# Percent

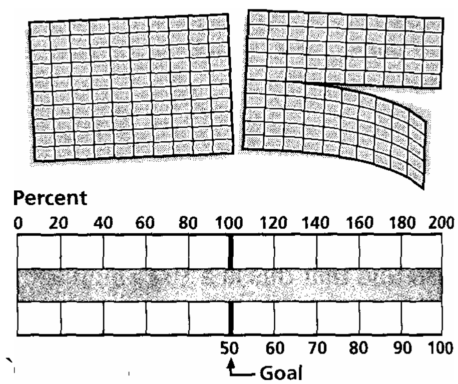
# PERCENT

Percent is an important mathematical topic. It is used frequently in real life situations, particularly in business when working with discounts, interest, commission and changes in price. The sales tax that you pay on purchases is given as a percent. We frequently use percents to compare two quantities, e.g. the literacy rates in two countries. Schools often use percent to indicate how well students do on tests.

**Percent** literally means "per hundred”.

**Ex. 1:** If you have 100 restaurant coupons and 25 of them are for Pizza Hut, then 25% of the coupons are Pizza Hut coupons.

**Ex. 2:** If you used 50 out of 200 postcard stamps, then you used 25% of your stamps.




**Ex. 3:** Your child's school is holding a car wash. If the school's goal was to wash 50 cars in one day, and the students actually washed, then they achieved 200% of their goal!

***What Is Percent?***

1. Percent can be thought of as a special **fraction** where 100 is the denominator. For example, you can say that it was found that $\frac{9}{100}$ of the computers had a virus (as a fraction), or 9% of the computers were infected with the virus (as a percent).

2. Percent is also a special **ratio** where 100 is the size of the group being studied. For example, you can say that out of 100 computers tested, only 9 were infected with the virus (9:100 as a ratio), or 9% of the computers were infected with the virus (as a percent).

3. A percent can always be converted to a **decimal** where the percent value is expressed as hundredths. For example, you can say that it was found that 0.09 of the computers tested had the virus (as a decimal), or 9% of the computers were infected with the virus (as a percent).

A number followed by a percent symbol (%) is the same as writing a ratio with a denominator of 100, so you can easily write it as either a fraction or a decimal.

| PERCENT | FRACTION | DECIMAL |
|---|---|---|
| 25% | 25/100 = 1/4 | 0.25 |
| 2% | 2/100 = 1/50 | 0.02 |
| 0.5% | 0.5/100 = 5/1000 = 1/200 | 0.005 |
| $33\frac{1}{3}$% | $33\frac{1}{3}$/100 = 100/300 = 1/3 | 0.333 … |

# RELATING PERCENT TO DECIMALS

Since the word percent means "per cent" or "per hundred," the “%” sign represents hundredths. Many people rely solely on moving the decimal point two places to the right or to the left to convert decimals to percent or vice versa. Often, learners move the decimal point without understanding so that 0.5% becomes 50 instead of 0.005 or 0.9 becomes 9% instead of 90%. This activity focuses on identifying the number of hundredths in a given number to determine the percent representation.

**Example:** How many hundredths are there in each of the following numbers?

| | | |
|---|---|---|
| 0.25 | ➔ | 25 hundredths or 25% |
| 0.8 | ➔ | 80 hundredths or 80% |
| 2.74 | ➔ | 274 hundredths or 274% |
| 0.005 | ➔ | 0.5 hundredths or 0.5% |

Since hundredths is the second decimal place, the “%” sign takes the place of two decimal places.

**EXAMPLES:** 85% contains two decimal places, so the 85 must be written using two places: 0.85.

2% contains two decimal places, so the 2 must be written using two decimal places: 0.02.

0.5% contains three decimal places (0.5 has 1 and % has 2), so the 5 must be written using three decimal places: 0.005.

0.98 contains two decimal places, so it equals 98%.

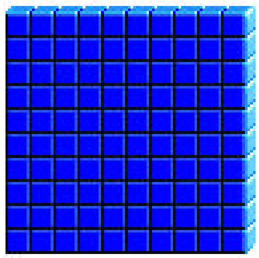
0.5 only contains one decimal place, but 0.5 = 0.50, and 0.50 equals 50%.

0.042 contains three decimal places; since the “%” sign uses two of them, it equals 4.2%.

# PERCENT MODELS

Many people confess that they are sometimes confused about percent and do not understand the basics of percent computations. To avoid this, models should be used so that it is easy to visualize concepts pertaining to percents. Instruction should emphasize context and the reasonableness of answers.

Several models can be used to help visualize percents. One of the most useful is the 100-unit square. Each small square represents 1% and the whole 100-unit square represents 100%.

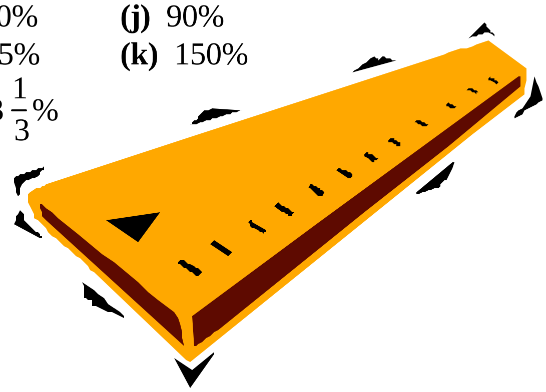


You can estimate percents using reference percents: 1%, 25%, 50%, 75%, and 100%.

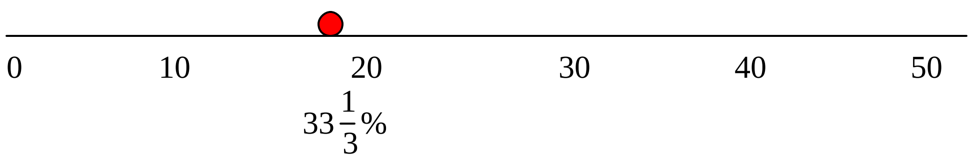
- 1% is close to zero. The indicated quantity is very small compared to the whole amount, such as 1 to 100, 3 to 300, and 25 to 2500.
- 25% equals $\frac{1}{4}$. The indicated quantity is substantially less than half of the whole amount, such as 5 to 20, 9 to 36, and 40 to 160.
- 50% equals $\frac{1}{2}$. The indicated quantity equals half of the whole amount, such as 12 to 24, 30 to 60, and 200 to 400.
- 75% equals $\frac{3}{4}$. The indicated quantity is substantially more than half of the whole amount, such as 30 to 40, 60 to 80, and 300 to 400.
- 100% equals 1. The indicated equals the whole amount, such as 50 to 50, 25 to 25, and 200 to 200.

