Numeracy Boost: Background Materials for Adult Learners in Mathematics

These materials were developed as part of a 4-day conference sponsored by Human Resources Development Canada for adult literacy facilitators working with adult learners of mathematics.

The materials provide background to make concepts and procedures involved in learning about fractions, decimals, whole numbers, algebra and percent more meaningful for learners.

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Whole numbers and decimals

Number Sense –
wholes and decimals

WHAT IS NUMBER SENSE?

Number sense refers to an intuitive feeling for numbers and their various uses and interpretations, an appreciation for various levels of accuracy when calculating, the ability to quickly detect arithmetical errors, and a common-sense approach to using numbers.

Number sense is characterized by a desire to make sense of numerical situations. Number sense is a way of thinking that must permeate all aspects of mathematics teaching and learning if mathematics is to make sense. When highlighting number sense, students focus on their solution strategies rather than on a "right answer", on thinking rather than on the mechanical application of rules, and on student-generated solutions rather than on teacher-supplied answers. Learners will acquire a good sense of number if they are engaged in purposeful activities requiring them to think about numbers and number relationships and to make connections to quantitative information seen in everyday life. Learners should be encouraged to develop their own techniques for finding the answer, to consider many ways to work out each problem, to consider the form their answers might take, and to share their reasoning. Where number sense is a priority, learners are active participants who share their hypotheses, reasoning and conclusions and who are given opportunities to create their own procedures.

Consider the Following Examples:

When asked to find the product of 48 and 0.5 mentally, one person converted 0.5 to $\frac{1}{2}$, and then took $\frac{1}{2}$ of 48 to get an answer of 24. In contrast, another person mentally "wrote" 48 above 0.5 in his mind and multiplied 5 and 8, remembering the 0 and "carrying" the 4. Then he calculated 5 times 4, getting 20, and added the 4 he carried, getting 24. Counting the number of decimal places in his problem, he knew to insert the decimal point in his answer between the 4 and 0, thus arriving at the answer, 24.0 or 24. Which person displayed number sense?

In computing the product of 4.5 and 1.2, one learner carefully lined up the decimals and then multiplied, bringing the decimal point straight down and reporting a product of 54.0. Reflection on the answer should have caused her to realize the product was too big. Multiplying 4.5 by a number slightly more than 1 produces an answer a little more than 4.5. Instead, the learner applied an incorrect procedure (line up the decimals in the factors and bring the decimal point straight down) and did not reflect on whether the resulting answer was reasonable.

Two individuals were using their calculators to calculate sales tax on a set of items. For one problem, tax was calculated to be \$54 on a \$12 purchase. The failure to enter the decimal form of the tax rate of 4.5% (0.045) yielded a totally unreasonable answer. Someone without number sense might never stop to question his answer.

A group of learners was discussing the fraction $\frac{2}{5}$. They made statements such as the following. " It's about the same as a half. It's less than $\frac{1}{2}$. It's more than $\frac{1}{4}$. It's more than $\frac{1}{3}$. It's twice as much as $\frac{1}{5}$. Two "two-fifths" would almost make a whole. It can be named as 0.4. It's the same as 40 percent. It's a proper fraction. It's a rational number. It's the same as 8/20. Monday and Tuesday make $\frac{2}{5}$ of our work week."

This group is displaying a good sense for fractions – relating them to their own experiences, creating extensions of those experiences, and exhibiting a sense for the size, meaning and variety of expressions associated with the fraction $\frac{2}{5}$. Notice that the talk was open-ended and learners were allowed opportunities to verbalize their knowledge, to listen to each other and build on each other's ideas, and to create their own meaningful representations.

Within the past few years, growing attention has been directed toward what is now being called number sense. A conference for researchers, *Establishing Foundations for Research on Number Sense and Related Topics*, was sponsored by the National Science Foundation in 1989. The *Principles and Standards for School Mathematics* (NCTM, 2000) includes number sense as a major theme throughout its recommendations:

> Equally essential is computational fluency – having and using efficient and accurate methods for computing. Fluency might be manifested in using a combination of mental strategies and jottings on paper or using an algorithm with paper and pencil, particularly when the numbers are large, to produce accurate results quickly. (p. 32)
>
> Part of being able to compute fluently means making smart choices abut which tools to use and when. (p. 36)

ESTABLISHING BENCHMARKS FOR WHOLE NUMBERS

Adults often use benchmarks or common referents to process numerical information. For example, knowing the population of your town might help you judge the size of a crowd attending a concert. Or, if the high school stadium holds 1000 people and a report says that 150 000 people attended a rock concert, you might think of the size of the concert crowd as being the same as the stadium filled 150 times. This activity is designed to see if learners are aware of the size of some commonly used referents.

First, find the number that represents each of the following; estimate and then do research to determine the value that answers each question.

1. Population of the world
2. Population of Canada
3. Population of your community
4. Canadian government budget
5. The number of tons of garbage generated every day by your community

EXACT AND APPROXIMATE NUMBERS

Think about situations when exact values are needed and other situations when approximate values are necessary and useful.

Investigate the use of approximate and exact numbers in newspaper articles. Examine copies of the front page of various newspapers. Use a marker to circle numbers in headlines and articles. Review the context for the use of each circled number to determine if it is an exact or an approximate value. For example, do the numbers in the headlines below refer to exact or approximate values?

Canadian population tops 220 million
Lottery winner earns over $200 000 annually
Stocks fall 5.4%

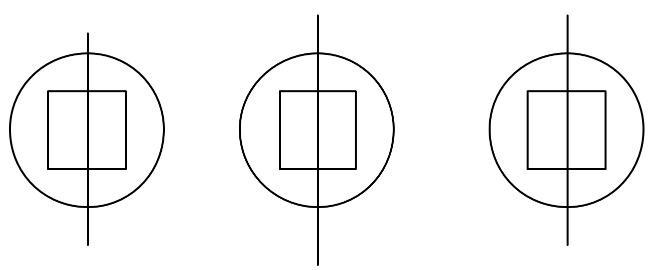
MENTAL MATH

Being able to calculate mentally is an asset in everyday life. There are many tricks to help you to do mental calculations.

For example, 82×3 can be difficult to do in your head, but if you change your thinking process and think that $80 \times 3 = 240$ and $2 \times 3 = 6$, then $80 \times 3 = 240 + 6 = 246$. Here, 82 is closer to 80 than 90, therefore it makes sense to use 80 and than add the extra later.

If the number ends in a digit such as 8 or 9, it makes sense to go up to a multiple of 10 and then find the difference as opposed to going down. For example, $4 \times 78 = 4 \times 80 = 320$, but this is too large because originally, there were 4 groups of 78 and now there are 4 groups of 80 so you have to subtract 8 from 320 to get 312, which is the answer for 4×78.

