Improving the Numeracy Component of the Essential Skills Research Project (ESRP) Methodology

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ABSTRACT

One means of helping Canadian workers have the necessary essential skills entails ensuring that HRSDC's numeracy framework reflects key development in numeracy research and that this framework is compatible with comparable established international assessment frameworks. The study reported next involved a comparative analysis of five international numeracy assessment frameworks and of the current ESRP framework. Several recommendations are proposed for HRSDC's consideration.

We find that the ESRP definition of numeracy should highlight declarative and procedural knowledge; emphasize use of knowledge and abilities; describe purposes for numeracy related activities; be context-related; and promote critical engagement with numerical information and activities. The ESRP framework should thus be organized around two domains, namely declarative mathematical knowledge and procedural mathematical knowledge. In addition to these domains, other components should be included in the numeracy framework including: (a) context; (b) cognitive enabling processes; (c) non-cognitive enabling processes; and (d) meta-cognitive processes. Finally, three complexity scales should be considered for the ESRP methodology: one for declarative knowledge complexity, one for procedural knowledge complexity, and one for mathematical representation.

These recommendations should be validated with subject matter experts, educators, and employers across the country. Furthermore, existing essential skills profiles should be aligned with the revised ESRP framework.

EXECUTIVE SUMMARY

Numeracy is among the fundamental skills required of individuals to function successfully in personal, educational and professional settings. As Canada evolves toward an increasingly high skills economy requiring advanced levels of literacy and numeracy, the importance of employees having essential skills to secure and retain employment as well as advance in the workplace continues to grow. Employees across labour market sectors require numeracy skills to perform the tasks required by their occupations, to develop more advanced occupational and life-long learning skills, to cultivate the ability to innovate, and to adapt to workplace changes (HRSDC, 2005).

To ensure that Canadian workers have relevant essential skills, the existing numeracy framework used by HRSDC should be updated to reflect key developments in numeracy research and to make it compatible with established international assessments. Making the current Essential Skills framework and its definitions, and especially those features pertaining to numeracy, compatible and comparable with existing national and international frameworks and definitions will increase the credibility, clarity and usability of measurement instruments used by decision-makers, employers and employees, as well as help inform the development of relevant training tools and curricula.

Directions Evidence and Policy Research group was contracted by HRSDC to assist with these tasks and to conduct the following activities: a review of assessment frameworks that measure numeracy in general and adult numeracy in particular; a comparative analysis of the definitions and facets of numeracy used by these international large-scale assessments with a view to evaluating their compatibility with the functions and intended uses of the essential skill profiles developed by HRSDC; and the development of a set of recommendations to revise the ESRP methodology (including its definition, facets and complexity levels and rating scale) to align it with the international assessments. These recommendations were intended to include a suitable definition of numeracy, an explanation of the numeracy concepts underpinning the definition and an outline of suggested complexity levels (and a corresponding rating scale) for each numeracy facet included in the definition.

Methodology. In consultation with HRSDC and the renowned Canadian numeracy expert Dr. David Robitaille, we identified five assessment and curriculum/conceptual frameworks that describe and/or measure numeracy. The following frameworks were selected on the basis of their internationally recognized value as well as their conceptual and measurement comprehensiveness: the OECD Programme for the International Assessment of Adult Competencies (PIAAC), the Adult Literacy and Lifeskills (ALL), the Tertiary Education Commission (New Zealand) Learning Progressions for Adult Numeracy and Assessment Tools, the Programme for International Student Assessment (PISA), and the UK Adult Numeracy Core Curriculum. Documents that described the conceptual and assessment frameworks were located and reviewed in full and are included in the reference list. Two researchers reviewed the frameworks listed above. Several matrices were developed to document the content of frameworks for comparative analysis. Key conclusions and recommendations are presented next.

Definition of numeracy. While the definitions of numeracy included in the international frameworks are similar, all of them focus on adult numeracy *in general*, rather than paying specific attention to the workplace context. Based on our review of numeracy definitions included in the selected international frameworks and other numeracy definitions discussed in the literature, we propose two numeracy definitions for HRSDC's consideration: a general definition of adult numeracy and a workplace-specific numeracy definition. We have formulated both definitions of numeracy to highlight the role of ***declarative and procedural knowledge***; to emphasize the ***use*** of the knowledge and related abilities; to describe the ***purpose*** of numeracy related activities; to reflect the importance of being ***situated*** in context; and to underscore the importance of ***critical engagement*** with numerical information and activities. The proposed definitions of general adult numeracy and workplace numeracy are the following:

> *Adult numeracy is the ability to identify, understand, critically interpret and apply mathematical concepts, processes, and representations in order to engage in and answer the mathematical demands of a variety of situations in adult life.*
>
> *Workplace numeracy is the ability to identify, understand, critically interpret and apply mathematical concepts, processes, and representations that are appropriate to the work context in which the individual is engaged in order to effectively manage/perform work activities.*

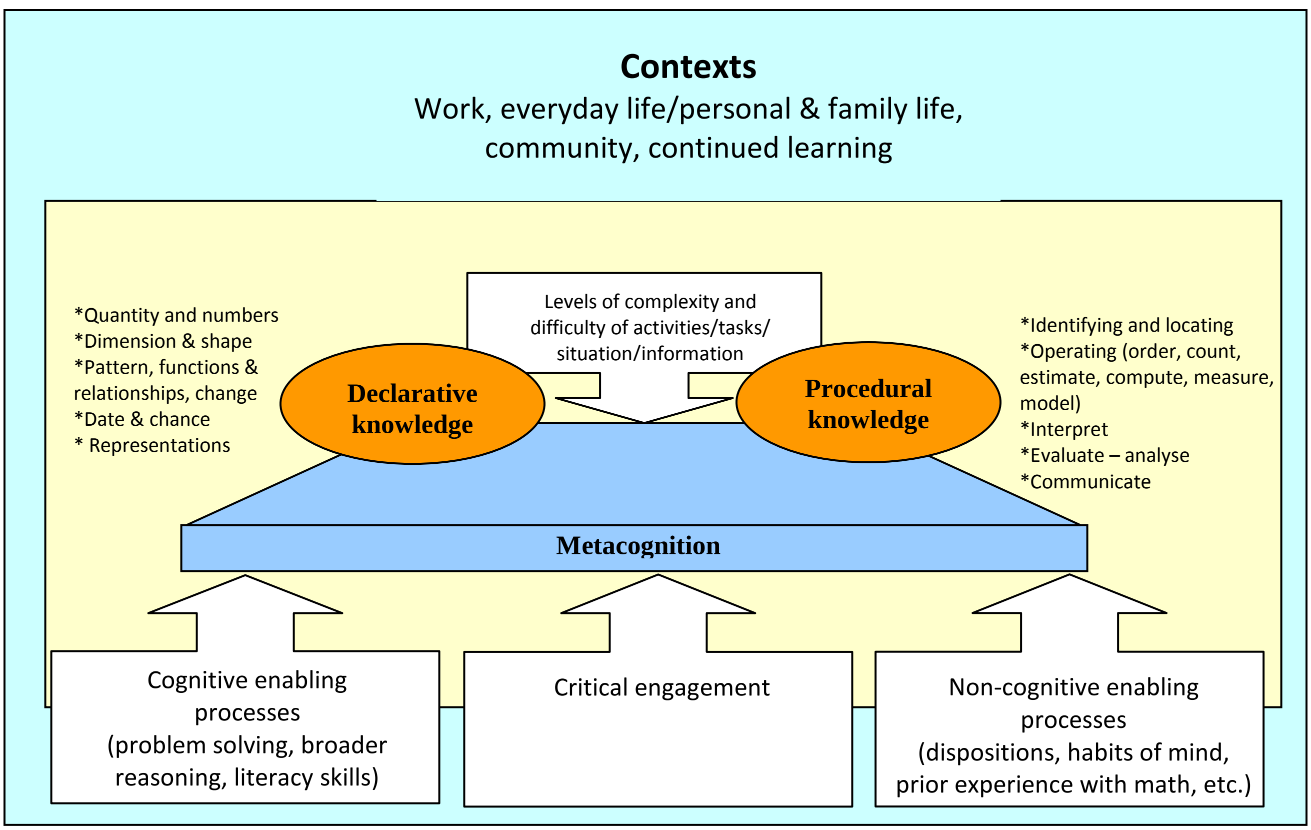
Numeracy facets. The numeracy frameworks we reviewed included a variety of numeracy-related concepts and behaviors. The frameworks were similar with regard to types of mathematical concepts included (e.g., number sense and estimation, quantity, shape and dimensions, pattern, functions and relationships, data and change, location and change). Furthermore, the frameworks listed a variety of actions and numerate behaviours that adults are expected to perform to complete tasks such as counting, estimating, measuring, interpretation, and analysis among others. After reviewing all the frameworks and facets identified in each document, we recommend organizing the revised ESRP numeracy framework into two domains: (1) declarative mathematical knowledge (content); and (2) procedural mathematical knowledge (i.e., responses/actions performed or required).

Other components of the numeracy framework. In addition to the two facets proposed above, several other components should be included in the numeracy framework: (a) context; (b) cognitive enabling processes; (c) non-cognitive enabling processes; and (d) meta-cognitive processes. While these components are not conceptualized here as separate facets or domains, they are deemed prerequisites of effective numerate behaviours, serving as enabling processes and factors for all three facets.

Complexity levels. Conducting a comparative analysis of the complexity levels and ratings proved challenging, as the frameworks included in this review used different factors to assess the complexity of numeracy tasks. However, several complexity ratings and factors were included in most of the frameworks. These reflected the number and type of operations required; the complexity of procedures; familiarity with the context; and the amount of information provided or required to complete the task. It should however be emphasized that understanding the complexity of the information as well as the variety and complexity of representations of mathematical information should be a key task in aligning the Essential Skills profile to established international assessments. Tasks should therefore be analyzed not only in terms of their mathematical complexity but also in terms of the extent to which employees would need to work with the textual or other types of representations of mathematical information to successful complete an activity.

After comparing and analyzing the complexity ratings used in international assessments described above, we propose to include three complexity scales in the ESRP methodology: (1) a declarative knowledge complexity rating scale (which will look at the complexity of mathematical concepts involved in the task and context familiarity); (2) a procedural knowledge complexity rating scale (which will assess the number of mathematical operations required to complete the task; the number of steps required/number of stages in iterative processing; the types/complexity of skills required; the number of factors (variables) that need to be taken into account; the consequence of error; and the degree of precision required); and (3) a mathematical representation complexity rating scale (which will evaluate the amount of information to be represented; the number and plausibility of distractors; and the forms of representations of information).

Figure 1. Framework for application of numeracy



Further steps and recommendations. While the recommendations proposed above are based on the analysis of the five international numeracy assessment and conceptual frameworks mentioned earlier, we recommend that these recommendations be validated with subject matter experts and employers across the country. Specifically, we strongly recommended that HRSDC pilot test the scales to ensure their validity and reliability.

Furthermore, we recommend revising the existing essential skills profiles to align them with the revised ESRP framework. In conducting this revision, it will be important to define key numeracy-related competencies as well as identify indicators that can be used to assess these competencies for each occupational profile or groups of occupations. One useful approach to consider would be the creation of a DACUM chart for each of the profiles or groups of related occupations.

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INTRODUCTION

The Importance of Numeracy

Numeracy is among the fundamental skills required of individuals to function successfully in society. Numeracy skills are required in contexts such as the workplace, at home, in the community at large, and to promote and support learning across the lifespan and in a variety of settings ranging from formal to informal. The work of Kindler et al. (1996), for instance, underscores the conclusion that numeracy needs in contemporary societies range from numeracy for practical purposes to numeracy for interpreting society, personal organization, and knowledge. This finding is mirrored by Steen (1990), who views modern numeracy as comprising five main dimensions, namely the practical, professional, civic, recreational and cultural.

Research into the impacts resulting from lack of basic numeracy (and literacy) skills on quality of life suggests that the lack of such skills can have notable impacts on individuals' ability to manage various aspects of their personal lives, such as their finances and their health. There is in fact good reason to believe that individuals who do not possess these skills find themselves at greater risk of socio-economic, professional and personal marginalization and that failure to acquire them can, over time, reinforce the dynamics that lead to social exclusion (Williams, Clemens, Oleinikova, & Tarvin, 2003).

