, as the whole number that represents the number of shares.

Common Denominator Division

Ex.: How many $\frac{3}{4}$-cups are needed to fill a 2-cup container?

If the 2-cup container were measured using quarter-cups, the problem could be easily solved. Two cups are equivalent to 8 quarter-cups. Therefore how many $\frac{3}{4}$ cups are there in $\frac{8}{4}$ cups, or how many threes are there in eight? Eight divided by three equals $2\frac{2}{3}$. Therefore, $2 \div \frac{3}{4} = \frac{8}{4} \div \frac{3}{4} = 8 \div 3 = \frac{8}{3} = 2\frac{2}{3}$.

In general, $\frac{a}{c} \div \frac{b}{c} = \frac{a}{b}$. Notice that $\frac{4}{5} \div \frac{2}{5} = \frac{4}{7} \div \frac{2}{7} = \frac{4}{10} \div \frac{2}{10} = \frac{4}{2}$.

This is because once the unit sizes are equal, we are really just asking how many times the *b* pieces fit into the *a* pieces; the size of the pieces no longer matters. In each case above, the question is, "How many '2 of the pieces' fit into '4 of the pieces'?"

Using Subtraction to Divide

When dividing a fraction by a fraction, you can use a subtractive approach.

Ex.: To calculate $\frac{3}{4} \div \frac{1}{4}$, you can think, “There is $\frac{3}{4}$ of a chocolate bar; how many shares of $\frac{1}{4}$ can be distributed?”

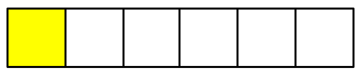
So, you can think :
“3 quarters – 1 quarter – 1 quarter – 1 quarter”, or $\frac{3}{4} \div \frac{1}{4} = 3$.

Invert and Multiply

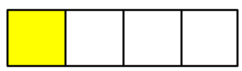
Another approach is shown below.

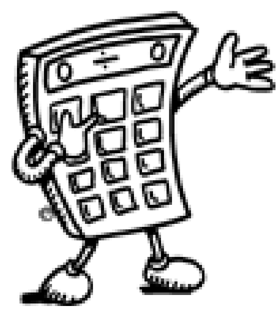
Ex. 1: $\frac{2}{3} \div \frac{1}{6}$

Step 1: How many sixths are in 1? The answer is 6.



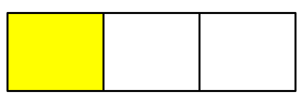
Step 2: Then, how many sixths are there in $\frac{2}{3}$? The number of sixths in $\frac{2}{3}$ must be $\frac{2}{3}$ of the number of sixths in 1. Therefore, $\frac{2}{3}$ of 6, or $\frac{2}{3} \times 6 = \frac{12}{3} = 4$.



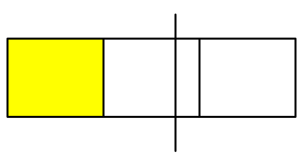


Ex. 2: $\frac{3}{5} \div \frac{1}{3}$

Step 1: There are 3 one-thirds in 1.



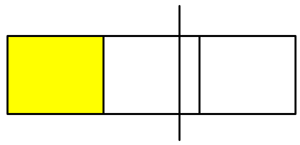
Step 2: There are $\frac{3}{5}$ as many one-thirds in $\frac{3}{5}$ as in 1.



Therefore, $\frac{3}{5} \div \frac{1}{3} = \frac{3}{5} \times 3 = \frac{9}{5}$ (almost 2).

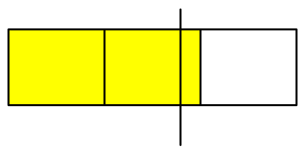
Ex. 3: $\frac{3}{5} \div \frac{2}{3}$

Step 1: Find $\frac{3}{5} \div \frac{1}{3}$ (as above as $\frac{3}{5} \times 3$).