Another mental tip when multiplying, is to double one number and halve the other. This can be used when one number is even, such as in 44×5. Looking at this problem, you can see that doubling the 5 to get 10 will make the multiplication easier. If you double one number you must halve the other in order to achieve the same answer. This is because if you have several groups of an item and split each group in half, you will have the same total, but there will be twice as many groups with half as much in each group.



This is easily done when multiplying with an even number:

44×5 — Half of 44 is 22, therefore I will multiply 22×10, which gives 220.

There are many more tips that you can use to aid in doing mental math.

Compensation Method

Example: 198 + 64
Think: 200 + 64 = 264 (Oops! 2 too many)
Reasoning: 264 – 2 = 262

Add-Up Method

Example: 63 – 27
Think: 27 + ? = 63
Reasoning: 27 + 3 = 30 (3 more to get to 30; add ones to multiple of 10)
30 + 33 = 63 (33 more to get to 63)
63 – 27 = 3 + 33 = 36

Left-to-Right Method

Example: 87 + 35
Think: 80 + 30 = 110, 7 + 5 = 12 (10 + 2)
Reasoning: 110 + 10 = 120, 120 + 2 = 122

Creating a Simpler, But Equivalent, Computation

Examples: 58 + 36 = 60 + 34 (move 2 from the 36 to the 58)
400 – 168 = 399 – 167 (take 1 away from 400 first, then take away the other 167)
38 × 50 = 19 × 100 (half as many groups, but twice as much in each group)
48 + 16 = 24 + 8 (if 16 people share 48, half of them share half of 48)

Doing "Too Much" and Fixing It Up

Examples: 199 + 386 = (200 + 386) – 1
423 – 199 = (423 – 200) + 1
17 × 19 = 17 × 20 – 17
24% of 80 = (25% of 80) – (1% of 80) = 20 – 0.8 = 19.2

Doing the Computation in Pieces

Examples:

$217 + 358 = 217 + 300 + 50 + 8$
$423 – 218 = 423 – 200 – 10 – 8$
$17 \times 22 = 17 \times 20 + 17 \times 2$
$586 + 5 = 555 + 5 + 31 + 5$
12% of 900 = (10% of 900) + (1% of 900) + (1% of 900) =
90 + 9 + 9 = 108

Looking for Compatibles

Examples:

$482 + 75 + 218 + 20 + 5 = (482 + 18) + 200 + (25 + 75)$
$4 \times 11 \times 25 = (4 \times 25) \times 11$

ESTIMATION

Estimation is an important skill in order to check the reasonableness of results, especially when using calculators and computers. Estimation uses mental math techniques. Thus, instruction and practice on both estimation and mental math should be integrated into as many topics as possible. Often a combination of both mental strategies and paper-and-pencil strategies are used to perform complicated computations. It is extremely effective to have students develop strategies using mental arithmetic and estimation. The use of mental computation and estimation helps learners to develop a more realistic view of computation. It also empowers them to select from several computing methods. The need for these basic skills has increased in our technological society.

Computational estimation, where an individual finds an approximate answer for a computation, is one aspect of estimation. Following is a list of computational estimation strategies that you might wish to present to learners. This is not a comprehensive list and you may want to include other estimation techniques. Examples use addition and subtraction, but the estimation techniques can be used with other operations.

Rounding Method

There are many variations of rounding. Whether you round up or down or simply make adjustments depends on the circumstance.

Example:	3662 – 1180
Think:	Round 3662 to 3700.
	Round 1180 to 1200.
Then:	3700 – 1200 = 2500, therefore the exact answer should be approximately equal to 2500.

Front-End Method

Example:	345 + 175	
Think:	High estimate:	400 + 200 = 600
	Low estimate:	300 + 100 = 400

Front-End Method with Adjustments

This method is especially good with computations involving money.

Example: $1.26 + $4.79 + $0.99 + $1.37 + $2.58
Think: I have $1 + $4 + $1 + $2 = $8, and I can round off the cents to (25¢ + 75¢ = $1) + (99¢ = $1) + (40¢ + 60¢ = $1) and then make an adjustment of $3 for the cents.
Therefore, the estimate is $11.

Clustering Method

This method is used when numbers cluster around a central value.

Example: 72 250 + 63 819 + 73 180 + 67 490
Think: Three numbers are around 70 000 and I know that 63 819 is closer to 60 000 but it is easier to multiply 70 000 $\times$ 4 which is approximately 280 000.

GROUPING IN COLUMN ADDITION

Suppose you want to add a few numbers in your head. You can group them in the way that makes it easiest for you to add them.

EXAMPLE 1

You are traveling through Arizona. It is 42 kilometres on Route 66 from Interstate 40 to Grand Canyon Caverns. It is 58 kilometres from the caverns on to Hackberry and 39 kilometres farther to where you can pick up Interstate 40 again. If you want to take the scenic route along Route 66, how many kilometres will you travel?

Solution:

Group the numbers in the way that makes adding easiest.

42	100	In this problem, it's easier to add 42 and 58 first,
58		then add on 39 afterwards, giving a total of 139.
<u>+39</u>	<u>+39</u>	However, if it's easier to add the second and third
	139	numbers first, you should do so.

EXAMPLE 2

Use mental math to add 1.5 + 7.3 + 2.7 + 8.0.

Solution:

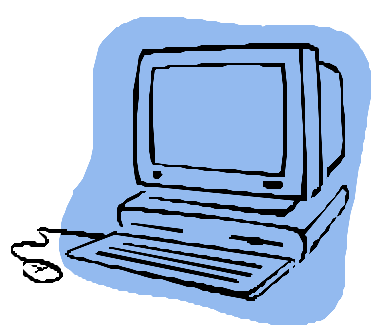
1.5		7.3 + 2.7 = 10, and 1.5 + 8.0 = 9.5
7.3	10.0	so, 1.5 + 7.3 + 2.7 + 8.0 = 19.5
2.7		
<u>+8.0</u>	<u>+9.5</u>	
	19.5	

A sum is the result of adding two or more numbers. Sometimes you don't need an exact sum. In these cases, you can estimate. You can use estimates to check computation.

EXAMPLE 1

Is $2000 enough money to buy a computer and printer?

ESTEY'S COMPUTER SALE	
COMPUTERS	PRINTERS
$1595.00	$819.00



Solution:
Do you need an exact sum or an estimate? Since you only need to know whether 1595 + 819 is less than or equal to 2000, you do not need an exact sum. In this case, an estimate is enough. Therefore, 1595 + 819 is approximately 1600 + 800, and 1600 + 800 = 2400, so $2000 is not enough money.

EXAMPLE 2

A grand piano weighs 610 kilograms. An upright piano weighs 480 kilograms. Could you put both pianos in an elevator with a weight limit of 1000 kilograms?

Solution:

Since the question only asks if 610 + 480 is greater than 1000, you do not need an exact sum. In this case, an estimate is enough. Therefore, 610 + 480 is approximately 600 + 500, and 600 + 500 = 1100. That's over the limit.