In the specific domain of work, the lack of adequate numeracy skills is known to create barriers to employment, advancement and labour market retention. It is also associated with low productivity, higher error rates, low participation in work-related activities, and reduced workplace safety, among other challenges. Research also suggests that those who lack basic literacy and numeracy skills are likely to encounter reduced employment options (often limited to unskilled and semi-skilled work) as well as be subject to lower wages. For example, a study conducted by the UK Department for Education and Skills suggests that seven out of ten full-time workers with numeracy skills at Level 2 or above earn on average £8,000 more than those with lower numeracy skills (Grinyer, 2005).

Canadian society, and the national economy in particular, have over the last two decades remained on a trajectory characterized by demands ever increasing levels of literacy and numeracy. This persistent trend underscores the importance to those currently employed, as well as those seeking entry into the labour market, of securing the essential numeracy skills that will help them acquire employment and/or advance in the workplace. As noted by Human Resources and Social Development Canada (HRSDC), employees across all sectors require numeracy skills to perform the tasks required by their occupations, to develop more advanced occupational and life-long learning skills, to cultivate the ability to innovate, and to adapt to workplace changes (2005).

This conclusion notwithstanding, the available data suggests that the current Canadian workforce has insufficient numeracy skills. Fifty-five percent of the Canadian population older than 16 years of age performs below Level 3 on the numeracy scales of the 2003 International Adult Literacy and Skills Survey (IALSS), a level of numeracy proficiency deemed necessary to function in contemporary “knowledge” economies (Murray, Clermont, & Binkley, 2005).

In Canada, numeracy skills are considered part of a larger skill set, defined as essential skills, that people need to fully participate in workplace and community activities (HRSDC, 2009). According to HRSDC, these are divided into nine categories of fundamental skills, namely reading text, writing, document use, thinking skills, numeracy, oral communication, continuous learning, working with others, and computer use. However, only three of the nine essential skill categories (i.e., reading text, document use and oral communication) are currently deemed compatible with other assessment and measurement frameworks and methodologies used in Canada and in the international stage. While numeracy has been the focus of a number of national and international surveys and research projects, there is no direct comparison between the Essential Skills Research Project (ESRP) and international and national frameworks and definitions.

In working towards ensuring that its workers have high level of essential skills, including literacy and numeracy, Canada requires a numeracy framework compatible and comparable with existing national and international frameworks and definitions that will increase the credibility, clarity and usability of measurement instruments used by decision-makers, employers and employees, as well as help inform the development of relevant training tools and curricula. This consideration has motivated HRSDC to update its existing numeracy framework in such a way that it reflects key development in numeracy research and is compatible with international assessments.

This report presents the results of a comparative analysis of the Essential Skills Research Project framework developed by HRSDC and selected international numeracy assessment and conceptual frameworks. The report begins with a review of numeracy definitions mentioned in the literature and goes on to describe and analyze five international numeracy frameworks. The comparative analysis examines definitions, facets, and complexity levels included in the frameworks, with the aim to align the ESRP methodology with the international approaches. A set of recommendations is provided on how to revise the ESRP framework to ensure its compatibility with the international assessments and frameworks.

Conceptualizing Adult Numeracy

The notion of numeracy is not encompassed by a single, universally accepted definition. The challenge of coming up with a single definition is complicated by the existence, in the domain of adult numeracy, of a number of overlapping terms and definitions that have been equated with numeracy. These include *mathematical literacy, numerical literacy, quantitative literacy,* and *functional mathematics*. For instance, HRSDC defines numeracy as “the workers' use of numbers and their being required to think in quantitative terms" (HRSDC, 2009). Earlier versions of the International Adult Literacy Survey (IALS) that used the term quantitative literacy defined it as "the knowledge and skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials, such as balancing a checkbook, calculating a tip, completing an order form, or determining the amount of interest on a loan from an advertisement" (NCES, n.d. b). The Adult Literacy and Lifeskills Survey (ALL) refers to numeracy as "the ability to interpret, apply, and communicate mathematical information" (NCES, n.d. a). Another international assessment, the Programme for International Student Assessment (PISA), uses the term "mathematical literacy" to define an individual's capacity to "identify and understand the role that mathematics plays in the world, to make well founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (OEDC, 2003).

While it was beyond the scope of this project to analyze definitions of adult numeracy in general, we believe it is important to include some of the definitions mentioned in the literature in order to better situate the findings provided in the upcoming sections. However, the list of definitions included here is not meant to be exhaustive; it is, rather, an illustration of the variety of approaches and conceptual frameworks used in the field.

The term "numeracy" was first defined by the Crowther Report as "not only the ability to reason quantitatively but also some understanding of scientific method and some acquaintance with the achievement of science" (Ministry of Education, 1959, p. 282). The authors who were describing numeracy for adolescents (ages 15 to 18) saw numeracy as a "mirror image" of literacy. Several years later, the Newsom report developed a more extensive definition which aligned numeracy with basic arithmetic and its use in everyday life (Ministry of Education, 1963).

As definitions of numeracy evolved, the need arose to analyze and compare different definitions and frameworks. In their analysis of existing definitions and conceptual frameworks of numeracy, Maguire and O'Donoghue (2002) proposed a continuum to organize the existing definitions. Their continuum presents a progression from "formative" definitions which focus on basic arithmetic skills to more complex and multifaceted "integrative" definitions that incorporate numeracy in a variety of contexts (See

Figure 2).

The formative group of definitions views numeracy as basic arithmetic skills needed by adults to be able to perform and to function in society. Here the skills are described as “simple”, primarily numerical or quantitative, operations. An example of this group of definitions is that included in the UK’s National Numeracy Project in 1993, which described numeracy as a series of autonomous basic numerical skills and knowledge of number facts such as multiplication tables (ALBSU, 1993, as described in Brown, Askew, Baker, Denvirs & Millett, 1998).

Figure 2: Continuum of numeracy definitions (Source: Maguire & O'Donoghue, 2002).

FORMATIVE	MATHEMATICAL	INTEGRATIVE
(basic arithmetic skills)	(mathematics in context of everyday life)	(mathematics integrated with the cultural, social, personal and emotional aspects of individual's life)
Examples: "A … set of autonomous basic numerical skills, emphasising mental and written calculations and knowledge of number facts such as multiplication tables." (ALBSU, 1993, p. 13, described in Brown, Askew, Baker, Denvir & Millett, 1998)	Examples: "The knowledge and skills required to apply arithmetic operations, either along or sequentially, to numbers embedded in printed materials (such as balancing a check book, figuring out a tip, completing an order form, or determining the amount of interest on a loan)." – IALS framework (Gal, et al, 2002, p. 11) Numeracy "involves using some mathematics to achieve some purpose in a particular context." (AAMT, 1997) "The competence and inclination to use number concepts and skills to solve problems in everyday life and employment." (Brown, 2002)	Examples: "To be numerate is more than being able to manipulate numbers, or even being able to succeed in school or university mathematics. Numeracy is a critical awareness, which builds bridges between mathematics and the real world, with all its diversity." (Johnston, 1995, p. 34) "To be numerate means to be competent, confident, and comfortable with one's judgments on *whether* to use mathematics in a particular situation and if so, *what* mathematics to use, *how* to do it, what *degree of accuracy* is appropriate, and what the answer means in relation to the context." (Coben, 2000, p. 10) "The aggregation of skills, knowledge, beliefs, dispositions, and habits of mind as well as general communicative and problem solving skills that people need in order to effectively handle real-world situations or interpretive tasks with embedded mathematical or quantifiable elements. (Gal, 1995) "The ability to situate, interpret, critique and perhaps even create mathematics in context, taking into account all the mathematical as well as social and human complexities which come with that process." (Johnston & Yasukawa, 2001)

The second "mathematical" group includes broader definitions of numeracy that extend numeracy beyond simple autonomic calculations. Here, numeracy is conceptualized as a broad set of mathematical knowledge and capabilities that can be applied in everyday situations. A number of earlier national and international frameworks adopted this type of numeracy definitions. For example, the International Adult Lifeskills Survey presented what the authors called a "quantitative literacy" definition that included "the knowledge and skills required to apply arithmetic operations, either along or sequentially, to numbers embedded in printed materials (such as balancing a check book, figuring out a tip, completing an order form, or determining the amount of interest on a loan)" (Gal, van Groenestijn, Manly, Schmitt, & Tout, 2002, p 11). Similarly, the Cockcroft report from the Department of Education and Science/Welsh Office (1982), defined numeracy as

> … an "at-homeness" with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life…[and] an ability to have some appreciation and understanding of information, which is presented in mathematical terms, for instance graphs, charts or tables or by reference to percentage increase or decrease (p. 11).

Finally, the third "integrative" group of numeracy definitions describe numeracy as a multifaceted, complex concept that, in addition to mathematical capabilities and knowledge, also incorporates other aspects and contexts of individuals' lives including the cultural, emotional, social, personal, technological, and communication elements. Individuals are also expected to be able to critically review and analyze mathematical information. For instance, according to Johnston (1995),

> To be numerate is more than being able to manipulate numbers, or even being able to succeed in school or university mathematics. Numeracy is a critical awareness, which builds bridges between mathematics and the real world, with all its diversity. (p. 34)

Of the definitions reviewed in preparing our report, two recent definitions proposed by Van Croenestijn (2002) and Coben's (2000) were found to be of special importance, as they identify key elements of numeracy that we believe should be included in the modern numeracy definition for it to reflect the requirements of the modern age. Van Groenestijn (2002) emphasizes the role of personal, societal and work contexts, as well as the need to adjust to growing informational and technological requirements of today's society:

> Numeracy encompasses the knowledge and skills required to effectively manage mathematical demands in personal, societal and work situations, in combination with the ability to accommodate and adjust flexibly to new demands in a continuously rapidly changing society that is highly dominated by quantitative information and technology. (p. 37)

Finally, Coben (2000) emphasizes the importance of individual's ability to exercise judgements about how to use mathematics in various contexts:

> To be numerate means to be competent, confident, and comfortable with one's judgments on *whether* to use mathematics in a particular situation and if so, *what* mathematics to use, *how* to do it, what *degree of accuracy* is appropriate, and what the answer means in relation to the context." (p. 10)

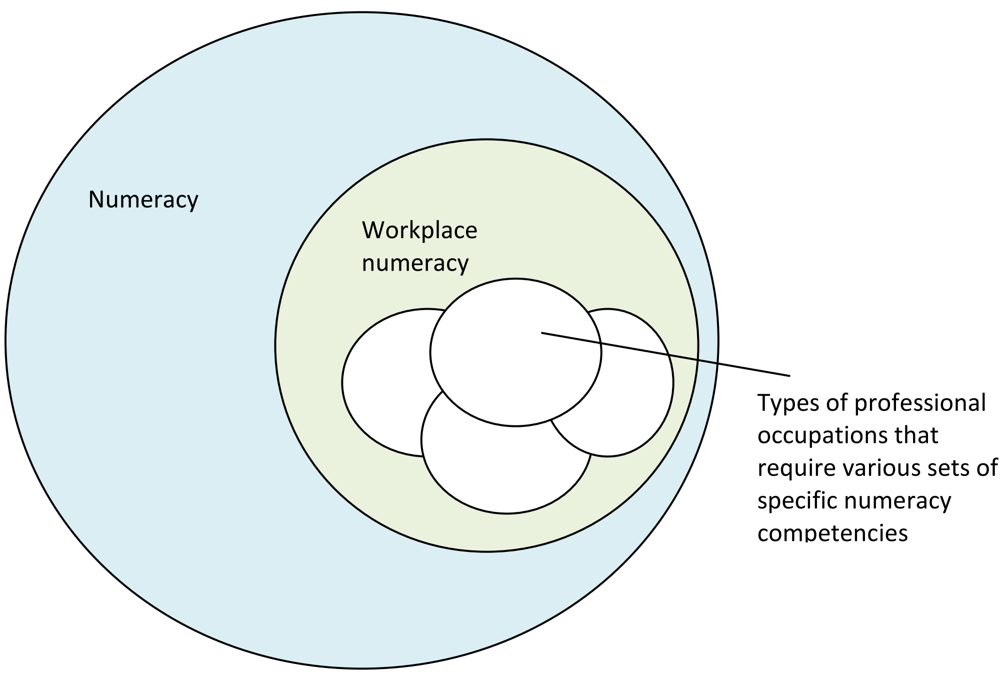
Workplace Numeracy

The numeracy definitions presented above focused on various aspects of adult numeracy. However, definitions of workplace numeracy might differ slightly from the broader, generic definitions described earlier. Coben (2008) indicates that workplace numeracy should focus on “effective use” rather than knowledge and skills alone. Context is therefore intimately linked to numeracy in the workplace. Furthermore, numeracy is linked to other skills and capabilities including literacy. As Townsend and Waterhouse (2008) argue,

> To address a work problem or complete a workplace task might entail gathering and analysing information; using number or mathematical skills; reading, writing and reporting (verbally and/or in writing); using a computer or another piece of equipment; working with other people, perhaps in a team; and quite possibly demonstrating some initiative. In this way, language, literacy, numeracy and generic or employability skills are linked with notions of employability and work performance. (p. 13)

In general, workplace numeracy can be considered a subset of the wider numeracy concept encompassing varied contexts. Furthermore, different jobs or professions might require slightly different sets of numeracy competencies.