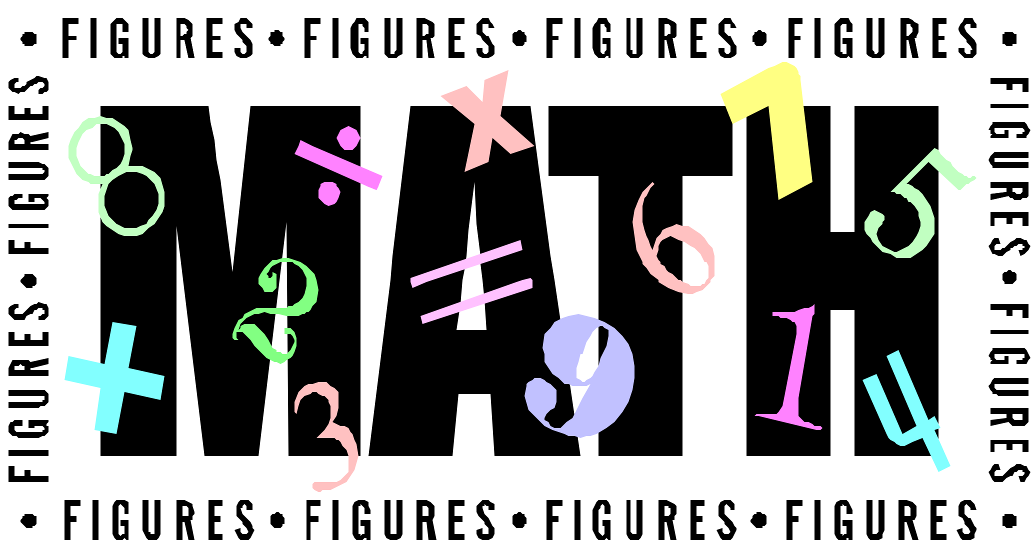


Step 2: But $\frac{2}{3}$ is twice as long as $\frac{1}{3}$, so only half as many fit in.

Therefore, $\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$ (almost 1).



In general, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.



Fraction Activities

ACTIVITY SHEET 1

EXERCISE 1 Everyday Fractions

List some measurements in everyday situations that involve fractions.

EXERCISE 2 Pattern Blocks

Pattern blocks are used to model fractions as part of a whole. When completing the following exercise, keep in mind what the whole is in each case.

Color Key: Hexagon = Yellow
Rhombus = Blue
Trapezoid = Red
Triangle = Green

(a) If a hexagon is 1, what is a triangle? a rhombus? a trapezoid?

(b) If a hexagon is 1, show $\frac{2}{3}$, $\frac{4}{6}$ and $\frac{7}{6}$.

(c) If a trapezoid is 1, what is a triangle? a rhombus? a hexagon?

EXERCISE 3 Drawing Patterns

Draw pictures to represent each of the following fractions.

(a) $\frac{1}{3}$ **(b)** $\frac{4}{2}$ **(c)** $\frac{1\frac{1}{2}}{3}$

EXERCISE 4 Building Patterns

Use pattern blocks to model each of the following.

(a) Build a triangle that is $\frac{1}{3}$ green and $\frac{2}{3}$ red.

(b) Build a parallelogram that is $\frac{3}{4}$ blue and $\frac{1}{4}$ green.

(c) Build a parallelogram that is $\frac{2}{3}$ blue and $\frac{1}{3}$ green.

(d) Build a trapezoid that is $\frac{1}{2}$ red and $\frac{1}{2}$ blue.

Further activities using pattern blocks can be found at the following web site:
www.arcytech.org/java/patterns

GAMES

FRACTION BAR GAME I

Object of the game: To arrange fractions in increasing order.
Players: 2-4

Rules:

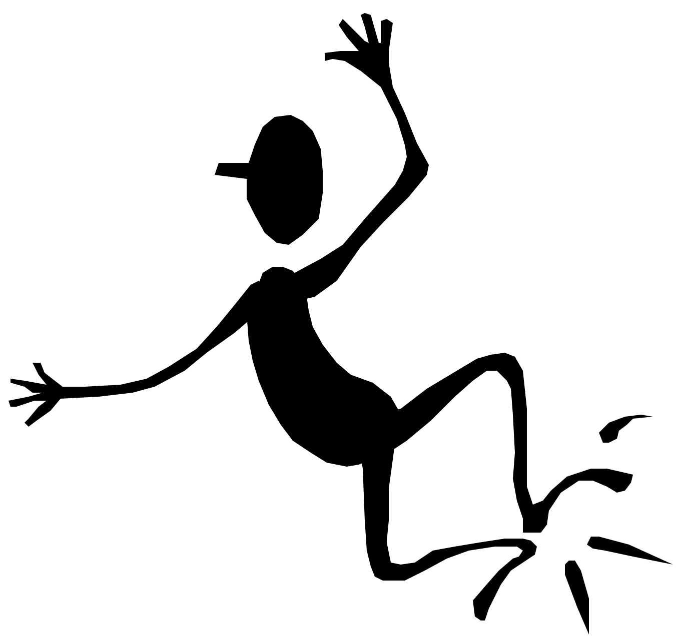
Each player is dealt 5 bars in a row face up.
These bars are left in the order that they are dealt.
The remaining bars are placed in a stack face down.
Each player, in turn, takes a bar from the stack and uses it to replace any of their dealt bars.
The first player to get their bars in order wins the game!

