ADULT NUMERACY:

Taking Mathematics from the Real World into the Classroom and Back

by

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CONTENTS

INTRODUCTION

Preamble
A Definition of Numeracy

THE NATURE OF THE NUMERACY LEARNER

The Need for Numeracy Tuition
Past Experience with Mathematics
The Use of Mathematics by Adults

THE NATURE OF MATHEMATICS LEARNING

The Learning of Mathematical Concepts
The Mathematical Code
Mathematics as a Tool for Problem-Solving
Using Mathematics to Interpret the Real World

IMPLICATIONS FOR NUMERACY TUITION

A Balanced Approach
Assessing the Learner's Needs
Facilitating Learning in Numeracy
A Model for Numeracy Tuition

CONCLUSION

BIBLIOGRAPHY
INTRODUCTION

Preamble
Mathematics is a topic that evokes wonder, excitement and satisfaction in some adults, but anxiety, frustration and inadequacy in others. Yet even this second group acknowledges the need to learn mathematics. It is mainly for the latter group that we seek to explore the subject of this paper: the learning of mathematics by adults.

We get encouragement to proceed in our study from the great mentor of adult education, Eduard Lindeman (1961), who proclaims that "because adult education is free from the yoke of subject-tradition, it's builders are able to experiment boldly even in the sacrosanct sphere of pedagogical method. Indeed, if adult education is to produce a difference in quality in the use of intelligence, its promoters will do well to devote their major concern to method and not content." (p114)

This paper is essentially divided into three parts. First, we will look at the nature of the adult learner who needs and seeks instruction in mathematics. Secondly, we will examine the nature of mathematics learning and its role in interpreting the real world. Finally, we will see what implications these two areas of study have for instruction of adults in mathematics.

A Definition of Numeracy
Until now we have avoided a word that appears in the title of this paper. "Numeracy", as distinct from "arithmetic" or "mathematics", is a relatively new word in the jargon of adult education. Like literacy, there are many ways to look at numeracy, but as we proceed through this research paper, its meaning should become more clear. For the sake of a common starting point, however, let us adopt the definition of numeracy given by Penny (1984): "the ability to understand and use mathematics as a means of communication; to interpret a situation given in mathematical terms or to employ mathematics
to represent a situation and, if necessary, use mathematical symbols to obtain further information." (p24)

THE NATURE OF THE NUMERACY LEARNER

The Need for Numeracy Tuition

Is the ability to do mathematics necessary in everyday life? Although there may be differing opinions about the degree to which we need mathematics, few will deny its usefulness. To determine those needs, let us look at the self-expressed motivations of those who seek numeracy tuition. From information gathered at the Adult Literacy Centre in Nottingham, U.K. (Riley, 1984) and Friends' Centre Project in Brighton, U.K. (Traxler & Gabony, 1982), four clear areas of demand may be identified:

1. coming to terms with the math that one encountered unsuccessfully at school (feeling of having "missed out") and thus improving one's self-image;

2. vocational math for non-specific reasons or for employment entrance exams;

3. social or "survival" maths including the wish to help children with school work;

4. math for enjoyment or for further study.

An important conclusion drawn from these observations is that people seek numeracy tuition for reasons other than assistance with everyday problems and that success in mathematics is as great a motivating factor for learning as relevance.

Past Experience with Mathematics

Mathematics seems to invoke the extremes in sentiment. Whenever I tell someone that I was a math teacher, the response is either: "Oh, I never did good in math", or "Oh, I used to love doing math". Those who seek tuition in numeracy usually fall into the first category. From their public school experience, many express the sense of confusion, frustration and boredom. The Cockroft Report (1982), Mathematics Counts, makes it clear
that schools have continued to be dominated by an examination system which reinforces "feelings of inadequacy and failure" (Marks, 1984). These negative feelings about mathematics provide a poor foundation for the practical math of adult life. These observations are further corroborated by the two studies that follow.