Figure 3: General and workplace numeracy



Some researchers argue that the identification of numeracy in workplace environments is not straightforward. It might for instance be represented indirectly through artefacts (tools or procedures (Evans, 2000; Wedege, 1999, 2002a, 2002b). Workers' ability to "recognize" numeracy activities can also be affected by individuals' prior math-related experiences and dispositions. These researchers emphasize the importance of making numeracy visible in workplaces by explicitly describing competencies required for a particular job.

Canadian Definitions

While the analysis of Canadian definitions is not the purpose of this project, the publications we reviewed as part of our work indicate that Canadian organizations and reports tend to refer to a variety of international definitions, without giving a preference to one in particular. Among international definitions, the most frequently used are the definitions from the International Adult Literacy and Skills Survey (IALSS), the Adult Literacy and Life Skills Survey (ALL), the Programme for the International Assessment of Adult Competencies (PIAAC), and the Programme for International Student Assessment (PISA).

The Canadian definition of workplace numeracy was developed by HSRDC and "refers to the workers' use of numbers and their being required to think in quantitative terms" (HRSDC, 2007). This definition appears to have been adopted by provincial ministries as well as other education and employment/labour market organizations. For instance, Workplace Education Manitoba (n.d.) defines numeracy as **"**the ability to use numbers and think in quantitative terms. We use this skill when doing numerical estimating, money math, scheduling or budgeting math and analyzing measurements or data". Similarly, Workplace Learning PEI (n.d.), with a reference to HRSDC, suggests that "in the workplace you use forms of numeracy such as: estimating, calculating and/or measuring".

The current project will help define both a broad and more workplace specific definition of numeracy for application in the Canadian context.

PROJECT OBJECTIVES

HRSDC is working on improving the current numeracy measurement framework underlying the development of its Essential Skills Occupation Profiles by moving toward better alignment with prominent national and international numeracy measurement scales and frameworks, such as the International Adult Literacy and Skills Survey (IALSS). Specifically, the project involved:

- Revising the present ESRP numeracy definition to reflect current workplace requirements and to ensure better compatibility with available large-scale numeracy skills data.
- Revising the ESRP facets of numerate behaviour, or numeracy concepts used to describe the numeracy skills in the profiles, to ensure better alignment with the revised definition, current Canadian workplace requirements and available numeracy skills data from large scale assessments.
- Revising the complexity levels and descriptors to reflect the revised definitions and numeracy concepts ensuring consistent and reliable measurement parameters, and simple interpretation of the ESRP numeracy information.

The *Directions* Evidence and Policy Research Group (hereafter *Directions*) was contracted by HRSDC to assist with the following activities:

(1) to conduct a review of assessment frameworks that measure numeracy;
(2) to realize a comparative analysis of the definitions and facets of numeracy used by the international large-scale assessments listed above, with a view to evaluating their compatibility with the functions and intended uses of the essential skill profiles developed by HRSDC; and
(3) to complete the development of a set of recommendations to revise the ESRP methodology to align it with the international assessments, including recommendations regarding:
    - a suitable definition of numeracy;
    - numeracy concepts underpinning the definition; and
    - complexity levels (and a corresponding rating scale) for each numeracy facet included in the definition.

The full report that follows presents a summary of the findings and recommendations on how to align the ESRP framework with the international assessments approaches.

METHODOLOGY

As a first step in this project, and in consultation with HRSDC and the renowned Canadian numeracy expert Dr. David Robitaille, *Directions* identified five assessment and curriculum/conceptual frameworks that describe and/or measure numeracy. The following frameworks were selected based on their recognition by the international community of numeracy researchers and the comprehensiveness of their conceptual and measurement aspects:

- the OECD Programme for the International Assessment of Adult Competencies (PIAAC) (2009),
- the Adult Literacy and Lifeskills (ALL) (2003),
- the Tertiary Education Commission (New Zealand) Learning Progressions for Adult Numeracy and assessment tools (2008),
- the Programme for International Student Assessment (PISA) (2003, 2009), and
- the Adult Numeracy Core Curriculum (UK, 2001).

Documents that describe the conceptual and assessment frameworks were located and reviewed in full text and are included in the reference list. Two researchers reviewed the frameworks listed above. Several matrices were developed to document the content of frameworks for comparative analysis. The following information was extracted from each document:

- the definition(s) and/or components of numeracy,
- the definition(s) and/or components of numerate behaviour,
- the facets and concepts included in each facet, and
- the assessment approaches proposed (if any), including the complexity levels used to describe each of the facets of numerate behaviors, scoring ratings, and item complexity information.

For each framework, a separate analytical matrix was completed. The data were later entered into aggregate matrices, and similarities and differences across frameworks were identified and described. Upon completion of the review, analytical summaries and tables were reviewed by Dr. David Robitaille, who served as subject matter expert on this project.

COMPARATIVE ANALYSIS: RESULTS

This section describes, in narrative and matrix format, the specific numeracy content extracted from the set of five international frameworks selected for review. The content is compared to the ESRP numeracy content in order to develop recommendations of how to adapt the current ESRP numeracy framework. This section begins by providing context and background information on the numeracy component of the HRSDC Essential Skills Research Project and its occupation skills profiles as well as other selected international frameworks. The following sections focus on the definitions, numeracy facets and complexity levels included in the reviewed frameworks. Each section includes information extracted from the reviewed frameworks as well as basic conclusions or recommendations concerning their adaptation in the ESRP context.

Framework Background

Essential Skills Research Project

The Essential Skill Research Project was launched in 1994 by Human Resources and Skills Development Canada. The goal was to examine what fundamental skills were found across virtually all occupations and how these essential skills were used in various occupations across the country to address the differences in skill requirements between occupations (HRSDC, 2009).

In the ESRP framework, essential skills are defined as “skills needed for work, learning and life” and enable people to secure and retail employment, evolve with their jobs and adapt to changes in the workplace. Nine essential skill areas are identified in the ESRP, including reading text, document use, **numeracy**, writing, oral communication, working with others, continuous learning, thinking skills and computer use. The definition of numeracy presented in the ESRP is rather specific and “refers to the workers' use of numbers and their being required to think in quantitative terms” (HRSDC, 2007).

Over 350 essential skills profiles were developed as part of the ESRP, based on interviews with practitioners working in each occupation, and validated with subject matter experts across the country. Each essential skills profile includes a description of the occupation and lists the key skills that are used in that occupation.

Numeracy skills in each profile were rated using two complexity ratings:

- a complexity of **numerical calculation tasks**, and
- a complexity of tasks involving **numerical estimation** (HRSDC, 2007)**.**

The ***Numerical Calculation Rating Scale*** includes two dimensions, each having five levels of complexity. The first dimension, called ***Operations Required***, describes the "actual math operations used", such as subtraction, division or multiplication. Specifically, operations are described in terms of the number of operations used, the number of steps in the calculation and the difficulty of the operations required. The second dimension of the Numerical Calculation Rating Scale, called ***Translation***, refers to the process of "turning a work problem into a set of mathematical operations so that math may be applied to obtain an answer." The levels of complexity mention several indicators, such as the amount of information available to make a decision about the calculations needed, the level of uncertainty and ambiguity of the task, and the completion of formulae required (if any).

Numeracy calculation tasks are rated in four work settings and include (1) money math, (2) scheduling, budgeting or accounting math, (3) measurement and calculation math, and (4) data analysis math. Each of the settings is expected to require a slightly different set of competencies. The Essential Skills profiles therefore provide ratings for all four areas.

The ***Numerical Estimation Complexity Rating Scale*** includes five dimensions, each having four levels of complexity. The dimensions included in the scale are the following:

- the availability of a set procedure,
- the number of factors comprising the item being estimated,
- the amount of information available,
- the consequence of error, and
- the degree of precision required.

In addition to the complexity rating scales presented above, the ESRP methodology also describes the core declarative and procedural knowledge used in a particular occupational group. These are divided into three groups:

- <u>Mathematical foundations used</u>. This group refers to the specific declarative and procedural knowledge used in the occupation, and includes number concepts, pattern and relationships, space and spatial sense, and statistics and probability;
- <u>The means for performing calculation</u>. This group provides four possible means of conducting calculations, including "in the worker's head", "using pen and paper", "using a calculator" and "using a computer."
- <u>Measurement instruments used</u>. This group describes the type of measurements performed, the instruments used and the type of measurement systems employed by workers.

Selected Numeracy Frameworks

The Adult Literacy and Lifeskills (ALL) Survey

The Adult Literacy and Lifeskills (ALL) Survey (formerly known as the International Adult Lifeskills Survey) is an international assessment initiative aimed at assessing the performance of youth and adults (between 16 and 65 years of age) in three broad domains: prose and document literacy, numeracy and problem solving (Gal, van Groenestijn, Manly, Schmitt, & Tout, 2002). It was developed by Statistics Canada and by the US National Center for Education Statistics (NCES), in partnership with the Organization for Economic Cooperation and Development (OECD) and the United Nations Educational, Scientific, and Cultural Organization (UNESCO (OREALC)). The survey is a follow up to the International Adult Literacy Survey (IALS), the first international literacy assessment.

Using household survey methods, the ALL survey examined adult literacy performance in three literacy-related domains (prose and document literacy, numeracy and problem solving) and investigated the respondents' experiences with technologies. Furthermore, the survey aimed to compare the outcomes across participating countries, to correlate these with various social and demographic factors, and to explore the relationship between different types of literacies.

The ALL documents provide a brief definition of numeracy whereby numeracy is conceptualized as "the knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations" (Gal, et al., 2002, p. 15). In addition to this brief definition, the ALL documents also include a more elaborate definition of numerate behavior. Five key facets of numeracy behaviour were identified: (1) context, (2) responses, (3) mathematical information, (4) representations of mathematical information; and (5) other enabling factors and processes. Finally, the documents also discuss an assessment framework that can be used to assess the complexity and difficulty of numeracy tasks. These will be described in the sections that follow.

The Programme for International Assessment of Adult Competencies (PIAAC)

The Programme for the International Assessment of Adult Competencies (PIAAC) is another international assessment initiative undertaken by the Organization for Economic Co-operation and Development (OECD) in 27 countries including Canada. The objective of the PIAAC is to examine literacy competencies that are required to succeed in the 21st century workplace and society, including the "interest, attitude, and ability of individuals to access, manage, integrate, and evaluate information, construct new knowledge, and communicate with others" (Gal, et al., 2009, p. 7). PIAAC assesses four areas of literacy competencies: (1) problem-solving in a technology-rich environment, (2) literacy, (3) reading, and (4) numeracy. Several objectives are put forth for this initiative: to examine the "differences between individuals and countries in competencies believed to underlie both personal and societal success" (OECD, n.d.); to assess the impact of these competencies on various economic and social outcomes; and to provide information for educators and policy makers to improved education and training system nationally and internationally.

PIAAC was developed based on two previous international assessments: the International Adult Literacy Survey (IALS) and the Adult Literacy and Lifeskills Survey (ALLS), adopting their framework, facets and complexity levels (with some modifications). As a result, it adopted a number of elements of earlier assessment frameworks to enable continuity.

In PIAAC, conceptualizations of numeracy are built upon the idea of competency, which reflects the ability of workers to carry out their activities and tasks successful meeting the demands of their employers and organizations. This approach emphasizes the fact that competency is a "complex action system" that is comprised of cognitive and non-cognitive components. In congruence with the competency view, PIAAC defines numeracy and numerate behavior in terms cognitive elements (i.e., mathematical declarative and procedural knowledge) and "non-cognitive or semi-cognitive" (i.e., attitudes, habits of mind, dispositions, and beliefs) (Gal, et al., 2009, p. 10).