FRACTION BAR GAME II

Object of the game: To recognize equivalent fractions.
Players: 2-4

Rules:

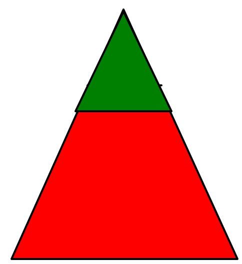
All bars are dealt face down to the players.
Each player, keeping the bars face down, must stack them.
When the dealer says “flip,” each player flips over the top bar and the length of the shaded bars are compared.
The player with the greatest shaded length wins the bars that have been flipped.
This player arranges the collected bars into piles of the same shaded length.
The first player to have 3 stacks of 3 bars each wins!



EQUIVALENT FRACTION EXERCISE SHEET

EXERCISE 1 Pattern Blocks

The triangle shown is $\frac{3}{4}$ red.



Build a triangle that is $\frac{2}{3}$ red, $\frac{1}{9}$ green, and $\frac{2}{9}$ blue.

EXERCISE 2 Pattern Blocks

A tangram is a square that has been cut into certain specified shapes. It can be used to teach geometrical as well as fractional concepts. A copy of a tangram is included in the appendix of this unit. Photocopy the tangram and cut out the pieces to assist you with the following problem.

If the tangram itself represents one whole unit, what fractions do each of the other pieces in the tangram represent?

EXERCISE 3

Use pattern block models to show that $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent fractions.

PROBLEM

How many fractions are there equivalent to $\frac{1}{2}$, where the denominator is a number between 10 and 100?

EXERCISES ON COMPARING FRACTIONS

EXERCISE 1

Without changing fractions into equivalent fractions, explain how each of the following pairs of fractions can be compared.

(a) $\frac{11}{12}$ and $\frac{3}{4}$

(b) $1\frac{3}{4}$ and $2\frac{1}{3}$

(c) $\frac{99}{100}$ and $\frac{999}{1000}$

EXERCISE 2

Compare these fractions. In each group, which one is the greatest? Explain your choices.

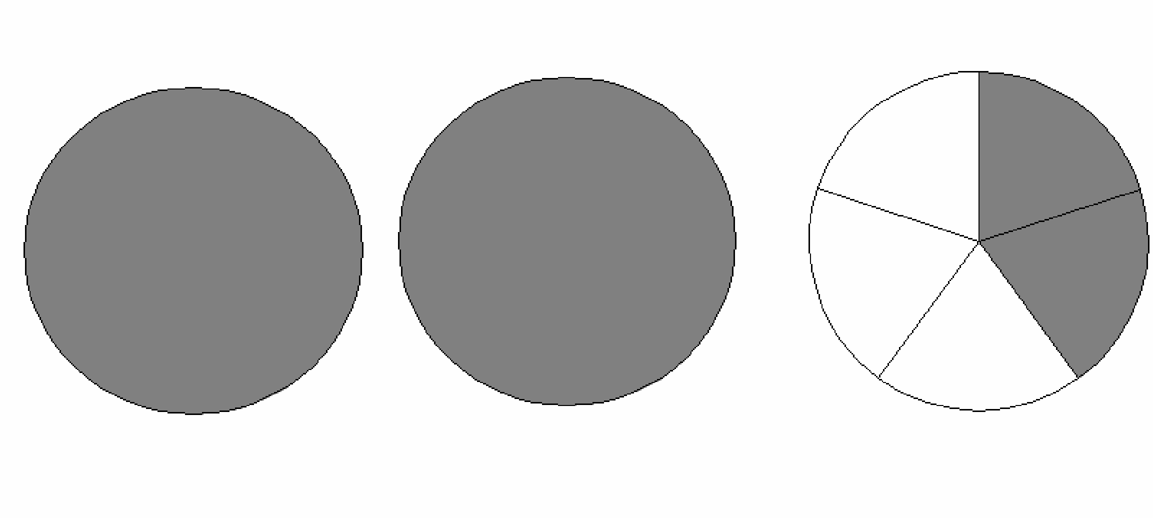
(a) $\frac{16}{24}, \frac{2}{3}, \frac{5}{8}$

(b) $\frac{4}{18}, \frac{12}{27}$

MIXED NUMBER EXERCISES

EXERCISE 1

The following fraction circles illustrate the mixed number $2\frac{2}{5}$. Write this mixed number in fraction form with only a numerator and a denominator.