**The Use of Mathematics by Adults**

In 1980 a two-stage study was completed in the Reading - Berkshire (U.K.) area concerning adults' use of mathematics (*Use of Mathematics by Adults in Daily Life, ACACE*). Among the conclusions reported by Warburton (1982) are the following:

1) It is not easy to define everyday mathematical needs; they depend on the mathematical skills possessed and, more importantly, on the positive personal attitudes towards the use of mathematics.

2) Many people, especially women and the less educated, are inhibited about using mathematics and they try to avoid doing mathematical calculations.

3) There are many reasons for these inhibitions, and these can include the unsatisfactory experience of the formal learning of mathematics at school with its apparent irrelevance to everyday life.

4) Since the mathematics used varied widely according to each adult's lifestyle, it suggests that mathematics might be more usefully taught and learned through a functional approach.

5) Many of those interviewed used very simple and rather clumsy methods, suggesting that much of their school mathematics had been forgotten or its relevance never perceived.

6) The most frequent difficulties were experienced dealing with percentages, ratios, graphs and charts, timetables and metric units.

In order to assess adults mathematical ability and to validate the results of the Reading-Berkshire study, a survey was taken in 1981 with 2890 people covering a representative sample of England, Scotland and Wales over sixteen years of age. (*Adult Numeracy Study: Tabulated Results of a Survey, ACACE*). The findings, cited by Warburton (1982), showed that:
1) One adult in ten cannot cope with simple addition.
2) Three adults in ten cannot cope with simple subtraction.
3) Three adults in ten cannot cope with simple multiplication.
4) Three adults in ten cannot cope with simple division.
5) Three adults in ten cannot cope with simple percentages.
6) Four adults in ten cannot cope with a simple timetable.
7) Six adults in ten do not understand the meaning of the rate of inflation.

The results of this survey are clear. In Canada, no similar study has ever been undertaken. For the purpose of this paper, let us assume that the results of this study may be generalized, and that the findings in Britain can be a beacon shedding light on the numeracy needs in other developed countries. While the second study establishes clearly the need for numeracy tuition, the first study gives some direction on how this tuition may be best provided.

THE NATURE OF MATHEMATICS LEARNING

The Learning of Mathematical Concepts
"For many, mathematics represents a series of random facts linked to calculation performed at speed and comprehensible only to possessors of giant intellect and incredible memory" (Penny, 1984, p.26). Unfortunately this represents a common belief - and there is a speck of truth in it. The power of mathematics lies in its abstract nature; however, this is not the domain of only "giant intellects", but abstract thinking is something we all do.

Abstracting is the activity that makes us aware of similarities among our experiences. The product of this classification activity is called a concept. For example, when we see a person with two arms, two legs, two eyes and two ears, the concept of
"two" is established. From primary concepts derived from our direct sensory experience, we may abstract higher order concepts. Experience with many red objects, brings us to the concept of "red"; then experiencing "green", "blue" and so on establishes the second order concept of "colour".

In his extensive study of the subject, Skemp (1986) presents two first principles of the learning of mathematics:

1) Concepts of a higher order than those which people already have cannot be communicated to them by a definition, but only by arranging for them to encounter a suitable collection of examples.

2) Since in mathematics these examples are almost invariably other concepts, it must first be insured that these are already formed in the mind of the learner.

Though these principles may be known to the learner, it is the teacher or communicator of mathematical concepts for whom they have much relevance. As a consequence of the second principle, in building up the structure of successive abstractions, if a particular level is imperfectly understood, the learning of any other concept is in jeopardy.