EXAMPLE 3

You add 457 804 + 369 750 on your calculator. The display shows **627554**. To check this answer, you can estimate the sum and compare. If the number on the display is not close to your estimate, you should re-compute the sum.

Using Compatible Numbers to Estimate Sums

Compatible people are people who get along. Compatible numbers are numbers that get along, too. Number pairs that are easy to add are compatible.

Fives are compatible:

75 + 25 = 100   **15 + 35 = 50**

Tens and any numbers that make tens are compatible:

30 + 40 = 70   **33 + 47 = 80**

When estimating sums, you can replace a pair of addends with compatible numbers before adding.

EXAMPLE

A double-decker train car has 77 people on the top level and 27 people on the bottom level. Approximately how many people are in the train car?

Solution:

To solve the problem, you can estimate.

77 + 27

Use compatible numbers 75 and 25, which are easy to add and close to 77 and 27.

75 + 25 = 100

Since 75 + 25 = 100, 77 + 27 is a bit more than 100. So, there are approximately 100 people in the train car.

Front-End Estimation of Sums

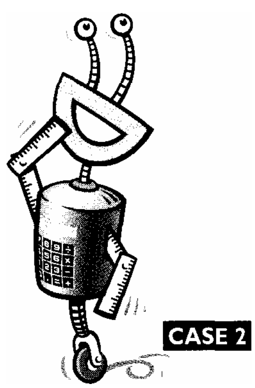
Another way to estimate sums is to add the front digits.

EXAMPLE 1

Paul's father takes a lot of trips for business. He traveled 9964 kilometres during April, 9816 kilometres during May, and 8640 kilometres during June. As a Frequent Flyer member, he earns a free airplane ticket for every 25 000 kilometres he travels. Did these trips earn him a free ticket?

Solution:

To solve the problem, you can estimate. Using front-end estimation, add the values of the digits in the front place (thousands), and you will see that he traveled at least 9 + 9 + 8 = 26 thousand kilometres.



Front-end estimation always gives a sum less than the actual sum. Therefore he earned a free ticket, because he traveled at least 25 000 kilometres.

You can still use front-end estimation when the addends have a different number of digits.

EXAMPLE 2

Estimate the value of 2106 + 742 + 895 + 309.

Solution:

One of the front digits is in the thousands place. The others are in the hundreds place. To estimate the sum, you can use the thousands and hundreds digits. Add the values of the digits in those two places.

$$\begin{array}{rcr} 2106 & \longrightarrow & 2100 \\ 742 & \longrightarrow & 700 \\ 895 & \longrightarrow & 800 \\ 390 & \longrightarrow & \underline{+300} \\ & & 3900 \end{array}$$

Therefore, the sum is greater than 3900.

A **product** is the result of multiplication. Sometimes when you multiply, you don't need an exact product. In these cases you can estimate products. You can also use estimates to check exact answers you've found.

EXAMPLE 1

You can rent a car for \$69 per day, with no extra charge for the number of kilometres driven. Is \$300 enough to rent the car for 5 days?

Solution:

Since you only need to find out whether 5×69 is less than or is equal to 300, you can use an estimate to answer this question: $5 \times 70 = 350$. Therefore, you don't have enough money.

EXAMPLE 2

You want to find 43 $\times$ 621. Your calculator displays **25703** as the product. You can estimate to check whether your exact answer is reasonable. If your estimate isn't even close, you compute again, and get 40 $\times$ 600 = 24 000. Your answer is reasonable.

Front-End Estimation of Products

To estimate products, you can multiply the front digits.

EXAMPLE 1

The air distance between Chicago and New York is 1150 kilometres. Suppose you made the trip 52 times (26 round trips) in one year and you earned one point for each kilometre flown. Did you earn enough for a bonus that requires 30 000 points? (Source: *World Almanac and Book of Facts*)

Solution:

Since you only need to know whether you've flown more than or less than 30 000 kilometres, you can estimate 52 $\times$ 1150, as follows:

1150	——>	1100	The exact product is greater than 35 000, so
$\underline{\times 52}$	——>	$\underline{\times 50}$	you can be sure you earned enough points.
		55 000	

EXAMPLE 2

You're heading for the checkout stand with 5 cans of orange juice priced at $1.35 apiece when you see a magazine you really want to read. You have $10 in your pocket. Can you buy the orange juice and the $3 magazine?

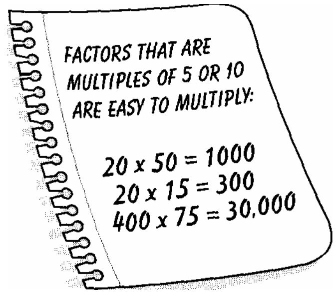
Solution:

To solve the problem, you can estimate the cost of the orange juice to see whether paying for the juice will leave you with at least an extra $3. Front-end estimation would be 5 $\times$ 1 = 5. Since $5 + $3 is only $8, you'd pick up the magazine, assuming you trusted the estimate. But, you know that a front-end estimate is less than the exact product. Therefore, to be a little more certain, you might adjust your estimate:

$$5 \times 0.30 = 1.5, \qquad \text{another \$1.50 for the orange juice}$$

This is getting awfully close to $10, which is all you have in your pocket, so an estimate might not really be enough to help you make your decision.

Using Compatible Numbers to Estimate Products



Compatible numbers are numbers that work well together. In multiplication, they are number pairs that are easy to multiply. To estimate products, replace one or both factors with compatible numbers.

EXAMPLE 1

You need to plan snacks for 26 weekly meetings for a staff of 40. About how many snacks will you need?

Solution:

Find compatible numbers for 26 and 40 and use them to estimate the product. Try 25×40.

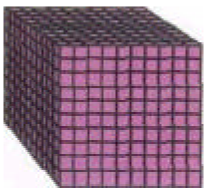
$$\begin{array}{r} 26 \\ \underline{\times 40} \end{array} \longrightarrow \begin{array}{r} 25 \\ \underline{\times 40} \\ 1000 \end{array}$$

So, 26×40 is close to 25×40, which means you'll need to order over 1000 snacks!

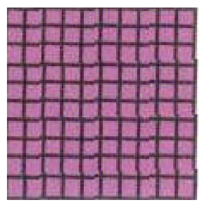
WHOLE NUMBERS: PLACE VALUE

You probably use place value all the time without even knowing it. Place value tells you the value of each digit in a number. In our numeration system, each place has 10 times the value of the place to its right. The pattern makes it easier to use our number system:

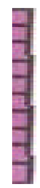
<u>1 Thousand</u>
10 times greater than one hundred



<u>1 Hundred</u>
10 times greater than one ten



<u>1 Ten</u>
10 times greater than one one



We also arrange numbers into groups of three places called periods. Note that the places within periods repeat (hundreds, tens, ones; hundreds, tens, ones; and so on). Knowing the place *and* period of a number will help you find values of digits in any number as well as helping you read and write numbers.