Try to draw a number line or to use a metre stick to estimate the position of percents such as:

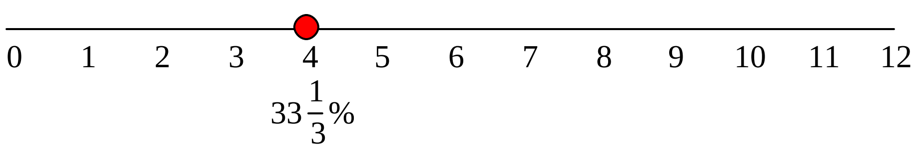
**(a)** 1% **(d)** 20% **(g)** 50% **(j)** 90%
**(b)** 0.5% **(e)** 25% **(h)** 75% **(k)** 150%
**(c)** 10% **(f)** $66\frac{2}{3}\%$ **(i)** $33\frac{1}{3}\%$



For example, on a number line ranging from 0 to 50 with increments of 10 placed on the line, $33\frac{1}{3}\%$ or $\frac{1}{3}$ of the distance of 50 can be estimated as shown:




As another example, $33\frac{1}{3}\%$ of 12 can be represented as shown:

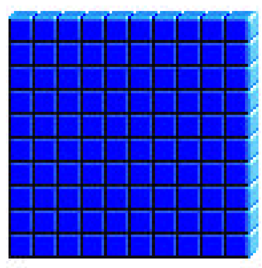



# EXPRESSING A FRACTION, DECIMAL OR RATIO AS A PERCENT

It is useful to be able to relate equivalent expressions for fractions, decimals, and percents.

A baseball player has 3 hits for 5 times at bat. Express his average as a fraction and as a percent.

You know that each square on the chart represents $\frac{1}{100}$ or 1%, while the entire grid represents 100%. Clearly, $\frac{3}{5}$ must be less than 100%.



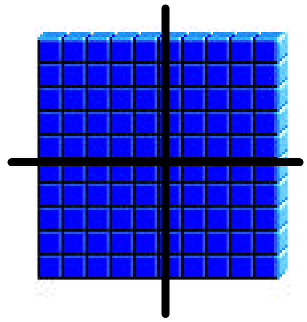
Since there are 100 squares and $\frac{1}{5}$ of 100 is 20, $\frac{1}{5}$ of 100% is 20%. $\frac{3}{5}$ must be $3 \times 20\% = 60\%$.

The situation can also be represented by the proportion 3:5 = $h$:100, where $h$ represents the number of hits:

$$\frac{3}{5} = \frac{h}{100}$$

You can use a variety of techniques to solve the proportion but, in this case, it might be solved by mentally multiplying 3 by 20 to produce $\frac{60}{100}$ or 60%.

Similarly, $\frac{3}{4}$ is 75%. You can split the 100-unit square into fourths and use 3 of the fourths (3 sets of 25 squares) to model $\frac{3}{4}$,



or you can solve a proportion.

$$\frac{3}{4} = \frac{h}{100}$$

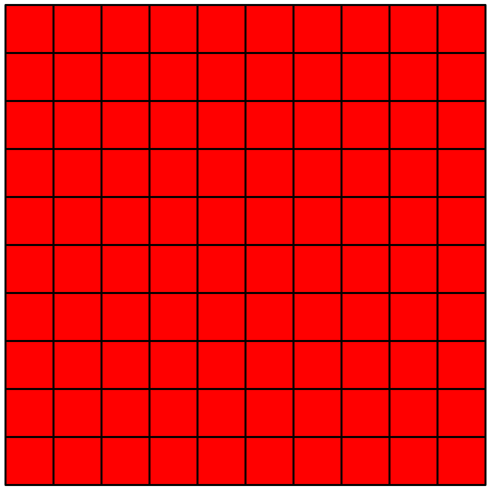
A ratio or fraction with a whole number for the second term or denominator that is a factor of 100 (1, 2, 4, 5, 10, 20, 25, 50, or 100) is easy to express as a percent. It is slightly more difficult to change a fraction to a percent when the denominator is not a factor of 100.

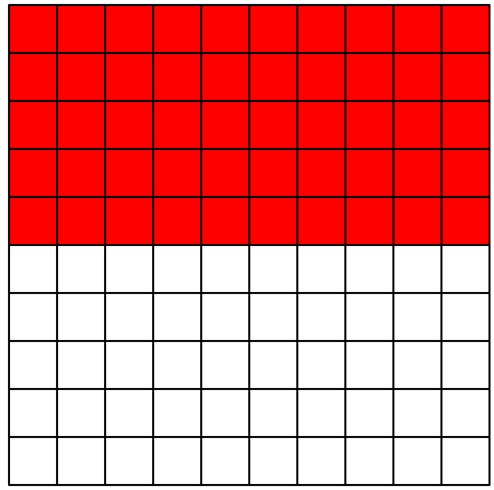
A baseball player has 3 hits for 7 times at bat. Express his average as a fraction and as a percent. $\frac{1}{7}$ of 100 squares can be found by dividing 100 by 7, which gives $14\frac{2}{7}$%. Therefore, $\frac{3}{7}$ must be $3 \times 14\frac{2}{7}\% = 42\frac{6}{7}\%$.

**Note:** In general, initially use a $10 \times 10$ grid to convert a fraction to a percent only when the denominator is a factor of 100; a proportion is probably more convenient to use to convert other fractions.

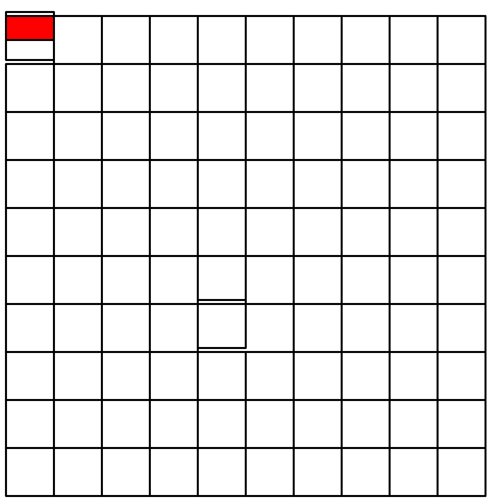
A $10 \times 10$ grid can also be used to represent a percent less than 1 and a percent greater than 100.