Similar to the ALL, the PIAAC documents describe the definitions of numeracy and numerate behavior; facets of numerate behavior and complexity levels that can be used to assess the complexity/difficulty of numeracy activities, tasks or items. The description of these will be presented in the later sections of this report.

The Tertiary Education Commission

The third numeracy framework reviewed for the report is the Learning Progressions for Adult Numeracy developed by the Tertiary Education Commission (TEC), New Zealand. The framework adopts a competency-based approach similar to the international frameworks described above. According to the framework, the term "competencies" includes "the knowledge, the cognitive and practical skills and the attitudes (including motivation) needed to meet demands or carry out tasks successfully" (Britt, et al., 2008, p. 6).

The framework is organized around "learning progressions" that represent a continuous, sequential movement that describes how adult learners progress towards expertise in a particular area of numeracy. Thirteen learning progressions are organized into three strands: (1) Making sense of number to solve problems; (2) Reasoning statistically; and (3) Measuring and interpreting shape and space. According to the TEC, progressions define the key steps numeracy learning. The progressions also illustrate the cumulative nature of learning, as learners start their journey at different points along a continuum.

Overall, the TEC's intent was to develop a framework that could illustrate what adult learners should know and do at successive points as they develop their expertise in numeracy learning. Specifically, the term "progression" used in the framework implies "a continuous, sequential movement towards expertise rather than a series of separate tasks to be mastered in order to move 'up'" (Britt, et al., 2008, p. 6). Furthermore, the steps in the learning progressions included in the framework are seen as "signposts" that indicate a significant stage of development.

It should be noted, however, that the learning progressions are not meant to be an assessment tool. Rather they are intended to inform the development of curricula, assessment tools and learning activities; to gain basic picture of adults' numeracy knowledge and skills; and/to identify numeracy-related demands of specific activities, problems or information (Britt et al., 2008).

The Programme for International Student Assessment (PISA)

The fourth numeracy framework reviewed for this project was that of the OECD's Programme for International Student Assessment (PISA). The PISA assessment takes a broad approach to measuring knowledge, skills and attitudes that reflect current changes in curricula, moving beyond school-based approaches towards the use of knowledge in everyday tasks and challenges.

In contrast to the other frameworks reviewed for this project, which were devised primarily to conceptualize the numeracy competencies of adult learners, PISA is intended as an assessment of 15-year-old students (an age selected because it presupposes the occurrence of potentially sophisticated numerical competency development in a population that, across participating international jurisdictions, is still in the process of completing compulsory education). Despite its distinctive population focus, it was selected because of its international use and validity. Furthermore, it was selected because adolescents leaving school are expected to perform key numeracy-related skills in a variety of workplace and everyday contexts, skills that are included in the adult numeracy frameworks.

PISA focuses on real-world problems, moving beyond the kinds of situations and problems typically encountered in school classrooms. Consequently, it seeks to assess the competencies that participating students will need in the future along with what they are able to accomplish with what they have learned. By asking students to apply what they learn in school to non-school environments, to evaluate their choices and to make decisions, the PISA framework presupposes a conception of learning as dynamic and lifelong, in which new knowledge and skills necessary for successful adaptation to a changing world are continuously acquired throughout life.

The PISA assessment involves participation by OECD member countries and typically involves the participation of between 4,500 and 10,000 students from 150 schools in each participating country. Since its introduction in 1997, the assessment has been conducted every three years, which each assessment examining three domains: reading, numeracy, and science. However, each domain is given particular focus in every third iteration of the assessment (i.e. every nine years).

The framework puts forth as its foci three main facets of numeracy, respectively that of (A) situations and contexts, (B) mathematical content, and (C) mathematical processes. These facets are in turn operationalized into eight characteristic cognitive numerical competencies:

(1) thinking and reasoning, (2) argumentation, (3) communication, (4) modeling, (5) problem posing and solving, (6) representation, (7) use of symbolic, formal and technical language and operations, and (8) use of aids and tools. These competencies are in turn developed for each of three specific “clusters”: the Reproduction Cluster, the Connections Cluster, and the Reflection Cluster. These reflect increasing levels of complexity, abstraction, and numeracy sophistication.

The 2003 PISA framework briefly describes a process by which the numeracy complexity of an assessment item might be assigned to one of the competency clusters. It suggests analyzing the demands of the item, rating each of the eight competencies mentioned in the preceding paragraph for that item, and determining which of the three aforementioned clusters provides the most fitting description of the item’s demands in relation to each competency. The assessment item is ultimately assigned to a complexity cluster on the basis of the aggregated level of complexity required for all competencies in the item.

The Adult Numeracy Core Curriculum

The United Kingdom Adult Numeracy Core Curriculum (2001) outlines generic skills and knowledge elements that are perceived to act as the basic building blocks that everyone needs in order to use numeracy skills effectively in everyday life. The ANCC assumes that differences in skill usage are related to the contexts for their use and the often vastly differing past experiences that learners bring to their learning. Moreover, in contrast to PISA, the ANCC focuses on adult learners and, specifically, on those adult learners who struggle with numeracy.

Although it addresses issues of assessment, the ANCC framework places particular emphasis on the curricular elements to be mastered by adult learners and therefore seeks to draw the attention of teachers providing numeracy instruction to adult learners to the progression of capabilities that can be expected and should be sought. The inclusion of explicit curriculum targets in the framework is deliberate and is built on the assumption that it will result in a better formulation of desirable outcomes by learners as well as by teachers, and in better attendance and progression by learners. It is, in short, intended as an important guide to curriculum and program development to respond to the needs of adult learners with limited (even very limited) numeracy skills.

The ANCC’s curricular elements are organized according to five pre-set levels of numeracy competency ranging from the most basic to the highest level of sophistication specified for the framework: an “entry level” set of numeracy competencies, itself divided into three sublevels, a higher Level 1 and the highest level in this framework, Level 2. These last two levels correspond, respectively, to level 1 and level 2 of the national qualifications framework developed by the UK’s Qualifications and Curriculum Authority (QCA, 2000).

The ANCC identifies numeracy as encompassing the ability to understand and use mathematical information, calculate and manipulate mathematical information and interpret results and communicate mathematical information. It is organized around four “progressions” that appear to mirror what other frameworks identify as "facets" of numeracy: (1) capabilities (2) number, (3) measures, shapes and space, and (4) handling data. These are in turn operationalized into specific knowledge, skills and competencies for each of the aforementioned five levels. The framework also presents six contexts in which adults might apply numeracy skills: citizen and community, economic activity (paid and unpaid work), domestic and everyday life, leisure, education and training, and using ICT in social roles. These contexts, however, are not described in-depth in the ANCC framework, which also doesn't contain information about *complexity ratings*.

Framework Definitions of Numeracy

Comparison of Numeracy Definitions

As described in the Methodology section, two *Directions* researchers reviewed the frameworks described previously. The information was extracted in comparative matrices, and the definitions were contrasted. Research on numeracy and workplace numeracy definitions reviewed by *Directions* suggest that a numeracy definition should (1) go beyond knowledge and skills and emphasize effective ***use*** of these ***declarative and procedural knowledge***, (2) indicate the ***purpose*** of using mathematical declarative and procedural knowledge; (3) be ***situated*** in contexts; and (4) emphasize ***critical engagement*** on the part of numeracy "agent" (e.g., see Coben et al, 2003, 2007, 2008; Condelli et al., 2006). We used these criteria to analyze numeracy definitions from ESRP and international frameworks (see Table 1). The key findings from the comparative analysis of the definitions are described below.

Table 1. Numeracy definitions: Comparative Analysis

Framework	Definition	Declarative knowledge	Procedural knowledge	Effective use	Purpose - Making sense of use	Situatedness - Variety of contexts	Critical engagement
ESRP	**Numeracy** refers to the workers' use of numbers and their being required to think in quantitative terms.	YES	YES	YES			
PIAAC	**Numeracy** is the ability to access, use, interpret, and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life.	YES	YES	YES	YES	YES	
ALL	**Numeracy:** The knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations.	YES	YES	YES	YES	YES	
TEC	The **view of numeracy** that is used for these learning progressions places an emphasis on the need for learners to gain: (1) knowledge and understanding of mathematical concepts; and (2) the ability to use their mathematical knowledge to meet the varied demands of their personal,	YES	YES	YES	YES	YES	

Framework	Definition	Declarative knowledge	Procedural knowledge	Effective use	Purpose - Making sense of use	Situatedness - Variety of contexts	Critical engagement
	study and work lives. Numeracy is about knowing and understanding: it is therefore both broad and contextualised.						
PISA	This framework utilizes the term **mathematical literacy**, which it defines as the capacity of individual "to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen."	YES	YES	YES	YES	YES	YES
ANCC	**Numeracy** covers the ability to understand and use mathematical information, calculate and manipulate mathematical information and interpret results and communicate mathematical information.	YES	YES	YES			

The comparative analysis indicates that definitions used by international frameworks are similar. All five international definitions (PIAAC, ALL, TEC, PISA, and ANCC) refer to declarative and procedural knowledge, albeit with different terminology, and the ability to effectively use this information to manage and solve numeracy-related problems and tasks. Also, four of the definitions (PIAAC, ALL, TEC, and PISA) indicate that the purpose of using the mathematical knowledge and abilities reflects the need to engage and manage demands of various situations. These frameworks also emphasize the importance of various contexts in which numeracy competencies are used. These are variously referred to as "diverse situations", "a range of situations in adult life", "personal, study and work lives". The UK ANCC framework does not include contexts in the definition of numeracy; however, the document mentions six contexts in which adults might apply numeracy skills: citizen and community, economic activity (paid and unpaid work), domestic and everyday life, leisure, education and training, and using ICT in social roles.

Unlike the international numeracy definitions, which are more general in nature, the ESRP's numeracy definition focuses specifically on work-related context. Although other contexts such as everyday life and further learning are mentioned in the ESRP documents, these are not explicitly included in the numeracy definition. Furthermore, while the definition provides specific reference to the use of numbers and ability to think in qualitative terms, which can be interpreted as declarative and procedural knowledge, these references are rather limited. Specifically, other frameworks provide more detailed description of procedural knowledge including such activities as calculation, interpretation, accessing, and managing among others. This strongly suggests that it would be important to extend the ESRP definition beyond the use of numbers.

Recommendations Regarding the ESRP Numeracy Definition

Upon analysis of the numeracy definitions included in the ESRP and selected international frameworks, as well as other numeracy definitions discussed earlier, we propose two numeracy definitions for HRSDC's consideration: a generic definition of adult numeracy and a workplace numeracy definition. We believe a definition of numeracy (whether generic or workplace oriented) should include five key considerations. It should:

- highlight ***declarative and procedural knowledge***;
- entail the ***use*** of knowledge and abilities;
- indicate the ***purpose*** of numeracy related activities;
- be ***situated*** in context; and
- emphasize the importance of ***critical engagement*** with numerical information and activities.

Potential formulations of recommended numeracy definitions are presented next.

> *Adult numeracy is the ability to identify, understand, critically interpret and apply mathematical concepts, processes, and representations in order to engage in and answer to the mathematical demands of a variety of situations in adult life.*
>
> *Workplace numeracy is the ability to identify, understand, critically interpret and apply mathematical concepts, processes, and representations that are appropriate to the work context in which the individual is engaged in order to effectively manage/perform work activities.*

Framework Numeracy Facets

In addition to the review of definitions, the *Directions* team examined the facets or dimensions of numeracy included in the ESRP and international numeracy assessment frameworks. Each framework was first reviewed to identify the number and type of facets included. The information was extracted into individual, framework-specific matrices then added to the aggregate matrix and compared across the framework. We used the ALL/PIAAC categories as a basis for the matrix. Where additional dimensions were identified, these were added as additional columns in the table.

While the frameworks use a variety of terms to define the key components of numeracy, these seem to fit within four general categories: (1) mathematical content with which an adult needs to be familiar (knowledge, key topics, mathematical information); (2) the processes and operations required; (3) the contexts in which numerate activities are taking place; and (4) various representations of mathematical information that are needed to complete the task or communicate the results. Some frameworks also refer to “enabling” processes and factors that are more generic in nature and serve as a foundation for numeracy activities.

Both the ALL and PIAAC frameworks list four of the five facets described above: Contexts, Facet Responses, Mathematical Information, and Representations of Mathematical Information. While ALL includes enabling processes as the fifth facet, PIAAC identifies these as important but supporting components. Similarly, the PISA documents mention three facets of numeracy: Situations (Contexts), Mathematical Content, and Mathematical Competencies and Processes.