EXAMPLE

What is the value of the digit 9 in the number 71 905 346 521?

Solution:

Billions Period			Millions Period			Thousands Period			Ones Period		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
	7	1,	9	0	5,	3	4	6,	5	2	1

The digit 9 is in the *hundred millions* place. Its value is 9 hundred million, or 900 000 000. A stack of 900 000 000 dollar bills would reach 61 kilometres into space!

INTRODUCTION TO DECIMALS

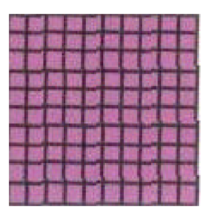
Decimals, like fractions, can name a part of a whole. Decimals are numbers that contain a **decimal point** such as 1.5, 2.83 and 0.2. When we write numbers, we use a system called **base 10**. In base 10, the first number to the left of the decimal point represents the number of ones; the second number represents the number of tens; the third number represents the number of hundreds, and so on.

The place values can be explained using the following example.

5	**7**	**8**	**6**	**9**	**.**	**8**	**6**	**9**	**5**
10 000	1000	100	10	1		0.1	0.01	0.001	0.0001

Notice that as you move to the right, each place value is 10 times less that the one before it. You need to understand this to work with decimals. If you continue the pattern of dividing the previous place value (the one on the left) by 10, then the numbers to the right of the decimal represent the number of tenths (0.1), hundredths (0.01), thousandths (0.001), and so on.

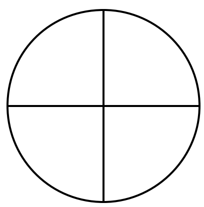
As you moved to the right, you divided by 10, therefore 8 tenths can be written as 0.8, as shown by the division 8 ÷ 10.



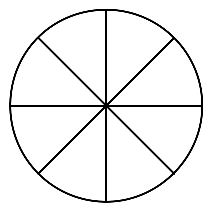
In the above diagram, there are 100 squares to equal the whole region. Therefore each square represents one one-hundredth or 0.01 of the region. One row has 10 squares, so it represents ten-hundredths or one-tenth or 0.10 or 0.1 of the region. If you shade in 30 blocks, you have a fraction of 30 hundredths that would be shown as a decimal to the hundredths place and would be written as 0.30. As an example, you can look at money to show the relationship between decimals and fractions. The cent part is written to the right of the decimal point.

Both fractions and decimals can be used to represent the same number. You see that half the 100 squares can be represented by 50 hundredths or 0.50 (0.5). Any fraction equivalent to 50 hundredths ($\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots$) can also be represented by the decimal 0.5. This is also true for $\frac{1}{4}$ which, as a decimal, is written as 0.25. Therefore, $\frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots$, for example, as decimals, are all written as 0.25. You can continue with the information that you already know to help us find other connections between fractions and decimals.

EXAMPLE



$\frac{1}{4} = 0.25$



$\frac{1}{8} = 0.250 \div 2 = 0.125$

In changing from $\frac{1}{4}$ to $\frac{1}{8}$, there is still one piece, but the piece is only half the size. What is half of 0.25? The number 0.25 can also be written as 0.250; if this is shared equally by 2 people , each gets 0.125. Using this information, you can find a decimal to represent any number of eighths.

$\frac{1}{8} = 0.125$

$\frac{2}{8} = 0.125 \times 2 = 0.250$ (the same as $\frac{1}{4}$)

$\frac{3}{8} = 0.375$

$\frac{4}{8} = 0.500$ (0.5 or $\frac{1}{2}$)

$\frac{5}{8} = 0.625$

$\frac{6}{8} = 0.750$ (0.75 or $\frac{3}{4}$)

$\frac{7}{8} = 0.875$

$\frac{8}{8} = 1.000$

Each decimal mentioned so far is called a **terminating** decimal, that is, when examining its equivalent fraction, dividing the denominator into the numerator will produce a result that has a last digit.

However, not all fractions can be written as terminating decimals. Fractions whose decimal representations do not terminate are called **repeating** decimals. For example, just as $\frac{1}{10}$ meant that you divide 1 by 10, $\frac{1}{9}$ means that you divide 1 by 9. This division, however, will result in an answer that continues to repeat itself, namely, 0.11111… . Using the same technique as above,

$$\frac{2}{9} = 0.11111\ldots \times 2 = 0.22222\ldots, \text{ which is slightly more than } 0.2$$
$$\frac{3}{9} = 0.11111\ldots \times 3 = 0.33333\ldots, \text{ which is slightly more than } 0.3$$

For each example using 9 as a denominator, the fraction and its decimal equivalent are both greater than $\frac{1}{10}$. The denominator is less than 10, so the part of the wholes that $\frac{1}{9}$ represents is greater than $\frac{1}{10}$. What about fractions that are less than $\frac{1}{10}$, such as $\frac{1}{12}$? In relating it to the whole, you see that the part of the whole is smaller as the denominator increases.

Because $\frac{1}{12}$ is less than $\frac{1}{10}$, there is no number in the tenths place when you find the decimal equivalent, so you should start the decimal with a zero. If you follow the pattern established previously, you see that

$$\frac{1}{12} = 0.0833333\ldots$$
$$\frac{2}{12} = 0.0833333\ldots \times 2 = 0.166666\ldots$$

The decimal for $\frac{2}{12}$ has a non-zero number in the tenths place because it is now greater than $\frac{1}{10}$.

While you can easily see the repeating decimal in certain fractions, others are more difficult to see. For example, if you convert $\frac{1}{7}$ to a decimal, it takes longer before it begins to repeat.

Example 1: $\frac{1}{7} = 0.142857142857\ldots$

What relationship can you see? Seven is a prime number (it only has two factors, 1 and 7). The numbers in the decimal do not begin repeating until the seventh place after the decimal point.

Example 2: $\frac{1}{17} = 0.05882352941176470588235294117647\ldots$

Seventeen is a prime number and the sequence does not begin repeating until the seventeenth place to the right of the decimal. The repeating period for a decimal is never longer than the denominator of the fraction being converted, but as shown above, it could be that long.

If decimals are another way of representing the value of a fraction, why do you need to use decimals? Decimals are good for many reasons:

1. They are easy to work with. To add $\frac{1}{5}$ and $\frac{1}{2}$ for example, you first have to find a common denominator. That isn't too hard in this case, but it would be easier if to add 0.2 and 0.5, which is 0.7.

2. Decimals are usually found with the metric part of measurement on the packaging of food (for example, 453.5 grams).

Decimals are numbers that are expressed using a decimal point (4.75, for example). A whole number is also a decimal. For example, 235 can also be written as 235.0 or 235.00.

You can think of money to help you understand decimals and their place values.