For example, here's how $\frac{3}{2}$ can be represented as 150%. Each $\frac{1}{2}$ represents 50 squares or 50%. Therefore, $\frac{3}{2}$ can be represented by 150 squares or 150%.





$\frac{1}{2}$ of 1% can also be represented using a $10 \times 10$ grid. Notice that in the diagram below, the shaded $\frac{1}{2}$ of one square represents $\frac{1}{2}$ of 1% or 0.5%. You can also see that 0.5% must be equivalent to the fraction $\frac{1}{200}$ since 200 half squares would make the whole grid. Notice that, because one square represents 0.01, $\frac{1}{2}$ of 1% equals the decimal 0.005.



# ESTIMATING PERCENTS

Decide whether each of the following statements is reasonable. Use an S for sense and an N for nonsense.

1. _____ When the Ames family went out to eat, 50% of them ordered hamburgers, 50% of them ordered hot dogs, 50% of them ordered French fries, and 50% of them ordered juice.
2. _____ The bank charges an annual percentage rate of 75% for a car loan.
3. _____ Shannon missed three problems on a quiz and scored 75%.
4. _____ Jamie missed four questions on a test and scored 80%.
5. _____ The Lincoln High Lions won 120% of their games.
6. _____ Ali paid $4.00 sales tax for an $8.00 purchase.
7. _____ The price of gasoline has increased more than 200% since 1972.
8. _____ A 50% chance of rain on Saturday and a 50% chance of rain on Sunday means that there is a 100% chance of rain during the weekend.
9. _____ A sale item marked 50% off with an additional 50% off is free.
10. _____ The stock value decreased 110%.
11. _____ Sean averaged his grades of 77%, 72%, 81%, and 73%, giving a result of 80%.
12. _____ Al was so hungry, he ate 125% of the cake.
13. _____ Adam correctly answered 45 of 50 questions and received a grade of 90%.
14. _____ The class survey showed that 75% of the class liked football, 50% of the class liked basketball, but only 25% of the class liked baseball.
15. _____ The class survey showed that 30% of the class had math second period, 40% had physical education second period, and 40% had English second period.
16. _____ Adam gave an effort of 110% on the hockey team.

Decide whether or not each of the following situations is reasonable. If not, state the error that is made, and correct the situation.

1. A $90.00 watch on sale for 30% off is marked at $60.00.

2. Mark can buy a $100 bicycle on sale at 20% off. If he pays cash, he receives an additional 5% off. Mark figures that he will pay $75.00 in cash for the bike.

3. Jo can buy two $50 skirts on sale for 25% off or she can take advantage of the "Buy One, Get One Off" special. Jo figures that the money saved is the same in both cases.

4. Ali can open a $500 CD account for 3 months at a rate of 6% followed by 9 months at a rate of 5%. Another bank has a 12-month $500 CD for 5.5%. Ali figures the interest earned in both cases will be the same since the average of 5% and 6% is 5.5%.

5. The news report states that the cost for a litre of gasoline was 110% of last year's price. Mary said that it could not be correct since 100% is the total price.

6. The club membership went from 75 to 90 members and then dropped down to 75 again. Tom reasoned that the percent of decrease was the same as the percent of increase.

Often it is easy to eliminate unreasonable answers to help determine the correct answer.


**WHAT ARE SOME WAYS TO FIND PERCENT?**

**METHOD 1:** To find a percent, you can write it as a fraction and then solve using fraction computational procedures.

**Ex.:** What percent is 23 of 92?

What you are being asked is, "What would be the equivalent fraction, using 100 as the denominator, of $\frac{23}{92}$?"

If $\frac{23}{92} = \frac{x}{100}$, there are several ways to solve for $x$.

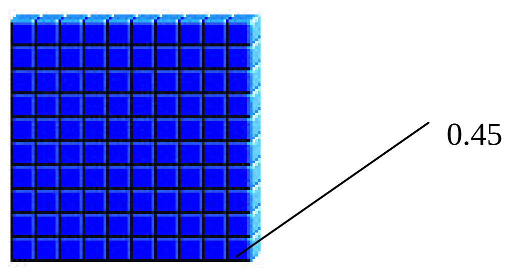
**(a)** Use equivalent fractions, as below:

$\frac{23}{92} = \frac{23 \cdot 100}{92 \cdot 100}$ *and* $\frac{x}{100} = \frac{92x}{92 \cdot 100}$, so 2300 = 92$x$ and $x$ = 25.

**(b)** Notice that $\frac{23}{92} = \frac{1}{4}$, which is 25%.

Therefore 25% of 92 is 23.

**METHOD 2:** Another way that you could solve a percent problem is by using the unit method. This method uses a grid of 100 squares and assigns a value to each square. For example, each square would be worth 1 if the entire grid were worth 100. Therefore, to show 20%, you would need 20 of the small squares. If the whole grid were worth 45, then each square would be worth 0.45. Therefore, to show 30% of 45, you would multiply $30 \times 0.45$ value/square to get 12 squares.



**Ex.:** What percent of 400 is 60?

If the whole grid is worth 400 and there are 100 squares, each square is worth 4. So, you ask the question, 60 is the value of how many squares?

Since $60 \div 4 = 15$, 60 is 15% of 400.

Remember that each square represents 1% and that the value assigned per square is worth 1%.

**METHOD 3**: Percents can be solved by representing them as decimals. Since you know that to make a percent into a decimal, all you have to do is write it in terms of hundredths, you can multiply the number by the decimal representation of the percent to find the answer.

**Ex.:** What is 33% of 40?

You know that 33% is 33 hundredths or 0.33. So, to find 33% of 40, you can multiply $0.33 \times 40$ to get 13.20.