In their numeracy framework, TEC defined 13 learning progressions organized into three strands: Strand #1—Making sense of number to solve problems (Additive Strategies progression, Multiplicative Strategies progression, Proportional Reasoning Strategies progression, Number Sequence progression, Place Value progression and Number Facts progression); Strand #2—Reasoning statistically (Preparing Data for Analysis progression, Analysing Data for Interpretation progression, Interpreting Data to Predict and Conclude progression, and Probability progression); and Strand #3—Measuring and Interpreting shape and space (Shapes and Transformations progression, Location progression, and Measurement progression). While contexts were not included as a separate facet, these were included in the definition of numeracy in the introductory sections of the document.

The ANCC framework does not specifically identify “facets” of numeracy or numerate behaviour. However, the curriculum is organized around four progressions. The first progression is conceptualized as focused on capabilities that describe adults’ abilities to read, understand, calculate, manipulate, interpret and communicate mathematical information. The other three progressions are focused on specific curricular themes: (1) Numbers, (2) Measures, Shape and Space, and (3) Handling Data. These in turn are structured around five levels of complexity with respect to math concepts and operations. In addition, the document presents six contexts in which adults might apply numeracy skills: citizen and community, economic activity (paid and unpaid work), domestic and everyday life, leisure, education and training, and using ICT in social roles.

The ESRP framework uses a different approach to conceptualizing numeracy. The document describes three main numeracy components:

- Numerical Calculation (which examines tasks performed in four application settings: money math; scheduling or budgeting and accounting math; measurement and calculation math; and data analysis math);
- Numerical Estimation (which describes the complexity of operations involved) ; and
- Math Skills (which summarizes 15 math-related content topics).

We analyzed the definitions, descriptions and examples included in the ESRP documents to identify implicit references to other numeracy-related contexts, knowledge and processes. The results are presented in Appendix A. The sections that follow provide a summary and comparison of each of the facets and include recommendations of how to align the facets. For some of the frameworks, the link to the proposed facet classification was *inferred* from the text of the documents, numeracy definitions, examples of activities provided in the documents, as not every framework had an explicit description of numeracy facets or dimensions.

Domain 1: Context

This section presents the comparative analysis of the ESRP and selected international frameworks. Our suggestions and recommendations on how to adjust the structure and content of the current ESRP framework will be presented in a separate section.

All frameworks reviewed for this study emphasized that the ability to perform mathematical/ numeracy-related activities in a variety of contexts is an important aspect of numeracy. However, while some frameworks included contexts as a separate facet (ALL and PIAAC), others mentioned these in the definition or emphasized the importance of contexts in the text of the document.

Describing contexts in which numeracy activities and tasks might be performed, all five international frameworks reviewed included references to everyday life/personal, work, community/society, and study contexts (Table 2). The PISA framework refers to contexts as situations and defines them as “the part of students’ world in which the tasks are placed”. These include the personal, educational/occupational, public and scientific, ranging from the situations closest to the student (personal) to those that are arguably the most remote (scientific). The ANCC curriculum presents six contexts in which adults might apply numeracy skills: citizen and community, economic activity (paid and unpaid work), domestic and everyday life, leisure, education and training, and using ICT in social roles. These are, however, not well described. In the ANCC framework, these contexts are not mutually exclusive and involve the same declarative and procedural knowledge as well as the same underlying mathematical themes.

While ESRP does not have a separate facet related to contexts, the facet entitled Numerical Calculation describes numerical calculation tasks performed in four application settings: money math, scheduling or budgeting and accounting math, measurement and calculation math, and data analysis math. All examples and illustrations provided relate to work-related situations, such as activities performed by chip stand operators, tour directors, bank tellers or investment analysts, thus allowing us to draw a conclusion about the importance of the work related context. While the ESRP document itself mentions other situations (such as everyday life, workplace, community and family contexts), there are no explicit references to these in the numeracy definition or descriptions of the facets and complexity levels (HRSDC, n.d., 2009).

Table 2. Domain 1: Contexts & Situations

Framework	Work	Everyday life	Societal/community	Further learning	Scientific*
ESRP	Yes (#1 Numerical calculation - Application setting - work)	Yes (mentioned in the text of the document)	Yes (mentioned in the text of the document)	Yes (mentioned in the text of the document)	
PIAAC	Yes (included as a Contexts facet)	Yes (included as a Contexts facet)	Yes (included as a Contexts facet)	Yes (included as a Contexts facet)	
ALL	Yes (included as a Contexts facet)	Yes (included as a Contexts facet)	Yes (included as a Contexts facet)	Yes (included as a Contexts facet)	
TEC	Yes (mentioned in definition)	Yes (mentioned in definition)	Yes (mentioned in the text)	Yes (mentioned in definition)	
PISA	Yes (occupational/educational)	Yes (personal)	Yes (public)	Yes (occupational/educational)	Yes
ANCC	Yes (economic activity, including paid and unpaid work)	Yes (domestic and everyday life; leisure)	Yes (citizen and community; using ICT in social roles)	Yes (education and training)	

* While this context was included in PISA framework, it was not mentioned in any of other international assessments. Thus, it was not included in the numeracy framework developed by *Directions* and presented in the remaining sections of this report.

Domain 2: Mathematical Processes (Procedural Knowledge)

In addition to the numeracy definitions discussed in earlier sections, the frameworks also described different types of responses or behaviours through which numeracy is revealed. Two frameworks (ALL and PIAAC) included definitions of numeracy behavour. According to the ALL definition, “numerate behaviour is observed when people manage a situation or solve a problem in a real context. It involves responding to information about mathematical ideas that may be represented in a range of ways. It requires the activation of a range of enabling knowledge, factors, and processes.” PIAAC provides a brief and similar definition that mentions “managing a situation or solving a problem in a real context, by responding to mathematical content, information, ideas represented in multiple ways.”

In general, numerate behaviour can be exhibited through a variety of actions or processes. These range from generative processes (e.g., counting, quantifying, computing, calculating, estimating) to interpretive responses (e.g., making sense of verbal or text-based information to interpret, evaluating or analyzing quantitative data) to “decisions” situations that require people to locate and consider a variety of information to draw conclusions or predictions.

While other frameworks do not include explicit definition of numerate behavior, examples of numerate behaviors can be extracted from the numeracy definitions, and examples of tasks and activities included in the frameworks. Thus, three frameworks reviewed here included a separate facet that described numerate behaviours: ALL and PIAAC (Responses facet), and PISA (Mathematical competency and processes facet) (see Table 3). Table 3 summarizes various numerate behaviours/responses that were mentioned in the frameworks. We used the ALL/PIAAC as the basis of classification and augmented it with additional behaviours mentioned in the texts of the frameworks.

The ALL and PIAAC Responses facet explicitly include such behaviours as identifying or locating, using/acting upon (ordering/sorting, counting, estimating, computing, measuring, and modeling), interpreting and communicating. In the later PIAAC version, this facet also includes evaluating /analyzing behavior. Other frameworks, however, mention other behaviours in the text trough descriptions and examples, including thinking and reasoning, argumentation, predicting and making conclusions.

Similarly, the ANCC curriculum document includes a Capabilities Progression that summarizes three types of mathematical actions that adults are expected to perform: understanding and using mathematical information [reading and understanding, specifying and describing]; calculating and manipulating math information [generating results]; and interpreting results and communicating math information [presenting and explaining results]. Additional mathematical activities/responses (such as counting, ordering, estimating, locating information, predicting, and evaluating the results) are included in the three progressions of the curriculum elements.

TEC's learning progressions include references to numerate behaviours throughout all thirteen progressions, including such behaviours as using strategies to solve problems, locating mathematical information, manipulating numbers; performing basic and advanced mathematical operations; identifying and describing potentially relevant attributes; sorting organizing and representing data; drawing conclusions and making predictions; communicating information; and carrying out various measurements among others.

The ESRP framework includes descriptions and references to mathematical processes in all three facets. For example, the Numerical Calculations facet refers to measurement and calculation and data analysis processes. The Numerical Estimation facet mentions such activities as involving estimation of set procedures, factor estimation, estimation of the amount of information, degree of precision and predicting the consequences of errors. The Math Skills facet refers to activities related to measurements.

While some frameworks include problem solving, thinking and reasoning, argumentation, predicting and making conclusions, the use of these capabilities extends beyond the frame of reference on which we have chosen to focus. In the PIAAC framework, these broader capabilities are seen as enabling processes and skills that support numerate behavior.

Table 3. Domain 2: Mathematical Processes (Responses/Actions)

Framework	Identifying, locating or accessing	Using						Interpreting	Evaluating/ analysing	Communicating	Thinking and reasoning; Argumentation	Predicting/ Making conclusions
		Order	Count	Estimate	Compute	Measure	Model					
ESRP*	Yes	Yes[1]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	?	?	Yes
PIAAC	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
ALL	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes (in the text)	Yes	Yes (in the text)	Yes (in the text)
TEC	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes (in the text	Yes (in the text
PISA	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
ANCC	Yes	Yes	Yes	Yes	Yes	Yes		Yes	Yes	Yes	Yes	

* Some information regarding mathematical processes was inferred from examples provided, but they are not explicitly mentioned.

[1] As classifying.

Domain 3: Mathematical Concepts or Declarative Knowledge

Mathematical concepts or content (so called "big ideas") refer to the fundamental mathematical knowledge that adults require to successfully perform tasks. These differ slightly from framework to framework (see Table 4). Earlier research conducted by Ginsburg, Manly, and Schmitt (2006), who compared 29 numeracy frameworks and identified four mathematical content strands: number and operational sense; patterns, functions, and algebra; measurement and shape; and data, statistics, and probability.

We also found that all the frameworks reviewed for this study included topics that could be grouped into four broad categories: (a) quantity and number, (b) dimension and shape, (c) patterns, functions, relationships and change, (d) data and chance. The ALL, PIAAC and PISA frameworks provide explicit references to these topics. In other frameworks, the links to some of the topics are included indirectly through examples and descriptions. For example, the ANCC curriculum document explicitly mentions numbers, measures, shape and space, data and probability, while other concepts such as change and patterns are included in the descriptions of examples.

The ESRP framework mentions 15 specific mathematical topics grouped in four primary categories: number concepts, shape and spatial sense, patterns and relationship, and statistics and probability. Number concepts include topics such as integers, whole and rational numbers, fractions, other real numbers such as powers and roots. Patterns and relations focused on the use of rates, ratios and proportions, and knowledge of equations and formulae. The shape and spatial sense category includes measurement conversions: areas, perimeters and volumes; and geometry and trigonometry. Finally, the statistics and probability strand includes two subtopics: summary calculations (e.g., calculations of averages, proportions or ratios) and statistics and probabilities.

Declarative knowledge includes as much the mathematical representations as the roles, systems and structures that govern each of the math concepts noted above. Mathematical representations are basic declarative knowledge from which numeracy content can be acquired. They include a wide range of media and types of information such as numbers, objects, maps, diagrams, charts, graphs, tables, and texts.

Table 4. Domain 3: Mathematical Concepts/Knowledge (Content)

Framework	Quantity & number	Dimension & shape/location	Pattern, functions, relationships & change	Data & chance
ESRP*	Yes (number concepts, measurements)	Yes (shape and spatial sense)	Yes (patterns and relations)	Yes (statistics and probability)
PIAAC	Yes	Yes	Yes	Yes
ALL	Yes	Yes	Yes**	Yes
TEC	Yes (measurement, number sequence, number facts, place value)	Yes (shape and transformation; location)	Yes (shape and transformation, proportional reasoning)	Yes (probability)
PISA	Yes (quantity, number sense)	Yes (shape and space)	Yes (Change and relationship, patters are included in Quantity section)	Yes (uncertainty)
ANCC	Yes (Number **-** numbers, number system [fractions, decimals and percentage])	Yes (Measures, shape and space **-**money, rime, temperature, distance, length, weight, capacity, perimeter, area, volume, shape, position)	Yes (Pattern*, relationships, number, measures and shapes and handling data)	Yes (Handling data**-**data, statistical measures, probability)

*Some information regarding mathematical processes was inferred from examples provided, but they are not explicitly mentioned.
** In the ALL framework, change is included as a separate strand in the mathematical Content facet.

Domain 4: Representation of Mathematical Information

The last domain included in this review is that of representation of mathematical information. As workers in contemporary societies are faced with the need to work with a variety of media and forms of information, the ability to locate, interpret, use and represent mathematical information in various forms is an important competency for numeracy.