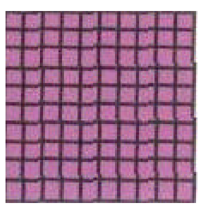
A dime is $\frac{1}{10}$ of a dollar, or 0.1 dollar.

A penny is $\frac{1}{100}$ of a dollar, or 0.01 dollar.

You can also understand decimals by using the place value pattern. Place value tells you the value of each digit in a number. In our decimal system, each place has ten times the value of the place to its right.

1 One

10 times greater than one-tenth



1 Tenth

10 times greater than one-hundredth

1 Hundredth

10 times greater than one-thousandth

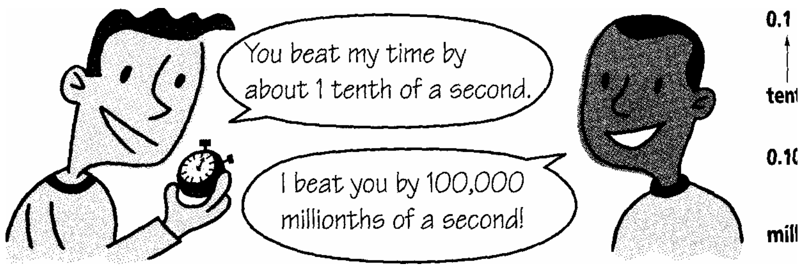
You can use place value to read a decimal or find the value of a digit in a decimal. For example, 13.578 is read, "thirteen and five hundred seventy-eight thousandths."

In general, to read a decimal

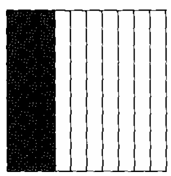
- read the whole number part, if there is one;
- read the decimal point as "and";
- read the number to the right of the decimal point as you would if it were a whole number; and
- read the place value of the last digit.

EQUIVALENT DECIMALS

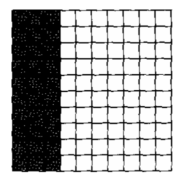
Decimals that name the same amount are equivalent decimals.



The following diagram shows decimals that are equivalent.

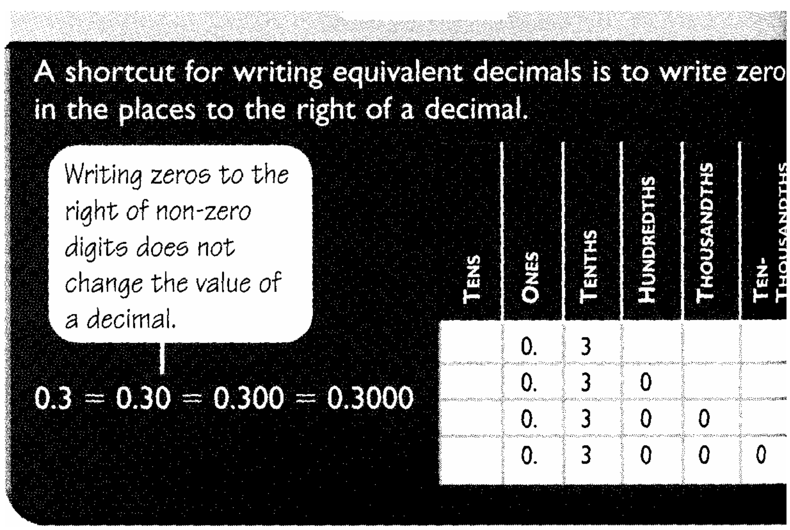


0.3 (three tenths) of the square is shaded



0.30 (30 hundredths) of the square is shaded

0.3 = 0.30
three tenths = thirty hundredths
3/10 = 30/100



Adding no hundredths or no thousandths or no ten-thousandths does not change the total value of a decimal.

ESTIMATING WITH DECIMALS

Problem: Without calculating, tell all you know about the solution to each of these problems. Use the following two examples to help you.

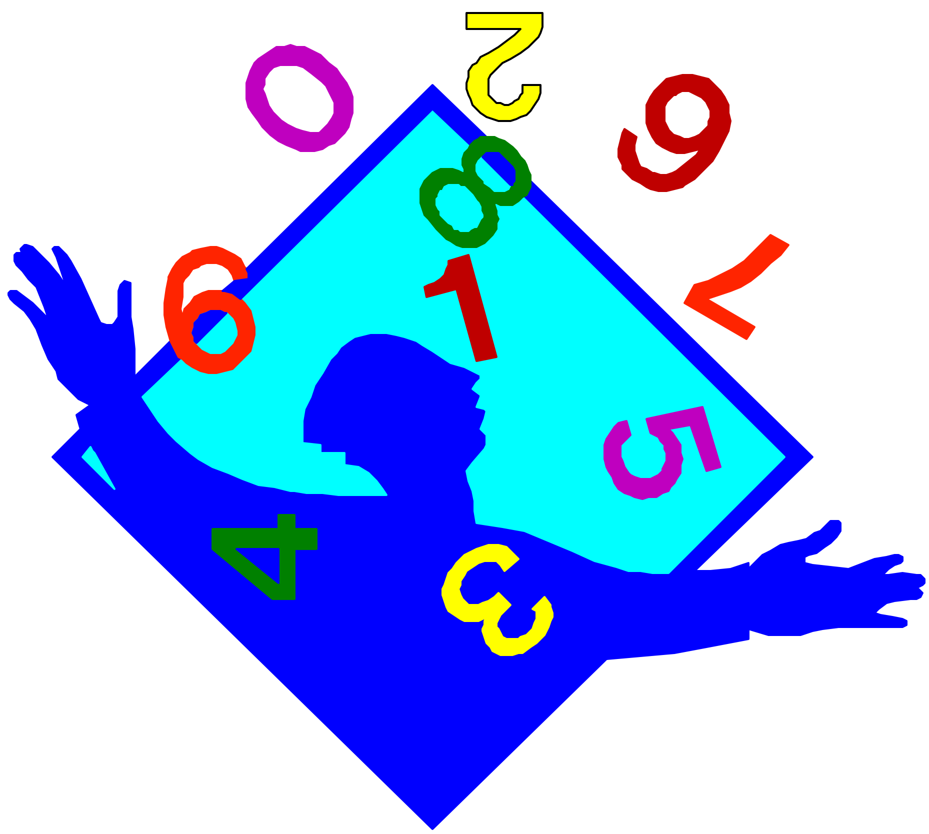
EXAMPLE 1

For 132×0.3, you might say, "Less than 132, since 132×1 is 132; more than 13, since 132×0.1 is about 13; or about 40, since 0.3 is about one third, and one third of 120 is 40."

EXAMPLE 2

For $152 \div 1.5$, you might say, "Less than 152, since $152 \div 1 = 152$; or more than 76, since $152 \div 2 = 76$."