You should remember that if you changed the 40 to 0.40 and the 0.33 to 33 and multiplied them, you would get the same answer:

$$33 \times 0.40 = 13.20$$

Thinking in this manner may be helpful in solving a problem such as finding 48% of 50.

$$48\% \text{ of } 50 = 0.48 \times 50 = 0.50 \times 48$$

If you changed the question to 50% of 48, you could easily solve the question mentally, because 50% of 48 means half of 48, so

$$0.50 \times 48 = 24$$

**Note:** In using a calculator with a "%" button, the calculator automatically converts the percent to a decimal. To use this feature to find 25% of 77, for example, enter $77 \times 25\%$ then "=" button. (Some calculators don't require you to press the "=" button.) The answer is 19.25.

**METHOD 4:** There is another way that uses percent as a fraction and is similar to the first method and the third method as follows.

**Ex:** Find 72% of 16.

$$\frac{72}{100} \cdot 16 = \frac{1152}{100} = 11.52$$

**METHOD 5:** Some percents are easy to calculate mentally and you can use combinations of these special percents to solve certain problems. Some of the easy to use percents include 1%, 10%, 50% and 100%.

One percent is one-hundredth of a number, or 0.01, if expressed as a decimal. Therefore to find 1% of a number, you multiply it by 0.01.

**Ex. 1:** 1% of 34 = $0.01 \times 34 = 0.34$

Ten percent is 10 hundredths or 0.10 as a decimal. Therefore, to find 10% of a number you multiply it by 0.10.

**Ex. 2:** 10% of 34 = $0.10 \times 34 = 3.4$

Fifty percent is 50 hundredths, which is 0.50 as a decimal or $\frac{1}{2}$ as a fraction. Therefore, to find 50% of a number, you could divide it in half. This method would also work for 25% and 75% if you thought of 25% as "a half of a half (or a quarter)" and 75% as "a half plus a quarter." One hundred percent is 100 hundredths, which is 1.00 or $\frac{100}{100}$, and that means that the product of a whole number and 100% is equal to the whole number.

**Ex. 3:** 100% of 63 = 63

This is useful to think of when dealing with percent values greater than 100, such as in the following example:

**Ex. 4:** 120% of 63 = (100% of 63) + (10% of 63) + (10% of 63) = 63 + 6.3 + 6.3 = 75.6

***Some Examples of Percent Problems***

1. What percent is 125 of 75?

**<u>Solutions</u>:**

**METHOD 1:** $\frac{125}{75} = \frac{x}{100}$ ➔ $\frac{125 \cdot 100}{75 \cdot 100} = \frac{12500}{7500}$ *and* $\frac{x \cdot 75}{100 \cdot 75} = \frac{75x}{7500}$

Therefore, $75x = 12\,500$ and $x = 166\frac{2}{3}$. The answer is $166\frac{2}{3}$%.

**METHOD 2:** If the whole grid were worth 75, then each square would be worth 0.75. Then how many squares would you need to make 125? The answer is 125 ÷ 0.75 value/square, which equals $166\frac{2}{3}$%.

**METHOD 3:** $x \times 75 = 125$ implies that $x = 1\frac{2}{3}$ which, as a decimal is $166\frac{2}{3}$%.

**METHOD 4:** 1% of 75 is 0.75. You need to find how many times 0.75 goes into 125 ➔ $125 \div 0.75 = 166\frac{2}{3}$%.

**METHOD 5:** 125 is more than 100% of 75 but less than 200% of 75, which is 150. Since 125 is closer to 150 than 75, 125 is closer to 200% of 75 than to 100% of 75. 125 is 25 away from 200% of 75 and 50 away from 100% of 75. Therefore, you could say 125 is 100% of 75 and another $\frac{2}{3}$ of the way to 200%. The fraction two-thirds is $66\frac{2}{3}$%. If you add 100% + $66\frac{2}{3}$%, you get $166\frac{2}{3}$%.

2. What is $\frac{1}{10}$% of 40?

<u>**Solutions**</u>**:**

**METHOD 1:** Begin by changing one-tenth to its decimal form, which is 0.10. This means that you must solve the equation $\frac{0.1}{100} = \frac{x}{40}$. Since $\frac{0.1}{100} = \frac{1}{1000}$ *and* $\frac{x}{40} = \frac{25x}{1000}$, this implies that $25x = 1$ and $x = 0.04$.

**METHOD 2:** If the whole grid is worth 40, then each square is worth 0.4. To show one-tenth of a square, you would find $\frac{1}{10}$ of 0.4, which is 0.04 $(0.1 \times 0.4 = 0.04)$.

**METHOD 3:** $0.001 \times 40 = 0.04$

**METHOD 4:** $\frac{0.1}{100} \times 40 = 0.04$

**METHOD 5:** You know that 1% of 40 is 0.4, and one-tenth of 1% is $\frac{1}{10}$%. Therefore, if you take $\frac{1}{10}$ of 0.4, you get 0.04.

3. Nine is 30% of what?

**Solutions**

**METHOD 1:** You would solve the equation $\frac{30}{100} = \frac{9}{x}$. Since $\frac{30}{100} = \frac{30x}{100x}$ *and* $\frac{9}{x} = \frac{900}{100x}$, it follows that $30x = 900$ and $x = 30$.

**METHOD 2:** Nine is represented as 30% of the grid or 30 squares. This means each square is worth 0.3. (Since 30 squares = 9, then 1 square = $9 \div 30 = 0.3$.) If you have 100 squares each worth 0.3, then the whole grid is worth $100 \times 0.3 = 30$.