The frameworks reviewed here mention a variety of mathematical representations ranging from objects and numbers, to charts, tables and texts. Overall, eight types of representations across the frameworks were identified: objects and pictures, numbers and symbols, formulae, diagrams and maps, graphs, tables, texts, and technology-based displays (see Table 5). Three frameworks (PIAAC, PISA, and ANCC) mention all of the representations while the ALL documents referred to all representation types except for technology-based representations. ANCC devotes specific attention to the topic of representation of mathematical information and the ability of adults to locate, read, interpret and use a variety of different media and representations including numbers, objects, maps, diagrams, charts, graphs, tables, and texts.

ESRP explicitly mentioned the use of technologies such as computers and calculators to carry out calculations as well as the use of formulae. While other representations are not described in the numeracy section, the document refers readers to the Document Use section.

Although mathematical representations are identified as a separate facet in some of the frameworks, they can be considered a form of declarative knowledge. We therefore propose to combine mathematical representations with the Declarative Knowledge domain, as presented below.

Table 5. Domain 4: Representations of Mathematical Information

Framework	Objects & pictures	Numbers & symbols	Formulae	Diagrams & maps	Graphs	Tables	Texts	Technology-based displays
ESRP	?		Yes	Yes*	Yes*	Yes*	Yes*	Yes (use of technology for calculations)
PIAAC	Yes	Yes	Yes	Yes	Yes (combined with diagrams)	Yes (combined with diagrams)	Yes	Yes
ALL	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
TEC	Yes	Yes		Yes	Yes	Yes		
PISA	Yes (math competencies)	Yes (math competencies)	Yes (math competencies)	yes (math competencies)	Yes (math competencies)	Yes (math competencies)	Yes (math competencies)	Yes (math competencies)
ANCC	Yes (progression)	Yes (progression)	Yes ((in Number, measures and shapes and handling data)	Yes (progression)	Yes (progression)	Yes (progression)	Yes (progression)	mentions calculators

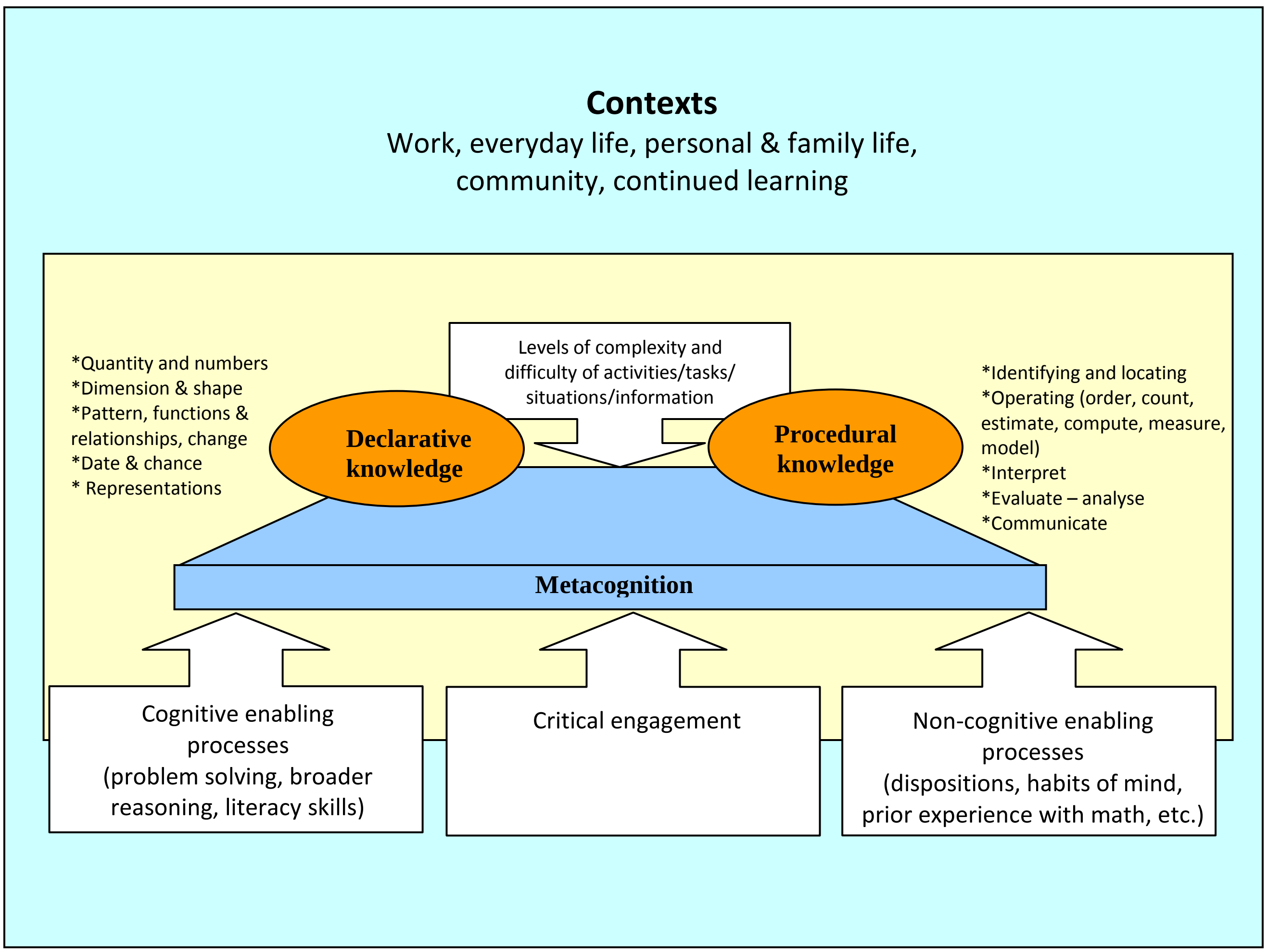
*These are included in the Document use Essential Skills, and mentioned in the numeracy section.
?: These were inferred from examples provided, but were not explicitly mentioned in the framework.

Recommendations Regarding ESRP Framework Facets

After reviewing all the frameworks and facets identified in each document, we recommend organizing the revised numeracy framework intended to inform the ESRP into two domains:

1. Declarative mathematical knowledge (content)
2. Procedural mathematical knowledge (i.e., responses/actions performed or required) (see Figure 4).

Figure 4. Framework for application of numeracy



In addition to the two facets proposed above, several other components should be included in the numeracy framework: (a) context, (b) cognitive enabling processes, (c) non-cognitive enabling processes, and (d) meta-cognitive processes. While these components are not conceptualized here as separate facets/domains, they are seen as prerequisites of effective numerate behaviours, serving as enabling processes and factors for both facets.

The framework proposed here emphasizes that numeracy exists within contexts. When we describe adult numeracy in general, these contexts include a range of situations, including everyday life, work, learning, and community/societal situation. However, workplace numeracy focuses specifically on work-related situations. Moreover, the situations in which numeracy may be required can range from the generative (conducting simple or complex mathematical operations and generating a response to a problem); to the interpretive (making sense of quantitative information without actually conducting calculations); through to decision-making situations (identifying and evaluating multiple pieces of information to determine a course of action).

It should also be noted that metacognition plays an important role in the successful application of numeracy. The ability of individuals to be aware of their thought processes and progress as they solve numeracy-related tasks fosters independent self-directed learning and builds numeracy capacity.

Finally, individuals' application of numeracy is influenced by other cognitive and non-cognitive processes. Some of the cognitive processes include broader reasoning skills, argumentation and problem solving skills, as well as more general literacy skills. Among non-cognitive enablers, prior experiences with mathematics, habits of mind, and attitudes and dispositions to, and beliefs about, mathematics should be considered.

Framework Numeracy Assessment Approaches

This final section of the comparative analysis describes and compares the complexity ratings used in ESRP and international frameworks and provides recommendations on possible modifications that could be made to better align the ESRP methodology with international assessment approaches.

It should be noted that conducting a comparative analysis of the complexity levels and ratings proved the most challenging of this research, as the frameworks included in the review used different factors to assess the complexity of the tasks. Moreover, making reasonable comparisons proved difficult in light of the differences in scope and intended purpose of the frameworks reviews (e.g. the use of PISA to evaluate numeracy competencies of secondary school learners still engaged in formal education relative to the ANCC as a framework primarily intended to guide the instruction and competency development of adult learners with low levels of numeracy).

The matrix summarizing the comparative analysis of the complexity levels is included in Appendix B.

ESRP Complexity Rating Scales

In the ESRP framework, essential skills are given two types of complexity ratings: one for the complexity of the **numerical calculation tasks**; and another for the complexity of tasks involving **numerical estimation.** The **complexity rating scale for** the **numerical calculation tasks** contains five levels and two dimensions. The first dimension, called ***Operations Required***, describes the "actual math operations used", such as subtraction, division or multiplication. Specifically, operations are described in terms of the number of operations used, the number of steps in the calculation and the difficulty of the operations required. The second dimension of the numerical calculation rating scale, called ***Translation***, refers to the process of "turning a work problem into a set of mathematical operations so that math may be applied to obtain an answer." The levels of complexity mention several indicators such as the amount of information available to make a decision about the calculations needed, the level of uncertainty and ambiguity of the task and the completion of formulae required, if any. The Operations Required levels range from the simplest operations to operations involving multiple steps and advanced techniques. Translation levels also range from minimal translation to situations involving complex formulae and abstract calculations.

The ***Numerical Estimation Complexity Rating Scale*** includes five dimensions, each having four levels of complexity. The dimensions included in the scale are the following:

- the availability of a set procedure,
- the number of factors comprising the item being estimated,
- the amount of information available,
- the consequence of error, and
- the degree of precision required.

ALL and PIAAC complexity factors

The ALL and PIAAC frameworks both use the same approach to complexity rating. Developers of these frameworks aimed to expand the assessment approach beyond merely assessing the number and type of operations and problem transparency (translation). An attempt was also made to include other factors in the rating scale, such as the type of math information, in order to reflect the broad definition of numeracy in a variety of contexts.

Both frameworks assess two aspects of numeracy tasks: their textual aspects, such as type of match/problem transparency and the plausibility of distractors; and their mathematical aspects, such as the complexity of mathematical information/data, the type of operation/skill and the expected number of operations. That ranking scale ranges between 3 and 5 scores for each factor.

PISA item difficulty levels

In PISA, the assessment of item difficulty reflects conceptual complexity, familiarity with the context, and recent opportunity to learn and practice.

Student performance is summarized based on a six-level performance scale. At **Level 1**, students can answer clearly defined questions containing all the relevant information in familiar contexts. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli. At **Level 2**, students can interpret and recognise situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and making literal interpretations of the results. Students deemed to be performing at **Level 3** can execute clearly described procedures, including those that require sequential decisions. They can select and apply simple problem solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They can develop short communications reporting their interpretations, results and reasoning.

In turn, the three upper levels of student performance reflect advanced numeracy competencies of growing sophistication and flexibility. **Level 4** students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic ones, linking them directly to aspects of real- world situations. Students at this level can utilise well-developed skills and reason flexibly, with some insight, in these contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments, and actions. At **Level 5,** students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare, and evaluate appropriate problem solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterisations, and insight pertaining to these situations. They can reflect on their actions and formulate and communicate their interpretations and reasoning. At the highest level of numeracy performance (**Level 6**), students can conceptualise, generalise, and utilise information based on their investigations and modelling of complex problem situations. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situations.

Four factors are deemed to inform the performance scale and level difficulty associated with PISA items. These include: (1) the kind and degree of interpretation and reflection required; (2) the nature of the representational skills reflected in item difficulty and the corresponding numeracy competency required by an item; (3) the kind and level of mathematical skill required; and (4) the sophistication, type and degree of mathematical argumentation required. These are explained further next.

The *kind and degree of interpretation and reflection required* includes: (a) the nature of demands arising from the problem context; (b) the extent to which the mathematical demands of the problem are apparent or to which students must impose his or her own mathematical construction on the problem; and (c) the extent to which insight, complex reasoning and generalisation are required. The *nature of the representation skills required* range from, at the lower end of the complexity spectrum, problems where only one mode of representation is used, to problems where students must alternate between different modes of representation or make independent choices regarding appropriate modes of representation.