1. 146×0.76
2. 7.8×0.98
3. 45.1×1.05
4. $16 \div 0.5$
5. $39.5 \div 0.95$
6. $436.2 \div 0.63$
7. $82.5 \div 1.2$



MULTIPLICATION

Multiplying decimals is the same as multiplying whole numbers, except that you must take the decimals into account. There are several ways to approach this problem, some of which will be discussed here.

Before you multiply, you should estimate the answer.

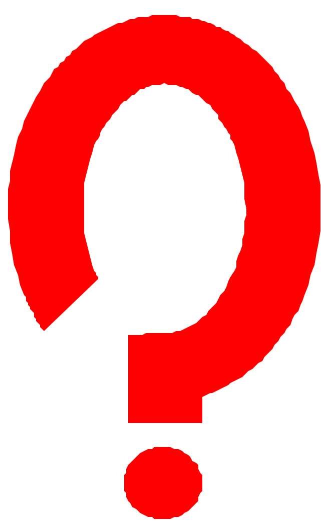
For example, 2.8×3.7 is about $3 \times 4 = 12$. The exact answer can be found by realizing that you are multiplying 28 tenths by 37 tenths, so the answer is found by multiplying 28×37 and dividing by 100 (tenths $\times$ tenths). Since $28 \times 37 = 1036$, the answer is 10.36 (which is approximately 12).

It might be useful to think like the following.

A rectangle that is 2.8 cm $\times$ 3.7 cm has an area of 2.8×3.7 square centimetres. But this is really 28 mm $\times$ 37 mm. So the area is 1036 square millimetres, or 10.36 square centimetres.

When working with decimals less than 1, students need to realize that they are taking only a part of the other number. For example 0.1×10 is one-tenth of 10, or 10 tenths, which is 1.0. Therefore 0.3×0.6 is 3 tenths of 6 tenths or 18 hundredths.

What approximately should the answer be, given the problem 2.3×4.9? Rounding off the factors gives 2×5, giving a product of 10. Therefore, the result should be approximately 10. Multiplying the original factors gives a number sequence of 1127 (23×49), and my approximation tells me that the decimal place should be after the 11, or 11.27.



DIVISION

Dividing by decimals is just like dividing by whole numbers. When you calculate 2.5 ÷ 0.5, the question is, "How many 5 tenths are in 25 tenths?" which is the same as asking, "How many fives are in 25?" Notice that each part of the question is multiplied by 10, yielding the same result, but the question is now easier to solve.

It's tricky if the question is something like 2.05 ÷ 0.5. This time, the question is, "How many 5 tenths are in 205 hundredths?" The units are not the same, so, instead, think of the question as, "How many 50 hundredths are in 205 hundredths?" Then, solve 205 ÷ 50.

Zeros in the quotient can also be a problem when dividing decimals. For example, 606 ÷ 6 will often result in an answer of 11 rather than 101; the same will occur when you encounter 6.06 ÷ 6 and find a solution of 1.1 instead of 1.01.

Decimals can also be thought of in terms of money. The following illustrates an example of division involving money. Alan bought a sweater for $19.50. Betty bought a sweater for $32.20. How many times as expensive as Alan's sweater was Betty's? First estimate. Discuss how to round the answer to a reasonable amount. To solve this problem, you can divide 322 dimes by 195 dimes to get approximately 1.7. Therefore, Betty's sweater is approximately 1.7 times as expensive then Alan's sweater. When you divide 32.2 by 19.5, the same answer is achieved as if one divides 32.20 by 19.50, the increase is the same no matter how you write the price, as long as each price is multiplied by the same number.

Decimal Activities

REASONABLE ANSWERS

Problem: Suppose Ms. Jones, an experienced painter, can paint a wall in three hours, whereas a rookie painter, Mr. King, paints the same wall in seven hours. If the two painters work together to paint the wall, how long will it take them to complete the job? Choose the most reasonable answer. Explain your decision.

- **(a)** 21 hours
- **(b)** 10 hours
- **(c)** between 5 and 10 hours
- **(d)** 5 hours
- **(e)** 4 hours
- **(f)** 3 hours
- **(g)** 2 hours
- **(h)** $\frac{1}{2}$ hour

ESTIMATING THE SIZE OF AN ANSWER

Encourage students to verbalize their thinking. For example, in Exercise 1 below, a student might think, "I know the answer will have three digits because it can't be as large as 200 + 700, or 900." This type of discussion provides information to the teacher regarding students' global understanding of each operation and communicates to the students the value of estimation.

The idea illustrated in this activity might be used before assigning a group of computation problems. Choose several problems from those to be assigned and ask students to think about how large the answer will be. Ask questions such as these: Will the answer be a two-digit number? Will the answer be a three-digit number?

Provide problems such as those listed here. After examining each number represented in the problem, ask students to identify how many digits the answer will contain. Ask them to explain their thinking.

1. $134 + 689$
2. $134 + 989$
3. $1246 - 348$
4. $2054 - 128$
5. 12×234
6. 5×689
7. $2344 \div 4$
8. $2338 \div 14$

MENTAL MATH EXERCISES

Group A

1. $8 \times 12 =$ __________
2. $24 \times 25 =$ __________
3. $5 \times 23 \times 4 =$ __________
4. $\frac{1}{4}$ of $484 =$ __________
5. $2446 \times 20 =$ __________
6. $120 \times 9 =$ __________
7. $900 - 292 =$ __________
8. $3 - \frac{1}{4} =$ __________
9. $8236 \times 0.5 =$ __________
10. $32.1 \times 0.1 =$ __________
11. $66 \times 50 =$ __________
12. $4250 \div 5 =$ __________

Group B

1. $4 \times 13 \times 25 =$ __________
2. $1.98 + 2.99 =$ __________
3. $38 \times 5 =$ __________
4. $600 - 187 =$ __________
5. $1836 \div 20 =$ __________
6. $4614 \times 0.5 =$ __________
7. $423 \times 20 =$ __________
8. $12 \times 8 =$ __________
9. $36 \times 25 =$ __________
10. $402 \times 25 =$ __________
11. 10% of $43.1 =$ __________
12. 9% of $43.1 =$ __________

Group C

1. $4.2 \times 20 =$ __________
2. $41 \times 30 =$ __________
3. 22% of $50 =$ __________
4. $36 \times 25 =$ __________
5. $48 \div 16 =$ __________
6. $52 \times 25 =$ __________
7. $4 \times 9 \times 8 \times 5 =$ __________
8. $900 - 289 =$ __________
9. $32 \times 11 =$ __________
10. $2^3 =$ __________
11. $199 + 256 =$ __________
12. $12 \times 7 =$ __________

Group D

1. $7 \times 12 =$ ________
2. $1350 \div 5 =$ ________
3. $1200 - 683 =$ ________
4. $0.3 \times 32 =$ ________
5. 15% of 50 = ________
6. $\frac{3}{4}$ of 420 = ________
7. \$10.94 + \$6.98 = ________
8. $4 \times 18 \times 5 \times 5 =$ ________
9. $36 \times 25 =$ ________
10. $83 \times 0.01 =$ ________
11. 15% of 50 = ________
12. $\frac{1}{5}$ of 610 = ________