A conceptual structure, known as a schema, integrates our existing knowledge and acts as a tool to assimilate future learning. In a study reported by Skemp (1986) comparing schematic and rote learning, the differences were clear:

<table>
<thead>
<tr>
<th></th>
<th>Immediate</th>
<th>After 1 day</th>
<th>After 4 weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schematic</td>
<td>69</td>
<td>69</td>
<td>58</td>
</tr>
<tr>
<td>Rote</td>
<td>32</td>
<td>23</td>
<td>8</td>
</tr>
</tbody>
</table>

The schematically learned material was not only better learned but better retained.
After certain schemas have been learned, new experiences may be encountered that do not fit these schemas. In this case, they need to be restructured. A person who understands the multiplication of whole numbers as rapid addition, will need to restructure that conceptual structure to understand the multiplication of fractions.

**The Mathematical Code**

In trying to understand the nature of mathematics, some say that it is a language, while others say it is not; however, all will agree that it is an instrument of communication. Moss (1984) refers to the mathematical code which, like a language, needs to be standardized in order to be understood. Although a relevant concept may be learned by encountering an example which draws it out intuitively and involuntarily, the use of an associated symbol makes it possible to record and communicate the concept. These associated symbols, including words and graphics, make up the code.

According to Skemp (1986), visual codes, which are intuitive and integrative, represent more individual thinking and are harder to communicate than verbal–symbolic codes, which are logical and analytical, and tend to represent more socialized thinking. For instance, gathering together two pencils and three pencils to make five pencils requires little discussion compared to "2 + 3 = 5". This theory is validated by mathematics and numeracy educators, such as Liedtke (1988) and Willis (1984), who suggest that encouraging students to talk and listen to their responses will give valuable insight about their level of understanding of a concept.

**Mathematics as a Tool for Problem Solving**

In the first part of this paper, we looked at the adult learner and the need for tuition in numeracy. Although not every adult expressed the need to learn mathematics to solve everyday problems, the learner lives in the real world and mathematics, if it is to be a means of communication, also starts there. We have come to see that learning mathematics involves acquiring
Using Mathematics to Interpret the Real World

Maintaining that numeracy, in practice, is concerned with numbers that have a direct application in real life, Traxler and Gabony (1982) expand their definition of numeracy to include four inter-related parts:

1. Exact manipulation of abstract quantities. From basic arithmetic to algebra, this implies no literacy or cultural context, but precise and definite facts.

2. Approximating and estimating. This skill grows out of experience and understanding of the number system, and emphasizes numbers as quantities and not subjects of arbitrary arithmetic.

3. A feel for numbers as part of the real world. This involves a sense of weight, volume, distance, etc. and joins the world of number with the world of experience.

4. Selecting the appropriate strategy to deal adequately with problems. Developing this skill depends on literacy, social and cultural cues, as well as an insight into personal background.
IMPLICATIONS FOR NUMERACY TUITION

A Balanced Approach
Having looked at the nature of the numeracy learner and the nature of learning mathematics, we may now look at what are the implications for tuition or instruction. Mortiboys (1984) warns against the two extremes: (1) teaching mathematical skills without a context or (2) adopting a purely functional approach with myriads of timetables, menus, advertisements, etc. "We need to adopt a balanced approach: one in which mathematical rules are understood and practiced, and where appropriate, used in situations deemed to be relevant to the student by the student," concludes Riley (1984). The numeracy tutor, then, must establish an open relationship with the learner in order to be aware of the individual's needs and at the same time must be familiar with the learning of mathematical concepts and the structure of the hierarchy of skills in order to determine an appropriate agenda of instruction.

Assessing the Learner's Needs
To provide any kind of tuition, the tutor must know about what the learner can do and what the learner wants to do. Initial assessment, according to Penny (1984), should include the following elements:

+ Motivation
+ Literacy and/or language assessment
+ Perceptual difficulties
+ Hand-eye co-ordination and spatial concepts
+ Discussion of feelings about learning
+ An initial math diagnosis to establish a starting point

Although such a procedure looks formidable, we should avoid any type of formal testing session, because many adult learners, as we have studied earlier, have feelings of failure and inadequacy from previous experiences with school exams. By good
communication, listening and observing, the tutor can gain much information.