Similarly formulated to reflect a range of potential numeracy competencies, *the kind and level of mathematical skill required* include, at the lower levels, single-step problems requiring students to reproduce basic mathematical facts and perform simple computation processes through to multi-step problems involving more advanced mathematical knowledge, complex decision-making, information processing, and problem solving and modelling skills. Finally, the *sophistication, type and degree of mathematical argumentation required* cover problems where no arguing is necessary, problems where students may apply family arguments, and problems where students have to independently produce their own mathematical arguments, understand other people's argumentation or assess the appropriateness and accuracy of given arguments or proofs.

ANCC Framework

The ANCC framework relies on a unique approach to describing the complexity levels associated with numeracy concepts and skills. Each of the four progressions (the capabilities progression and the three progressions between curricular elements, namely (A) number, (B) measures, shapes and space, and (C) handling data) are built around five levels of complexity: one entry level that is itself divided into three sublevels (entry levels 1, 2, and 3), and two higher levels labelled Level 1 and Level 2.

The complexity ratings for curriculum elements are operationalized for different aspects of each of the "progressions" or numeracy facets noted above. For instance, the *capabilities progression* describes adults' ability to read and understand information which ranges from understanding information given by numbers and symbols in simple graphical, numerical and written material to understanding math information written for different purposes and in a variety of formats. Adults are also assessed on their ability to describe practical problems using textual and other types of representative (from numbers at entry level 1 to textual and mathematical representations at the highest level).

The *number progression* involves skills related to the manipulation and understanding of whole numbers as well as fractions and decimals. With respect to whole numbers, the levels for this progression range from the ability to count up to ten items (entry level 1) to the ability to count, read, write, order and compare numbers up to 1000 (entry level 3), on to the ability to read, write, order and compare positive and negative numbers of any size in a practical context (Level 2). At the highest level of complexity, adults are expected to be able to perform more complex tasks and calculations, interpret and solve practical problems (compared to reading and describing at the entry levels), and work with abstract and generalized information. With respect to fractions and decimals, no skills are expected at entry level 1.They then progress gradually from basic abilities to identify, read, write, and compare simple fractions (halves and quarters) of quantities, small quantities of items or shapes (entry level 2); understanding common fractions, using equivalent fraction forms (including decimals) and manipulating fractions through the use of tools such as calculators (entry level 3); through to understanding and manipulating of percentages and increasingly complex fractions and decimal representations, developing capabilities to approximate values through rounding up and down (Level 1) and performing arithmetic manipulations of fractions in various modes of representation and of increasing detail (Level 2).

Recommendations Regarding ESRP Complexity Factors

After comparing and analyzing the complexity ratings used in the international assessments described preciously, we propose that three complexity scales be included in the ESRP methodology.

<u>A Declarative Knowledge Complexity Rating Scale</u>. This scale will include two dimensions:

- the complexity of mathematical concepts involved in the task (ranging from simple concrete to more complex abstract concepts), and
- context familiarity (ranging from familiar to novel situations).

At the lower levels, the tasks might require the use of simple concepts (e.g., small simple numbers or percentages) in familiar situations, while at higher levels an individual might require knowledge of and the ability to use more abstract complex concepts and apply those to novel unfamiliar situations.

Procedural Knowledge Complexity Rating Scale. This scale will include six dimensions and will assess the complexity of the procedures required to complete the numeracy task effectively. The dimensions will be the following:

- the number of mathematical operations required to complete the task;
- translation[2] or the number of steps required/number of stages in iterative processing;
- the types/complexity of skills required (count, measure, communicate, evaluate, etc.);
- the number of factors (variables) that need to be taken into account;
- the consequence of error; and
- the degree of precision required.

A Mathematical Representation Complexity Rating Scale. This scale will evaluate the complexity of the information and variety and complexity of representations of mathematical information. This scale will have three dimensions including:

- the amount of information to be represented,
- the number of plausibility of distractors, and
- the forms of representations of information.

The tasks in workplace contexts might differ with regard to the amount of information required and available to complete the task, amount of additional relevant or irrelevant information provided as well as complexity and number of various representations of information.

Table 6 summarizes the proposed complexity scales, their dimensions and possible values.

Table 6. Proposed Complexity Factors/Indicators

Complexity Scale	Dimensions	Scale values
Complexity of declarative knowledge required	Conceptual complexity	Simple - complex/concrete - abstract
	Familiarity of context	Familiar - novel
Complexity of operations (procedural)	Number of mathematical operations required to complete the task	Few - many
	Translation -number of steps required/number of stages in iterative processing	Few - many
	Types/complexity of skills required (count, measure,	Simple - advanced/concrete - abstract

[2] Turning a work problem into a set of mathematical operations so that math may be applied to obtain an answer

Complexity Scale	Dimensions	Scale values
	communicate, evaluate, etc.)	
	Number of factors (variables) that need to be taken into account	Few - many
	Consequence of error	Low importance - high importance
	Degree of precision required	Estimation - precise quantification
Use of math representations	Amount of information to be represented	All information is available - Little or no information is available
	Number of plausibility of distractors	Few - many
	Forms of representations of information	Few - many

Tables 7 through 9 describe the proposed levels of complexity for each of the scales. These were developed based on complexity levels included in the ESRP, PIAAC, ALL, and PISA frameworks.

Table 7. Scale 1: Complexity of required declarative knowledge

Dimensions	Level 1	Level 2	Level 3
Conceptual complexity	**Simple, concrete**	**Relatively simple**	**Complex, abstract**
	Limited knowledge of very simple mathematical concepts is required to perform the tasks (e.g., simple whole number relations and patters; time, common everyday measures of length).	Knowledge of relatively simple concepts including some generalization is required (e.g., rates and ratios; relations and patterns including written everyday generalizations; large whole numbers; area and volume formulae).	Knowledge of complex and abstract concepts is required (e.g., complex ratios, relations, patterns; formal mathematical information such as more complex formulae, knowledge of relationships between dimensions or variables, complex, abstract and generative reasoning or explanation required; trigonometry; algebraic and graphical conventions).
Familiarity of context	**Familiar**	**Somewhat familiar**	**Novel**
	Tasks are based on very concrete, real-life activities, familiar to most in daily life.	Based on real-life but less often undertaken activities.	Based on abstract ideas or unfamiliar activities in a context new to most.

Table 8. Scale 2: Complexity of required procedural knowledge

Dimensions	Level 1	Level 2	Level 3
Number of mathematical operations required to complete the task	**Few**	**Several**	**Many**
	One operation, action or process.	Application of two or three steps, the same or similar operation, action or process.	Integration of several steps covering more than one different operation, action or process.
Translation - number of steps required/number of stages in iterative processing	**Few**	**Several**	**Many**
	Only minimal translation is required to turn the task into a mathematical operation. Few steps are required. Procedure is known and well developed.	Some translation may be required or the numbers needed for the solution may need to be collected from several sources. Several steps are needed to be performed to complete the task. Procedure is known and might need to be adjusted.	Considerable translation is required. Numbers needed for calculations may need to be derived or estimated; approximations may need to be created in cases of uncertainty and ambiguity. Complex formulae, equations or functions may be used. There is no set procedure that can be used to complete this task, and workers need to develop the procedure.
Types/complexity of skills required (count, measure, communicate, evaluate, etc.)	**Simple, concrete**	**Relatively simple**	**Complex, abstract**
	Only the simplest operations are required and the operations to be used are clearly specified. Only one type of mathematical operation is used in a task.	Only relatively simple operations are required. Tasks may require a combination of operations or multiple applications of a single operation.	Advanced mathematical techniques involved abstract concepts and complex formulae may be required.
Number of factors (variables) that need to be taken into account	**Few**	**Several**	**Many**
	One factor comprises the item being estimated.	Small number of factors is involved in calculations.	Many factors involved and the method for making the estimate must be developed by the workers.
Consequence of error	**Low importance**	**Medium importance**	**High importance**
	Little or no consequence of error; estimation errors can be easily and quickly rectified with little or no work plan required or costs incurred.	Estimation errors have consequences, e.g., loss of money or time, but can be rectified.	Estimation errors have significant consequences that are not rectifiable or only rectifiable at significant cost.
Degree of	**Estimation**	**Relatively precise**	**Very precise**

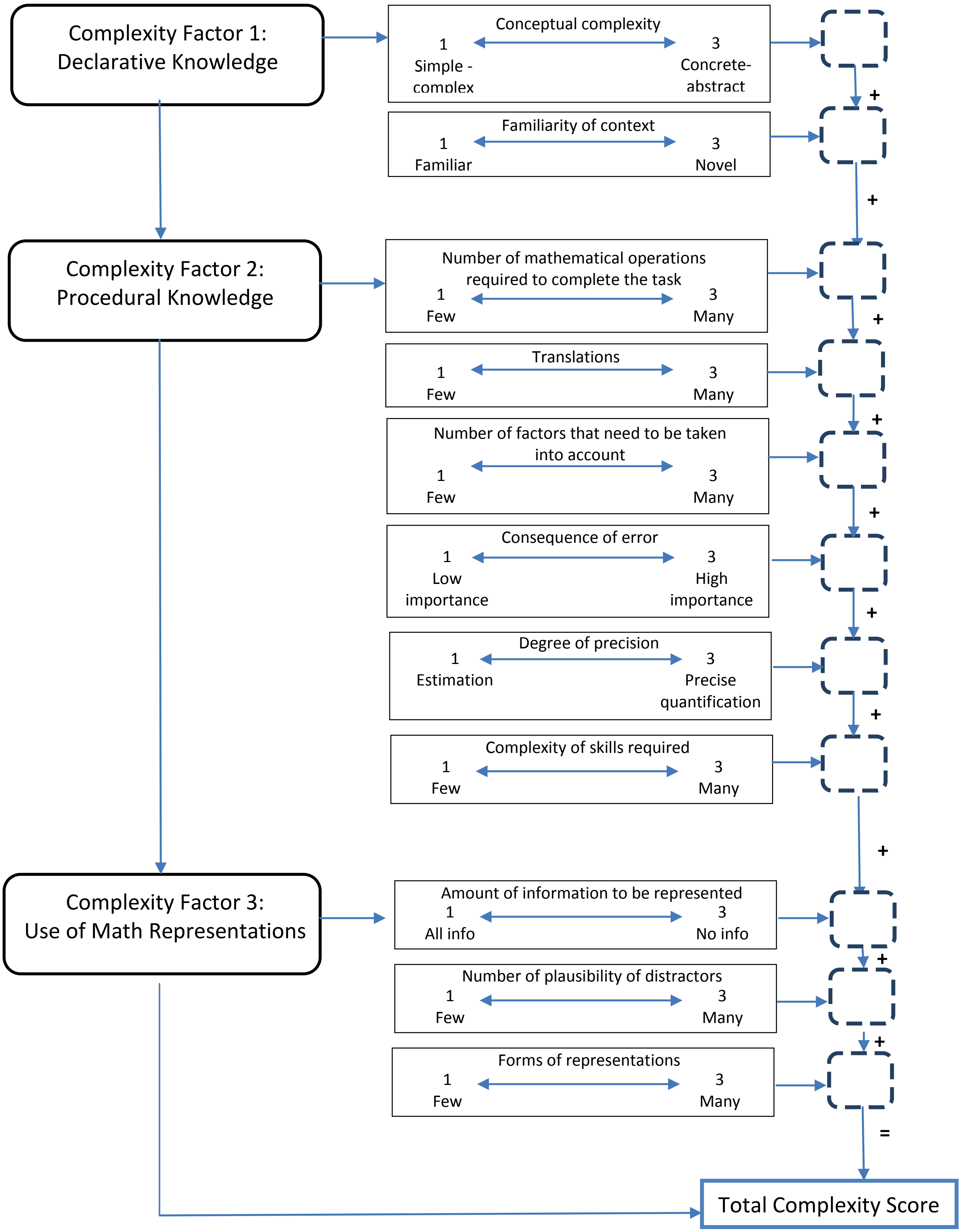
Dimensions	Level 1	Level 2	Level 3
precision required		**quantification**	**quantification**
	No or litter precision is needed.	Precision required within relatively wide range of values.	High degree of precision is required.

Table 9. Scale 3: Use of required mathematical representations

Dimensions	Level 1	Level 2	Level 3
Amount of information to be represented	**All information is available**	**Some information is available**	**Little or no information is available**
	All information about the factors that make up the estimate and how to combine them is known.	Some information is available, but worker may need to bring to the problem simple information or knowledge from outside the problem.	Necessary information or knowledge is missing, so outside information or knowledge needs to be brought in.
Number of plausibility of distractors	**Few/No**	**Several**	**Many**
	No other mathematical information is present apart from that requested – no distractors.	There is some mathematical information included that is not relevant to the topic	A lot of other irrelevant mathematical information appears – many distractors.
Forms of representations of information	**Few**	**Several**	**Many**
	The information required to perform the task is specified in little or no text, using familiar objects and/or photographs or other clear, simple visualizations.	The information required to perform the task is given using clear, simple sentences and/ or visualisations where some translation or interpretation is required.	The information required to perform the task is embedded in text as well as included in a variety of complex reprehensions/ visualizations (tables, charts etc.) where considerable translation or interpretation is required.