**METHOD 3:** $x \times 0.30 = 9 \Rightarrow x = \frac{9}{0.30} = 30$

**METHOD 4:** This problem is probably solved more easily using one of the other methods.

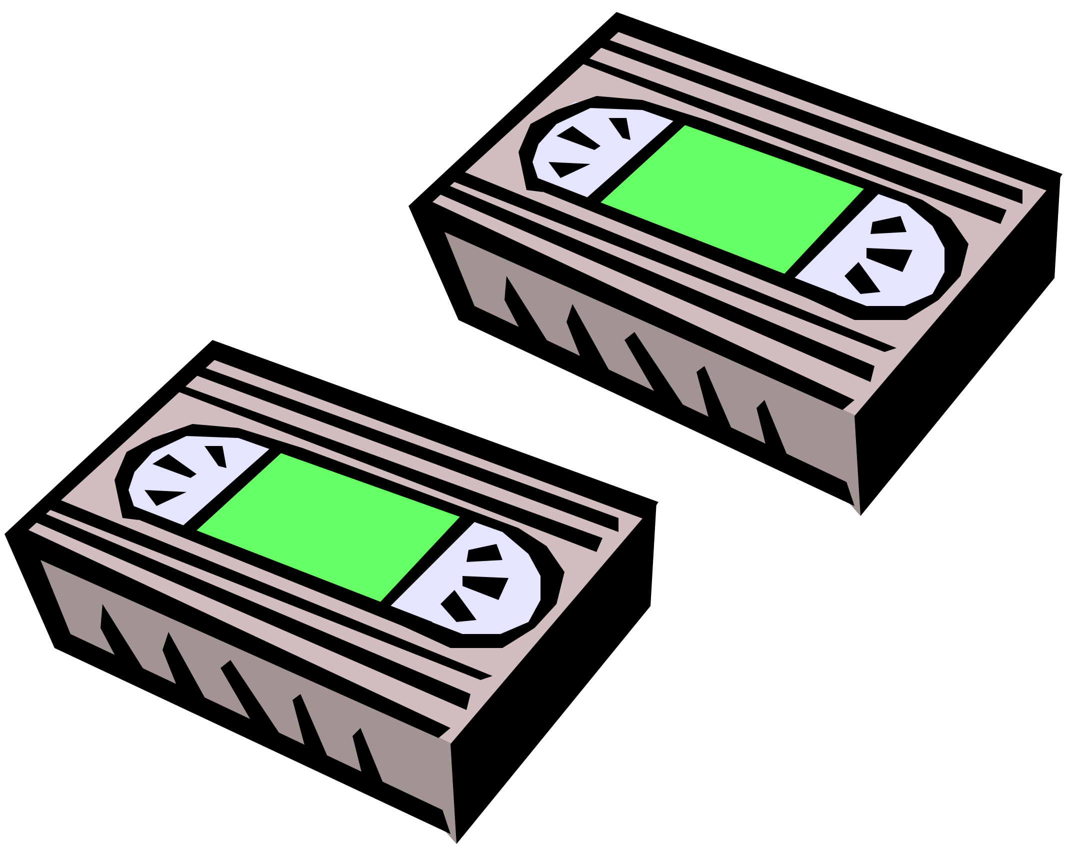
**METHOD 5:** If 30% of the number is 9, then 10% of the number is one third of that, or 3, and 90% of the number is $9 \times 10\%$ of the number, or 27. Then you can add 10% of the number (3) + 90% of the number (27), which equals 100% of the number, or 30.

Following is a real world problem that incorporates percents:

The VHS edition of a movie normally sells for $22.99. Blockbuster has the video on sale for $15.99. What was the percent decrease in price?

**Solution:**
First you have to determine what percent $15.99 is of $22.99. Therefore, you must solve the equation $\frac{15.99}{22.99} = \frac{x}{100}$.

Since $\frac{15.99}{22.99} = \frac{1599}{2299}$ *and* $\frac{x}{100} = \frac{22.99x}{2299}$, $22.99x = 1599$ and $x = 69.55\%$.
From this information, you can deduce the percent decrease.
If the new cost is 69.55% of the old cost, there must have been a 30.45% decrease in the original price, since 100% – 69.55% = 30.45%.



# FINDING A NUMBER WHEN A PERCENT OF IT IS GIVEN

Finding a number when the percent is given has been historically a difficult process for students to understand. But again the 100 unit square grid and the solution of a proportion can help students develop an understanding of the problem-solving process required to solve this problem.

Thirty percent of a price is \$16. What is the price?

You know that 30 squares or 30% of the grid represents \$16. Therefore, to determine the amount represented by 100% would be a way to know the full price.

If \$16 is represented by 30 squares, then each square, or each 1%, is worth one-thirtieth of \$16 or \$0.533. To find the full price, multiply 1% by 100.

$100 \times \$0.533 = \$53.33.$

You can check as follows: $30\% \times \$53.33 = \$16.00$.

This process can also be represented by the proportion:

$$\frac{30}{100} = \frac{16}{a} \Rightarrow 30 \times a = 16 \times 100 \Rightarrow a = 1600 \div 30 = 53.33$$

**EXERCISE 1**
Use the $10 \times 10$ grid to find a number when 12% of it is 25. Then check your solution by using another approach.

**EXERCISE 2**
Bev paid \$13 340.00 for a used automobile. This price included the 15% sales tax. What was the advertised purchase price of the automobile before tax?

# INTEREST

Interest is the amount that someone pays to use someone else's money. If you invest money in a savings account, the bank pays interest to you. If you borrow money from the bank, you pay interest on that money to the bank. The amount of money borrowed or invested is called the principal. The interest rate is the percent charged or paid during a given period of time.

## *Simple Interest*

There are two different ways of calculating interest: simple and compound. When you pay simple interest, you pay interest only on the principal, not on interest that has already been paid.

**EXAMPLE:** Suppose you borrow \$2000 at a simple interest rate of 12% per year. You agree to repay the loan at the end of 2 years. How much interest will you pay? How much will you pay to the bank in all?

The interest depends on how much you borrow, the interest rate at which you borrow and how long you borrow. It makes sense, therefore, that the formula that tells you how to compute interest involves all three amounts.

To find the amount of interest that you will pay, you can use this formula:

**Interest (*I*) = Principal (*P*) • Annual Rate of Interest (*r*) • Time in Years (*t*)**

The next few sentences talk about why this formula makes sense.

It seems reasonable that if you borrow twice as much, the interest should be twice as much. With this formula, that would happen.

$$I = prt$$

If the principal is $2p$, then $I = (2p)rt = 2(prt)$

It seems reasonable that if the rate is twice as high, you should pay twice as much. With this formula, that would happen.

$$I = prt$$

If the rate is $2r$, then $I = p(2r)t = 2(prt)$

It seems reasonable that if you borrow for twice as long, the interest would be twice as much. With this formula, that would happen.

$$I = prt$$

If the time is $2t$, then $I = pr(2t) = 2(prt)$

This should help you have some confidence in the formula.

This is how the formula is used:

$$\begin{aligned} I &= prt \\ &= (2000)(0.12)(2) \quad \text{[Remember that 12\% = 0.12.]} \\ &= 480 \end{aligned}$$

You will pay $480 in interest. Since the principal is $2000, you will pay $2000 + $480, or $2480, to the bank.