Group E

1. $9 \times 12 =$ ________
2. $5^2 =$ ________
3. $5 \times 43 \times 20 =$ ________
4. $53 \div 100 =$ ________
5. $901 - 596 =$ ________
6. $0.99 \times 300 =$ ________
7. $0.2 \times 413 =$ ________
8. $7 \times 28 =$ ________
9. 25% of 44 = ________
10. $2460 \div 5 =$ ________
11. \$4.18 + \$5.82 = ________
12. $5050 \div 25 =$ ________

Group F

1. $6 \times 12 =$ ________
2. $42 \times 30 =$ ________
3. $6 \times 32 \times 5 =$ ________
4. $800 - 286 =$ ________
5. $12 \times 102 =$ ________
6. $\frac{1}{3}$ of 297 = ________
7. $4680 \div 20 =$ ________
8. \$3.91 + \$6.97 = ________
9. $32 \times 25 =$ ________
10. $0.3 \times 33 =$ ________
11. $4 \times 16 \times 25 =$ ________
12. $\frac{3}{4}$ of 416 = ________

ONE HUNDRED NINETY-EIGHT

Materials: Deck of regular playing cards for groups of 2-5 players
Objective: To avoid playing a card that makes the total greater than 198.

Directions:

1. Deal three cards to each player. Put the remaining cards in the middle.
2. Each player puts down a card when it is his/her turn. After putting down the card, the player must say aloud the total sum of the numbers in the pile, as shown below:

   Special cards:
   (a) Ace = 1
   (b) Cards 2-10 = face value
   (c) King = 10
   (d) Queen = 0
   (e) Jack = -10 (subtract 10 from the total)

   For example, the first player puts down a seven and says "seven"; the second player puts down a five and says "twelve" because seven plus five sums to twelve; the third player puts down a Jack and says "2" since 12 – 10 = 2.

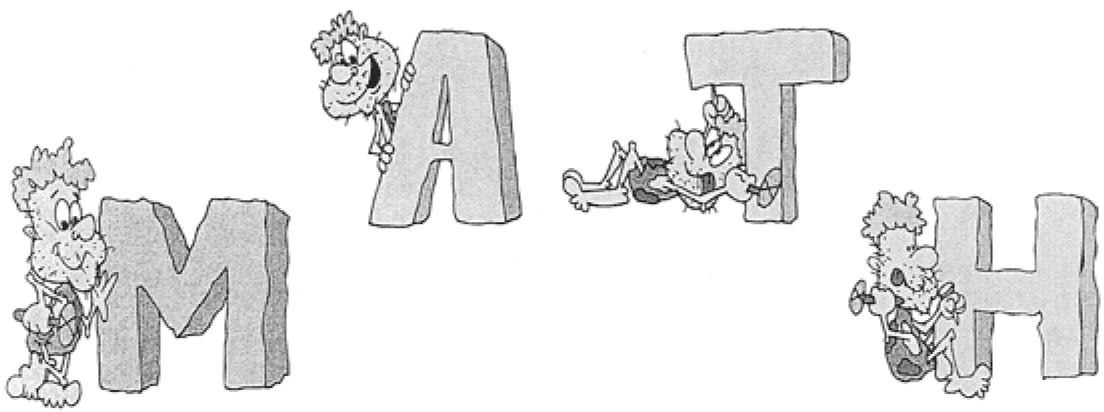
3. After a player puts down a card, he/she must draw another from the pile in the middle.

4. Play continues in this manner until a player goes over 198. Each player who goes over 198 is eliminated from the game. The last player remaining is the winner.

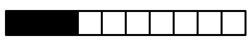
MODELING BASE TEN RELATIONSHIPS

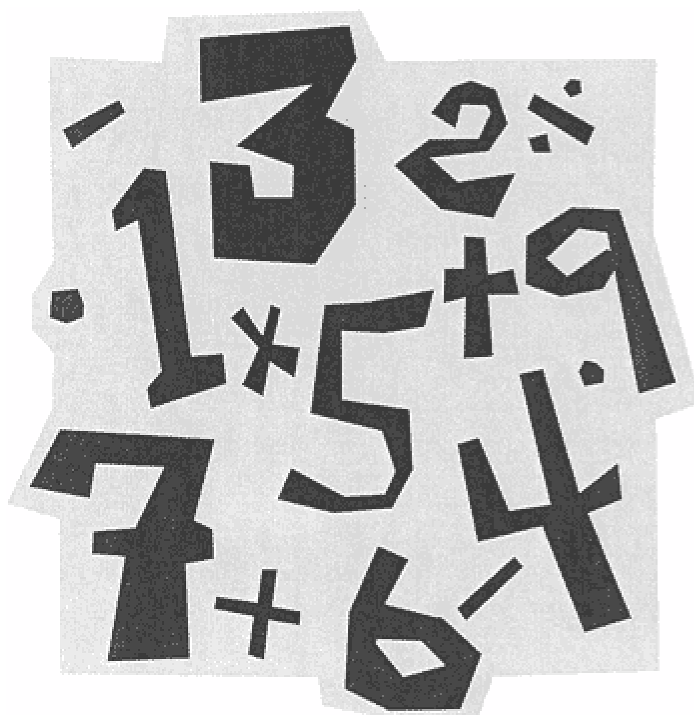
Model using base ten blocks to show each of the following:

- 2 units
- 3 tens
- 1 hundred
- 1 thousand
- 1132
- 113.2
- 11.32
- 1.132
- 0.1132



CONNECTING DECIMALS AND FRACTIONS

WORD NAME	DECIMAL NAME	FRACTION NAME	PICTURE OR MODEL
17 hundredths			
	0.008		
		9/100	
			
	3.55		
		1/4	
34 thousandths			
	1.6		
			



ESTIMATING DECIMAL PRODUCTS

For: 2 players
Equipment: calculator

Four-Decimals-in-a-Row

Take turns choosing two factors, one from the circular factor board and one from the square factor board. If the product of those numbers is displayed on the grid, the player captures that cell. You can check each other with the calculator. The first one to capture four cells in a row, vertically, horizontally, or diagonally, is the winner.

Grid

221.4	**88.2**	**82.8**	**110.7**
107.8	**9**	**60.27**	**135.3**
2.45	**176.4**	**41.4**	**48.02**
4.5	**50.6**	**5.5**	**22.54**

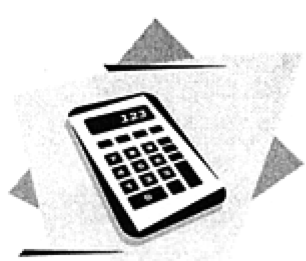
Factor Boards

46 98

123 5

1.8 0.9

1.1 0.49



DECIMAL REPLACEMENT

Estimate the value of the product by approximating each factor. For example, in Exercise 2, one might think, "It would be about half of 500, or 250." The decimal must go after the 1 to make the answer 291.357. Note that all ending zeroes have been eliminated from the decimal representation.