Similar to what was done in the international frameworks we reviewed, it should be possible to estimate the overall complexity score of the tasks by scoring and summing up each of the dimensions of the Complexity Factors 1, 2 and 3, and then combining the scores for each factor. The diagram below provides a possible representation of this process.

Figure 5: Complexity flowchart



The proposed approach is based on a review of only five international frameworks, which differ in their approaches to estimating the complexity of tasks. We therefore strongly recommend undertaking a two-stage validation process of the proposed dimensions and scales. The first step would seek to determine the face validity of the proposed dimensions and scales with experts and practitioners. The second would involve evaluating the construct and content validity of the framework and determining the weights for each of the elements and factors.

CONCLUSIONS AND RECOMMENDATIONS

For this study, *Directions* reviewed five international numeracy assessment and conceptual frameworks and compared those with the existing ESRP methodology, with the goal of providing recommendations on how to align the exiting ESRP framework with international approaches. We present below our key conclusions and recommendations.

Definition of numeracy. While the definitions of numeracy included in the international frameworks are similar, all of them focus on adult numeracy in general, rather than specifically on the workplace context. Furthermore, each of the frameworks included in this review has a particular approach to defining numeracy and to the identification of skills considered to be important. In particular, the PISA assessment has a specific focus on the mathematical skills and problems that are related to formal school curricula, with less attention given to the real-life context of tasks. While we can argue that assessments like PISA still cover skills required to be successful in everyday life, these differ in focus from essential skill frameworks that focus on basic adult numeracy skills and aim to assess adults who, in some cases at least, may have been out of school for extended period of time.

After reviewing numeracy definitions included in the selected international frameworks and other numeracy definitions discussed in the literature, we propose two numeracy definitions for HRSDC's consideration: a general definition of adult numeracy and a workplace numeracy definition. Specifically, we recommend that such definition of numeracy (whether generic and workplace) highlight ***declarative and procedural knowledge***; emphasize the ***use*** of knowledge and abilities; describe the ***purpose*** of numeracy related activities; be ***situated*** in context; and emphasize the importance of ***critical engagement*** with numerical information and activities. Thus, the proposed definitions of general adult numeracy and workplace numeracy are the following:

> *Adult numeracy is the ability to identify, understand, critically interpret and apply mathematical concepts, processes, and representations in order to engage in and answer to the mathematical demands of a variety of situations in adult life.*
>
> *Workplace numeracy is the ability to identify, understand, critically interpret and apply mathematical concepts, processes, and representations that are appropriate to the work context in which the individual is engaged in order to effectively manage/perform work activities.*

Numeracy facets. The analysis indicates that numeracy frameworks include a variety of concepts and behaviors. The frameworks are similar with regard to types of mathematical concepts included (e.g., number sense and estimation, quantity, shape and dimensions, pattern, functions and relationships, data and change, location and change). Furthermore, the frameworks list a variety of actions/numerate behaviours that adults are expected to perform to complete tasks such as counting, estimating, measuring, interpretation, and analysis among others. Based on our review of the framework documents and the numeracy facets comprised in each of these, we recommend organizing the revised ESRP numeracy framework into two domains: (1) that of declarative mathematical knowledge (content); and (2) that of procedural mathematical knowledge (i.e. responses/actions performed or required).

Other components of numeracy framework. In addition to the two domains proposed above, several other components should be included in the numeracy framework, namely: (a) contexts, (b) cognitive enabling processes, (c) non-cognitive enabling processes, and (d) meta-cognitive processes. While these components are not conceptualized here as separate facets/domains, they are seen as prerequisites of effective numerate behaviours, serving as enabling processes and factors for all three facets.

Complexity levels. A comparative analysis of the complexity levels and ratings proved challenging, as the frameworks included in this review used different factors to assess the complexity of the tasks. However, there were several complexity ratings/factors included in most of the frameworks. These included the number and type of operations required; the complexity of procedures; familiarity with the context; and the amount of information provided or required to complete the task. Assessing the complexity of information and the variety and complexity of representations of mathematical information associated with different tasks will remain important. Tasks can be analyzed not only in terms of their mathematical complexity but also in terms of the extent to which employees will need to work with textual or other types of representations of mathematical information to successful undertake their activities. As numeracy is often mediated through written/printed or electronic text forms, difficulty in reading and low ability to extract necessary information will likely impede numeracy performance.

Moreover, numeracy is related to other types of essential skills and other types of literacy, including document use, computer literacy, reading and writing. Some numeracy tasks might not require manipulation of numbers, but instead require workers to interpret mathematical information in text or media. International numeracy assessments tend to integrate numeracy, literacy, and technological literacy components in their numeracy items. After comparing and analyzing the complexity ratings used in international assessments described above, we propose to include three complexity scales in the ESRP methodology: (1) a declarative knowledge complexity rating scale (which will look at complexity of mathematical concepts involved in the task and likely degree of familiarity embedded in the context); (2) an operations/procedural complexity rating scale (which will assess number of mathematical operations required to complete the task; the number of steps required/number of stages in iterative processing; the types/complexity of skills required; the number of factors (variables) that need to be taken into account; consequence of error; and degree of precision required); and (3) a mathematical representation complexity rating scale (which will evaluate the amount of information to be represented; the number of plausible distractors; and forms of representations of information).

“Hidden math.” Some of the activities that adults encounter in real life might not be considered as “mathematics” but still reflect aspects of workplace numeracy. It is therefore important to workers to be able to have transferable and generalizable numeracy skills to be able to deal with more abstract tasks.

NEXT STEPS

HRSDC might consider a number of activities to validate and disseminate the results of the current study.

While the recommendations proposed above are based on the analysis of the five international numeracy assessment and conceptual frameworks, we recommend that these be validated with subject matter experts and employers across the country. Specifically, we strongly recommended that HRSDC pilot the suggested scales to ensure their validity and reliability.

Furthermore, we recommended that the existing essential skills profiles be revised to align them with the revised ESRP framework. For each occupational profile or groups of occupations, it will be important to define key numeracy-related competencies as well as identify indicators that can be used to assess these competencies. A useful approach to consider may be the creation of a DACUM chart for each of the profiles/or groups of related occupations.

Finally, to engage stakeholders in the review and validation of the revised ESRP methodology, the following Knowledge Exchange and Stakeholder Engagement Activities can be considered:

- The preparation of a brief outline of the findings of the present review and dissemination among targeted audiences.
- Consultations with targeted sectoral representatives to determine suitability of proposed revised framework, identify opportunities for and obstacles to the introduction of the new framework.
- A communications plan or initiative to foster knowledge exchange about the revised frameworks and the need to support numeracy skill development.

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APPENDIX A. FACETS: COMPARATIVE ANALYSIS

Appendix Table 1. Facets: Comparative Analysis

Framework	Context	Mathematical Processes (Responses/Actions)	Math Concepts/Knowledge/Content	Representation of Math Information	Other
ESRP	**#1 Numerical Calculation** Application setting - work	**#1 Numerical Calculation** data analysis math, measurement and calculation **#2 Numerical Estimation** **# 3 Math skills** calculating/measuring	**#3 Math Skills** Mathematical foundations used - Number concepts; patterns and relations; shape and spatial sense; statistics and probability	**#3 Math Skills** Mathematical foundations used: how calculations are performed [e.g., computer use]	
PIAAC	**# 1 Contexts** everyday life, work, societal or community, further learning	**# 2 Responses** identifying, locating or accessing; acting upon, using order, count, estimate, compute, measure, model, interpreting, evaluating /analysing, communicating about	**#3: Mathematical information** quantity & number; dimension & shape; pattern, relationships, change; data & chance	**# 4: Representations of mathematical information** objects & pictures; numbers & math symbols; formulae; diagrams & maps, graphs, tables; texts; technology-based displays	**NOT a facet = - ENABLING process** Other enabling factors and processes
ALL	**# 1 Contexts** everyday life, work, societal or community, further learning	**# 2 Responses** (identifying or locating; acting upon (order/sort; count; estimate; compute; measure; model); interpreting; communicating about)	**#3: Mathematical information** (quantity & number; dimension & shape; pattern, functions & relationships; data & chance; change)	**# 4: Representations of mathematical information** (objects & pictures; numbers & symbols; formulae; diagrams & maps; graphs; tables; Texts)	**# 5: Other enabling factors and processes**
TEC	No facet, but mentioned in the definition	**#1 Make Sense of Number to Solve Problems** Additive Strategies progression, Multiplicative Strategies progression; Proportional Reasoning Strategies progression	**#1 Make Sense of Number to Solve Problems** (Number Sequence progression; Place Value progression; Number Facts progression) **#2 Reason Statistically**	**#2 Reason Statistically** (Preparing Data for Analysis progression) **#3 Measure and Interpret Shape and Space** Location progression	**Other enabling factors** Meaningful contexts and examples; understanding and reasoning; degree

Framework	Context	Mathematical Processes (Responses/Actions)	Math Concepts/Knowledge/Content	Representation of Math Information	Other
		#2 Reason Statistically Preparing Data for Analysis progression, Analysing Data for Interpretation progression, Interpreting Data to Predict and Conclude progression, Probability progression **#3 Measure and Interpret Shape and Space** Location progression; Measurement progression	Probability progression **#3 Measure and Interpret Shape and Space** Shapes and Transformations progression		of precision; algorithms
PISA	**#1 Situations (Contexts)** personal, educational, occupational, public, scientific	**#1 Mathematical competencies & processes** Cognitive-thinking and reasoning; argumentation, communication; modeling, problem posing and solving, representation, using symbolic, formal and technical language and operations, mathematization	**#2 Mathematical Content** quantity/numbers/patterns, shape & space, change & relationships, uncertainty	**#3 Mathematical competencies & processes** Representation; using symbolic, formal and technical language and operations, mathematization	
ANCC	**Contexts** citizen and community, economic activity (paid and unpaid work), domestic and everyday life, leisure, education and training; using ICT in social roles)	**Capabilities Progression** Understanding and using mathematical information [read and understand, specify and describe]; calculate and manipulate math information [generate results]; interpreting results and communicating math information [present and explain results])	**Numbers** Numbers, number systems [fractions, decimals and percentage], calculations **Measures, Shape and Space** Money, time, temperature, distance, length, weight, capacity, perimeter, area, volume, shape, position **Handling Data** Data, statistical measures, probability	**Capabilities Progression** Communicating math information [present and explain results] Using ICT in social roles	**Problem-Solving**

APPENDIX B. COMPLEXITY RATINGS: COMPARATIVE ANALYSIS

Appendix Table 2. Complexity ratings: Comparative analysis

	Operations required		Problem transparency/translation	complexity of mathematical information	Textual aspects - distractors	Interpretation/analytical skills required
	numerical calculations	Numerical estimates				
ESRP	**Numerical calculation tasks** operations required	**Numerical Estimation Complexity** Whether there is a set procedure; number of factors comprising the item being estimated; consequence of error; segree of precision required	**numerical calculation tasks** translation	**Numerical Estimation Complexity** Amount of information available		
ALL /PIAAC	**Mathematical aspects** expected number of operations	**mathematical aspects** type of operation/skill	**textual aspects** type of match/problem transparency	**mathematical aspects** complexity of mathematical information/data	**textual aspects** plausibility of distractors	
PISA	The kind and level of mathematical skill required	The kind and level of mathematical skill required	kind and degree of interpretation and reflection required	The kind of representation skills required	The kind of representation skills required	kind and degree of mathematical argumentation
ANCC	**Complexity of curriculum elements** Math concepts and operations	**Complexity of curriculum elements** Math concepts and operations Capabilities progression (from making sense of math information to generation of results if appropriate level of accuracy)	**Complexity of curriculum elements** from simple math tasks to more complex practical problems	**Complexity of curriculum elements** Math concepts and operations – simple concepts vs. more complicated concepts	**Capabilities progression** Ability to read, understand, interpret and use a variety of representations	**Complexity of curriculum elements** Counting and describing vs. interpreting, solving problems