### *Compound Interest*

Unlike simple interest, compound interest is paid on the principal and on interest that has already been paid. You can calculate compound interest by making a table.

**EXAMPLE:** Suppose you put $500 in a bank account that pays an 8% annual interest rate and is compounded every month. After each 1-month period, the interest is added to the principal and you earn interest on the new total in your account. How much money will you have in the account at the end of 3 months?

Three months is equal to three 1-month periods.
The rate ($r$) is 8%, or 0.08 per year.
Since the time period is 1 month, which is 1/12 year, $t = 0.083$ (8 ÷ 12)

| PERIOD | PRINCIPAL | INTEREST | NEW TOTAL |
|---|---|---|---|
| 1st month | 500.00 | (500.00)(0.08)(0.083) = 3.32 | 503.32 |
| 2nd month | 503.32 | (503.32)(0.08)(0.083) = 3.34 | 506.66 |
| 3rd month | 506.66 | (506.66)(0.08)(0.083) = 3.36 | 510.02 |

So, at the end of 3 months, you will have $510.02 in the account.

You might compare this to simple interest. In this case, the interest would be found by using the formula:

$$\begin{aligned} I &= prt \\ &= 500(0.08)(\frac{1}{4}) = \$10 \end{aligned}$$

The total amount is \$500 + \$10, or only \$510.

Another way to calculate the interest using the formula above would have been to use the monthly interest for *r* and the number of months for *t*.

$$500(0.08 \div 12)(3) = \$10$$

***Credit Cards***

When you use credit cards, you may pay compound interest without realizing it. So, your annual effective interest — the interest rate you actually pay for the year — may be greater than the simple annual percentage rate listed on the card. How can that be? The answer comes from how your finance charges are computed. If you don't pay off your balance in full each month, you pay interest both on the unpaid balance and on the finance charges that are applied each day. When you pay interest on interest, interest is compounded. So, although the daily rate listed on the bill is accurate, the interest you pay over a year ends up being greater than the annual rate listed on the bill.

# PERCENT INCREASE AND DECREASE

Sports statistics can be used to demonstrate percent increase and decrease Here are a few questions that may help you to understand the concept:

**EXAMPLE 1:**

In the last two games that a basketball team played, they shot at the basket 115 times, but only 63 shots were successful. In their second game, they made 57 of 118 baskets. The coach, as well as the rest of the team, would like to know if they are playing better (that is, a percentage increase) or if they are actually getting worse (that is, a percentage decrease). To figure out whether the team has increased its playing skill from game 1 to game 2 you first must find the success percent for each individual game:

Game 1: $\frac{63}{115} = 0.547 = 54.7\%$

Game 2: $\frac{57}{118} = 0.483 = 48.3\%$

These numbers represent the percents in each of the games.

Since the percent of successful shots in the second game is less than in the first, this implies that there was a percent decrease in the success rate. The percent decrease is the difference between the two percents: $54.7\% - 48.3\% = 6.4\%$ decrease. You must be careful not to assume that because there were more baskets made in the second game that it would be a higher percent. The answer is not only dependant on how many field goals were made, but also on how many were attempted.

An example where the larger number of successes does not result in a larger percent would be as follows: In game 1, 118 attempts were made but only 57 went in, while in game 2, 63 baskets went in out of the 135 attempted.

Game 1: $\frac{57}{118} = 48.3\%$

Game 2: $\frac{63}{135} = 46.7\%$

In this example, there were fewer baskets made in game 1, but in proportion to the shots attempted, there is a higher percent than in game two. In this situation there was a percent decrease of 1.6% in success rate (that is, $48.3\% - 46.7\%$). You must be careful not to assume that because there were more field goals made in the second game that it would be a higher percent. The answer is not only dependant on how many field goals were made, but also on how many were attempted.

**EXAMPLE 2:**

In 1968, the women's first prize in the U.S. Open in tennis was only \$6000, but by the year 1974, women's tennis had attracted a wider audience and the first prize had risen to \$22 500, which is a significant increase in prize money.

To find what the percent increase was from 1968 to 1974, you could divide \$6000 into \$22 500, which is $22\ 500 \div 6000 = 3.75$. Since 3.75 equals 375%, this means that the new prize was 375% of the old prize. The increase, though, was 275%, since 275% = 375% – 100%.

This is a huge increase. An increase of 275% tells us that the prize money is almost four times as great as it was only six years earlier. It might also have been helpful to estimate the answer first.

**Estimated Method:**

$\$6000 \times 3 = \$18\ 000$, which is quite a bit less then \$22 500, while $\$6000 \times 4 = \$24\ 000$, which is a little more then \$22 500. This would tell you that the newer prize is somewhere between 3 and 4 times as much as the older one, but closer to 4 then to 3. Using this approach, you should be able to estimate that the percent increase is somewhere between 200% and 300%, and since it is closer to 4 times as much, we know that the percent increase is closer to 300%.

# FUN WITH NUMBERS

Here is a little trick that may be of some interest:

> Increase a number by 10%, then increase the new number by 10%. How many increases of 10% must you make before you double your original number? Remember that after you increase the number by 10%, you then must take 10% of the new number not the original number.
>
> Repeat the process with another number. Do you get the same results?
>
> **EXAMPLE:** If your original number is 10, how many increases of 10% will you have to make to double 10 to get to the number 20?
>
> 10% of 10 = 1 ➔ The new number is now 10 + 1 = 11.
> 10% of 11 = 1.1 ➔ The new number is now 11 + 1.1 = 12.1.
> 10% of 12.1 = 1.21 ➔ The new number is now 12.1 + 1.21 = 13.31.
> 10% of 13.31 = 1.331 ➔ The new number is now 13.31 + 1.331 = 14.641.
> 10% of 14.641 = 1.4641 ➔ The new number is now 16.1051.
> 10% of 16.1051 = 1.61051 ➔ The new number is now 17.71561.
> 10% of 17.71561 = 1.771561 ➔ The new number is now 19.487171.
> 10% of 19.487171 = 1.9487171 ➔ The new number is now 21.4358881.

# POPULATION AND PERCENTS

Percents are used to determine the number of people that might have moved into and/or out of an area over a time period in relation to the population at that present time.