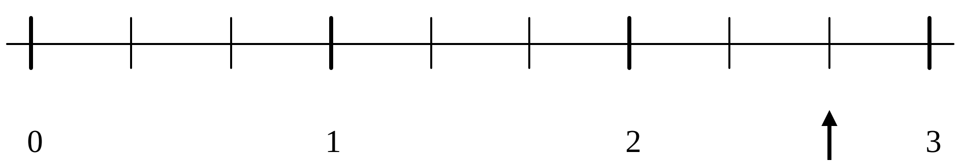


EXERCISE 2

Using fraction circles, illustrate the improper fraction $\frac{23}{5}$.

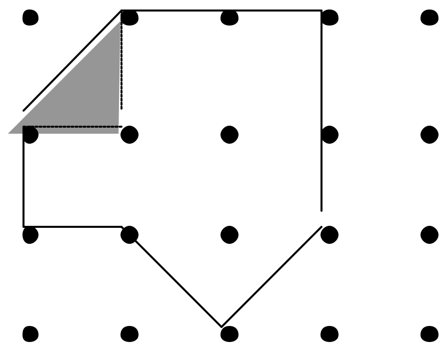
EXERCISE 3

The following number line represents a mixed number. What is it? Express this number as an improper fraction.



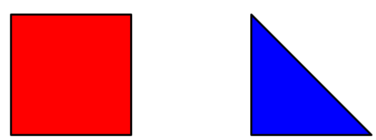
EXERCISE 4

If the shaded area has a value of $\frac{1}{3}$, what is the value of the whole shape?



EXERCISE 5

Use a geoboard or dot paper to form the fractions described below. Assume that a square and a triangle refer to the shapes shown below.



(a) Draw a shape to represent $2\frac{1}{2}$, if the unit (whole) is 2 squares.

(b) Draw a shape to represent $2\frac{1}{2}$, if the unit is 4 squares.

(c) Draw a shape to represent $2\frac{1}{3}$, if the unit is a square and a triangle

EXERCISE 6

Write each mixed number as an improper fraction:

(a) $6\frac{3}{7}$ **(b)** $5\frac{2}{3}$ **(c)** $34\frac{5}{7}$

EXERCISE 7

Write each improper fraction as a mixed number:

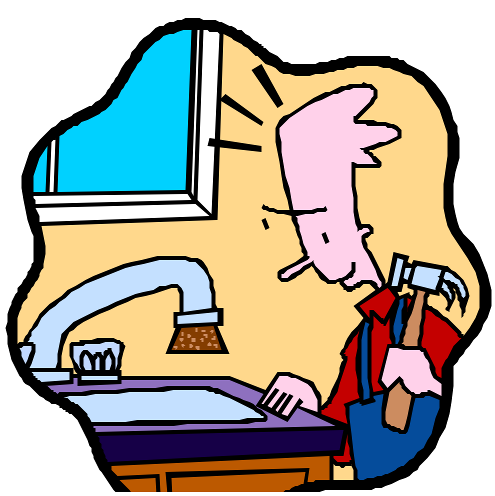
(a) $\frac{17}{3}$ **(b)** $\frac{32}{6}$ **(c)** $\frac{435}{56}$

EXERCISE 8

Jaimie needs $2\frac{2}{3}$ cups of flour for a zucchini cake, but only has the $\frac{1}{3}$-cup measure. How many of the $\frac{1}{3}$-cups are needed for the recipe? ***Hint:*** Visualize fraction circles.

EXERCISE 9

Pat needs a new plug for the bathroom sink. Before going to the hardware store, Pat measures the diameter of the sink drain with a ruler marked off in tenths. The drain measures $1\frac{1}{5}$ inches. At the hardware store, the two plugs available measure $1\frac{1}{8}$ to $1\frac{2}{8}$ inch and $1\frac{3}{8}$ to $1\frac{5}{8}$ inches. Which one should be purchased? ***Hint:*** Use estimation first, and then verify using equivalent fractions.