Assessment is not a "one-shot deal", but is an integral and continuous part of instruction. In order to ensure that the programme is meeting the learner's needs, the tutor should be continually monitoring his/her progress and encouraging the learner to do self-assessment. In Numeracy Training (Austin et al, 1986), they use self selection assessment cards, otherwise known as "can do" and "can't do" cards, to determine how the learners perceive their own problem and how they actually deal with the material. To deal with math anxiety, Taylor and Brooks (1986) suggest that students look at their own personal math life history. No matter what form assessment takes, the tutor should have up-to-date records of what work the group or individual has done, for consultation with the learner and for use in future planning. In fact, assessment will become automatic if a time for revision work and for a preview of the next lesson is planned for every lesson.

**Facilitating Learning in Numeracy**

From what we have come to know about the adult numeracy student and how mathematics is best learned, we may now develop some guidelines for the numeracy tutor to help facilitate learning. Many of these ideas are outlined in the introduction to Working With Numbers – Ideas and Examples for Numeracy Worksheets (Gabony and Traxler, 1982).

1) Because mathematics is based on conceptual structures, each small step in teaching a skill must be **consolidated** before the next can be dealt with.

2) If a student already knows a certain method (for example, long division), it is always better to **reinforce** that method than introduce a new or "better" one.

3) Since adult learners bring with them their past experience and particular needs, the tutor must be **flexible** in programme planning in order to capitalize on those areas.
4) Because attendance can be irregular for the adult student who has many responsibilities, lessons and learning materials need to be autonomous and self-contained.

5) The tutor should be aware of the learners' reading ability and cultural background that might affect their ability to learn the mathematical code.

6) If the learner's past learning environment was academic and authoritarian, informal learning, especially games and puzzles, must be introduced with sensitivity.

7) The best way to clarify one's understanding of a concept is to explain it to someone else; hence, we should encourage peer-group collaboration.

8) If the learning of basic skills is the main need of the learners, one can get bogged down in abstract details. So these skills should be related as often as possible to overall objectives and possible applications.

9) Although we know that understanding concepts will produce better learning than rote, many learners prefer to master a procedure first and then reflect afterwards on the reason why it works.

10) If mathematics represents the real world, we should be able to find, especially for the most basic skills, a visual or concrete explanation.

11) Whether to work individually or in groups depends on both the learner and the skills being learned. In general, abstract skills are best learned on an individual basis with a clear hierarchy of skill development, while group work provides a better environment for social or applied math.

A Model for Numeracy Tuition
From all that we have studied about the adults learning mathematics in the real world and numeracy tuition, to go out now and do it requires yet more thought and struggle. It is still confusing to me the interplay of abstract concepts and symbolic codes and real life applications. To get a grip on the implications for instruction, I developed a model for tuition that may be helpful to others.
We start from the real world to give us motivation to learn a certain concept. Since this concept is very abstract, we use a concrete example to explain it. Then we work to assimilate the new concept through practice. Finally we go back to the real world to apply what we have learned. For example: learning percentages may be motivated by interest rates on a loan; the idea of percent is then explained by a big pie cut up into one-hundred equal slices; after the skill has been acquired through discussion and practice, it is used to calculate sales tax or something else depending on the learners' concerns. The main point here is not to confuse "concrete" and "applications". A concrete example may have nothing to do with the real world, but may be a fabricated "pie" diagram that aides in visualization of a concept; on the other hand, an application of $\sqrt{1}$ in electrical theory is hardly a concrete situation.

CONCLUSION

We have seen that adults need numeracy tuition and want it for a variety of reasons. Since the learning of mathematics has been a negative experience for many who seek tuition, it is important that the numeracy tutor choose instructional strategies that encourage positive learning. Having studied that mathematics is learned through conceptual structures and symbolic codes, the tutor can then design a programme of skills acquisition that is based on an appropriate developmental process. Therefore, by
starting with the needs of the learner and by following a sound but flexible methodology, we can hope for much success in promoting numeracy among adults.