> For example, let's say that Koho had a population of 100 000 people last year. This year, they had a population increase of 5% and are aware that some new companies are setting up headquarters in the city. The city officials hope that this will draw in new residents. They want to make up a sign letting everyone know how many people live in their city, but the sign is for next year. The companies say that their businesses will bring about a 7% increase in population. What is the population that they should put on the sign?
>
> The original number of people in Koho was 100 000, but 5% more people moved in this year. This means that there are 5000 (100 000 × 5%) new residents in Koho this year. This brings the total number of people in the town to 105 000. However the companies claim that their businesses will increase the population of Koho by 7%. Since the businesses expect that next year there will be 7350 (105 000 × 7%) more people, the total number of people that you would expect to see in Koho next year would be 112 350 (105 000 + 7350). Should the city officials decide to include the new potential residents in their sign, they could estimate that there would be 112 350 people living in Koho next year.

Another example relating to population, is below:

> Dodleda has the population of 3.2 million people. Last year, it had a population of 3.3 million people, which means that 0.1 million (or 100 000) people moved out of Dodleda in a year. That seems like a lot of people, but not if thought of as a percent decrease.
>
> To find the percent decrease, you need to find out what percent 0.1 million is of 3.3 million (the original population).
>
> $0.1 \div 3.3 = 0.03 = 3\%$ ➔ Written as a percent, this number seems very small.
>
> However, in the much smaller city of Numbchecka, which has a population of 200 000, if one hundred thousand people moved out, then the percent decrease would be much greater (100 000 ÷ 200 000 = 0.5 = 50% decrease). It is the same number of people moving out of Numbchecka as moved from Dodleda, but because the population of Dodleda was so much larger its decrease seemed small.

# SHOPPING, SALES PRICES & TAXES

Here are examples of percent problems that deal with buying items.

**EXAMPLE 1:** A Tommy Hilfiger sweater costs \$79.99 and there is a sales tag attached to the sweater saying that it is 25% off. In addition to that, there is a special in the store today. The special says that the store will pay the tax (15%) for any item(s) purchased today. What percent is the buyer saving from the original price of \$79.99? In other words, what is the percent decrease in price?

The sweater would normally cost \$79.99 plus tax, which is \$79.99 + (\$79.99 $\times$ 15%) = \$79.99 + \$12.00 = \$91.99. On sale, the sweater costs \$79.99 – (25% $\times$ original price) + no tax = \$79.99 – (\$79.99 $\times$ 25%) + 0 = \$79.99 – \$20.00 = \$59.99.

By buying the sweater on this particular day, you would save \$32 (\$91.99 – \$59.99). This would be a percent decrease of 34.8% (32 $\div$ 91.99 $\times$ 100%).

This is a good type of question to show that you cannot simply add the two given percentages (that is, 40% = 25% + 15%) to get the results; if that had been the case, then the sweater would cost \$47.99 (\$79.99 $\times$ 40%).

**EXAMPLE 2:** George wants to buy a pair of Ikeda jeans. Both Jeans Experts and Bootlegger sell the jeans that he wants. The jeans are ticketed at both stores as costing \$64.99, however at both stores, the jeans are on sale. Bootlegger has the jeans for 15% off the ticketed price while Jeans Experts is going to pay the sales tax of 15%. At which store should George buy his jeans?

At Jeans Experts, you would pay \$64.99 (you would save \$9.75, because \$64.99 $\times$ 15% = \$9.75). At Bootlegger, you would pay \$63.53 (you would save \$11.21; \$64.99 + \$9.75 [tax]) $\times$ 15% = \$11.21).

Although it may appear that the jeans should be the same price at both stores, that is not the case. George will get his jeans for a better price at Bootlegger. Be very careful - things are not always as they appear.

**EXAMPLE 3**: Marjorie is at a store and the sign says 30% off. The dress she wants is $79.99 at the regular price. What **one** calculation can she do to find the sale price?

If a price is reduced by 30%, the actual price is 70% of the original cost, so she can calculate $79.99 × 70% = $55.99. This is simpler than calculating 30% and then subtracting from the full price.

# Percent Activities

## ACTIVITY

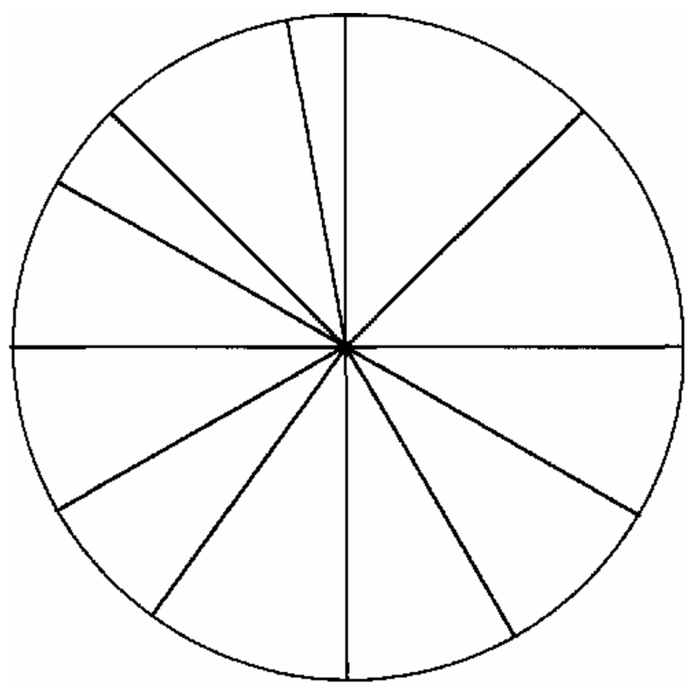
**Circle Graphs:** Estimate the relative size of regions of a circle as a percent. Use the given data to calculate percents and draw circle graphs.

**Optional:** Transparent circle protractor
If possible, use a transparent circle protractor and consider the relationship between degree measure and percent. These activities require you to estimate percent regions. For example, you should realize that 25% is $\frac{1}{4}$ of a circle, which is 90 degrees. Other reference percents to consider include: 10% (36 degrees), 50% (180 degrees), and 75% (270 degrees).

The Circle Graph activity could be expanded to include estimating the number of degrees in each region divided by the whole quantity (base).