ADDITION AND SUBTRACTION EXERCISES

EXERCISE 1

Use a model to find $\frac{2}{3} + \frac{1}{2}$. Explain your solution.

EXERCISE 2

Solve the following problem:

Three 12-inch pizzas were ordered for a birthday party. After the party, $\frac{1}{2}$ of the pepperoni pizza, $\frac{2}{3}$ of the vegetarian pizza, and $\frac{3}{4}$ of the mushroom pizza were left over. Do the leftovers add up to 2 pizzas? If not, do the leftovers add up to more or less than two pizzas?

EXERCISE 3

Estimate $\frac{7}{8} + \frac{12}{13}$.

EXERCISE 4

(a) Find the pattern in the sums below:

$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$ $\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$ $\frac{1}{5} = \frac{1}{6} + \frac{1}{30}$

(b) Use the pattern to write each of the following fractions as the sum of two fractions:

(i) $\frac{1}{4}$ **(ii)** $\frac{1}{7}$ **(iii)** $\frac{1}{10}$

(c) Use models to explain the above pattern.

EXERCISE 5

Two companies advertising for different items offer these special deals. Which is the better deal and why?

(a) Vogue optical, your second pair is free!
(b) Buy one, get the second half off!

EXERCISE 6

A pie that had been cut into 8 pieces had 3 pieces missing. John would like to have $\frac{1}{4}$ of the entire pie. If John gets $\frac{1}{4}$ of the pie, what fraction of the entire pie will be left?

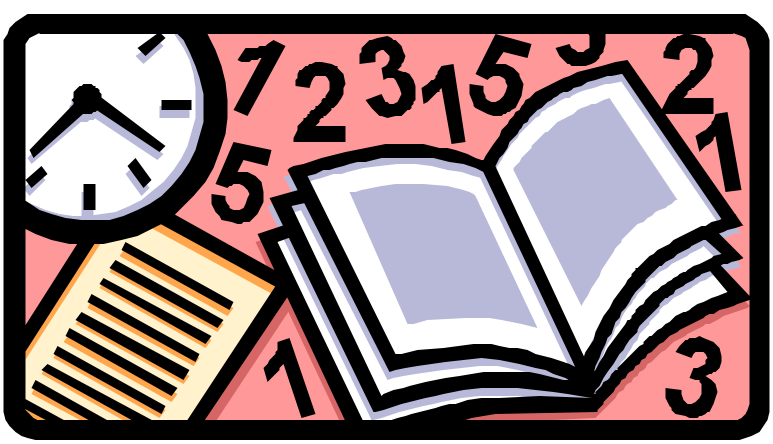
EXERCISE 7

After getting her allowance for the week, June spent $\frac{1}{3}$ of her allowance on a magazine, $\frac{1}{6}$ of her allowance on candy and $\frac{1}{4}$ of her allowance on a toy. What fraction of her allowance was left?

EXERCISE 8

Calculate each of the following:

(a) $\frac{5}{7} - \frac{1}{3}$ **(b)** $\frac{11}{12} - \frac{3}{4}$ **(c)** $\frac{2}{3} - \frac{2}{8}$



MULTIPLICATION EXERCISES

EXERCISE 1

Use paper folding to model $\frac{3}{4} \times \frac{1}{4}$.

EXERCISE 2

Pat likes to shop for bargains. A shop has just advertised a half of a half off sale ($\frac{1}{2} \times \frac{1}{2}$). What fraction of the original price will Pat pay for purchases made? Use an array to illustrate the answer.

EXERCISE 3

Use a model to illustrate $\frac{2}{3} \times \frac{4}{5}$.

EXERCISE 4

Pat & Jaimie found $\frac{1}{3}$ of a cake in the kitchen. Pat ate $\frac{1}{2}$ of it.

(a) How much of it was left for Jaimie?

(b) If Jaimie ate $\frac{1}{2}$ of the remainder, how much of the entire cake was left?

DIVISION EXERCISES

EXERCISE 1

Use a model to show how many fourths there are in $1\frac{1}{2}$.

EXERCISE 2

After a party, $2\frac{1}{2}$ pizzas were left. If Pat can eat $\frac{1}{2}$ of a pizza each day, for how many days can Pat eat pizza?

EXERCISE 3

Each of the following is a recipe for a large number of cupcakes. If you wanted to make only $\frac{1}{2}$ of the recipe, how much of each ingredient would be required?