Think holistically about each problem to determine the placement of the decimal by what seems reasonable. Each of the multiplication and division problems below has been carried out except for placing the decimal point. Place each decimal point using estimation.

1. $7.836 \times 4.92 = 3855312$
2. $534.6 \times 0.545 = 291357$
3. $5.03 \times 17.6 = 88528$
4. $49.05 \times 6.044 = 2964582$
5. $4.436 \times 0.49 \times 29.5 = 6412238$
6. $68.64 \div 4.4 = 156$
7. $400.14 \div 85.5 = 468$
8. $0.735 \div 0.7 = 105$
9. $51.1875 \div 1.05 = 4875$
10. $3.773 \div 0.98 = 385$

MULTIPLYING AND DIVIDING BY NUMBERS NEAR ONE

What do you think happens when you multiply:

- two numbers that are both greater than 1?
- a whole number by a fraction between 0 and 1?
- two numbers that are both between 0 and 1?

Without performing any calculation, decide whether $<$, $=$, or $>$ will correctly complete each sentence. Ask the students to justify their thinking.

1. $246 \times 1.3 \quad \square \quad 246$

2. $920 \times 0.8 \quad \square \quad 20$

3. $98 \times 1.001 \quad \square \quad 98$

4. $32 \times 0.5 \quad \square \quad 32$

5. $0.5 \times 0.86 \quad \square \quad 0.5$

6. $0.3 \times 5.8 \quad \square \quad 2.9$

PERCENT BENCHMARKS

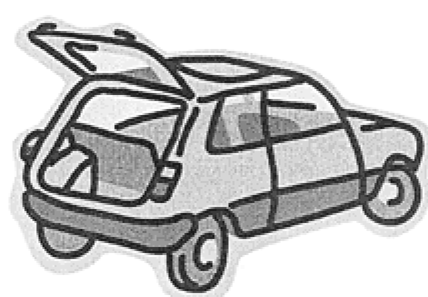
Complete each statement below using one of the following choices:

0 percent
Fewer than 10 percent
Approximately 25 percent
Fewer than 50 percent
Approximately 50 percent
More than 50 percent
Approximately 75 percent
At least 90 percent
100 percent

When finished, ask students to give an example of a statement that would be answered with each of the statements that were not used.

Statements:

1. _________ of the students in my class are left-handed.
2. _________ of the students in my class have red hair.
3. _________ of the students in my class like baseball.
4. _________ of the people in my town are over ninety years old.
5. _________ of the people in my town own a car.
6. _________ of the people in my town are female.



QUIZ

Part 1

For each of the following questions, place the correct answer in the space provided.

1.	$4.99 + 3.99 =$	________	7.	$\$10.94 + \$6.98 =$	________
2.	$30 \times 8000 =$	________	8.	$234 + 199 =$	________
3.	$\frac{1}{2} \times 26 =$	________	9.	$43 \times 0.01 =$	________
4.	$(2\times6) + (3\times6) =$	________	10.	$16 \times 50 =$	________
5.	$8 \times 25 =$	________	11.	$800 - 351 =$	________
6.	$0.5 \times 18 =$	________	12.	$2 \times 4 \times 7 \times 5 =$	________

Part 2

For each of the following, place a circle around **the letter** that best answers the question.

1. Which of the following is the correct order from *least to greatest*?

(a) 5/6	12/11	0.38	70%
(b) 12/11	5/6	70%	0.38
(c) 0.38	70%	5/6	12/11
(d) 0.38	5/6	70%	12/11

2. Out of a total attendance of 2397 for the week, 602 went to the fair on Friday. Approximately what percent attended on Friday?
(a) 15% **(b)** 25% **(c)** 30% **(d)** 40%

3. Which fraction is approximately the same as 35%?
(a) 1/5 **(b)** 3/8 **(c)** 3/5 **(d)** 9/16

4. A pair of pants regularly sells for $50. They were on sale at a 20% discount. What is the final price if 15% tax is added to the discounted price?
(a) $40 **(b)** $46 **(c)** $51 **(d)** $59

5. What number is 0.01 more than 2.76?
(a) 3.76 **(b)** 2.86 **(c)** 2.78 **(d)** 2.77

6. Of these numbers, which one is the greatest?
(a) 1/2 **(b)** 3/4 **(c)** 5/6 **(d)** 3/10

7. Give a fraction that is less than 0.8. __________

8. If you knew that 10% of 440 is 44, how would that help you determine what 15%

of 440 is?

9. List 3 fractions between $\frac{1}{2}$ and 1.

10. How would you solve the following problem, using a mental strategy?

    $55 + $10.25 + $4.75

11. Write a fraction less than 0.6.

12. Give a good estimate for 29×93. Why is it a good estimate?

13. When Tom was playing basketball on Saturday, he made $\frac{2}{5}$ of the baskets he shot. Written as a decimal, what would it be?

14. Shawn bakes 2000 bagels in an 8-hour work day. On the average, how many bagels does he bake in a half-hour.
    (a) 125 bagels **(b)** 250 bagels
    (c) 8000 bagels **(d)** 16 000 bagels

15. If the choir has 36 members, and the ratio of women to men is 4:5, then the number of men in the choir is
    (a) 5 **(b)** 9 **(c)** 16 **(d)** 20

16. What is 0.5% of 120?
    (a) 0.6 **(b)** 6.0 **(c)** 60 **(d)** 600

17. Which one of these expressions has the same value as 22% of 3800?
    (a) 44% of 1900 **(b)** 11% of 1900
    (c) 44% of 7600 **(d)** 11% of 7600

18. Which one of these is the same as 4.23 million?
    (a) 423 hundred thousands **(b)** 423 ten thousands
    (c) 423 thousands **(d)** 423 hundreds

19. Which one of these numbers is least?
    (a) 25% of 824 **(b)** 40% of 596
    (c) 15% of 5000 **(d)** 80% of 420

20. Which one of these problems gives an answer of approximately 200?
    (a) $43.6 \div 0.2$ **(b)** 38.9×7.3 **(c)** $417 \div 0.2$ **(d)** 52.9×7.6

21. Jeff's salary was increased from $6.25 an hour to $6.75 an hour. What was the percent increase?
    (a) 5% **(b)** 6% **(c)** 7% **(d)** 8%

22. Lyle is driving from home to his friend's house, a distance of 720 km. He plans to drive at an average speed of 80 km per hour, take an hour for lunch, and take two 15-minute breaks. He must arrive at his friend's house by 7:00 p.m. What is the latest possible time that Lyle could leave home in order to arrive at his friend's house on time?

23. The regular price of a video camera is $849.95. If it goes on sale for 25% off, what will be the sale price?