***Pie Chart Meter***

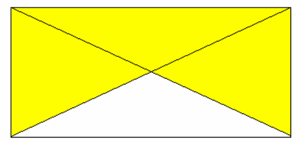
Using the circle below; shade in each of the following percentages: 90%, 100%, 83.3%, 75% and 66.6%.



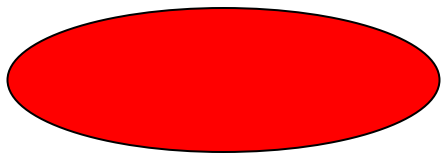
Choose one of the given percents to **estimate** the required area or solve the problem posed. Do not calculate exact amounts.

1% 25% 50% 75% 100% 150%

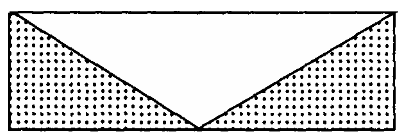
1. The shaded region:



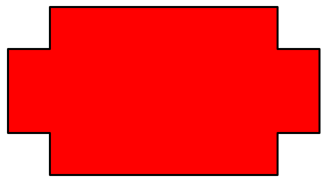
2. The shaded region:



3. The unshaded region:



4. The shaded region:



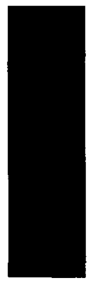
5. Twelve of 22 students in a class are girls. What percent of the students are girls?
6. What percent of 62 is 46?

7. Three of the 297 students were state winners. What percent of the students were state winners?

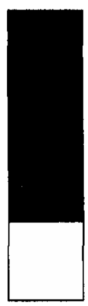
8. Last year, the jacket cost $70. Now it costs $100. By what percent did the price increase?

9. The Jacobsons left a $5.00 tip for a $22.74 bill. What percentage of the bill was the tip?

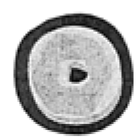
10. Let the following shape represent 100%.



What percent does the following shape represent?



## EXERCISES



1. Shawn bakes 2000 bagels in an 8-hour work day. On the average, how many bagels does he bake in a half hour?
   **(a)** 125 bagels **(b)** 250 bagels **(c)** 8000 bagels **(d)** 16 000 bagels

2. If the choir has 36 members, and the ratio of women to men is 4:5, then the number of men in the choir is
   **(a)** 5 **(b)** 9 **(c)** 16 **(d)** 20

3. What is 0.5% of 120?

**(a)** 0.6 **(b)** 6.0 **(c)** 60 **(d)** 600

4. What has the same value as 22% of 3800?
   **(a)** 44% of 1900 **(b)** 11% of 1900 **(c)** 44% of 7600 **(d)**11%of 7600

5. Which is the same as 4.23 million?
   **(a)** 423 hundred thousands **(b)** 423 ten thousands
   **(c)** 423 thousands **(d)** 423 hundreds

6. Which is least?
   **(a)** 25% of 824 **(b)** 40% of 596 **(c)** 15% of 5000 **(d)**80% of 420

7. Which one of these numbers is approximately 200?
   **(a)** 43.6 ÷ 0.2 **(b)** 38.9 x 7.3 **(c)** 417 ÷ 0.2 **(d)** 52.9 x 7.6

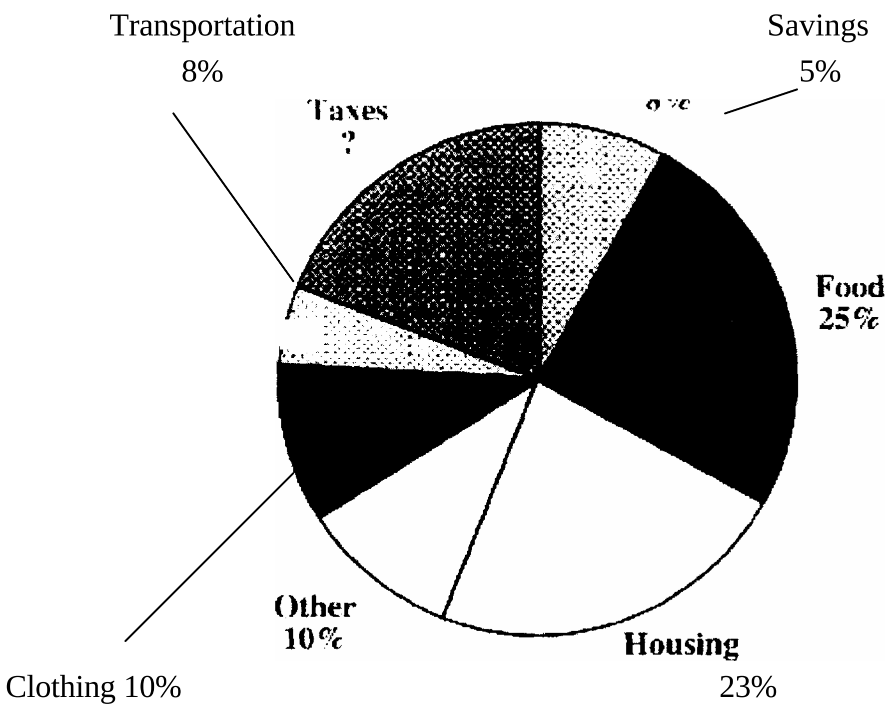
8. Jeff's salary was increased from $6.25 an hour to $6.75 an hour. What was the percent increase?
   **(a)** 5% **(b)** 6% **(c)** 7% **(d)** 8%

9. Lyle is driving from home to his friend's house, a distance of 720 km. He plans to drive at an average speed of 80 km per hour, take an hour for lunch, and take two 15-minute breaks. He must arrive at his friend's house by 7:00 p.m. What is the latest possible time that Lyle could leave home in order to arrive at his friend's house on time?

10. The regular price of the video camera shown in an ad is $849.95. What is the sale price?

Use the following information when answering questions A and B.

# Family Budget




The Johnson family's income was $34 000 in 2000. The graph above shows how the family budgeted their money.

1. Calculate the amount that the Johnson family spent for food in one year.
   **(a)** $250 **(b)** $850 **(c)** $2500 **(d)** $8500

2. How much did the Johnson family pay in taxes in 2000?
   **(a)** $646 **(b)** $2754 **(c)** $6460 **(d)** $27 540