EVERYDAY CUPCAKES

$\frac{1}{2}$ **cup shortening**
$1\frac{3}{4}$ **cups sifted all-purpose flour**
1 cup sugar
$2\frac{1}{2}$ **teaspoons baking powder**
$\frac{1}{2}$ **teaspoon salt**
1 egg
$\frac{3}{4}$ **cup milk**
1 teaspoon vanilla

FUDGE CUPCAKES

$\frac{2}{3}$ **cup brown sugar**
$\frac{1}{3}$ **cup milk**
2 1-ounce squares unsweetened chocolate
$\frac{2}{3}$ **cup brown sugar**
$\frac{1}{3}$ **cup shortening**

* * *

1 teaspoon vanilla
2 eggs
$1\frac{1}{3}$ **cups sifted all-purpose flour**
1 teaspoon soda
$\frac{1}{2}$ **teaspoon salt**
$\frac{1}{2}$ **cup milk**

EXERCISE 4

In your cupboard, all you have is $\frac{1}{4}$ cup of cocoa, but you have a craving for a Red Devil's Food Cake. If you make this cake with only $\frac{1}{4}$ cup of cocoa, what will be the necessary amounts of each of the other ingredients?

RED DEVIL'S FOOD CAKE

$\frac{1}{2}$ cup shortening
$1\frac{3}{4}$ cups sugar
1 teaspoon vanilla
3 eggs, separated
$2\frac{1}{2}$ cups sifted cake flour
$\frac{1}{2}$ cup cocoa (regular -type, dry)
$1\frac{1}{2}$ teaspoons soda
1 teaspoon salt
$1\frac{1}{3}$ cups cold water

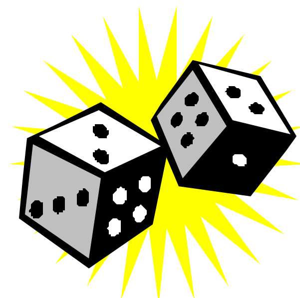
MATERIALS FOR FRACTION GAMES

Plain white cards (you could use index cards/recipe cards) with the following fractions:

0/2, 1/2, 2/2
0/3, 1/3, 2/3, 3/3
0/4, 1/4, 2/4, 34, 4/4,
0/6, 1/6, 2/6, 3/6, 4/6, 5/6, 6/6
0/12, 1/12, 2/12, 3/12, 4/12, 5/12, 6/12, 7/12, 8/12, 9/12, 10/12, 11/12, 12/12

2 of each fraction card

ICED DICE



You roll two dice and get a fraction.
Ex: 4 and 5 gives you 4/5.

Player A gets a point if the fraction is in lowest terms. (like 4/5)
Player B gets a point if it isn't. (like 4/6)
Play until someone has 10 points.

TWO

You get one card turned over and one turned up.
You can pass or be "hit".
Your goal is to be closest to a total of two (whole number) without going over.

FRACTION TRACKS

This is a cooperative game for two players. Start with a marker at the 0 box of each "fraction track." The goal is to get all of your markers to 1. On each turn, roll two dice to get a fraction. If you roll a 1, treat it as an 8. The lesser number is the numerator of a fraction. For example, if you roll a 1 and a 5, your fraction is 5/8. You may move one marker the full value of your fraction, or you may move any number of markers as long as their movements add to the fraction. You may use equivalent fractions. You may not "run over." To get to 1, you have to do so exactly.

Materials: Fraction tracks (below), dice or number cubes, six markers per pair
Work in pairs to play—for at least a few turns—a version of the fraction tracks game.

Some Questions:

1. When do you have to use equivalent fractions?
2. In what situations can't you move? What would be a good rule to use for such a situation?
3. What other techniques could you use to generate fractions?
4. This game has no tenths. Does that create any problems? Are there advantages or disadvantages to having tenths?

0/2--1/2--2/2

0/3-----------------------------------1/3--2/3-----------------------------3/3

0/4---------------------------1/4-----------------------------2/4---------------------------3/4-----------------------4/4

0/5------------------1/5-------------------2/5------------------------3/5------------------------4/5----------------5/5

0/6------------1/6--------------------2/6--------------------3/6--------------------4/6------------------5/6----------6/6

0/8----------1/8-----------2/8------------3/8------------4/8------------5/8------------6/8-----------7/8--------8/8

start **finish**